

The role of orbital angular momentum in the proton spin

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Abstract

The orbital angular momenta L^u and L^d of up and down quarks in the proton are estimated as functions of the energy scale as model-independently as possible, on the basis of Ji's angular momentum sum rule. This analysis indicates that $L^u - L^d$ is large and negative even at low energy scale of nonperturbative QCD, in contrast to Thomas' similar analysis based on the refined cloudy bag model. We pursue the origin of this apparent discrepancy and suggest that it may have a connection with the fundamental question of how to define quark orbital angular momenta in QCD.

1 Introduction

The so-called “nucleon spin puzzle” is still one of the most fundamental problems in hadron physics [1]. The recent precise measurements of the deuteron spin structure function by the COMPASS and HERMES groups established that about 1/3 of the nucleon spin is carried by the intrinsic quark polarization [2],[3], so that the missing spin fraction is now believed to be of order of 2/3. However, there is no widely-accepted consensus on the decomposition of the remaining part. (Still, it should be kept in mind that a lot of recent attempts to directly measure the gluon polarization Δg were all led to the conclusion that Δg is likely to be small or at least it cannot be large enough to resolve the puzzle of the missing nucleon spin based on the $U_A(1)$ anomaly scenario [4]-[7].)

Recently, Thomas claims that the modern spin discrepancy can well be explained in terms of standard features of the nonperturbative structure of the nucleon, i.e. relativistic motion of valence quarks, the pion cloud required by chiral symmetry, and an exchange current contribution associated with the one-gluon-exchange hyperfine interaction [8]-[11]. His analysis starts from an estimate of the orbital angular momenta (OAM) of up and down quarks based

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on the improved (or fine-tuned) cloudy bag model taking account of the above-mentioned effects. Another important factor of his analysis is the observation that the angular momentum is not a renormalization group invariant quantity, so that the above predictions of the model should be associated with a very low energy scale, say, 0.4 GeV. Then, after solving the QCD evolution equations for the up and down quark angular momenta, first derived by Ji, Tang and Hoodbhoy [12], he was led to a remarkable conclusion that the orbital angular momenta of up and down quarks cross over around the scale of 0.5 GeV. This crossover of L^u and L^d seems absolutely necessary for his scenario to hold. Otherwise, the prediction $L^u - L^d > 0$ of the improved cloudy bag model given at the low energy scale is incompatible with the current empirical information or lattice QCD simulations at the high energy scale, which gives $L^u < 0, L^d > 0$.

Actually, the importance of specifying the scale when discussing the nucleon spin contents, has been repeatedly emphasized in a series of our papers [13] -[18]. (The observation on the scale dependence of the nucleon spin matrix elements has much longer history. See [19] and [20], for instance.) In particular, we have recently carried out a semi-empirical analysis of the nucleon spin contents based on Ji's angular momentum sum rule, and extracted the orbital angular momentum of up and down quarks as functions of the scale. (See Fig.6 of [18].) Remarkably, we find no crossover of L^u and L^d when Q^2 is varied, in sharp contrast to Thomas' analysis. This difference is remarkable, since if there is no crossover of L^u and L^d , Thomas' scenario for resolving the proton spin puzzle is not justified. The purpose of the present paper is to pursue further the cause of this discrepancy, which is expected to provide us with a valuable insight into a very fundamental physical question, i.e. the role of orbital angular momentum in the nucleon spin.

2 Semi-empirical extraction of quark orbital angular momenta in the proton

There is no doubt about the fact that the nucleon spin consists of quark and gluon parts as $J^Q + J^g = 1/2$. (Here, $Q = u + d + s$ for three quark flavors.) The point is that this decomposition can be made experimentally through the GPD (generalized parton distribution) analysis of high energy deeply-virtual Compton scatterings and of deeply-virtual meson productions. Our semi-phenomenological estimate of J^Q starts with Ji's angular momentum sum rule [21],[22] given as $J^Q = \frac{1}{2} [\langle x \rangle^Q + B_{20}^Q(0)]$, where $\langle x \rangle^Q$ is the net momentum fraction carried by all the quarks, while $B_{20}(0)$ is the net quark contribution to the anomalous gravitomagnetic moment of the nucleon. For flavor decomposition, we also need flavor nonsinglet combinations, i.e. $J^{(NS)} = \frac{1}{2} [\langle x \rangle^{(NS)} + B_{20}^{(NS)}(0)]$, with $J^{(NS)} = J^{u-d}$, or J^{u+d-2s} etc. The quark momentum fractions and the angular momentum fractions are both scale dependent quantities. Ji showed

that they obey exactly the same evolution equations. At the leading order (LO), the solutions of the flavor singlet part J^Q is given by

$$2 J^Q(Q^2) = \frac{3 n_f}{16 + 3 n_f} + \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{2(16+3 n_f)/9\beta_0} \times \left(2 J^Q(Q_0^2) - \frac{3 n_f}{16 + 3 n_f} \right), \quad (1)$$

with $\beta_0 = 11 - \frac{2}{3} n_f$ and similarly for $\langle x \rangle^Q$. On the other hand, the scale dependence of the flavor nonsinglet combinations is given by

$$2 J^{(NS)}(Q^2) = \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{32/9\beta_0} 2 J^{(NS)}(Q_0^2), \quad (2)$$

and similarly for $\langle x \rangle^{(NS)}$.

A key observation now is that the quark and gluon momentum fractions are basically known quantities at least above $Q^2 \simeq 1 \text{ GeV}^2$, where the framework of perturbative QCD is safely applicable. For instance, the familiar MRST2004 and CTEQ5 fits give almost the same quark and gluon momentum fractions below 10 GeV^2 [23],[24]. In the following analysis, we shall use the values corresponding to the scale $Q^2 = 4 \text{ GeV}^2$ from MRST2004 fits.

Neglecting small contribution of strange quarks, which is not essential for the present qualitative discussion, we are then left with two unknowns, i.e. $B_{20}^{u+d}(0)$ and $B_{20}^{u-d}(0)$. For these quantities, we need some theoretical information, for example, from lattice QCD simulations. (One must remember the fact that the lattice QCD simulations at the present stage have a lot of problems, for instance, the omission of disconnected diagrams, the estimate of the finite volume effects, and the difficulty of simulations in the realistic chiral region. The problem is then to judge to what extent we can trust the predictions of the lattice QCD at the present stage. This point will be discussed later.) Fortunately, the available predictions of lattice QCD corresponds to the renormalization scale $Q^2 \simeq 4 \text{ GeV}^2$, which is high enough for the framework of perturbative QCD to work. Then, assuming that all the necessary quantities for the decomposition of the proton spin are prepared at this high energy scale, an interesting idea is to use the QCD evolution equations to estimate the corresponding values at lower energy scales. This is just the opposite to what was done in Thomas' analysis [8],[11] as well as in our previous analyses [17],[18]. As already mentioned, Thomas uses the predictions of the improved cloudy bag model as initial values given at the low energy scale, i.e. $\sqrt{Q^2} = 0.4 \text{ GeV}$. Strictly speaking, there is no rigorous theoretical basis for this choice of starting energy. It is basically motivated by the fact that a similar scale is needed to match parton distribution functions calculated in various modern quark models to high energy experimental data. An advantage of starting from high energy scale and using downward evolution is that we can avoid this problem, although the precise matching energy with the low energy models are left

undetermined. Keeping this in mind, one may continue the downward evolution to the scale μ^2 , where $\langle x \rangle^Q = 1$, and $\langle x \rangle^g = 0$. (Numerically, we find that $\mu^2 \simeq 0.070 \text{ GeV}^2$ in the case the leading-order evolution equation is used, while $\mu^2 \simeq 0.195 \text{ GeV}^2$ if the next-to-leading order evolution equation is used.) This scale may be regarded as a matching scale with the low energy effective quark models as advocated in [25] and [26]. Or, one may take a little more conservative viewpoint that the matching scale would be between μ^2 and somewhere below 1 GeV^2 . At any rate, it is at least obvious that the use of the evolution equation below this scale, i.e. the unitarity violating limit, is meaningless.

Now we concentrate on getting reliable information for two unknowns, i.e. $B_{20}^{u+d}(0)$ and $B_{20}^{u-d}(0)$. The situation is better for the isovector quantity $B_{20}^{u-d}(0)$. One finds that the newest predictions of two lattice QCD groups given at the scale $Q^2 = 4 \text{ GeV}^2$, i.e. $B_{20}^{u-d}(0) = 0.274 \pm 0.037$ from the LHPC Collaboration [27] and $B_{20}^{u-d}(0) = 0.269 \pm 0.020$ from the QCDSF-UKQCD Collaboration [28], are remarkably close to each other. There also exists an estimate based on the chiral quark soliton model (CQSM). Its prediction evolved to the scale $Q^2 = 4 \text{ GeV}^2$ from the starting energy scale $\mu^2 = 0.30 \text{ GeV}^2$ with the next-to-leading (NLO) evolution equation gives $B_{20}^{u-d}(0) \simeq 0.289$ [18], which is also close to the lattice QCD estimates of two groups. To avoid initial scale dependence of the CQSM estimate, we simply use here the central value of the LHPC Collaboration, $B_{20}^{u-d}(0) = 0.274$ given at the scale $Q^2 = 4 \text{ GeV}^2$.

In contrast to the isovector case, the situation for the isoscalar combination $B_{20}^{u+d}(0)$ is not very satisfactory, because the predictions of the lattice QCD simulations are quite sensitive to the adopted method of chiral extrapolation and dispersed. The result of the LHPC group obtained with covariant baryon chiral perturbation theory is $B_{20}^{u+d}(0) = -0.094 \pm 0.050$, while the result of the same group obtained with heavy baryon chiral perturbation theory is $B_{20}^{u+d}(0) = 0.050 \pm 0.049$. On the other hand, the result of the QCDSF-UKQCD group is $B_{20}^{u+d}(0) = -0.120 \pm 0.023$. Fortunately, from an analysis of the forward limit of the unpolarized generalized parton distribution $E^{u+d}(x, \xi, t)$ within the CQSM, the 2nd moment of which gives $B_{20}^{u+d}(0)$, we were able to give a reasonable theoretical bound for this quantity, i.e. $0 \geq B_{20}^{u+d}(0) \geq -0.12 (= \kappa^{p+n})$ with κ^{p+n} being the isoscalar magnetic moment of the nucleon [18], which works to exclude some range of lattice QCD predictions. In the following, we therefore regard $B_{20}^{u+d}(0)$ as an unknown constant within this bound. (Note that it is a conservative bound since it is actually given at the low energy model scale and the magnitude of $B_{20}^{u+d}(0)$ is a decreasing function of the scale parameter Q^2 .)

The information on the quark orbital momenta can be obtained from J^u , J^d and J^s by subtracting the corresponding intrinsic spin contributions, $\Delta\Sigma^u$, $\Delta\Sigma^d$ and $\Delta\Sigma^s$. Basically, they are all empirically known quantities. (Note that, at the leading order, any of these three are scale independent.) Among the three combinations $\Delta\Sigma^Q$, $\Delta\Sigma^{u-d}$, and $\Delta\Sigma^{u+d-2s}$, the flavor singlet one has a largest uncertainty. For simplicity, here we use the central value of the recent

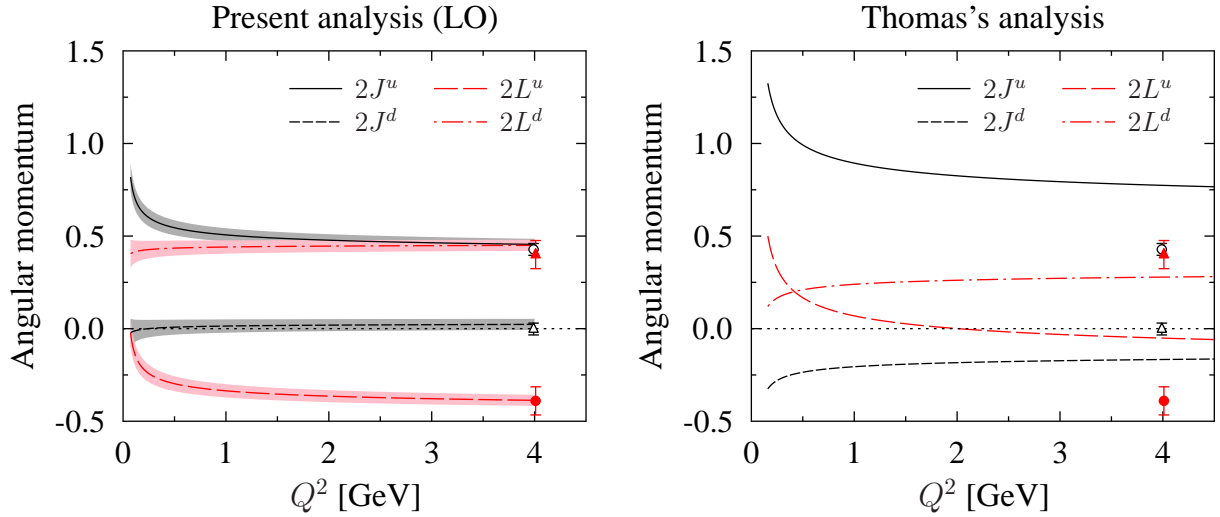


Figure 1: The left panel shows the results of the present semi-phenomenological extraction of the total angular momenta as well as the orbital angular momenta of up and down quarks, while the right panel shows the corresponding results of Thomas [11]. In both panels, the open circle, open triangle, filled circle, and filled triangle respectively represent the predictions of the LHPC lattice simulations for $2J^u$, $2J^d$, $2L^u$, and $2L^d$ [27].

HERMES analysis, i.e. $\Delta\Sigma^Q = 0.33$, by neglecting the error-bars.

For completeness, we list below all the initial conditions at $Q^2 = 4 \text{ GeV}^2$, which we shall use in the present analysis :

$$\langle x \rangle^Q = 0.579, \quad \langle x \rangle^{u-d} = 0.158, \quad \langle x \rangle^s = 0.041, \quad (3)$$

$$B_{20}^{u-d} = 0.274, \quad 0 \geq B_{20}^Q = B_{20}^{u+d-2s} \geq -0.12, \quad (4)$$

$$\Delta\Sigma^Q = 0.33, \quad \Delta\Sigma^{u-d} = 1.27, \quad \Delta\Sigma^{u+d-2s} = 0.586. \quad (5)$$

(The inclusion of the strange quark contributions to the momentum fractions and the longitudinal quark polarization appears inconsistent with the neglect of the corresponding contribution to B_{20} . It is however clear that the influence of the strange quark components are so small that they never affect the main point of the present analysis.)

After preparing all the necessary information, we now evaluate the total angular momentum as well as the orbital angular momentum of any quark flavor as functions of Q^2 . The answers for $2J^u$, $2J^d$ as well as for $2L^u$, $2L^d$ are shown in the left panel of Fig.1, respectively by the solid, short-dashed, long-dashed, and dash-dotted curves with shaded areas. The open circle, open triangle, filled circle, and filled triangle in the same figure represent the predictions of the

latest LHPC Collaboration for $2J^u$, $2J^d$, $2L^u$, and $2L^d$. For comparison, the corresponding predictions of Thomas' analysis [8] are shown in the right panel. One immediately notices that the difference between our analysis and Thomas' one is sizable. The most significant qualitative difference appear in the orbital angular momenta. As already mentioned, Thomas' analysis shows that the orbital angular momenta of up and down quarks cross over around the scale of 0.5 GeV. In contrast, no crossover of L^u and L^d is observed in our analysis : L^d remains to be larger than L^u down to the scale where the gluon momentum fraction vanishes. Comparing the two panels, the cause of this difference seems obvious. Thomas claims that his results are qualitatively consistent with the empirical information as well as the lattice QCD data at high energy scale. (We recall that the sign of L^{u-d} at the high energy scale is constrained by the asymptotic condition $L^{u-d}(Q^2 \rightarrow \infty) = -\frac{1}{2}\Delta\Sigma^{u-d}$, which is a necessary consequence of QCD evolution [18],[8].) However, the discrepancy between his results and the recent lattice QCD predictions seems to be never small as is clear from the right panel of Fig.1.

It can also be convinced from a direct comparison with the empirical information on J^u and J^d . In Fig.2, we compare the prediction of our semi-empirical analysis, that of Thomas' analysis, and that of the recent LHPC Collaboration, with the HERMES [29],[30] and JLab [31] determinations of J^u and J^d . One sees that, by construction, the result of our analysis is fairly close to that of the lattice QCD simulation. A slight difference between them comes from the fact that we use the empirical information (not the lattice QCD predictions) for the momentum fractions and the longitudinal polarizations of quarks. On the other hand, Thomas' result considerably deviates from the other two predictions. Although it is consistent with the HERMES data, it lies outside the error-band of JLab analysis. The latter observation is mainly related to the fact that his estimate for J^d is sizably larger than the lattice QCD data or our estimate and his estimate for J^u is smaller in magnitude than the other two. (One must be careful about the fact, however, that experimental extraction of J^u and J^d has a large dependence on the theoretical assumption of the parametrization of relevant GPDs and it should be taken as qualitative at the present stage.)

So far, to avoid introducing inessential complexities into our simple analysis, we did not pay enough care to the errors of the empirical and semi-empirical information given at the scale $Q^2 = 4 \text{ GeV}^2$, except for the quantity $B_{20}^{u+d}(0)$ having the largest uncertainty. One may worry about how strongly the conclusion of the present analysis depends on the ambiguities of the other quantities prepared at $Q^2 = 4 \text{ GeV}^2$. Fortunately, for the isovector quantity $L^{u-d} \equiv L^u - L^d$, which is of our primary concern in the present paper, one can convince that our central conclusion is not altered by these uncertainties. To see it, let us first recall the relation

$$2L^{u-d} = \left[\langle x \rangle^{u-d} + B_{20}^{u-d}(0) \right] - \Delta\Sigma^{u-d}. \quad (6)$$

Here, $\Delta\Sigma^{u-d} = g_A^{(I=1)}$ is scale independent and known with high precision, i.e. within 0.3 %.

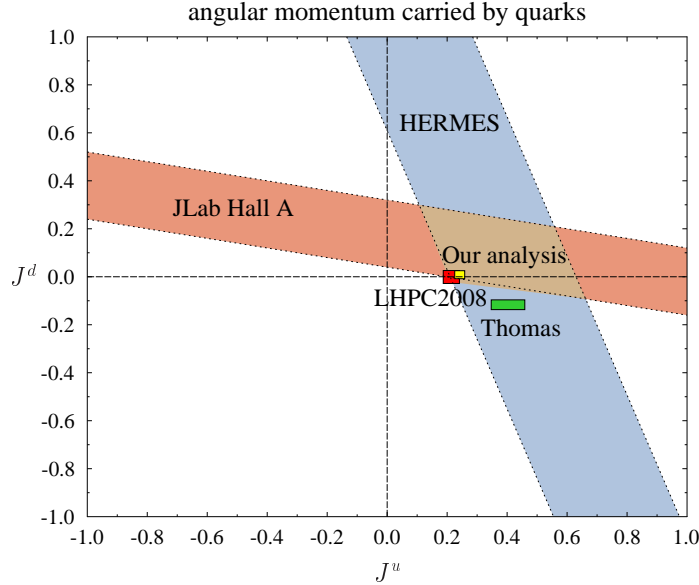


Figure 2: The HERMES and JLab Hall A determination of the quark angular momentum J^u and J^d [29],[30], [31] in comparison with our semi-empirical prediction. Also shown for comparison are the recent lattice QCD prediction by the LHPC Collaboration [27] and the result of Thomas' analysis [11].

The momentum fraction $\langle x \rangle^{u-d}$ is also known with fairly good precision. In fact, the difference between the familiar MRST2004 and CTEQ5 fits at $Q^2 = 4 \text{ GeV}^2$ turns out to be within 1%. The main uncertainty then comes from the isovector anomalous gravitomagnetic moment of the nucleon $B_{20}^{u-d}(0)$. We recall again the predictions of the two lattice QCD collaborations at $Q^2 = 4 \text{ GeV}^2$, i.e. $B_{20}^{u-d}(0) = 0.274 \pm 0.037$ from the LHPC Collaboration and $B_{20}^{u-d}(0) = 0.269 \pm 0.020$ from the QCDSF-UKQCD Collaboration, and also the prediction of the CQSM evolved to the same energy scale $B_{20}^{u-d}(0) \simeq 0.289$. In the analysis so far, we have used the central value of the LHPC prediction by simply neglecting the error-bar. Now let us take account of the error-bar and see how large this uncertainty would propagate and affect the value of L^{u-d} at the low energy model scale. (Note that, the error estimate of the LHPC group is most conservative and the prediction of the QCDSF-UKQCD group and that of the CQSM are contained in the error-band of this LHPC analysis.)

The filled area with dark grey in Fig.3 show the result of this downward evolution of $2L^{u-d}$ by starting with the initial condition given at $Q^2 = 4 \text{ GeV}^2$ on account of this error-band. In consideration of the possibility of incomplete nature of the present-day lattice QCD predictions (and also small uncertainties of the other two quantities $\langle x \rangle^{u-d}$ and $\Delta\Sigma^{u-d}$), we

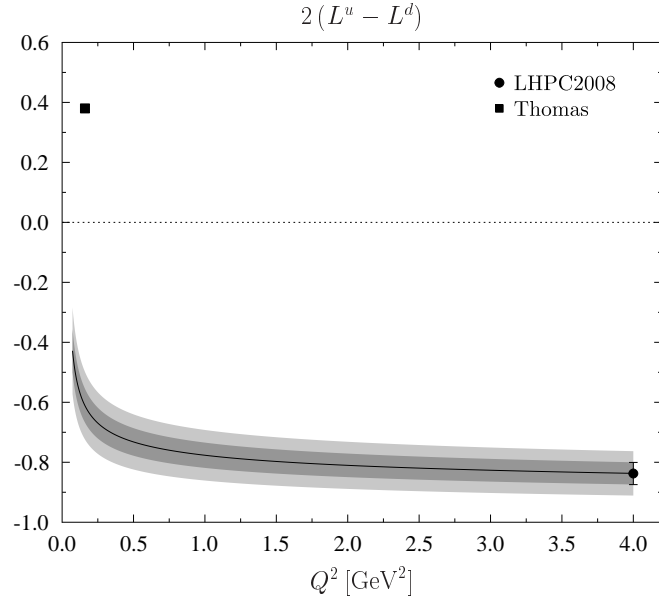


Figure 3: The sensitivity of the quark orbital angular momentum difference $2(L^u - L^d)$ to the initial condition given at $Q^2 = 4 \text{ GeV}^2$. The filled area with dark grey is obtained with the LHC prediction $B_{20}^{u-d}(0) = 0.274 \pm 0.037$ given at $Q^2 = 4 \text{ GeV}^2$, while filled area with light grey is obtained by artificially doubling the error of LHC prediction [27]. Also shown by the filled square is the prediction of the improved cloudy bag model corresponding to the scale $Q_0^2 = 0.16 \text{ GeV}^2$ [8].

have also carried out a similar analysis in which the error-bar of the LHC prediction is artificially doubled. The result of this latter analysis is shown by the filled area with light grey. One can clearly see that the quantity $2L^{u-d}$ remains negative even down to the lower energy scale close to the unitarity-violating bound, which appears to be very different from the prediction of the refined cloudy bag model shown by the filled square in the same figure.

In any case, our semi-phenomenological analysis, which is consistent with the empirical information as well as the lattice QCD data for J^u and J^d at high energies, indicates that $L^u - L^d$ remains fairly large and negative even at the low energy scale of nonperturbative QCD. If this is in fact confirmed, it may as well be called “new or another nucleon spin puzzle”. The observation is in fact a serious challenge to any low energy models of nucleon, since they must now explain small $\Delta\Sigma^Q$ and large and negative L^{u-d} simultaneously. The refined cloudy bag model of Thomas and Myhrer appears to be incompatible with this observation, since it predicts $2L^u \simeq 0.50$ and $2L^d \simeq 0.12$, or $2(L^u - L^d) \simeq 0.38$ at the model scale. (See TABLE I of [8].) Is there any low energy model which can reproduce this feature? Surprisingly, the CQSM

can explain both of these peculiar features of the nucleon observables at least qualitatively. It has been long known that it can explain very small $\Delta\Sigma^Q$ ($\Delta\Sigma^Q \simeq 0.35$ at the model scale) due to the very nature of the model, i.e. the nucleon as a rotating hedgehog object [32],[33]. Very interestingly, its prediction for L^{u-d} given in [34], i.e. $L^{u-d} \simeq -0.33$ at the model scale, also matches the conclusion obtained in the present semi-empirical analysis. (This could be anticipated from the fact that its prediction for $B_{20}^{u-d}(0)$ matches the lattice QCD predictions after account of the scale dependence.)

3 Note on the nucleon spin decomposition

To understand the cause of the apparent mismatch between our observation and the picture of standard quark models, typified by the refined cloudy bag model, it may be of some help to remember the important fact that the decomposition of the nucleon spin is not unique at all. There are two widely-known decompositions of the nucleon spin, i.e. the Ji decomposition [21] and the Jaffe-Manohar one [35],[36]. (See also recent yet another proposal [37],[38].) The Ji decomposition is given in the form

$$\frac{1}{2} = J^Q + J^g, \quad (7)$$

whose terms are defined as nucleon matrix elements of the corresponding operators

$$\hat{J}^Q = \int \psi^\dagger \left[\frac{1}{2} \boldsymbol{\Sigma} + \mathbf{x} \times (-i \mathbf{D}) \right] \psi d^3x, \quad (8)$$

$$\hat{J}^g = \int [\mathbf{x} \times (\mathbf{E} \times \mathbf{B})] d^3x, \quad (9)$$

with $\boldsymbol{\Sigma} = \gamma^0 \boldsymbol{\gamma} \gamma^5$. On the other hand, the Jaffe-Manohar decomposition is given in the form

$$\frac{1}{2} = J'^Q + J'^g, \quad (10)$$

whose terms are defined as nucleon matrix elements of the following operators

$$\hat{J}'^Q = \int \psi^\dagger \left[\frac{1}{2} \boldsymbol{\Sigma} + \mathbf{x} \times (-i \nabla) \right] \psi d^3x, \quad (11)$$

$$\hat{J}'^g = \int [(\mathbf{E} \times \mathbf{A}) - E_i (\mathbf{x} \times \mathbf{E}) A_i] d^3x. \quad (12)$$

Very recently, Burkardt and BC have studied both the Jaffe-Manohar as well as the Ji decomposition of angular momentum for two simple toy models, i.e. in the scalar diquark model as well as for an electron in QED in order $\alpha = e^2/4\pi$ [39]. They demonstrated that both decomposition yield the same numerical value for the fermion OAM in the scalar diquark model, but not in QED. They have also shown that the fermion OAM distributions in the Feynman- x space in both decompositions do not coincide even in the scalar diquark model.

Their investigation throws a renewed interest in the difference existing between the quark OAM resulting from the Jaffe-Manohar decomposition and that obtained from the Ji decomposition. It has been long recognized that the quark OAM in the Ji decomposition is manifestly gauge invariant, and accordingly it contains an interaction term with the gluon. On the other hand, the quark OAM appearing in the Jaffe-Manohar decomposition has simpler physical interpretation as a canonical orbital angular momentum in that it is given as a nucleon matrix element of free-field expression of quark OAM. Unfortunately, no reliable information exists on the difference between the magnitudes of these two definitions of the quark OAMs from lattice QCD simulations.

Since the CQSM is an effective quark theory that contains no gauge field, one might naively expect that there is no such ambiguity problem in the definition of the quark OAM. It turns out that this is not necessarily the case, however. The point is that it is a highly nontrivial interaction theory of quark fields. To explain it, we recall the past analyses of Ji's angular momentum sum rule within the framework of the CQSM. The analysis for the isoscalar combination was carried out by Ossmann et al. [40]. Starting with the theoretical expression for the unpolarized GPD $E_M^{u+d}(x, \xi, t) \equiv H^{u+d}(x, \xi, t) + E^{u+d}(x, \xi, t)$, they analyzed its 2nd moment, which is expected to give $2J^{u+d}$ on the basis of general argument of Ji. In fact, by using the equation of motion of the model, they could show that

$$\frac{1}{2} \int_{-1}^1 x E^{u+d}(x, 0, 0) dx = L_f^{u+d} + \frac{1}{2} \Delta \Sigma^{u+d}, \quad (13)$$

where the terms in the r.h.s are respectively given as proton (with spin up) matrix elements of the following operators within the model :

$$\hat{L}_f^{u+d} = \int \psi^\dagger(x) [\mathbf{x} \times (-i \nabla)]_3 \psi(x) d^3x, \quad (14)$$

$$\hat{\Sigma}^{u+d} = \int \psi^\dagger(x) \Sigma_3 \psi(x) d^3x. \quad (15)$$

As anticipated, the answer is given as a sum of the proton matrix element of the free-field expression for the quark OAM operator and that of the isoscalar quark spin operator. This is nice, but still we must be careful about the following fact. The net quark OAM distribution in x -space defined through the unintegrated version of Ji's sum rule written as

$$L^{u+d}(x) = \frac{1}{2} x E_M^{u+d}(x, 0, 0) - \frac{1}{2} \Delta \Sigma^{u+d}(x), \quad (16)$$

does not seem to coincide with the OAM distribution corresponding to the Jaffe-Manohar decomposition numerically evaluated within the same CQSM in [14]. This observation just corresponds to the recent finding by Burkardt and BC in the scalar diquark model [39].

A similar analysis for the isovector combination was carried out in [34]. It was found there that the 2nd moment of the isovector GPD $E_M^{u-d}(x, 0, 0)$ is now given as a sum of three pieces

Table 1: The CQSM predictions for L_f^{u-d} and δL^{u-d} as well as their sum at the leading order in the collective angular velocity Ω .

	L_f^{u-d}	δL^{u-d}	$L^{u-d} = L_f^{u-d} + \delta L^{u-d}$
Valence	0.147	- 0.289	- 0.142
Sea	- 0.265	0.077	- 0.188
Total	- 0.115	- 0.212	- 0.330

as

$$\frac{1}{2} \int_{-1}^1 x E_M^{u-d}(x, 0, 0) dx = \left(L_f^{u-d} + \delta L^{u-d} \right) + \frac{1}{2} \Delta \Sigma^{u-d}. \quad (17)$$

Here, L_f^{u-d} and $\Delta \Sigma^{u-d}$ terms are naively anticipated ones, i.e. a proton matrix element of free-field expression for the isovector quark OAM operator and that of the isovector quark spin operator respectively given as

$$\hat{L}_f^{u-d} = \int \psi^\dagger(x) \tau_3 [\mathbf{x} \times (-i \nabla)]_3 \psi(x) d^3x, \quad (18)$$

$$\hat{\Sigma}^{u-d} = \int \psi^\dagger(x) \tau_3 \Sigma_3 \psi(x) d^3x. \quad (19)$$

Somewhat embarrassingly, we found an extra piece represented as

$$\delta L^{u-d} = -M \frac{N_c}{18} \times \sum_{n \in occ} \langle n | r \sin F(r) \gamma^0 [\boldsymbol{\Sigma} \cdot \hat{\mathbf{r}} \boldsymbol{\tau} \cdot \hat{\mathbf{r}} - \boldsymbol{\Sigma} \cdot \boldsymbol{\tau}] | n \rangle. \quad (20)$$

Here, $|n\rangle$ stand for the eigenstates of the Dirac Hamiltonian $H = -i \boldsymbol{\alpha} \cdot \nabla + M \beta e^{i\gamma_5 \boldsymbol{\tau} \cdot \hat{\mathbf{r}} F(r)}$ with hedgehog mean field. The symbol $\sum_{n \in occ}$ denotes the sum over all the occupied eigenstates of H . This extra term is highly model-dependent and its physical interpretation is far from self-evident. It is nevertheless clear that there is no compelling reason to believe that the quark OAM defined through Ji's sum rule must coincide with the canonical one, i.e. the proton matrix element of the free-field OAM operator. Since the CQSM is a nontrivial interacting theory of effective quarks, which mimics the important chiral-dynamics of QCD, it seems natural to interpret this peculiar term as a counterpart of the interaction dependent part of the quark OAM in the Ji decomposition of the nucleon spin.

A natural next question is how significant the influence of this peculiar term is. For illustration, we show in table 1 the predictions of the CQSM for L_f^{u-d} and δL^{u-d} as well

as their sum. (The numerical values are from the leading-order prediction of the CQSM given in [34].) Here, shown in the 2nd and the 3rd rows are respectively the contributions of the three valence quarks and those of the negative energy Dirac-sea quarks, while shown in the 4th row are their sums. One sees that the valence quark contribution to L_f^{u-d} is positive but the Dirac-sea contribution to it is negative and larger in magnitude than the valence quark one, so that the net contribution to L_f^{u-d} is negative. Concerning the term δL^{u-d} , it is dominated by the valence quark contribution, which is large and negative. Adding up the two contributions, L_f^{u-d} and δL^{u-d} , we thus find that the CQSM prediction for the isovector quark OAM $L^u - L^d$ is sizably negative. We again emphasize that this prediction of the CQSM is totally different from the corresponding prediction of the refined CB model, which gives that $L^u - L^d$ is sizably large and positive at the model scale. A word of caution here. The calculation of the quark OAM in the CB model does not seem to use Ji's way of defining the quark OAM, although the detail is not clear from the papers. It would be interesting if one can check whether the two ways of calculating the quark orbital angular momenta make any difference also in the framework of the refined CB model or not.

4 Summary and Conclusion

To sum up, we have estimated the orbital angular momenta L^u and L^d of up and down quarks in the proton as functions of the energy scale, by carrying out a downward QCD evolution of available information at high energy, to find that $L^u - L^d$ remains to be large and negative even at the low energy scale of nonperturbative QCD, in remarkable contrast to Thomas' conclusion based on the refined cloudy bag model. Although the orbital angular momenta of quarks are not direct observables, they can well be extracted since J^u and J^d are measurable quantities from GPD analysis and since the intrinsic quark polarizations are basically known quantities by now. (One should not forget about the fact that the orbital angular momenta of quarks extracted in this way correspond to the Ji decomposition.) Then, what is required for future experiments is to determine J^u and J^d as precisely as possible including their scale dependence. Ideal would be to confirm the predicted strong scale dependence between 1 GeV and several hundreds MeV region. In practice, the GPD analysis far below 1 GeV may not be so easy because of uncontrollable higher-twist effects. However, the precise determination of J^u and J^d around 1 GeV region should give crucial information to judge which of the two scenarios, Thomas' one and the present one, for L^u and L^d , are close to the truth, thereby providing us with a valuable insight into unexpected role of quark orbital angular momenta as ingredients of the nucleon spin.

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