The growth of linear perturbations in the DGP model

Xiangyun Fu¹, Puxun Wu^{1,2,3} and Hongwei Yu^{1,2*}

¹Department of Physics and Institute of Physics,

Hunan Normal University, Changsha, Hunan 410081, China

²Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China

³Department of Physics and Tsinghua Center for Astrophysics,

Tsinghua University, Beijing 100084, China

Abstract

We study the linear growth of matter perturbations in the DGP model with the growth index γ as a function of redshift. At the linear approximation: $\gamma(z) \approx \gamma_0 + \gamma_0' z$, we find that, for $0.2 \leq \Omega_{m,0} \leq 0.35$, γ_0 takes the value from 0.658 to 0.671, and γ_0' ranges from 0.035 to 0.042. With three low redshift observational data of the growth factor, we obtain the observational constraints on γ_0 and γ_0' for the ΛCDM and DGP models and find that the observations favor the ΛCDM model but at the 1σ confidence level both the ΛCDM and DGP models are consistent with the observations.

PACS numbers: 95.36.+x, 98.80.Es

^{*} Corresponding author

I. INTRODUCTION

Various observations show that our universe is undergoing an accelerating expansion [1, 2, 3] and many models have been proposed to explain this mysterious phenomenon. There are basically two main classes of models. One is dark energy which yields sufficient negative pressure to induce a late-time accelerated expansion; the other is the modified gravity, such as the scalar-tensor theory [4], the f(R) theory [5] and the Dvali-Gabadadze-Porrati (DGP) braneworld scenarios [6, 7], et al. However, these models may predict the same late time accelerated cosmological expansion, although they are quite different physically. So an important task is to discriminate one from another. Recently, some attempts have been made [8, 9, 10, 11, 12, 13, 14] in this regard. An interesting approach is to differentiate the dark energy and the modified gravity with the growth function $\delta(z) \equiv \delta \rho_m/\rho_m$ of the linear matter density contrast as a function of redshift z. While different models give the same late time expansion, they may produce different growth of matter perturbations [15].

To the linear order of perturbation, the matter density perturbation $\delta = \delta \rho_m/\rho_m$ satisfies the following equation [16] at the large scales

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff} \,\rho_m \delta = 0,\tag{1}$$

where G_{eff} is the effective Newton's constant and the dot denotes the derivative with respect to time t. In general relativity, $G_{eff} = G_N$ where G_N is the Newton's constant. Defining the growth factor $f \equiv d \ln \delta/d \ln a$, one can obtain

$$\frac{df}{d\ln a} + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right)f = \frac{3}{2}\frac{G_{eff}}{G_N}\Omega_m,\tag{2}$$

where Ω_m is the fractional energy density of matter. In general, analytical solutions to Eq. (2) are hard to find, and we need to resort to numerical methods. It has been known for many years that there is a good approximation to the growth factor f, which is given by [17]

$$f \equiv \frac{d \ln \delta}{d \ln a} \simeq \Omega_m(z)^{\gamma},\tag{3}$$

where γ is the growth index and is taken as a constant. This parameterized approach has been studied in some works recently, see e.g. [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. For

example, substituting the above equation into Eq. (2) and then expanding around $\Omega_m = 1$ (a good approximation at the high redshift), one can obtain $\gamma_{\infty} \simeq 0.5454$ [18, 20] for the ΛCDM model and $\gamma_{\infty} \simeq 11/16 \approx 0.6875$ [18, 19] for the flat DGP model. Therefore, in principle, one can distinguish the dark energy model from the modified gravity model with observational data on the growth factor. However, taking the index γ as a constant is only an approximation although it is a very good one in certain circumstances. More generically, one should rewrite Eq. (3) as

$$f \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m(z)^{\gamma(z)} . \tag{4}$$

Defining a new quantity $\gamma' \equiv \frac{d\gamma(z)}{dz}$, we can expand γ at the low redshift, as follows

$$\gamma(z) \approx \gamma_0 + \gamma_0' z$$
 $0 \le z \le 0.5$. (5)

This approximation has been studied in Refs. [29, 30, 31], and it was found that γ'_0 is a quasi-constant and $\gamma'_0 \simeq -0.02$ for dark energy models with a constant equation of state. However, for modified gravity models, such as some scalar-tensor models, γ'_0 is negative and can take absolute values larger than those in models inside General Relativity [30], while for the f(R) model γ'_0 is also negative but its value is largely outside the range found for dark energy models in General Relativity [31]. Therefore, an accurate γ'_0 at the low redshift could provide another characteristic discriminative signature for these models.

In this paper, we will mainly focus on the observational constraints on γ_0 and γ'_0 from data on the growth factor. Firstly, we will study the linear growth index with the form $\gamma \approx \gamma_0 + \gamma'_0 z$ for the DGP model. Then, with the best fit value $\Omega_{m,0}$ from the observational data we will discuss the theoretical values of γ_0 and γ'_0 and the observational constraints on them.

II. GROWTH INDEX OF DGP MODEL

For the DGP model, in general, G_{eff} can be written as

$$G_{eff} = G_N \left(1 + \frac{1}{3\beta} \right), \tag{6}$$

where $\beta = 1 - 2r_c H \left(1 + \frac{\dot{H}}{3H^2}\right)$ [23, 32, 33, 34] and the constant r_c is a scale which sets a length beyond which gravity starts to leak out into the bulk. According to Ref. [33], $\frac{G_{\rm eff}}{G_N}$ can be rewritten as

$$1 + \frac{1}{3\beta} = \frac{4\Omega_m^2 - 4(1 - \Omega_k)^2 + \alpha}{3\Omega_m^2 - 3(1 - \Omega_k)^2 + \alpha},$$
(7)

where $\alpha \equiv 2\sqrt{1-\Omega_k} (3-4\Omega_k+2\Omega_m\Omega_k+\Omega_k^2)$, $\Omega_k \equiv -k/(a^2H^2)$, and $\Omega_m \equiv 8\pi G\rho_m/(3H^2)$. Here the spatial curvature k=0, k>0 and k<0 correspond to a flat, closed and open universe respectively.

For the DGP model, the modified Friedmann equation takes the form [7, 19]

$$H^{2} + \frac{k}{a^{2}} - \frac{1}{r_{c}} \sqrt{H^{2} + \frac{k}{a^{2}}} = \frac{8\pi G}{3} \rho_{m}.$$
 (8)

Defining $\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}$, we have

$$E^{2}(z) \equiv \left(\frac{H}{H_{0}}\right)^{2} = \left[\sqrt{\Omega_{m,0}(1+z)^{3} + \Omega_{r_{c}}} + \sqrt{\Omega_{r_{c}}}\right]^{2} + \Omega_{k0}(1+z)^{2}.$$
 (9)

Setting z = 0 in the above gives rise to a constraint equation

$$1 = \left[\sqrt{\Omega_{m,0} + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}\right]^2 + \Omega_{k0}.$$
 (10)

Therefore, there are only two model independent parameters out of $\Omega_{m,0}$, Ω_{r_c} and Ω_{k0} .

The matter density perturbation in the DGP model satisfies the equation [16, 25]:

$$\frac{d^2 \ln \delta}{d(\ln a)^2} + \left(\frac{d \ln \delta}{d \ln a}\right)^2 + \left(2 + \frac{d \ln H}{d \ln a}\right) \left(\frac{d \ln \delta}{d \ln a}\right) = \frac{3}{2} \left(1 + \frac{1}{3\beta}\right) \Omega_m. \tag{11}$$

Using

$$\frac{d \ln H}{d \ln a} = \frac{\dot{H}}{H^2} = -\frac{3}{2} + \frac{\Omega_k}{2} - \frac{3}{2} \frac{-1 + \Omega_k}{1 + \Omega_m - \Omega_k} \left(1 - \Omega_k - \Omega_m \right), \tag{12}$$

we obtain

$$\frac{d^2 \ln \delta}{d(\ln a)^2} + \left(\frac{d \ln \delta}{d \ln a}\right)^2 + \frac{d \ln \delta}{d \ln a} \left(\frac{1}{2} \left(1 + \Omega_k\right) - \frac{3}{2} \frac{-1 + \Omega_k}{1 + \Omega_m - \Omega_k} \left(1 - \Omega_k - \Omega_m\right)\right)$$

$$= \frac{3}{2} \left(1 + \frac{1}{3\beta}\right) \Omega_m. \tag{13}$$

Thus, according to the definition of f, we have the following differential equation

$$\Omega_{m} \left[\frac{3(-1+\Omega_{k})}{1+\Omega_{m}-\Omega_{k}} (1-\Omega_{k}-\Omega_{m}) - \Omega_{k} \right] \frac{df}{d\Omega_{m}} + f^{2}
+ f \left[\frac{1}{2} (1+\Omega_{k}) - \frac{3}{2} \frac{-1+\Omega_{k}}{1+\Omega_{m}-\Omega_{k}} (1-\Omega_{k}-\Omega_{m}) \right]
= \frac{3}{2} \left(1 + \frac{1}{3\beta} \right) \Omega_{m} .$$
(14)

Substituting the generic expression for f, Eq. (4), into the Eq. (14) we arive at an equation on $\gamma(z)$

$$\frac{1}{2}[(1+\Omega_k - 2\gamma\Omega_k) + \frac{3(-1+\Omega_k)}{1+\Omega_m - \Omega_k}(2\gamma - 1)(1-\Omega_k - \Omega_m)] - (1+z)\gamma' \ln \Omega_m + \Omega_m^{\gamma} = \frac{3}{2}(1+\frac{1}{3\beta})\Omega_m^{1-\gamma}.$$
(15)

If we only consider the linear expansion at the low redshift as given in Eq. (5), it is easy to derive

$$\gamma_0' = \left(\ln \Omega_{m,0}^{-1}\right)^{-1} \left[-\Omega_{m,0}^{\gamma_0} + \frac{3}{2} (1 + \frac{1}{3\beta}) \Omega_{m,0}^{1-\gamma_0} - \frac{1}{2} (1 + \Omega_{k,0} - 2\gamma_0 \Omega_{k,0}) - 3 \frac{-1 + \Omega_{k,0}}{1 + \Omega_{m,0} - \Omega_{k,0}} (1 - \Omega_{k,0} - \Omega_{m,0}) (\gamma_0 - \frac{1}{2}) \right].$$
(16)

This gives a constraint equation

$$g(\gamma_0, \gamma_0', \Omega_{m,0}, \Omega_{k,0}) = 0.$$
 (17)

So, for any given background parameters $\Omega_{m,0}$ and $\Omega_{k,0}$, the value of γ'_0 can be determined by that of γ_0 . For the sake of simplicity, we will only consider the case of a spatially flat universe in this paper ($\Omega_k = 0$). Thus from Eq. (16), we get

$$\gamma_0' = \left(\ln \Omega_{m,0}^{-1}\right)^{-1} \left[-\Omega_{m,0}^{\gamma_0} + \frac{3}{2} \frac{4\Omega_{m,0}^2 + 2}{3\Omega_{m,0}^2 + 3} \Omega_{m,0}^{1-\gamma_0} - \frac{1}{2} + \frac{3}{1+\Omega_{m,0}} (1-\Omega_{m,0}) (\gamma_0 - \frac{1}{2}) \right].$$
(18)

According to equation $f(z=0) = \Omega_{m,0}(0)^{\gamma_0}$, the value of γ_0 can be obtained by solving Eq. (14) numerically for an given value of $\Omega_{m,0}$. Then plugging this obtained γ_0 into Eq. (18), we can get the value of γ'_0 . The results are shown in Fig. 1. We find, from

the right panel, that the value of γ_0 increases from 0.658 to 0.671 for $0.2 \leq \Omega_{m,0} \leq 0.35$. This suggests that γ cannot really be regarded as a constant as Ω_m varies. Notice that our result is different from that obtained for the ΛCDM model where the value of γ_0 is found to decrease from 0.558 to 0.554 for $0.2 \leq \Omega_{m,0} \leq 0.35$ [29]. This feature of γ_0 also provides a distinctive signature for the DGP model from the ΛCDM model. From the right panel, we can see that the γ'_0 is positive and ranges approximately from 0.035 to 0.042, which is also different from the dark energy model, the scalar-tensor model and f(R) model. For example for the wCDM model with $\Omega_{m,0} = 0.3$, γ'_0 is negative and quasi-constant $\gamma'_0 \simeq -0.02$. So, in principle, we can discriminate the DGP model from the dark energy model merely through the sign of γ'_0 if we can have an accurate value of γ'_0 from the observation data. Now we will discuss the the observational constraints on γ_0 and γ'_0

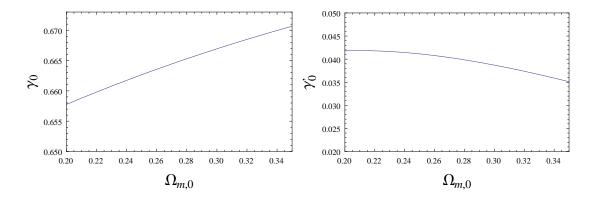


FIG. 1: γ_0 and γ'_0 are displayed as a function of $\Omega_{m,0}$ for DGP model respectively.

III. OBSERVATIONAL CONSTRAINTS

In order to obtain the observational constraints on γ_0 and γ'_0 , we firstly need to know the value of $\Omega_{m,0}$ determined by observations. Here we use the results in Ref. [28] where the author found $\Omega_{m,0} = 0.273 \pm 0.015$ for the ΛCDM model and $\Omega_{m,0} = 0.278 \pm 0.015$ for the DGP model respectively from the 307 Union Sne Ia data, the BAO from the SDSS

data, the shift parameter from the WMAP5 and the 11 Hubble parameter data. Using the best fit values $\Omega_{m,0} = 0.273$ for the Λ CDM model and $\Omega_{m,0} = 0.278$ for the DGP model respectively, we find $\gamma_0 = 0.665$, $\gamma'_0 = 0.04$ for theoretical vaules for the DGP model from Fig. (1) in this paper, and $\gamma_0 = 0.555$, $\gamma'_0 = -0.018$ for the Λ CDM model from Fig. (1) in Ref. [29].

To find the observational constraints on γ_0 and γ'_0 , only three observational data on f_{obs} given in Table I can be used, since the linear expansion is valid only at the low reshifts. With the best fit value of $\Omega_{m,0}$ we can obtain the constraints from the observations by using the following equation

$$\chi_f^2 = \sum_{i=1}^3 \frac{[f_{obs}(z_i) - \Omega_m^{\gamma_0 + \gamma_0' z_i}]^2}{\sigma_{fi}^2},\tag{19}$$

where σ_{fi} is the 1σ uncertainty of the f(z) data. The results are shown in Fig. (2). The best fit values are $\gamma_0 = 0.774$, $\gamma'_0 = -0.556$ for the ΛCDM model and $\gamma_0 = 0.767$, $\gamma'_0 = -0.732$ for the DGP model, which show that the observations imply an negative value of γ'_0 . Since the DGP model gives an positive γ'_0 , thus we can conclude that observations disfavor the DGP model. However, from Fig. (2), we find that at the 1σ confidence level both the ΛCDM and the DGP model are consistent with the observations.

z	f_{obs}	References
0.15	0.49 ± 0.1	[35]
0.35	0.7 ± 0.18	[36]
0.55	0.75 ± 0.18	[37]

TABLE I: The summary of the observational data on the growth factor f at low redshifts.

IV. CONCLUSION

In this letter, the growth factor of matter perturbations in the DGP model is studied and we find that the growth index, γ , should be treated as a function of time. With a

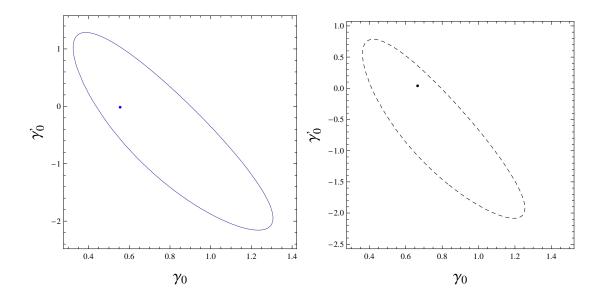


FIG. 2: The 1σ contours of γ_0 and γ'_0 by fitting the Λ CDM model and DGP model to the growth rate data. The points denote the theoretical values of γ_0 and γ'_0 with the $\Omega_{m,0}$ taking the best fit values.

linear expansion of $\gamma(z) \approx \gamma_0 + \gamma_0'z$, we obtain that γ_0 increases from 0.658 to 0.671 and γ_0' ranges approximately from 0.035 to 0.042, for $0.2 \leq \Omega_{m,0} \leq 0.35$. This is different from the results obtained for the ΛCDM model where γ_0 decreases from 0.558 to 0.554 and γ_0' is quasi-constant with $\gamma_0' \simeq -0.02$ for $0.2 \leq \Omega_{m,0} \leq 0.35$ [29]. These features provide distinctive signatures for the DGP model from the ΛCDM model. With the observational data on the growth factor, we analyze the observational constraints on γ_0 and γ_0' and find that the best fit values are $\gamma_0 = 0.774$, $\gamma_0' = -0.556$ for the ΛCDM model and $\gamma_0 = 0.767$, $\gamma_0' = -0.732$ for the DGP model. This seems to show that the observations favor the ΛCDM model since the theoretical value of γ_0' is positive for the DGP model. However, at 1σ confidence level both the DGP model and ΛCDM model are consistent with the observations as can be seen from Fig 2. It should be pointed out that our results are based upon merely three low redshifts data, since the linear approximation is valid only at the low redshiftes. To obtain stronger constraints which can clearly discriminate different models, we need a parametrized form of $\gamma(z)$, which is applicable for all the observational data, and hope to turn to this issue in the future [38].

Acknowledgments

X. Fu is grateful to Professor Yungui Gong for his very helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Grants No. 10575035, 10775050, 10705055, the SRFDP under Grant No. 20070542002, the Research Fund of Hunan Provincial Education Department, the Hunan Provincial Natural Science Foundation of China under Grant No. 08JJ4001, and the China Postdoctoral Science Foundation.

[1] A. G. Riess, A. V. Filippenko, P. Challis, et al., Astron. J. 116, 1009 (1998)[astro-ph/9805201];

S. J. Perlmutter, G. Aldering, G. Goldhaber, et al., Astrophy. J. 517, 565 (1999) [astro-ph/9812133];

J. L. Tonry et al., Astrophys. J. **594**, 1 (2003) [astro-ph/0305008];

R. A. Knop et al., Astrophys. J. **598**, 102 (2003) [astro-ph/0309368];

A. G. Riess *et al.*, Astrophys. J. **607**, 665 (2004) [astro-ph/0402512];

A. G. Riess *et al.*, Astrophys. J. **659**, 98 (2007) [astro-ph/0611572];

P. Astier et al., Astron. Astrophys. 447, 31 (2006) [astro-ph/0510447];

J. D. Neill *et al.*, Astron. J. **132**, 1126 (2006) [astro-ph/0605148];

W. M. Wood-Vasey et al., Astrophys. J. 666, 694 (2007);

T. M. Davis et al., Astrophys. J. 666, 716 (2007);

M. Kowalski et al., Astrophys. J. 686, 749 (2008).

[2] D. N. Spergel et al., Astrophys. J. Suppl. 170, 377 (2007) [astro-ph/0603449];

L. Page et al., Astrophys. J. Suppl. 170, 335 (2007) [astro-ph/0603450];

G. Hinshaw et al., Astrophys. J. Suppl. 170, 288 (2007) [astro-ph/0603451];

N. Jarosik et al., Astrophys. J. Suppl. 170, 263 (2007) [astro-ph/0603452].

[3] D. J. Eisenstein *et al.*, Astorphys. J. **633**, 560 (2005);

E. Komatsu et al., Astrophys. J. Suppl. 180, 330 (2009) [astro-ph/0803.0547] .

[4] Y. Fujii, Phys. Rev. D **62**, 044011 (2000);

- N. Bartolo and M. Pietroni, Phys. Rev. D 61 023518 (2000);
- F. Perrotta, C. Baccigalupi and S. Matarrese, Phys. Rev. D 61, 023507 (2000);
- V. Faraoni, Cosmology in Scalar-Tensor Gravity, Kluwer Academic, Dordrecht, 2004;
- R. Gannouji, D. Polarski, A. Ranquet, A. A. Starobinsky, JCAP 0609, 016 (2006);
- B. Boisseau, G. Esposito-Far'ese, D. Polarski and A.A. Starobinsky, Phys. Rev. Lett. 85, 2236 (2000).
- [5] S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002);
 - T. P. Sotiriou, V. Faraoni, [arXiv:0805.1726];
 - X. Yang, D. Chen, [arXiv:0812.0660].
- [6] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000) [hep-th/0005016];
 Z. Zhu, M. Sereno, Astro. Astrophys. 487 (2008) 831-835.
- [7] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [hep-th/0010186];
 - C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [astro-ph/0105068].
- [8] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003) [astro-ph/0207347];
 - V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000) [astro-ph/9904398];
 - S. M. Carroll, Living Rev. Rel. 4, 1 (2001) [astro-ph/0004075];
 - T. Padmanabhan, Curr. Sci. 88, 1057 (2005) [astro-ph/0411044];
 - S. M. Carroll, [astro-ph/0310342];
 - R. Bean, S. Carroll and M. Trodden, astro-ph/0510059;
 - A. Albrecht *et al.*, [astro-ph/0609591];
 - R. Trotta and R. Bower, [astro-ph/0607066];
 - M. Kamionkowski, [astro-ph/0706.2986];
 - B. Ratra and M. S. Vogeley, [astro-ph/0706.1565];
 - S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989);
 - S. Nobbenhuis, Found. Phys. **36**, 613 (2006) [gr-qc/0411093];
 - E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006) [hep-th/0603057];
 - T. Padmanabhan, Phys. Rept. **380**, 235 (2003) [hep-th/0212290];

- E. V. Linder, arXiv:0705.4102 [astro-ph];
- M. S. Turner and D. Huterer, arXiv:0706.2186 [astro-ph];
- J. Frieman, M. Turner and D. Huterer, arXiv:0803.0982 [astro-ph].
- [9] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005) [astro-ph/0505494];
 E. V. Linder, Phys. Rev. D 73, 063010 (2006) [astro-ph/0601052].
- [10] R. J. Scherrer, Phys. Rev. D **73**, 043502 (2006) [astro-ph/0509890].
- [11] T. Chiba, Phys. Rev. D **73**, 063501 (2006) [astro-ph/0510598].
- [12] E. V. Linder, Gen. Rel. Grav. 40, 329 (2008) [arXiv:0704.2064].
- [13] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. 77, 201 (2003) [astro-ph/0201498];
 - U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. 344, 1057 (2003) [astro-ph/0303009].
- [14] H. Wei and R. G. Cai, Phys. Lett. B 655, 1 (2007) [arXiv:0707.4526];
 Y. Gong, M. Ishak and A. Wang, [astro-ph/0903.0001].
- [15] A. A. Starobinsky, JETP Lett. **68**, 757 (1998).
- [16] L. M. Wang and P. J. Steinhardt, Astrophys. J. **508**, 483 (1998) [astro-ph/9804015].
- [17] J. N. Fry, Phys. Lett. B **158**, 211 (1985).
 - A. P. Lightman and P.L. Schechter, Astrophys. J. 74, 831 (1990);
 - L. Wang and P. J. Steinhardt, Astrophys. J.508, 483 (1998).
- [18] E. V. Linder and R. N. Cahn, Astropart. Phys. 28, 481 (2007) [astro-ph/0701317].
- [19] H. Wei, Phys. Lett. B **664**, 1 (2008). [astro-ph/ 0802.4122].
- [20] E. V. Linder, Phys. Rev. D **72**, 043529 (2005) [astro-ph/0507263].
- [21] D. Huterer and E. V. Linder, Phys. Rev. D **75**, 023519 (2007) [astro-ph/0608681].
- [22] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].
- [23] A. Lue, Phys. Rept. **423**, 1 (2006) [astro-ph/0510068].
- [24] C. Di Porto and L. Amendola, [astro-ph/0707.2686];
 - L. Amendola, M. Kunz and D. Sapone, [astro-ph/0704.2421];
 - D. Sapone and L. Amendola, [astro-ph/0709.2792].

- [25] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 77, 023504 (2008) [arXiv:0710.1092].
- [26] Y. Wang, Journal of Cosmology and Astro-Particle Physics 5, 21 (2008) [astro-ph/0710.3885];
 - Y. Wang, arXiv:0712.0041 [astro-ph].
- [27] B. Boisseau, G. Esposito-Farse, D. Polarski, A.A. Starobinsky, Phys. Rev. Lett. 85, 2236 (2000).
- [28] Y. Gong, Phys.Rev.D **78**, 123010 (2008) [arXiv:astro-ph/0808.1316].
- [29] D. Polarski and R. Gannouji, Phys. Lett. B 660, 439 (2008) [arXiv:0710.1510].
- [30] R. Gannouji and D. Polarski, JCAP **0805** 018 (2008) [astro-ph/0802.4196].
- [31] R. Gannouji, B. Moraes, D. Polarski, [arXiv:astro-ph/0809.3374].
- [32] M. S. Movahed, M. Farhang and S. Rahvar, astro-ph/0701339.
- [33] T. Chiba and R. Takahashi, Phys. Rev. D 75, 101301 (2007) [astro-ph/0703347].
- [34] K. Koyama and R. Maartens, JCAP **0601**, 016 (2006) [astro-ph/0511634].
- [35] L. Guzzo et al., Nature 451, 541 (2008);
 M. Colless et al., Mont. Not. R. Astron. Soc. 328, 1039 (2001).
- [36] M. Tegmark *et al.*, Phys. Rev. D **74**, 123507 (2006).
- [37] N.P. Ross *et al.*, Mont. Not. R. Astron. Soc. **381**, 573 (2007).
- [38] P. Wu and H. Yu and X. Fu, arXiv:0905.3444.