Chiral symmetry, nuclear forces and all that

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Dedicated to Gerry Brown on the occasion of his 85*th* birthday

These are personal recollections of how Gerry Brown's work and thinking influenced the development of the chiral effective Lagrangian based theory of nuclear forces.

1 The early days at Stony Brook

I first met Gerry on the occasion of the Erice school in 1981, where he and Mannque Rho were giving quite interesting lectures on the chiral bag model and related issues. At that point, I was close to finishing my diploma thesis under the supervision of Manfred Gari at Bochum, working on two-meson exchange currents in the framework of unitary transformations. More specifically, the goal was to find out the relative contribution of the isoscalar $\rho \pi^2 \gamma$ current compared to the well-established leading order isoscalar $\rho \pi \gamma$ current. This calculation turned out to be - to use Howard Georgi's words - an excellent exercise in self-torture, as one had to evaluate about 1000 time-ordered diagrams by hand. Also, the result that these corrections were already sizeable at momentum transfers below $q^2 \simeq 1 \,\text{GeV}^2$ did not quite fit into the standard lore which was so eloquently summarized by Gerry and Mannque in their chiral filter hypothesis [\[1\]](#page-15-0). This hypothesis that grew out of explicit meson-exchange current calculations including heavier hadrons like the $\Delta(1232)$ or the vector mesons ρ, ω was clearly emphasizing the role of the pion. Whenever present, the one-pion exchange current was supposed to dominate the pertinent response of light nuclei to electromagnetic probes, even up to momentum transfers as large as 1 GeV^2 , way beyond the soft-pion limit. In fact, it was clear to Gerry and Mannque – and others – that the understanding of the nucleon structure from QCD and the nuclear interactions were two intimately connected issues, firmly rooted in the chiral symmetry of QCD. It is worth quoting from Ref. [\[1\]](#page-15-0): "... that there must exist an intimate connection between what makes up a hadron and what induces a rich variety of interactions between hadrons. ..., it will be rather unlikely one will gain a full understanding of one without the other." Only then too much emphasis was put onto the quark structure of the nucleon - as we understand now, low-energy QCD is well approximated by an hadronic effective Lagrangian. When doing my calculations at Bochum, I did not know that, but learned about it quickly when I arrived at Stony Brook in April of 1982. The Erice lectures on the little bag model had aroused my interest and, honestly, I was fed up calculating meson-exchange currents. Therefore, I contacted Gerry about becoming a graduate student at Stony Brook. It did not take him long to accept me - as it turned out, I spent a short but very productive time in Gerry's group until my graduation in December 1984. But let me briefly come back to the meson-exchange current (MEC) story. In fact, only after some strong support from Dan-Olof Riska, one of the MEC pioneers [\[2\]](#page-15-1), my diploma work was finally published [\[3\]](#page-15-2) but largely ignored by the community. The crux of the matter was that in those days one did not have a power counting that allowed one to systematically address the corrections to the leading order one-pion contribution either in the nuclear forces or in the corresponding exchange currents, though a tremendous amount of phenomenology had been built up over the years. In particular, the Stony Brook and Paris approach to the two-nucleon force that used dispersion relations to connect the πN scattering amplitudes to the 2π -exchange in the NN force supplemented by some heavy mass Yukawa-type functions with parameters determined from a fit to the data where already very close to what would become later the "modern theory of nuclear forces". In any case, I had to learn the basis of the fundamentals of the Stony Brook potential by working through the book of Gerry and Andy Jackson [\[4\]](#page-15-3). This turned out to be a very painful exercise as the notation would change from chapter to chapter, but made me familiar with dispersion theory and the many intricacies of nuclear interactions. In fact, one of the early projects Gerry assigned to me was the coupling of ρ -mesons to the little bag, which I worked out within a few weeks and which was supposed to be the starting point for a more microscopic investigation of vector meson exchanges in the nuclear force, thus further contributing to the sought-after unified picture of nucleons and the nuclear interactions. When I presented to Gerry my notes in the form of a handwritten draft - LATEXwas not yet available he just told me that I was too fast for him and proposed to me that I work alone. So I teamed up with Andreas Wirzba - a fresh new graduate student from Münster - and Ismail Zahed, then a post-doc in Gerry's group to work on Casimir effects in chiral bag models and on many aspects of Skyrmions, which was a hot topic at that time. The Skyrme model promised to lead to the long wanted unification of nucleon and nuclear physics, and many impressive calculations revealed further deep connections, but again the lack of a power counting was the stumbling stone.

2 A first step: Studying pion-nucleon scattering in chiral perturbation theory

During my stay as a Heisenberg fellow at Bern, I was lucky to learn chiral perturbation theory (CHPT) from Jürg Gasser and Heiri Leutwyler, who had transformed Weinberg's ideas into an effective machinery to analyze low-energy QCD. With Véronique Bernard and Norbert Kaiser we systematically analyzed electroweak reactions on the nucleon and in particular neutral pion photoproduction off the proton, $\gamma p \to \pi^0 p$, triggered by measurements at Saclay, Mainz and Saskatoon to test the then believed low-energy

Figure 1: The LECs c_i (circles) in pion-nucleon scattering (left), the two-nucleon (NN) interaction (center) and the three-nucleon (NNN) interaction (right).

theorems based on soft-pion algebra (plus one extra assumption). The explanation of the data by a new one-loop effect helped to establish the method as a precision tool in the single nucleon sector. In the the late nineties and early years of the new millennium, with my students Sven Steininger and Nadia Fettes [\[5\]](#page-15-4) we investigated pion-nucleon scattering in the framework of heavy baryon chiral perturbation theory - this later turned out to be an important ingredient in the construction of the few-nucleon forces based on effective field theory (EFT). The underlying chiral Lagrangian of pions and nucleons coupled to external currents can be written as

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \dots,\tag{1}
$$

where the superscript denotes the chiral dimension (the power in small external momenta and/or pion masses). Tree level computations are done with $\mathcal{L}^{(1)} + \mathcal{L}^{(2)}$, a complete one-loop calculation involves all loop graphs with insertions from $\mathcal{L}^{(1)}$ and at most one insertion from $\mathcal{L}^{(2)}$ and so on. In its full glory, \mathcal{L}_{eff} can be found in [\[7\]](#page-15-5). Of particular importance in these studies are the finite dimension two low-energy constants (LECs) c_i , see Fig. [1,](#page-2-0) that also feature prominently in the description of the two- and three-nucleon forces - Gerry was one of the first who had understood this long before the advent of EFT. An even better determination of some of the c_i was possible by analytically continuing the amplitudes into the interior of the Mandelstam triangle, see Fig. [2](#page-3-0) - the ability to use dispersion relations as I learned it in Stony Brook turned out to be very valuable. All this work can be summarized by the following values for the *cⁱ* in units of GeV^{-1} :

$$
c_1 = -0.9^{+0.2}_{-0.5}, \quad c_2 = 3.3 \pm 0.2 \;, \quad c_3 = -4.7^{+1.2}_{-1.0} \;, \quad c_4 = 3.5^{+0.5}_{-0.2} \;.
$$

The values for *c*₂, *c*₃ and *c*₄ are larger than the expected (natural) values: $c_i \sim g_A/\Lambda_\chi \simeq$ 1.1 . . . 1.5, with $g_A = 1.267$ the nucleon axial-vector coupling and $\Lambda_{\chi} \simeq 1$ GeV the scale of chiral symmetry breaking. This was understood early in terms of resonance saturation - the fact that the heavier states integrated out from the EFT leave their imprint in the values of the LECs. This method was pioneered in the meson sector by Gerhard Ecker and collaborators and also by John Donoghue and collaborators [\[8\]](#page-15-6). In 1997, with Véronique Bernard and Norbert Kaiser, we had already determined the c_i by a comparison to scattering lengths and subthreshold parameters and extended

Figure 2: Mandelstam plane. The Mandelstam triangle is the inside of the thick lines.

the resonance saturation scheme to the one-baryon sector, see Fig. [3.](#page-3-1) Not surprisingly, the LECs c_2 and c_3 are largely dominated by the close-by $\Delta(1232)$ resonance, whereas *c*⁴ also receives an important contribution from the *ρ*-meson [\[9\]](#page-15-7). This leads to the following values of $c_{2,3,4}$ (in brackets the ranges obtained from resonance saturation) c_2 3.9 $[2 \dots 4], c_3 = -5.3 [-4.5 \dots -5.3], c_4 = 3.7 [3.1 \dots 3.7],$ whereas c_1 is given by scalar meson exchange ($\pi\pi$ correlations). These numbers are also consistent with the recent analysis of pion-nucleon scattering and subthreshold parameters derived from pionic hydrogen and deuterium, $c_1 = -1.2 \ldots -0.9, c_2 = 2.6 \ldots 4.0, c_3 = -6.1 \ldots -4.4$ [\[10\]](#page-15-8). Let me also stress that the uncertainties in the c_i are strongly correlated, as best seen from the combination $-2c_1 + c_2 + c_3 - g_A^2/(8m)$ that determines the small isoscalar pion-nucleon scattering length.

Figure 3: Resonance saturation for the dimension two (circles) and three (squares) LECs.

3 Flashback: The Texas revolution

Already in the early nineties a decisive step towards a systematic theory of the nuclear forces had been done. This was initiated by a series of papers by Steven Weinberg, who had already pioneered the use of effective chiral Lagrangians decades earlier under the nowadays somewhat strange-reading title "Phenomenological Lagrangians" [\[11\]](#page-15-9). In that

marvelous paper the idea of the power counting was developed in detail and further a very illuminating application of the renormalization group to work out the so-called chiral logarithms was presented. Years later, while teaching the effective field theory of pions and nucleons, Weinberg suddenly realized that there are additional terms consistent with the power counting, terms quadrilinear in the nucleon fields $\#^1$. He immediately realized that these must be related to the much discussed mechanism for the short-range repulsion of the nuclear forces. Here the fundamental principle of effective field theory works at it best - in the low-energy sector of a given theory one is not able to resolve the physics at large momentum scales but simply parameterizes it in terms of operators made of the fields one has at one's disposal - it just does not matter whether the nucleonnucleon repulsion is due to vector-meson exchange, quark rearrangement energies or whatever model has been invented before. Weinberg then worked out the consequences of the spontaneous chiral symmetry breaking for the forces between two, three and four nucleons using the method of phenomenological Lagrangians [\[12\]](#page-16-0), nowadays called chiral perturbation theory or chiral effective field theory. Remarkably, the first of these two papers contains only three references, one to the review Gerry had written in 1985 with Sven-Olaf Bäckman and Jouni Niskanen [\[13\]](#page-16-1) and also, he explicitly thanks Gerry for "enlightening conversations on nuclear forces". The important step taken by Weinberg was to realize that in the nuclear force problem one can not apply the power counting directly to the S-matrix, but rather to the effective potential - these are all diagrams without *N*-nucleon intermediate states. Such diagrams lead to pinch singularities in the infinite nucleon mass limit (the so-called static limit), so that e.g. the nucleon box graph is enhanced as m/Q^2 , with *m* the nucleon mass and $Q \ll \Lambda_{\chi}$ a small momentum. The beautiful power counting formula for the graphs contributing with the *ν*th power of *Q* or a pion mass to the effective potential reads (considering only connected pieces):

$$
\nu = 2 - N - 2L + \sum_{i} V_i \left[d_i + \frac{n_i}{2} - 2 \right] \ . \tag{3}
$$

Here, *N* is the number of in-coming and out-going nucleons, *L* the number of pion loops, V_i counts the vertices of type *i* with d_i derivatives and/or pion mass insertions and n_i is the number of nucleons participating in this kind of vertex. Because of chiral symmetry, the term in the square brackets is larger than or equal to zero and thus the leading terms contributing e.g. to the NN potential can easily be identified. These are the time-honored one-pion exchange and two four-nucleon contact interactions without derivatives. These contact interactions were indeed the missing part which were earlier modeled by heavy meson-exchanges or by some kind of fit function - but none of these approaches was controlled or systematic. Here again the awesome power of power counting becomes crystal-clear - it just took one person not biased by nuclear folklore to take this final step. The so-constructed effective potential is then iterated in the Schrödinger or Lippman-Schwinger equation, generating the shallow nuclear bound states as well as scattering states. The resulting contributions to the 2N, the 3N and the 4N forces are depicted in Fig. [4.](#page-5-0) I will come back to a detailed discussion of the various entries below, at this

 $\overline{f}^{\pm 1}$ I am grateful to Steve Weinberg for providing me with this recollection.

Figure 4: Contributions to the effective potential of the 2N, 3N and 4N forces based on Weinberg's power counting. Here, LO denotes leading order, NLO next-toleading order and so on. Dimension one, two and three pion-nucleon interactions are denoted by small circles, big circles and filled boxes, respectively. In the 4N contact terms, the filled and open box denote two- and four-derivative operators, respectively.

point it is important to observe that - consistent with phenomenological observations - three-nucleon forces appear only two orders after the dominant NN forces and fournucleon forces are even further suppressed, appearing only at $N³LO$. Let me stress that the beautiful analysis of few-nucleon forces in chiral EFT by Bira van Kolck [\[14\]](#page-16-2) was an important ingredient to set up the power counting as displayed in Fig. [4](#page-5-0) and that this extended power counting has been the major development on the way to the chiral theory of nuclear forces.

The first large-scale numerical evaluation of this chiral potential was done by Bira van Kolck and collaborators a few years after Weinberg's seminal papers, see Refs. [\[15\]](#page-16-3). They worked up-to-and-including N^2LO in a theory of pions, nucleons and also the Δ isobar, fitting the LECs of the contact interactions to low-energy NN phases separately for isospin $I = 0$ and $I = 1$ and properties of the deuteron. This work was certainly groundbreaking but it was also not immediately accepted in the nuclear physics community for the simple reason that the phenomenological approaches to the two-nucleon problem had achieved a much higher precision - at the expense of many more parameters and lack of systematicity. Still, the first important step was done. Before picking up this theme, let me stress that Weinberg extended his approach to pion reactions on light nuclei [\[16\]](#page-16-4), using at that time deuteron wave functions from phenomenological potentials. This so-

called "hybrid appraoch" laid the ground for systematic studies of pion scattering off light nuclei, pion photo- and electroproduction off light nuclei and of pion production in NN collisions.

4 The chiral theory of nuclear forces as a precision tool

4.1 The nucleon-nucleon interaction

When I was still at Bonn filling in for the vacant chair of Max Huber, then rector of Bonn University, I met a few times with Karl Holinde of the IKP Jülich. Karl had become famous for his work on the Bonn potential and was hired by Josef Speth at Jülich in 1984, which gave Karl a fantastic home base to continue his work. We talked about the chiral potential of the Texas group, and he was clearly interested but also aware that the precision was not yet competitive. So we decided to work together to try to improve the Texas potential. Unfortunately, before really getting started, this collaboration was terminated by Karl's untimely death in December of 1996. In fact, Gerry came to Jülich the next year and gave a wonderful talk about the roots, the physics and the achievements of the Bonn potential, focusing on Karl's contribution. Fortunately, a bit later Walter Glöckle from Bochum contacted me - he had acquired a brilliant young student named Evgeny Epelbaum, who had done some calculations of the chiral NN potential in his diploma thesis and was supposed to continue working on this topic for his doctoral thesis. Walter was (and still is) a world-leading expert in few-nucleon calculations but did not feel at ease with effective chiral Lagrangians. So we decided that we would team up to supervise Epelbaum, which turned out to be a true pleasure as we did not have to do much. Being aware of the short-comings of energy-dependent potentials in few-body calculations, we decided to recalculate the Texas potential using the method of unitary transformations - which I had not used for about 15 years. Based on Epelbaum's diploma thesis, we published a paper on the power counting adapted to this method in [\[17\]](#page-16-5). Before working out the phenomenological consequences of this, we decided to work out some simpler examples to get better acquainted with the many subtleties of the nuclear interaction and how they reflect on the corresponding effective theory. Interestingly, in two papers we developed a method to integrate out the highmomentum components from an effective theory based on a simple local potential of the Malfliet-Tjon type [\[18\]](#page-16-6). This preceeded the work of Achim Schwenk, Tom Kuo, Bengt Friman, Gerry and others at Stony Brook on the so-called *V*low−^k potentials based on elegant renormalization group techniques, see e.g. the review [\[19\]](#page-16-7) and references therein. Such parallel developments are a fine witness of Gerry's legacy - he has taught many generations of students a deep understanding of physics that will necessarily lead to progress in the field of nuclear physics. But back to the nuclear force problem. We published our N2LO results in [\[20\]](#page-16-8), which were markedly improved as compared to the Texas potential. The simple reason was that we did not make global fits but rather projected the chiral potential onto partial waves and determined the four-nucleons LECs by fitting to the S- and P-waves and the ³*S*1- ³*D*¹ mixing parameter in the neutron-proton system. An important paper that had been published earlier was the work by the Munich

Figure 5: Left: *np* S-waves at NLO (grid), N²LO (light shaded) and N³LO (dark shaded). Right: Differential cross section and vector analyzing power for *np* scattering at $E_{\rm lab} = 50$ MeV compared to the N²LO (light shaded) and N³LO (dark shaded) predictions.

group, where the role of the two-pion exchange to the peripheral partial waves was scrutinized [\[21\]](#page-16-9). They demonstrated that these peripheral waves are well described by one- and two-pion exchanges in the Born approximation, thus only the low partial waves were to be used in fitting the multi-nucleon LECs. However, the large values of the c_i , cf. Eq.[\(2\)](#page-2-1), lead to a very strong isoscalar central potential at short distances that generates unphysical deep bound states for the larger values of the cut-off scale ($\Lambda \simeq 1$ GeV) in the LS-equation. Again, earlier work done by Chemtob, Durso and Riska – that was taught at Stony Brook – came to our rescue, we simply adopted the so-called spectral function regularization method to deal with the unwanted short-distance behavior of the two-pion exchange [\[22\]](#page-16-10). So the ground was prepared for going to yet higher orders. While Evgeny had moved to Jefferson Lab as the first Nathan-Isgur-Distinguished-Fellow, we continued our collaboration and published the $N³LO$ results in [\[23\]](#page-16-11). Earlier on, Ruprecht Machleidt and David Entem had published $N³LO$ results using dimensional regularization in the two-pion exchange [\[24\]](#page-16-12) based on a series of papers from Norbert Kaiser - this story is told in the contribution of Machleidt to this volume. Some characteristic results of our $N³LO$ calculation are displayed in Fig. [5](#page-7-0) - these are not more accurate than the ones based on phenomenological potentials, however, they provide a measure of the theoretical uncertainty and, more importantly, they are based on the chiral symmetry of QCD. To close the circle, one can indeed show that the four-nucleon operators to a large extent are saturated by resonance excitations, as explicit calculations mapping boson-exchange models onto the chiral expansion of the effective potential showed, for details see [\[25\]](#page-16-13). In a way, EFT provides a reason why the meson-exchange models of the nuclear force

Figure 6: Topologies of the leading contributions to the chiral 3NF. From left to right: Two-pion exchange, one-pion-exchange and 6N contact interaction.

have been so successful.

4.2 Three-nucleon interactions

One of the biggest advantages of the chiral Lagrangian approach to the nuclear force problem - as stressed early in Weinberg's papers - is the consistent derivation of the 3- and 4-nucleon forces and also the meson-exchange currents (that is, the response to electroweak probes). It is worth mentioning that decades earlier Gerry was aware of the role of chiral symmetry in the description of three-body forces in nuclei - see his nice paper with Saul Barshay in 1972 [\[26\]](#page-16-14). But let me come back to the modern approach. As already noted, the three-nucleon force (3NF) only appears two orders after the leading NN interaction. At this order, there are only three topologies contributing, see Fig. [6.](#page-8-0) The two-pion exchange topology is given again in terms of the *cⁱ* , as discussed in detail in [\[27\]](#page-16-15). The so-called *D*-term, which is related to the one-pion exchange between a 4N contact term and a further nucleon, has gained some prominence over the last years, as many authors have tried to pin it down based on a cornucopia of reactions, such as $Nd \to Nd$, $NN \to NN\pi$, $NN \to d\ell\nu_{\ell}$ or $d\pi \to \gamma NN$. This demonstrates nicely the power of EFT - very different processes are related through the same LECs thus providing many different tests of chiral symmetry (as it is also the case with the LECs c_i , see Fig. [1\)](#page-2-0). The LEC *E* related to the 6N contact interaction can only be fixed in systems with at least three nucleons, say from the triton binding energy. Remarkably, although there are many diagrams at $\rm N^3LO$, there are no new LECs to be determined. The long-range terms of this force can be found in [\[28\]](#page-16-16) and the shorter ranged ones will be published soon (see also [\[29\]](#page-17-0)). Many applications of these forces and the testing of their structure can be found in the reviews [\[30,](#page-17-1) [31\]](#page-17-2).

As a very nice example that these forces make their way into nuclear structure calculations, I show here the results from Petr Navrátil and collaborators [\[32\]](#page-17-3), who performed large-basis no-core shell model (NCSM) calculations including the leading chiral 3N forces and demonstrated their necessity to describe the spectra of nuclei with $A = 10, \ldots 13$, see the entry NN+NNN in Fig. [7.](#page-9-0) Note, however, that in this calculation the 2N force was employed at N^3LO while the 3NF was only included at N^2LO . It will be interesting to see how the inclusion of the sub-leading 3NF terms will modify these results.

So far, I have only discussed the EFT with pions and nucleons. Clearly, the two-pion exchange diagram in Fig. [6](#page-8-0) is a controlled approximation to the time-honored Fujita-

Figure 7: Spectra of light nuclei using chiral NN and chiral NN+NNN forces in a NCSM calculation compared to the data. Figure adopted from [\[32\]](#page-17-3) and courtesy of P. Navrátil.

Miyazawa force, whose $50th$ anniversary was celebrated in Tokyo in 2007. For a light reading of how the Fujita-Miyazawa force is related to the EFT description of the 3NF, I refer to [\[33\]](#page-17-4) and references therein.

4.3 The nuclear matter problem

Another important question in nuclear physics is to understand the saturation properties of nuclear matter - an idealized infinite system of nucleons in which all Coulomb effects are switched off. From the properties of heavy nuclei using some sophisticated mass formula, one can extrapolate to nuclear matter - and determine its saturation properties. The binding energy per nucleon E/A is approximately -16 MeV at a Fermi momentum of about 1.3 fm−¹ . So what does chiral EFT have to say? A first important step was taken by Norbert Kaiser, Wolfram Weise and collaborators at München, who calculated the contribution of pion exchange(s) to the energy density of nuclear matter and showed that the energy density of isospin symmetric nuclear matter can be extremely well approximated by the simple form

$$
E/A = \frac{3k_F^2}{10m} - \alpha \frac{k_F^3}{m^2} + \beta \frac{k_F^4}{m^3} \tag{4}
$$

They then calculated the coefficients α and β from chiral dynamics. With some finetuning of the regulator, one finds an astonishingly good description of the energy density of nuclear matter [\[34\]](#page-17-5). This was later improved by including e.g. higher orders (sensitive again to the LECs c_i) and isobar degrees of freedom [\[35\]](#page-17-6). Interestingly, all this was based on a loop expansion with no explicit power counting - a power counting that indeed explained the success of these calculations was only set up much later, see [\[36\]](#page-17-7). Space forbids a discussion of this power counting and the resulting physics in detail - I only would like to mention that it could be shown that for many reactions the contributions

Figure 8: Left panel: Binding energy per particle of nuclear matter as a function of the Fermi momentum k_F for the N²LO potential using a Lippman-Schwinger cut off $\Lambda = 550$ MeV and a spectral function cut off $\tilde{\Lambda} = 600$ MeV (solid line). The binding energies obtained in the absence of three-nucleon interactions are shown by the dashed line. The black square gives the empirical nuclear matter properties. Right panel: Saturation points of nuclear matter. Downward triangle, upward triangle and rectangle: various phenomenological approaches. Circles: chiral EFT at N^2LO .

from the multi-nucleon interactions cancel to leading order which is at the heart of the success of the Munich group calculations.

Here, I briefly pursue a more "conventional" approach to the nuclear matter problem based on more standard nuclear many-body theory. For decades, the method of choice to analyze nuclear matter has been the G-matrix of Brueckner and others - leading to the problem that for phase-shift equivalent NN potentials the resulting saturation properties lie on the Coester line that does not pass the empirical values. Recently, Siegfried Krewald and collaborators have recalculated the properties of nuclear matter based on the chiral NN and 3N forces (see the v2 version of Ref. [\[38\]](#page-17-8)) utilizing the Rmatrix as proposed by Baker in 1971 [\[37\]](#page-17-9). As can be seen from the left panel of Fig. [8,](#page-10-0) the NN interaction alone only binds for Fermi momenta larger than 1.7 fm−¹ . However, including the 3NF with natural values for *D* and *E*, one finds binding with the proper strength at the proper density. In the right panel, the sensitivity to the two cut-offs in the NN force as compared to phenomenological determinations is shown, it is of comparable size. The dependence on the 3-body LECs *D* and *E* is also weak, see Fig. 3 in Ref. [\[38\]](#page-17-8). It will be interesting to push these calculations to N³LO and to include the $\Delta(1232)$ in the EFT.

4.4 Back to square one - meson-exchange currents

As told in section 1, I started my career calculating meson-exchange currents based on the method of unitary transformations. It is quite a nice turn of events that in chiral nuclear EFT one is now able to do this in a controlled and systematic manner, thanks to the power counting. In our approach, Stefan Kölling, Hermann Krebs and Evgeny Epelbaum

Figure 9: Predictions for the $n-d$ and $n-3He$ capture reactions. Chiral EFT: the curve labeled N3LO(S-L) includes pion loop corrections contact terms, while the curve labeled N3LO(LECs) includes in addition contributions from (higher order) pion exchanges and non-minimal contact currents. Pre-EFT approaches: SNPA and SNPA* denote up-to-date conventional calculations. Figure courtesy of Rocco Schiavilla.

Figure 10: Left panel: Euclidean space-time lattice with point-like nucleons on the lattice sites and a typical lattice spacing of $a = 2$ fm. Right panel: Nuclear phase diagram as accessible by lattice QCD and by nuclear lattice EFT. Figure courtesy of Dean Lee.

have taken up the task - the two-pion exchange electromagnetic current based on this method is published in [\[39\]](#page-17-10) and together with the Cracow group led by Henryk Witala and Jacek Golak a thorough investigation of electro-nuclear processes is underway, for a first application to deuteron photodisintegration see [\[40\]](#page-17-11). However, the group around Rocco Schiavilla, using old-fashioned time-ordered perturbation theory, has taken the lead and performed calculations of the MECs of one- and two-pion range and applied this to magnetic moments of light nuclei [\[41\]](#page-17-12). Most recently, they have calculated thermal neutron capture on *d* and ³He [\[42\]](#page-17-13). The resulting radiative capture cross sections σ_n^{γ} *nd* and $\sigma_{n^3He}^{\gamma}$ and the photon circular polarization parameter R_c , resulting from the capture of polarized neutrons on the deuteron, are shown in Fig. [9](#page-11-0) in comparison to two state-ofthe-art conventional approaches. As the authors note, these processes are not the best or simplest to illustrate the convergence pattern of chiral nuclear EFT. So much more work is needed here and will be done.

5 Flash forward: Combining nuclear EFT with lattice simulations

Having now constructed nuclear forces between two, three and four nucleons, one has a new tool to address the nuclear many-body problem. One venue is to combine standard many-body techniques with these forces, as exemplified by the NCSM calculation shown

Figure 11: Ground-state energy of ¹²C as a function of Euclidean time. For details, see [\[45\]](#page-17-14).

in Sec. [4.2](#page-8-1) or the discussion of nuclear matter in Sec. [4.3](#page-9-1) (see also the contribution by Josef Speth, Siegfried Krewald and Frank Grümmer to this Festschrift). Here, I briefly want to outline a very different and novel approach that combines the chiral EFT for the forces with the method of Monte Carlo simulations, that are so successfully utilized in the lattice approach to QCD. The basic idea is to formulate the chiral EFT on a Euclidean space-time lattice as depicted in the left panel of Fig. [10](#page-12-0) - here, the lattice spacing serves as an UV regulator and has to be chosen such that the nucleons - which are treated as non-relativistic point-like particles on the lattice sites - do not overlap. A lattice spacing of $a = 2$ fm entails an UV cut-off $\Lambda = \pi/a \simeq 300$ MeV. The pion and nucleon propagators as well as the one- and two-pion exchanges and the multi-nucleon contact interactions are written in terms of lattice variables, making use of Hubbard-Stratonovich fields to bring the 4- and 6-nucleon interactions into a quadratic form (for more details, see [\[43\]](#page-17-15) and references therein as well as the nice review by Dean Lee [\[44\]](#page-17-16)). Given this Euclidean formulation of the action, the generating functional of the theory can then be evaluated with stochastic methods, such as the hybrid Monte Carlo approach. It is important to stress that the approximate Wigner SU(4) symmetry of the nuclear interactions strongly suppresses the sign oscillations and thus makes such nuclear lattice simulations easier to handle than lattice QCD calculations. Within this scheme, a systematic study of nuclei up to $A \simeq 40$ will be possible using petascale computing as long as the total spin and isospin of the nucleus under consideration is zero. Also, the nuclear phase diagram can be studied for a wide range of temperatures and densities, as indicated in the right panel of Fig. [10.](#page-12-0) The state-of-the-art of these simulations is the calculation of the ground-state energy of 6 Li and 12 C [\[45\]](#page-17-14). In these works, for the first time the contributions from the Coulomb interaction between protons and strong isospin-breaking effects were included. A parameter-free prediction for the energy difference between the triton and ³He could be given,

$$
E(^{3}\mathrm{H}) - E(^{3}\mathrm{He}) = 0.78(5) \text{ MeV}, \qquad (5)
$$

in good agreement with the empirical value of 0*.*76 MeV. The ground-state energies of ⁶Li and ¹²C are calculated as −32*.*9(9) MeV and −99(2) MeV, respectively, not far from the empirical values of −32*.*0 and −92*.*2 MeV, cf. also Fig. [11.](#page-13-0) This accuracy is comparable to other so-called ab initio calculations (like the NCSM or Greens function Monte Carlo), that often are based on a less consistent formulation of the underlying nuclear forces.

Taking ¹²C as the benchmark, it is interesting to estimate the required computing time and storage needed for the calculation of larger nuclei^{$#2$}. With an improved algorithm, the required CPU time for the 12 C simulation on a BlueGene/P architecture and the necessary storage to make the configurations (about 3500) available is

$$
X_{12C}^{\text{CPU}} = 5 \times 10^{-5} \text{ PFlop} - \text{yr} , \quad X_{12C}^{\text{storage}} = 0.14 \text{ TB} . \tag{6}
$$

For a nucleus with *A* nucleons and with total spin *S* and isospin *I* the required CPU time and data storage place can be estimated as

$$
X^{\text{CPU}} \approx X_{12C}^{\text{CPU}} \times \left(\frac{A}{12}\right)^{3.2} \exp[0.10(A - 12) + 3(S \mod 2) + 4I],
$$

$$
X^{\text{storage}} \approx X_{12C}^{\text{storage}} \times \left(\frac{A}{12}\right)^2 \exp[0.10(A - 12) + 3(S \mod 2) + 4I],
$$
 (7)

so that e.g. the calculation of the ground-states energies of the magnesium isotopes ²⁴*,*25*,*26Mg would require 0*.*002*,* 0*.*064*,* 0*.*131 PFlop-yr and one would need 1*.*9*,* 73*.*8*,* 145*.*5 TB to store the configurations for the calculation of matrix elements for these nuclei. In many cases, neutron scattering off nuclei can also be calculated making use of Lüscher's formula that relates the continuum phase shift to the finite energy shift measured on the lattice. The CPU and storage requirements for neutron scattering of the isotopes ²⁴*,*26Mg would be 0*.*064*,* 5*.*43 PFlop-yr and 73*.*8*,* 5473 TB, respectively. All this work remains to be done but bears a lot of promise. The possibilities that would be offered in a future exascale era are simply breath-taking.

6 Some final words

The chiral effective Lagrangian of QCD offers a tool to systematically investigate the structure of the nucleon and the nuclear interactions with high accuracy - it appears that we are now on the right track to achieve what could only be dreamed about a few decades ago - see the discussion in Sec. 1. Nuclear physics will no longer be based on model-building and phenomenological approaches, although still quite a bit of work is ahead of us. But this work - even if hard and time-consuming - will be very rewarding at the end. Furthermore, strangeness nuclear physics can also be addressed within this

 $*^{2}I$ am grateful to Dean Lee for providing me with these estimates.

framework [\[46\]](#page-18-0), but here a sound data basis has yet to be established. Eventually, lattice QCD might also contribute significantly - but there are still quite a few obstacles to be overcome, see e.g [\[47\]](#page-18-1). It is fair to say that nuclear physics as a field, and I personally, owe a lot to Gerry - thank you.

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