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# RELATIVE PHASES IN DALITZ PLOT AMPLITUDES FOR $D^0 \to K_S\pi^+\pi^-$ AND $D^0 \to \pi^0 K^+ K^-$

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Relative phases of amplitudes for $D$ meson decays to a light pseudoscalar meson $P$ and a light vector meson $V$ decaying to two pseudoscalar mesons will lead to characteristic interferences on the three-body Dalitz plot. These phases may be compared with predictions of a flavor-symmetric treatment which extracts contributing amplitudes and their relative phases from a fit to $D \to PV$ decay rates. Good agreement was found previously for the cases of $B^0 \to K^+\pi^-\pi^0$ and $D^0 \to \pi^+\pi^-\pi^0$. The present work is devoted to the decays $D^0 \to K_S\pi^+\pi^-$ and $D^0 \to \pi^0 K^+K^-$, for which agreement is not found. Several suggestions are offered for this discrepancy.

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# I INTRODUCTION

The relative phases of amplitudes for decays of charmed mesons to three light pseudoscalar mesons $P$ are useful in extracting the phase $\gamma$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. (See, e.g., [1] and references therein.) These phases may be specified by amplitude fits to Dalitz plots. For amplitudes dominated by quasi-two-body final states such as $PV$, where $V$ denotes a light vector meson, such phases are also specified in fits to decay rates based on flavor symmetry. (For recent examples see Refs. [2, 3, 4].)

Good agreement between the two methods of extracting relative phases of $D \to PV$ amplitudes was found previously for $B^0 \to K^+\pi^-\pi^0$ [5] and $D^0 \to \pi^+\pi^-\pi^0$ [6]. In the present work we investigate a similar question for the decays $D^0 \to K_S\pi^+\pi^-$ [1, 7, 8, 9, 10] and $D^0 \to \pi^0 K^+K^-$ [11, 12]. For these two processes, we do not find agreement between phases based on Dalitz plot analyses and those calculated from our flavor-symmetric amplitude analysis. (The decays $D^0 \to K_S K^+K^-$ are also studied in many of these references, but the important role of scalar resonances, for which quark-model assignments are uncertain, puts a similar analysis beyond our reach for the moment.)

We recall notation for amplitudes in the flavor-symmetric analysis and quote their values obtained from previous fits [2, 4] in Sec. II. We then construct the amplitudes for relevant $D \to PV$ subprocesses in Sec. III, and compare them with those extracted from Dalitz plot fits in Sec. IV. We discuss possible reasons for the observed discrepancies in Sec. V, and conclude in Sec. VI. An Appendix discusses some phase conventions.

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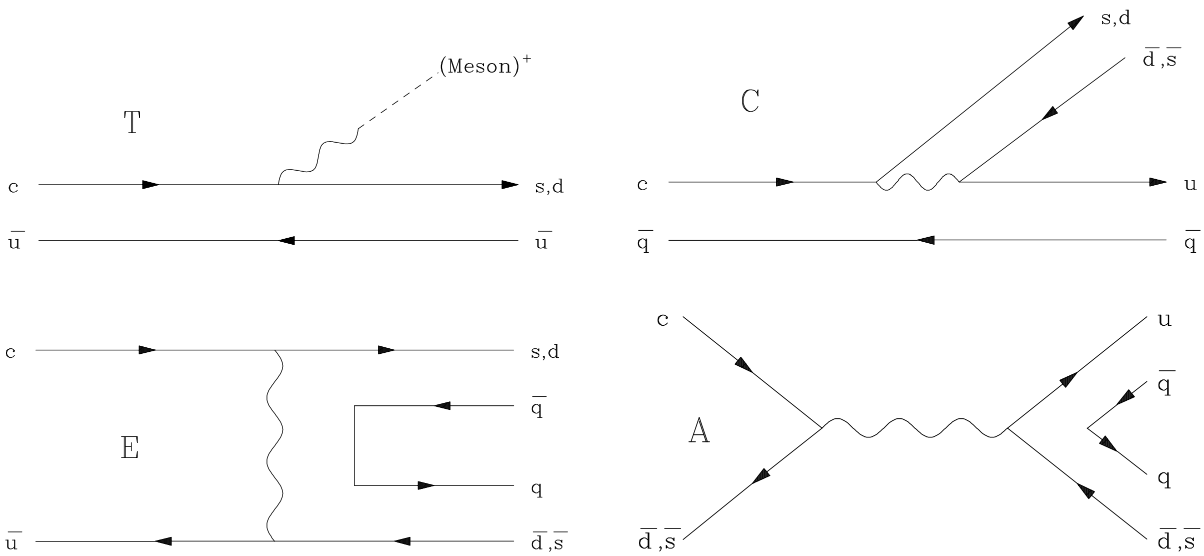


Figure 1: Flavor topologies for describing charm decays. $T$: color-favored tree; $C$: color-suppressed tree; $E$ exchange; $A$: annihilation.

# II AMPLITUDES FROM PREVIOUS FITS

We recall the notation from our previous analyses of $D \to PV$ decays [2]. The ratios of singly-Cabibbo-suppressed (SCS) amplitudes to Cabibbo-favored (CF) ones, and of doubly-Cabibbo-suppressed (DCS) to SCS ones, are SCS/CF = DCS/SCS = $\tan\theta_C \equiv \lambda = 0.2305$ [13], with $\theta_C$ the Cabibbo angle and signs governed by the relevant CKM factors.

For present purposes we shall be interested in amplitudes labeled as $T$ ("tree"), $C$ ("color-suppressed"), and $E$ ("exchange"), illustrated in Fig. 1. For $PV$ final states, a subscript on the amplitude denotes the meson ($P$ or $V$) containing the spectator quark. The partial width $\Gamma(H \to PV)$ for the decay of a heavy meson $H$ is expressed in terms of an invariant amplitude $\mathcal{A}$ as

$$\Gamma(H \to PV) = \frac{p^{*3}}{8\pi M_H^2}|\mathcal{A}|^2 \ , \tag{1}$$

where $p^*$ is the center-of-mass (c.m.) 3-momentum of each final particle, and $M_H$ is the mass of the decaying particle. With this definition, the amplitudes $\mathcal{A}$ are dimensionless.

Fits to rates for $D \to PV$ Cabibbo-favored decays not involving $\eta$ or $\eta'$ provide information on the amplitudes $T_V$, $C_P$, and $E_P$, as shown in Table I. For the amplitudes $T_P$, $C_V$, and $E_V$, one needs information on the $\eta$-$\eta'$ mixing, and Table II shows results for two values $\theta_\eta = 19.5^\circ$ and $11.7^\circ$.

# III CONSTRUCTION OF $D \to PV$ AMPLITUDES

The $D^0 \to PV$ amplitudes of interest for the present discussion are shown in Tables III ($\theta_\eta = 19.5^\circ$) and IV ($\theta_\eta = 11.7^\circ$), along with their representations in the flavor-SU(3) language and their values.

Two notable relations in these tables are

$$A(D^0 \to K^{*-}K^+) \ = \ \lambda A(D^0 \to K^{*-}\pi^+) \ \ , \tag{2}$$

Table I: Solution in Cabibbo-favored charmed meson decays to $PV$ final states favored by fits [2] to singly-Cabibbo-favored decays.

| $PV$ amplitude | Magnitude ($10^{-6}$) | Relative strong phase |
|---|---|---|
| $T_V$ | $3.95 \pm 0.07$ | — |
| $C_P$ | $4.88 \pm 0.15$ | $\delta_{C_P T_V} = (-162 \pm 1)^\circ$ |
| $E_P$ | $2.94 \pm 0.09$ | $\delta_{E_P T_V} = (-93 \pm 3)^\circ$ |

Table II: Solutions for $T_P$, $C_V$, and $E_V$ amplitudes in Cabibbo-favored charmed meson decays to $PV$ final states, for $\eta - \eta'$ mixing angle of $\theta_\eta = 19.5^\circ$ and $11.7^\circ$.

| | $\theta_\eta = 19.5^\circ$ | | $\theta_\eta = 11.7^\circ$ | |
|---|---|---|---|---|
| $PV$ ampl. | Magnitude ($10^{-6}$) | Relative strong phase | Magnitude ($10^{-6}$) | Relative strong phase |
| $T_P$ | 7.46±0.21 | Assumed 0 | 7.69±0.21 | Assumed 0 |
| $C_V$ | 3.46±0.18 | $\delta_{C_V T_V} = (172 \pm 3)^\circ$ | 4.05±0.27 | $\delta_{C_V T_V} = (162 \pm 4)^\circ$ |
| $E_V$ | 2.37±0.19 | $\delta_{E_V T_V} = (-110 \pm 4)^\circ$ | 1.11±0.22 | $\delta_{E_V T_V} = (-130 \pm 10)^\circ$ |

$$A(D^0 \to K^{*+}K^-) = -(1/\lambda)A(D^0 \to K^{*+}\pi^-) \ , \tag{3}$$

independent of the fitted values of these parameters. The phases of the first two amplitudes are expected to be equal, while the second pair should have a relative phase of $180^\circ$. Taking the quotient of the two, one finds

$$\frac{A(D^0 \to K^{*-}K^+)}{A(D^0 \to K^{*+}K^-)} = -\lambda^2 \frac{A(D^0 \to K^{*-}\pi^+)}{A(D^0 \to K^{*+}\pi^-)} \tag{4}$$

This relation can be checked using relative phases of Dalitz plot amplitudes, and will be one of the tests performed in Section. V.

# IV COMPARISON WITH MAGNITUDES AND PHASES IN DALITZ PLOT ANALYSES

The amplitudes for the $D^0 \to PV$ processes described in the previous Section do not have any information about the vector meson decay. In order to obtain information about how these amplitudes interfere on the $D^0 \to K_S\pi^+\pi^-$ or $D^0 \to \pi^0 K^+K^-$ Dalitz plots, we need to let the vector meson decay to two-pseudoscalar final states. In a $D^0 \to ABC$ Dalitz plot, if we consider the intermediate process $D^0 \to RC$ where $R$ is the intermediate $AB$ resonance, we need to multiply the amplitude for the intermediate process by the appropriate isospin Clebsch–Gordan factor governing the decay of the vector meson. The spin part of this amplitude on the Dalitz plot is $T = -2\vec{p}_A \cdot \vec{p}_C$ where $\vec{p}_i$ is the 3-momentum of the particle $i$ in the resonance rest frame. This implies that if we switch the order of the particles $A$ and $B$ in the vector meson decay, then the phase of the resulting amplitude

Table III: Amplitudes for $D^0 \to PV$ decays of interest for the present discussion (in units of $10^{-6}$). The first three processes contribute to $D^0 \to K_S\pi^+\pi^-$, while the last three contribute to $D^0 \to \pi^0 K^+ K^-$. Here we have taken $\theta_\eta = 19.5°$.

| $D^0$ final state | Amplitude representation | Amplitude $A$ Re | Im | $\|A\|$ | Phase (°) |
|---|---|---|---|---|---|
| $K^{*-}\pi^+$ | $T_V + E_P$ | 3.796 | –2.936 | 4.799 | – 37.7 |
| $\rho^0\overline{K}^0$ | $(C_V - E_V)/\sqrt{2}$ | –1.850 | 1.915 | 2.663 | 134.0 |
| $K^{*+}\pi^-$ | $-\lambda^2(T_P + E_V)$ | –0.353 | 0.118 | 0.373 | 161.5 |
| $K^{*-}K^+$ | $\lambda(T_V + E_P)$ | 0.875 | –0.677 | 1.106 | – 37.7 |
| $K^{*+}K^-$ | $\lambda(T_P + E_V)$ | 1.533 | –0.513 | 1.616 | – 18.5 |
| $\phi\pi^0$ | $\lambda C_P/\sqrt{2}$ | –0.756 | –0.246 | 0.795 | –162.0 |

Table IV: Same as Table III but for $\theta_\eta = 11.7°$.

| $D^0$ final state | Amplitude representation | Amplitude $A$ Re | Im | $\|A\|$ | Phase (°) |
|---|---|---|---|---|---|
| $K^{*-}\pi^+$ | $T_V + E_P$ | 3.796 | –2.936 | 4.799 | – 37.7 |
| $\rho^0\overline{K}^0$ | $(C_V - E_V)/\sqrt{2}$ | –2.219 | 1.486 | 2.671 | 146.2 |
| $K^{*+}\pi^-$ | $-\lambda^2(T_P + E_V)$ | –0.371 | 0.045 | 0.373 | 173.1 |
| $K^{*-}K^+$ | $\lambda(T_V + E_P)$ | 0.875 | –0.677 | 1.106 | – 37.7 |
| $K^{*+}K^-$ | $\lambda(T_P + E_V)$ | 1.608 | –0.196 | 1.620 | – 6.9 |
| $\phi\pi^0$ | $\lambda C_P/\sqrt{2}$ | –0.756 | –0.246 | 0.795 | –162.0 |

changes by $\pi$. Thus the order of the two pseudoscalar mesons in the vector meson decay is crucial for our understanding of interferences on the Dalitz plot. Here we are guided by a set of conventions kindly communicated to us by R. Andreassen [15] and K. Mishra [16]. These conventions are noted in Table V. The particle index numbers in column 5 of this table show the order of the two pseudoscalars from the vector meson decay, for each process with an intermediate vector resonance. The respective Clebsch–Gordan factors indicate the weight attached to the $D^0 \to PV$ amplitude to obtain its contribution to the interferences on the corresponding Dalitz plot.

We fix the amplitudes of $D^0 \to \rho^0 K_S$ for $D^0 \to K_S\pi^+\pi^-$ and $D^0 \to K^{*+}K^-$ for $D^0 \to \pi^0 K^+K^-$ to 1.0 and obtain the amplitudes and phases of the other processes relative to these. The results are shown for $\theta_\eta = 19.5°$ in Table VI for $D^0 \to K_S\pi^+\pi^-$ and in Table VII for $D^0 \to \pi^0 K^+K^-$. One does not see agreement between the relative phases predicted in the flavor -SU(3) approach and those implied by the Dalitz plot analysis. A similar set of results may be obtained using $\theta_\eta = 11.7°$. Even though there are slight changes in the relevant amplitudes and phases, they still do not agree with those obtained from the Dalitz plot analysis. In the next Section we shall discuss some possible reasons for this discrepancy.

Table V: Conventions for order of the two pseudoscalars in vector meson decay [15, 16]. Here we choose the CP-even state $K_S = \frac{K^0 - \overline{K}^0}{\sqrt{2}}$ following the convention in Ref. [17].

| Dalitz Plot | Bachelor Particle | | Vector Meson Decay | | |
|---|---|---|---|---|---|
| | Meson | Index | Process | Indices | Clebsch factor |
| | $K_S$ | 1 | $\rho^0 \to \pi^+\pi^-$ | 23 | 1 |
| $D^0 \to K_S\pi^+\pi^-$ | $\pi^+$ | 2 | $K^{*-} \to K_S\pi^-$ | 13 | $\sqrt{2/3}$ |
| | $\pi^-$ | 3 | $K^{*+} \to K_S\pi^+$ | 12 | $-\sqrt{2/3}$ |
| | $\pi^0$ | 1 | $\phi \to K^+K^-$ | 23 | $1/\sqrt{2}$ |
| $D^0 \to \pi^0 K^+K^-$ | $K^+$ | 2 | $K^{*-} \to K^-\pi^0$ | 31 | $-1/\sqrt{3}$ |
| | $K^-$ | 3 | $K^{*+} \to \pi^0 K^+$ | 12 | $-1/\sqrt{3}$ |

Table VI: Relative amplitudes and phases in $D^0 \to K_S\pi^+\pi^-$ Dalitz plot. We use $\theta_\eta = 19.5°$.

| Decay | Relative Theoretical | | Relative Experimental [10] | |
|---|---|---|---|---|
| mode | Amplitude | Phase (°) | Amplitude | Phase (°) |
| $D^0 \to \rho^0 K_S$ | 1.0 | 0.0 | 1.0 | 0.0 |
| $D^0 \to K^{*-}\pi^+$ | 1.472±0.187 | 188±7 | 1.735±0.005 | 133.5±0.2 |
| $D^0 \to K^{*+}\pi^-$ | 0.114±0.015 | 27±7 | 0.164±0.003 | –44.0±1.1 |

Table VII: Relative amplitudes and phases in $D^0 \to \pi^0 K^+K^-$ Dalitz plot. We use $\theta_\eta = 19.5°$.

| Decay | Relative Theoretical | | Relative Experimental [12] | |
|---|---|---|---|---|
| mode | Amplitude | Phase (°) | Amplitude | Phase (°) |
| $D^0 \to K^{*+}K^-$ | 1.0 | 0.0 | 1.0 | 0.0 |
| $D^0 \to K^{*-}K^+$ | 0.685±0.049 | –19±3 | 0.601±0.016 | –37 ±2.9 |
| $D^0 \to \phi\pi^0$ | 0.602±0.042 | 37±3 | 0.690±0.022 | –20.7±16.5 |

# V POSSIBLE SOURCES OF DISCREPANCIES

## A Inaccuracies or instabilities of the flavor-SU(3) approach

Although the flavor-SU(3) approach has had some success in fitting rates of charm decays to $PP$ and $PV$ [2, 3, 4, 17, 18], as well as in describing relative phases in $D^0 \to K^-\pi^+\pi^0$ [5] and $D^0 \to \pi^+\pi^-\pi^0$ [6] Dalitz plots, there are some notable shortcomings. Perhaps the most familiar is the prediction of equal rates for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, whereas the former rate is about 2.8 times the latter [14]. One possibility is that an earlier fit to Cabibbo-favored decay rates [19, 20] with $|C_P| < |T_V|$ ("Solution B") in Ref. [2], rejected because it did not fit SCS rates as well as the amplitudes quoted in Table I, nevertheless has some validity. One may use the B1 solutions quoted in Ref. [2] and check the relative amplitudes and phases as was done for the A1 solutions in the previous section. It turns out that the B1 solutions don't agree with relative amplitude predictions in the case of $D^0 \to \pi^0 K^+K^-$, indicating that $|C_P| < |T_V|$ is not very helpful. In the case of $D^0 \to K_S\pi^+\pi^-$, where the amplitudes do not explicitly depend on $C_P$, the B1 solutions give reasonable results for relative amplitudes, but fail to agree with the relative phases.

Even though a flavor-SU(3) approach might predict equal strong phases for a pair of amplitudes, it has been pointed out that this relation could be violated by SU(3)-breaking effects [21]. Thus, although flavor SU(3) predicts equal strong phases for $D^0 \to K^-\pi^+$ and $D^0 \to K^+\pi^-$, there might be no reason to expect such an equality. In fact, this is an experimental question, which can be attacked by a variety of means [22]. The CLEO Collaboration has addressed this problem using tagged $D^0$ mesons produced in pairs at the $\psi(3770)$ resonance, and finds a strong phase consistent with zero [23].

## B Dalitz plot conventions

It is notoriously tricky to specify conventions for Dalitz plot amplitudes for $D \to RC \to ABC$ when considering vector mesons $R = V$, because of the importance of choosing the order correctly in $V \to AB$ decay. This question was found to be non-trivial in both cases mentioned above in which agreement between flavor SU(3) and Dalitz plot analyses was eventually found. One cross-check which should be relatively airtight is the comparison of relative phases implied by Eqs. (2) and (3). Here we use a modified version of Eq. (4):

$$\frac{A(D^0 \to K^{*-}K^+)}{A(D^0 \to K^{*+}K^-)} = -\, r\, e^{i\,\phi}\, \lambda^2 \frac{A(D^0 \to K^{*-}\pi^+)}{A(D^0 \to K^{*+}\pi^-)}, \tag{5}$$

where $r$ and $\phi$ determine the amplitude and phase of the deviation from Eq. (4). [Eq. (4) corresponds to $r = 1$ and $\phi = 0°$.] We find the following results from the experimental values, taking into account the signs and magnitudes of the Clebsch-Gordan factors in Table V and remembering the present convention $K_S = (K^0 - \overline{K}^0)/\sqrt{2}$:

$$r_{\rm ex} = 1.069 \pm 0.034 \ ; \quad \phi_{\rm ex} = (-34.5 \pm 3.1)^\circ \ . \tag{6}$$

The experimental result differs from $0°$ by a phase $\Delta\phi_{\rm ex} = -34.5°$, which is small enough that any remaining discrepancy is probably at least not due to a sign convention.

Comparison of the experimental and theoretical values for the two sides of Eq. (4) can indicate whether a sign might be misplaced in the conventions for $D^0 \to \pi^0 K^+K^-$ (left-hand side) or $D^0 \to K_S\pi^+\pi^-$ (right-hand side). In Table VIII we compare the ratios in theory (based on Table III) and experiment (based on Tables V, VI, and VII).

Table VIII: Comparison between predicted and measured ratios in Eq. (4).

| Amplitude ratio | Predicted Magnitude | Predicted Phase (°) | Measured Magnitude | Measured Phase (°) |
|---|---|---|---|---|
| $A(K^{*-}K^+)/A(K^{*+}K^-)$ | 0.685±0.049 | –19.2±2.2 | 0.601±0.016 | –37.0±2.9 |
| $-\lambda^2 A(K^{*-}\pi^+)/A(K^{*+}\pi^-)$ | 0.685±0.049 | –19.2±2.2 | 0.562±0.010 | –2.5±1.1 |

We have checked the phase conventions for $D^0 \to \pi^0 K^+ K^-$ using the fact that in Ref. [11] the $\bar{K}^*$ and $K^*$ resonances are found to interfere destructively with one another. In that reference and in the BaBar analysis [8], this would follow from the conventions stated in Table V, which have been confirmed to be those used [16]. These involve cyclic permutations of the particle indices for the three $VP$ cases. On the other hand, approximate agreement of our relative phase prediction for $D^0 \to K^{*-}\pi^+$ and $D^0 \to K^{*+}\pi^-$ with that measured experimentally [10] suggested to us that the conventions for $D^0 \to K_S\pi^+\pi^-$ were as stated in Table V, and did *not* involve cyclic particle indices. This was also confirmed to be the case [15]. (See Appendix.)

## C Existence of multiple solutions to Dalitz plot fits

It is possible to find multiple solutions of relative phases on Dalitz plots. Usually one chooses the "best-fit" solution, but such a choice may depend on assumptions about other amplitudes in the fit. To that end, we suggest that fits be attempted starting from the relative phases we have proposed in Table VI and VII.

## D Alternative parametrization of S-wave $K\pi$ amplitudes

The relative phases of different $D^0 \to PV$ amplitudes in $D^0 \to K_S\pi^+\pi^-$ and $D^0 \to \pi^0 K^+ K^-$ Dalitz plots depend on the interference of these amplitudes with those involving $K\pi$ in an S-wave final state. This is particularly important for $D^0 \to K_S\pi^+\pi^-$, as the $PV$ bands in the Dalitz plot do not overlap with one another.

The BaBar analyses we have quoted (e.g., [8, 10, 12]) parametrize $K\pi$ S-wave amplitudes using a form consistent with LASS data [27]. The elastic scattering amplitudes in these data are expected to rise linearly with squared center-of-mass energy $s$ as a result of the Adler zero [28, 29] at $s \simeq m_K^2 - m_\pi^2/2$ [30, 31, 32]. Because of this behavior, a pole for a scalar resonance ("$\kappa$") will appear at a much lower mass than in a naïve Breit-Wigner parametrization [33]. An inelastic process such as $D \to \bar{K}\pi\pi$ does not involve the Adler zero, so a description of the S-wave $K\pi$ amplitude via the LASS amplitude is not appropriate. A similar difference between elastic $\pi\pi$ amplitudes consistent with current algebra, unitarity, and crossing symmetry [34, 35] (which contain the Adler zero) and inelastic processes such as $\gamma\gamma \to \pi^+\pi^-$ [36, 37] (which do not) is responsible for the peaking at very low dipion effective mass of the cross section for the latter process.

Although we cannot at this point indicate the quantitative effect a different $K\pi$ S-wave amplitude parametrization would have on the Dalitz plot analyses, it has been shown in fits to $D \to \bar{K}\pi\pi$ decays [31] that one obtains an improved description of the data by introducing the Adler zero into the $K\pi$ amplitude. A similar exercise would be worthwhile in the BaBar, Belle, and CLEO data, especially in light of the importance of relative phases in three-body $D^0$ decays for determining phases of the CKM matrix [1, 38].

# VI CONCLUSIONS

We have compared experimental determinations of relative phases in Dalitz plot amplitudes for $D^0 \to K_S \pi^+ \pi^-$ and $D^0 \to \pi^0 K^+ K^-$ with predictions from a flavor-SU(3) approach. In contrast to the previously-studied cases of $B^0 \to K^+ \pi^- \pi^0$ [5] and $D^0 \to \pi^+ \pi^- \pi^0$ [6], we do not find agreement. A simpler test, Eq. (4) relating the ratio $A(D^0 \to K^{*-} K^+)/A(D^0 \to K^{*+} K^-)$ to the ratio $A(D^0 \to K^{*-} \pi^+)/A(D^0 \to K^{*+} \pi^-)$ with coefficient $-\lambda^2 = -0.053$, is satisfied by magnitudes of amplitudes but fails in phase by $(34.5 \pm 3.1)^\circ$. The phase discrepancies in various relations seem to be limited to less than $60^\circ$, suggesting that at least sign conventions have been properly identified. Three remaining possibilities for these shortcomings include (a) inaccuracies of the flavor-SU(3) approach, (b) the possibility that other Dalitz plot solutions exist with phases closer to the flavor-SU(3) predictions, and (c) sensitivity to the parametrization of the S-wave $K\pi$ amplitudes, in particular to the distinction between elastic amplitudes which have an Adler zero and inelastic ones which do not.

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# APPENDIX: PHASE CONVENTIONS IN THREE-BODY DECAYS

Here we expand upon the phase conventions defined in Table V [15, 16]. For a decay $D^0 \to RC \to (AB)C$, where $R$ is the intermediate $AB$ resonance (here, a vector meson), the conventions for $D^0 \to K_S \pi^+ \pi^-$ and $D^0 \to \pi^0 K^+ K^-$ are illustrated in Figs. 2 and 3, respectively. The matrix element $T = -2\vec{p}_A \cdot \vec{p}_C$ for the vector meson contribution to the Dalitz plot amplitude for a decay $D \to ABC$ can also be expressed in terms of masses and two-body effective masses

$$m_{AB}^2 = (p_A + p_B)^2 \ , \quad m_{AC}^2 = (p_A + p_C)^2 \ , \quad m_{BC}^2 = (p_B + p_C)^2 \qquad (7)$$

in the form (see, e.g., [25, 26], containing also expressions for a spin-2 resonance $R$):

$$-2\vec{p}_A \cdot \vec{p}_C = \frac{1}{2}\left(m_{AC}^2 - m_{BC}^2\right) + \frac{1}{2m_{AB}^2}\left(m_B^2 - m_A^2\right)\left(m_D^2 - m_C^2\right) \ . \qquad (8)$$

The $K^*$ bands in $D^0 \to K_S \pi^+ \pi^-$ do not overlap with one another, so one cannot directly see the interference between the Cabibbo-favored decay $D^0 \to K^{*-} \pi^+$ and the doubly-Cabibbo-suppressed decay $D^0 \to K^{*+} \pi^-$. However, the $K^*$ bands do overlap in $D^0 \to$

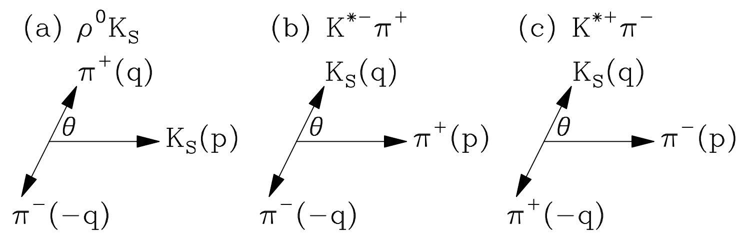


Figure 2: Phase conventions of Ref. [15] for the decays $D^0 \to K_S\pi^+\pi^-$.

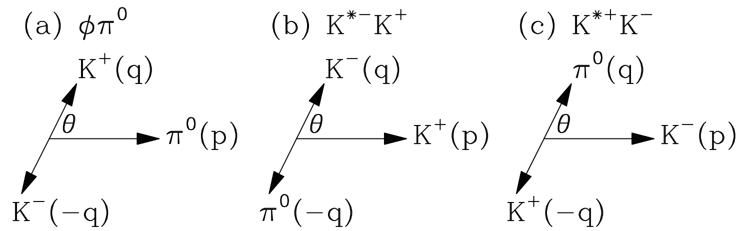


Figure 3: Phase conventions of Ref. [16] for the decays $D^0 \to \pi^0K^+K^-$.

$\pi^0K^+K^-$, and are of comparable strength, as both represent singly-Cabibbo-suppressed decays. It was noted in Ref. [24] that the sign of the interference can be readily diagnosed by inspecting the overlap region. In fact, in Ref. [11] the interference between the $K^{*+}$ and $K^{*-}$ bands was found to be destructive in the overlap region. This conclusion was based on enhancement of the $K^{*+}$ band in the low-$m(K^-\pi^0)$ region, but suppression of the $K^{*-}$ band in the low-$m(K^+\pi^0)$ region, indicating opposite signs of interference with a slowly-varying S-wave component.

Referring to Fig. 3, one sees that the low-$m(K^+\pi^0)$ region in the $K^{*-}$ band [illustrated by the configuration (b)] and the low-$m(K^-\pi^0)$ region in the $K^{*+}$ band [illustrated by the configuration (c)] are defined with the same sign. In that case, one expects the relative phase between the $D^0 \to K^{*+}K^-$ and $D^0 \to K^{*-}K^+$ amplitudes on the $D^0 \to \pi^0K^+K^-$ Dalitz plot, quoted in Table VII, to be near 180°, as predicted theoretically. Taking account of the relative sign of the Clebsch-Gordan coefficients $\pm1/\sqrt{3}$ in the last column and last two rows of Table V, this means that one expects the phase of the ratio on the left-hand side of Eq. (4), $A(D^0 \to K^{*-}K^+)/A(D^0 \to K^{*+}K^-)$, to be near zero, as predicted. (See Table VIII.)

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