# BRST analysis of topologically massive gauge theory: novel observations

R. Kumar<sup>(a)</sup>, R. P. Malik<sup>(a,b)</sup>

<sup>(a)</sup> Physics Department, Centre of Advanced Studies, Banaras Hindu University, Varanasi - 221 005, (U.P.), India

and

<sup>(b)</sup>DST Centre for Interdisciplinary Mathematical Sciences, Faculty of Science, Banaras Hindu University, Varanasi - 221 005, India e-mails: raviphynuc@gmail.com, malik@bhu.ac.in

Abstract: A dynamical non-Abelian 2-form gauge theory (with  $B \wedge F$  term) is endowed with the "scalar" and "vector" gauge symmetry transformations. In our present endeavor, we exploit the latter gauge symmetry transformations and perform the Becchi-Rouet-Stora-Tyutin (BRST) analysis of the four (3 + 1)-dimensional (4D) topologically massive non-Abelian 2-form gauge theory. We demonstrate the existence of some novel features that have, hitherto, not been observed in the context of BRST approach to 4D (non-)Abelian 1-form as well as Abelian 2-form and 3-form gauge theories. We comment on the differences between the novel features that emerge in the BRST analysis of the "scalar" and "vector" gauge symmetries.

PACS 11.15.Wx – Topologically massive gauge theories PACS 11.15.-q – Gauge field theories

Keywords: Dynamical non-Abelian 2-form theory; topological  $(B \wedge F)$  term; "vector" gauge symmetries; (anti-)BRST symmetries; Curci-Ferrari type restrictions; nilpotency and anticommutativity

# 1. Introduction

In recent years, there has been a renewed interest in the study of 4D topologically massive (non-)Abelian gauge theories. In such theories, there is an explicit coupling between the 1-form and 2-form gauge fields through the celebrated  $(B \wedge F)$  term. In fact, it has been shown that the 1-form gauge field acquires a mass, in a very natural fashion [1], for the above 4D topologically massive gauge theories. As a result, the above models [1-5] provide an alternative to the Higgs mechanism of the standard model of high energy physics as far as the mass generation of the 1-form gauge field is concerned.

In the above context, it may be mentioned that we have carried out the BRST analysis [6,7] of the 4D Abelian 2-form theory (with topological mass term) and obtained the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations by exploiting the geometrical superfield approach [8,9]. The latter formalism has also been applied to the 4D dynamical non-Abelian 2-form theory (with celebrated topological  $B \wedge F$ ) term) and we have exploited its "scalar" and "vector" gauge transformations (see, e.g., [10]) to derive the appropriate Lagrangian densities as well as proper (nilpotent and anticommuting) (anti-)BRST symmetry transformations.

In a very recent paper [11], we have performed the BRST analysis of the 4D non-Abelian topologically massive theory and shown that the conserved and nilpotent (anti-)BRST charges, corresponding to the "scalar" gauge symmetry of the theory, are unable to generate the (anti-)BRST transformations corresponding to the  $B_{0i}$  component of the antisymmetric tensor gauge field  $B_{\mu\nu}$  (with  $B_{\mu\nu} = B_{\mu\nu} \cdot T$ ) and the 1-form  $(K^{(1)} = dx^{\mu}K_{\mu} \cdot T)$  auxiliary field  $K_{\mu}$  (with  $K_{\mu} = K_{\mu} \cdot T$ ). This happens to be a novel observation in [11].

The central theme of our present paper is to exploit the "vector" gauge symmetry transformations of the above topologically massive non-Abelian theory and explore its details within the framework of BRST formalism. As it turns out, we observe yet another novel feature in the BRST analysis of the above topologically massive gauge theory. We find that the conserved and nilpotent (anti-)BRST charges are not able to generate the proper (anti-)BRST symmetry transformations *only* for the auxiliary field  $K_{\mu}$ . This is a new result that is quite different from the BRST analysis of the 4D (non-)Abelian 1-form [12,13] and Abelian 2-form and 3-form gauge theories [6,7,14]. We lay emphasis on the fact that the novel features, from the BRST analysis of "scalar" and "vector" gauge symmetry transformations of the *same* non-Abelian topologically massive theory, are quite different.

The material of our present paper is organized as follows. In our second section, we recapitulate the bare essential of the "vector" gauge symmetry transformations and derive the generator corresponding to them. In section three, we discuss the BRST symmetries corresponding to the above "vector" gauge symmetry transformations and deduce the BRST charge. Our section four is devoted to the derivation of anti-BRST symmetries and corresponding nilpotent and conserved charge. We deal with the ghost symmetries in section five where we also deduce the BRST algebra. Finally, we discuss our main results and make a few concluding remarks in section six.

In our Appendix, we briefly comment on the Stückelberg formalism, Abelian Higgs model and our present model of the dynamical non-Abelian 2-form gauge theory (i.e. a model of topologically massive gauge theory).

## 2. Preliminaries: symmetry transformations

Let us begin with the Lagrangian density of the 4D dynamical non-Abelian 2-form gauge theory<sup>1</sup>, that incorporates the celeberated  $(B \wedge F)$  term with the mass parameter m, as given below (see, *e.g.*, [3-5,11])

$$\mathcal{L}_{(0)} = -\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12} H^{\mu\nu\eta} \cdot H_{\mu\nu\eta} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa}.$$
 (1)

In the above, the curvature tensor  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - (A_{\mu} \times A_{\nu})$ , corresponding to the 1-form  $(A^{(1)} = dx^{\mu}A_{\mu} \cdot T)$  non-Abelian field  $A_{\mu}$ , has been derived from the curvature 2-form  $F^{(2)} = dA^{(1)} + i(A^{(1)} \wedge A^{(1)}) \equiv \frac{1}{2!} (dx^{\mu} \wedge dx^{\nu})F_{\mu\nu}$ . In exactly similar fashion, the curvature 3-form  $H^{(3)} = \frac{1}{3!} (dx^{\mu} \wedge dx^{\nu} \wedge dx^{\eta})H_{\mu\nu\eta}$  defines the totally antisymmetric third-rank tensor<sup>2</sup>

$$H_{\mu\nu\eta} = (\partial_{\mu}B_{\nu\eta} + \partial_{\nu}B_{\eta\mu} + \partial_{\eta}B_{\mu\nu}) - [(A_{\mu} \times B_{\nu\eta}) + (A_{\nu} \times B_{\eta\mu}) + (A_{\eta} \times B_{\mu\nu})] - [(K_{\mu} \times F_{\nu\eta}) + (K_{\nu} \times F_{\eta\mu}) + (K_{\eta} \times F_{\mu\nu})], \quad (2)$$

in terms of the 1-form  $(K^{(1)} = dx^{\mu}K_{\mu} \cdot T)$  auxiliary field  $K_{\mu}$ , 2-form  $(B^{(2)} = \frac{1}{2!}(dx^{\mu} \wedge dx^{\nu})B_{\mu\nu} \cdot T)$  antisymmetric tensor gauge field  $B_{\mu\nu}$  and the antisymmetric curvature tensor  $F_{\mu\nu}$  (defined earlier). Here the SU(N) generators  $T^{a}$   $(a = 1, 2..., N^{2} - 1)$  satisfy the Lie algebra  $[T^{a}, T^{b}] = if^{abc} T^{c}$  where the structure constants  $f^{abc}$  can be chosen to be totally antisymmetric in a, b and c for the semi-simple Lie group SU(N) under consideration (see, e.g., [13]).

<sup>&</sup>lt;sup>1</sup>We adopt the conventions and notations such that the background Minkowaskian 4D spacetime manifold is endowed with the flat metric  $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1, -1)$ . This entails upon two non-null vectors  $A_{\mu}$  and  $B_{\mu}$  to have:  $A_{\mu}B^{\mu} = \eta_{\mu\nu}A^{\mu}B^{\nu} = A_0B_0 - A_iB_i$  where the Greek indices  $\mu, \nu, \eta, \dots = 0, 1, 2, 3$  and Latin indices  $i, j, k, \dots = 1, 2, 3$ . We choose here the 4D Levi-Civta tensor  $\varepsilon^{\mu\nu\eta\kappa}$  to obey  $\varepsilon^{\mu\nu\eta\kappa}\varepsilon_{\mu\nu\eta\kappa} = -4!$ ,  $\varepsilon^{\mu\nu\eta\kappa}\varepsilon_{\mu\nu\eta\sigma} = -3! \, \delta^{\kappa}_{\sigma}$ , etc., and  $\varepsilon_{0123} = +1 = -\varepsilon^{0123}$ . In the SU(N) algebraic space, we follow  $P \cdot Q = P^a Q^a$  and  $(P \times Q)^a = f^{abc} P^b Q^c$  where  $a, b, c, \dots = 1, 2, 3, \dots (N^2 - 1)$ .

<sup>&</sup>lt;sup>2</sup>It is possible to eliminate the auxiliary field  $K_{\mu}$  from  $H_{\mu\nu\eta}$  by the shift transformation  $B_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} + (D_{\mu}K_{\nu} - D_{\nu}K_{\mu})$ . We comment on, some aspects of it, in our Appendix.

The above Lagrangian density (1), for the 4D topologically massive non-Abelian gauge theory, respects the following infinitesimal and continuous "vector" gauge symmetry<sup>3</sup> transformations ( $\delta_v$ ) [3,4]

$$\delta_v A_\mu = 0, \qquad \delta_v B_{\mu\nu} = -(D_\mu \Lambda_\nu - D_\nu \Lambda_\mu), \delta_v K_\mu = -\Lambda_\mu, \qquad \delta_v F_{\mu\nu} = 0, \qquad \delta_v H_{\mu\nu\eta} = 0,$$
(3)

because the Lagrangian density (1) transforms, under (3), as

$$\delta_{v}\mathcal{L}_{(0)} = -\partial_{\mu} \Big[ \frac{m}{2} \, \varepsilon^{\mu\nu\eta\kappa} \, F_{\nu\eta} \cdot \Lambda_{\kappa} \Big], \tag{4}$$

where  $\Lambda_{\mu} = \Lambda_{\mu} \cdot T$  is an infinitesimal Lorentz "vector" gauge parameter for the transformations  $\delta_v$ . It is evident from (3) that the action  $S = \int d^4x \mathcal{L}_{(0)}$ remains invariant under the "vector" symmetry transformations (3).

Noether theorem states that the continuous "vector" gauge transformations (3) would lead to the derivation of a conserved current. The precise expression for this current is as follows:

$$J^{\mu}_{(\nu)} = -H^{\mu\nu\eta} \cdot (D_{\nu}\Lambda_{\eta}) + \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot \Lambda_{\kappa}.$$
 (5)

It can be checked that  $\partial_{\mu}J^{\mu}_{(v)} = 0$  if we exploit the following Euler-Lagrange (E-L) equations of motion for all relevant fields, namely;

$$D_{\mu}H^{\mu\nu\eta} = \frac{m}{2} \varepsilon^{\nu\eta\kappa\sigma}F_{\kappa\sigma}, \qquad (H^{\mu\nu\eta} \times F_{\nu\eta}) = 0,$$
  
$$D_{\mu}\left[F^{\mu\nu} + (H^{\mu\nu\eta} \times K_{\eta}) - \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa}B_{\eta\kappa}\right] + \frac{1}{2} (H^{\nu\eta\kappa} \times B_{\eta\kappa}) = 0, \quad (6)$$

that emerge from the Lagrangian density  $\mathcal{L}_{(0)}$ . The above conserved Noether current leads to the derivation of conserved charge  $Q_{(v)} = \int d^3x J^0_{(v)}$  as

$$Q_{(v)} = \int d^3x \Big[ -H^{0ij} \cdot \Big( D_i \Lambda_j \Big) + \frac{m}{2} \varepsilon^{0ijk} F_{ij} \cdot \Lambda_k \Big]$$
  
$$\equiv \int d^3x \Big[ -\frac{1}{2} H^{0ij} \cdot \Big( D_i \Lambda_j - D_j \Lambda_i \Big) + \frac{m}{2} \varepsilon^{0ijk} F_{ij} \cdot \Lambda_k \Big].$$
(7)

The above conserved charge is the generator of the infinitesimal "vector" gauge transformations (3). It is interesting, however, to point out that it generates *only* the following infinitesimal transformations

$$\delta_{v}B_{ij} = -i[B_{ij}, Q_{(v)}] = -(D_{i}\Lambda_{j} - D_{j}\Lambda_{i}), \quad \delta_{v}A_{\mu} = -i[A_{\mu}, Q_{(v)}] = 0, \quad (8)$$

<sup>&</sup>lt;sup>3</sup>It is straightforward to check that  $\mathcal{L}_{(0)}$  also respects (*i.e.*  $\delta_{gt}\mathcal{L}_{(0)} = 0$ ) the "scalar" gauge transformations ( $\delta_{gt}$ ) corresponding to the usual 1-form non-Abelian gauge field:  $\delta_{gt}A_{\mu} = D_{\mu}\Omega, \ \delta_{gt}B_{\mu\nu} = -(B_{\mu\nu}\times\Omega), \delta_{gt}K_{\mu} = -(K_{\mu}\times\Omega), \delta_{gt}F_{\mu\nu} = -(F_{\mu\nu}\times\Omega), \delta_{gt}H_{\mu\nu\eta} = -(H_{\mu\nu\eta}\times\Omega)$  where  $\Omega = \Omega \cdot T \equiv \Omega^a T^a$  is the SU(N) valued infinitesimal gauge (Lorentz "scalar") parameter and the covariant derivative  $D_{\mu}\Omega = \partial_{\mu}\Omega - (A_{\mu}\times\Omega)$  [3,4,11].

which are a part of the total transformations (3) that corresponds to our "vector" gauge symmetry transformations. It can be seen that the transformations  $\delta_v B_{0i}$  and  $\delta_v K_{\mu}$  are *not* generated by the conserved charge  $Q_{(v)}$ . The former transformation can be derived by using the precise techniques of BRST formalism. We do the same in our next section.

## 3. BRST symmetries and BRST charge

We begin with the BRST invariant Lagrangian density  $\mathcal{L}_B$  (which is the generalization of the starting Lagrangian density  $\mathcal{L}_{(0)}$  (cf. (1))) such that the gauge-fixing and Faddeev-Popov ghost terms are incorporated in it. Such an appropriate (BRST-invariant) Lagrangian density is [10]

$$\mathcal{L}_{B} = -\frac{1}{4}F^{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12}H^{\mu\nu\eta} \cdot H_{\mu\nu\eta} + \frac{m}{4}\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu} \cdot F^{\eta\kappa} + B^{\mu} \cdot B_{\mu} - \frac{i}{2}B^{\mu\nu} \cdot (B_{1} \times F_{\mu\nu}) - (D_{\mu}B^{\mu\nu} - D^{\nu}\phi) \cdot B_{\nu} + D_{\mu}\bar{\beta} \cdot D^{\mu}\beta + \frac{1}{2}\left[(D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) - \bar{C}_{1} \times F_{\mu\nu}\right] \cdot \left[(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu}\right] + \rho \cdot (D_{\mu}C^{\mu} - \lambda) + (D_{\mu}\bar{C}^{\mu} - \rho) \cdot \lambda,$$
(9)

where  $B_{\mu} = B_{\mu} \cdot T$  and  $B_1 = B_1 \cdot T$  are the Nakanishi-Lautrup type bosonic auxiliary fields and  $\rho = \rho \cdot T$  and  $\lambda = \lambda \cdot T$  are fermionic auxiliary fields. The Lorentz vector fermionic (anti-)ghost fields  $(\bar{C}_{\mu})C_{\mu}$  (with  $C_{\mu}^2 = \bar{C}_{\mu}^2 =$  $0, C_{\mu}C_{\nu} + C_{\nu}C_{\mu} = 0, C_{\mu}\bar{C}_{\nu} + \bar{C}_{\nu}C_{\mu} = 0$ , etc.) and bosonic (anti-)ghost fields  $(\bar{\beta})\beta$  are required for the unitarity in the theory and they carry the ghost numbers ( $\mp$ 1) and ( $\mp$ 2), respectively. The bosonic scalar field  $\phi$  and the Lorentz scalar (anti-)ghost auxiliary fields  $(\bar{C}_1)C_1$  are also required for the BRST invariance in the theory. It can be explicitly checked that

$$s_{b}\mathcal{L}_{B} = -\partial_{\mu} \Big[ \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot C_{\kappa} - \lambda \cdot B^{\mu} + (C_{1} \times F^{\mu\nu}) \cdot B_{\nu} \\ - \rho \cdot D^{\mu}\beta - (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) \cdot B_{\nu} \Big], \qquad (10)$$

which shows that the action  $S = \int d^4x \mathcal{L}_B$  remains invariant under the following BRST symmetry transformations  $(s_b)$ :

$$s_{b}B_{\mu\nu} = -(D_{\mu}C_{\nu} - D_{\nu}C_{\mu}) + C_{1} \times F_{\mu\nu}, \quad s_{b}C_{\mu} = -D_{\mu}\beta, s_{b}\bar{C}_{\mu} = B_{\mu}, \quad s_{b}\bar{B}_{1} = i \lambda, \quad s_{b}\bar{C}_{1} = i B_{1}, \quad s_{b}\bar{B}_{\mu} = -D_{\mu}\lambda, s_{b}K_{\mu} = D_{\mu}C_{1} - C_{\mu}, \quad s_{b}\phi = \lambda, \quad s_{b}C_{1} = -\beta, \quad s_{b}\bar{\beta} = \rho, s_{b}[A_{\mu}, F_{\mu\nu}, H_{\mu\nu\eta}, \beta, B_{1}, \rho, \lambda, B_{\mu}] = 0.$$
(11)

It is pertinent to point out that the above BRST transformations have been obtained from the superfield approach to BRST formalism [10] which always produces the off-shell nilpotent ( $s_b^2 = 0$ ) BRST symmetry transformations for a given *p*-form (p = 1, 2, 3, ...) gauge theory in any arbitrary dimension.

Exploiting the basics of the Noether theorem, it turns out that the exact expression for Noether current is

$$J_{(b)}^{\mu} = \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot C_{\kappa} - \frac{1}{2} H^{\mu\nu\eta} \cdot \left[ (D_{\nu}C_{\eta} - D_{\eta}C_{\nu}) - C_{1} \times F_{\nu\eta} \right] + \left[ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \right] \cdot (D_{\nu}\beta) + (D^{\mu}\beta) \cdot \rho + \left[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \right] \cdot B_{\nu} + B^{\mu} \cdot \lambda.$$
(12)

The conservation of this current (*i.e.*  $\partial_{\mu}J^{\mu}_{(b)} = 0$ ) can be proven by exploiting the following set of E-L equations of motion<sup>4</sup>

$$D_{\mu}F^{\mu\nu} - \frac{m}{2} \varepsilon^{\mu\nu\eta\sigma} D_{\mu}B_{\eta\sigma} + D_{\mu}(H^{\mu\nu\eta} \times K_{\eta}) + i D_{\mu}(B^{\mu\nu} \times B_{1}) - D_{\mu}[(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) \times \bar{C}_{1}] + D_{\mu}[(D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) \times C_{1}] + \frac{1}{2} (H^{\nu\eta\sigma} \times B_{\eta\sigma}) - (B^{\mu\nu} \times B_{\mu}) + (B^{\nu} \times \phi) + (D^{\nu}\bar{\beta} \times \beta) + \bar{C}_{\mu} \times [(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu}] - (\bar{C}^{\nu} \times \lambda) + (C^{\nu} \times \rho) + (D^{\nu}\beta \times \bar{\beta}) - C_{\mu} \times [(D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu}] = 0, D_{\mu}H^{\mu\nu\eta} - \frac{m}{2} \varepsilon^{\nu\eta\kappa\sigma} F_{\kappa\sigma} - (D^{\nu}B^{\eta} - D^{\eta}B^{\nu}) - i (F^{\nu\eta} \times B_{1}) = 0, B_{\mu} = -(1/2) (D_{\mu}\phi - D^{\nu}B_{\nu\mu}), (B_{\mu\nu} \times F^{\mu\nu}) = 0, (H^{\mu\nu\eta} \times F_{\nu\eta}) = 0, D_{\mu}(D^{\mu}\bar{\beta}) = 0, \quad \rho = \frac{1}{2} (D_{\mu}\bar{C}^{\mu}), \quad \lambda = \frac{1}{2} (D_{\mu}C^{\mu}), \quad D_{\mu}B^{\mu} = 0, D_{\mu}[(D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu}] = -D^{\nu}\rho, \quad D_{\mu}(D^{\mu}\rho) = 0, [(D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) - \bar{C}_{1} \times F^{\mu\nu}] = -D^{\nu}\lambda, \quad D_{\mu}(D^{\mu}\lambda) = 0, [(D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) - \bar{C}_{1} \times F_{\mu\nu}] \times F^{\mu\nu} = 0.$$
(13)

The above equations emerge from the Lagrangian density  $\mathcal{L}_B$ .

The conserved current (13) leads to the derivation of the conserved ( $\dot{Q}_b = 0$ ) and nilpotent ( $Q_b^2 = 0$ ) BRST charge  $Q_b = \int d^3x J^0_{(b)}$  as

$$Q_b = \int d^3x \left[ \frac{m}{2} \varepsilon^{0ijk} F_{ij} \cdot C_k - \frac{1}{2} H^{0ij} \cdot \left( D_i C_j - D_j C_i - C_1 \times F_{ij} \right) \right]$$

<sup>&</sup>lt;sup>4</sup>The present theory is highly constrained because we have the conditions:  $F_{\mu\nu} \times B^{\mu\nu} = 0$ ,  $F_{\mu\nu} \times H^{\mu\nu\eta} = 0$ . However, the 2-form  $F^{(2)} = dA^{(1)} + iA^{(1)} \wedge A^{(1)}$  does respect zero curvature condition ( $F_{\mu\nu} = 0$ ) for the choice  $A^{(1)} = -i U d U^{-1}$  where  $U \in SU(N)$ .

$$+ \left( D_0 \bar{C}_i - D_i \bar{C}_0 - \bar{C}_1 \times F_{0i} \right) \cdot D^i \beta + B^0 \cdot \lambda + (D^0 \beta) \cdot \rho + \left( D_0 C_i - D_i C_0 - C_1 \times F_{0i} \right) \cdot B^i \right].$$
(14)

This charge is the generator of the BRST symmetry transformations (11).

We wrap up this section with the remarks that (i) one can derive the BRST transformation  $s_b B_{0i} = -i[B_{0i}, Q_b] = -(D_0 C_i - D_i C_0) + C_1 \times F_{0i}$ from the BRST charge (the analogue of which, we were unable to derive from the gauge symmetry generator  $Q_v$ ) (cf. section 2), (ii) one can derive *all* the BRST symmetry transformations for *all* the fields by various requirements (cf. section 4 below), and (iii) one is *not* able to derive, however, the BRST transformation  $s_b K_{\mu} = D_{\mu} C_1 - C_{\mu}$  from the conserved charge  $Q_b$ . Thus, we conclude that, except for the auxiliary field  $K_{\mu}$ , all the other fields have the usual off-shell nilpotent "vector" BRST symmetry transformations.

# 4. Anti-BRST symmetry transformations and anti-BRST charge

In addition to  $\mathcal{L}_B$ , there is yet another generalization of  $\mathcal{L}_{(0)}$  that includes the gauge-fixing and Faddeev-Popov ghost terms. Such an appropriate (anti-BRST invariant) Lagrangian density  $\mathcal{L}_{\bar{B}}$ , in its full blaze of glory, is [10]

$$\mathcal{L}_{\bar{B}} = -\frac{1}{4}F^{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12}H^{\mu\nu\eta} \cdot H_{\mu\nu\eta} + \frac{m}{4}\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu} \cdot F^{\eta\kappa} + \bar{B}^{\mu} \cdot \bar{B}_{\mu} + \frac{i}{2}B^{\mu\nu} \cdot (\bar{B}_{1} \times F_{\mu\nu}) + (D_{\mu}B^{\mu\nu} + D^{\nu}\phi) \cdot \bar{B}_{\nu} + D_{\mu}\bar{\beta} \cdot D^{\mu}\beta + \frac{1}{2}\left[(D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) - \bar{C}_{1} \times F_{\mu\nu}\right] \cdot \left[(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu}\right] + \rho \cdot (D_{\mu}C^{\mu} - \lambda) + (D_{\mu}\bar{C}^{\mu} - \rho) \cdot \lambda.$$
(15)

The above Lagrangian density respects the off-shell nilpotent  $(s_{ab}^2 = 0)$  anti-BRST symmetry transformations  $s_{ab}$  as listed below

$$s_{ab}B_{\mu\nu} = -(D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) + \bar{C}_{1} \times F_{\mu\nu}, \quad s_{ab}\bar{C}_{\mu} = -D_{\mu}\bar{\beta},$$
  

$$s_{ab}C_{\mu} = \bar{B}_{\mu}, \quad s_{ab}B_{\mu} = D_{\mu}\rho, \quad s_{ab}C_{1} = i \bar{B}_{1}, \quad s_{ab}\phi = -\rho,$$
  

$$s_{ab}\bar{C}_{1} = -\bar{\beta}, \quad s_{ab}B_{1} = -i \rho, \quad s_{ab}K_{\mu} = D_{\mu}\bar{C}_{1} - \bar{C}_{\mu},$$
  

$$s_{ab}\beta = -\lambda, \quad s_{ab}[A_{\mu}, F_{\mu\nu}, H_{\mu\nu\eta}, \bar{\beta}, \bar{B}_{1}, \rho, \lambda, \bar{B}_{\mu}] = 0, \quad (16)$$

because  $\mathcal{L}_{\bar{B}}$  transforms to a total spacetime derivative as

$$s_{ab}\mathcal{L}_{\bar{B}} = -\partial_{\mu} \left[ \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot \bar{C}_{\kappa} + \rho \cdot \bar{B}^{\mu} - (\bar{C}_{1} \times F_{\mu\nu}) \cdot \bar{B}_{\nu} + \lambda \cdot D^{\mu}\bar{\beta} + (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) \cdot \bar{B}_{\nu} \right].$$
(17)

As a consequence, the action  $S = \int d^4x \mathcal{L}_{\bar{B}}$  remains invariant under  $s_{ab}$ .

A few noteworthy points are in order. First, under the (anti-)BRST transformations, the kinetic terms  $\left(-\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} \text{ and } \frac{1}{12} H^{\mu\nu\eta} \cdot H_{\mu\nu\eta}\right)$ , owing their origin to the exerior derivative  $d = dx^{\mu}\partial_{\mu}$  (with  $d^2 = 0$ ), remain invariant. Second, the Nakanishi-Lauturp type auxiliary fields  $\bar{B}_{\mu}$  and  $\bar{B}_1$ , introduced in (15), are constrained to satisfy the Curci-Ferrari (CF) type restrictions as given below

$$B_{\mu} + \bar{B}_{\mu} = -D_{\mu}\phi, \qquad B_1 + \bar{B}_1 = i\phi.$$
 (18)

It should be recalled that, the fields  $B_{\mu}$  and  $B_1$ , were introduced in the definition of the BRST invariant Lagrangian density  $\mathcal{L}_B$ . Third, the above CF-type of restrictions have been derived from the superfield approach to BRST formalism for the dynamical non-Abelian 2-form gauge theory [10]. These are (anti-)BRST invariant as it can be checked that  $s_{(a)b}[B_{\mu} + \bar{B}_{\mu} + D_{\mu}\phi] = 0$ ,  $s_{(a)b}[B_1 + \bar{B}_1 - i\phi] = 0$  where  $s_bB_1 = 0$ ,  $s_b\bar{B}_1 = i\lambda$ ,  $s_bB_{\mu} = 0$ ,  $s_b\bar{B}_{\mu} = -D_{\mu}\lambda$ ,  $s_{ab}\bar{B}_1 = 0$ ,  $s_{ab}B_1 = -i\rho$ ,  $s_{ab}\bar{B}_{\mu} = 0$ ,  $s_{ab}B_{\mu} = D_{\mu}\rho$ . Fourth, it can be checked that  $s_{(a)b}$  obey off-shell nilpotency ( $s_{(a)b}^2 = 0$ ) and absolute anticommutativity (*i.e.*  $s_ss_{ab} + s_{ab}s_b = 0$ ) if we exploit appropriately the CF-type conditions (18). Fifth, both the Lagrangian density  $\mathcal{L}_B$  and  $\mathcal{L}_{\bar{B}}$  are coupled and equivalent (due to CF-type conditions (18)) as it can be checked that *both* of them respect the (anti-)BRST symmetry transformations. This statement can be corroborated by the following observations, namely;

$$s_{ab}\mathcal{L}_{B} = -\partial_{\mu} \Big[ \frac{m}{2} \, \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot \bar{C}_{\kappa} + (D^{\mu}\bar{\beta}) \cdot \lambda + \bar{B}^{\mu} \cdot \rho + B^{\mu\nu} \cdot (D_{\nu}\rho) \\ + \Big\{ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \Big\} \cdot (D_{\nu}\phi + \bar{B}_{\nu}) \Big] \\ + \frac{i}{2} \Big[ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \Big] \cdot \Big( B_{1} + \bar{B}_{1} - i\phi \Big) \times F_{\mu\nu} \\ + D_{\mu} \Big[ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \Big] \cdot \Big( B_{\nu} + \bar{B}_{\nu} + D_{\nu}\phi \Big) \\ + (D_{\mu}\rho) \cdot \Big( B^{\mu} + \bar{B}^{\mu} + D^{\mu}\phi \Big),$$
(19)

$$s_{b}\mathcal{L}_{\bar{B}} = -\partial_{\mu} \Big[ \frac{m}{2} \, \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot C_{\kappa} - (D^{\mu}\beta) \cdot \rho - B^{\mu} \cdot \lambda + B^{\mu\nu} \cdot (D_{\nu}\lambda) \\ - \Big\{ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \Big\} \cdot (D_{\nu}\phi + B_{\nu}) \Big] \\ - \frac{i}{2} \Big[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \Big] \cdot \Big( B_{1} + \bar{B}_{1} - i\phi \Big) \times F_{\mu\nu} \\ - D_{\mu} \Big[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \Big] \cdot \Big( B_{\nu} + \bar{B}_{\nu} + D_{\nu}\phi \Big) \\ - (D_{\mu}\lambda) \cdot \Big( B^{\mu} + \bar{B}^{\mu} + D^{\mu}\phi \Big).$$
(20)

The above equations, in addition to (10) and (17), establish the equivalence of  $\mathcal{L}_B$  and  $\mathcal{L}_{\bar{B}}$  as far as the validity of CF-type restrictions and the existence of the nilpotent (anti-)BRST symmetries are concerned.

The infinitesimal continuous anti-BRST symmetry transformations (16) lead to the derivation of Noether current  $J^{\mu}_{(ab)}$  as

$$J^{\mu}_{(ab)} = \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} F_{\nu\eta} \cdot \bar{C}_{\kappa} - \frac{1}{2} H^{\mu\nu\eta} \cdot \left[ (D_{\nu}\bar{C}_{\eta} - D_{\eta}\bar{C}_{\nu}) - \bar{C}_{1} \times F_{\nu\eta} \right] - \left[ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \right] \cdot \bar{B}_{\nu} - (D^{\mu}\bar{\beta}) \cdot \lambda - \left[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \right] \cdot (D_{\nu}\bar{\beta}) - \bar{B}^{\mu} \cdot \rho.$$
(21)

The conservation law  $\partial_{\mu} J^{\mu}_{(ab)} = 0$  can be proven by exploiting the E-L equations of motion derived from the Lagrangian density  $\mathcal{L}_{\bar{B}}$ . In fact, many equations of motion are common for the Lagrangian density  $\mathcal{L}_{B}$  and  $\mathcal{L}_{\bar{B}}$ . The ones that are different from (13) and derived from  $\mathcal{L}_{\bar{B}}$  are

$$D_{\mu}F^{\mu\nu} - \frac{m}{2} \varepsilon^{\mu\nu\eta\sigma} D_{\mu}B_{\eta\sigma} + D_{\mu}(H^{\mu\nu\eta} \times K_{\eta}) - i D_{\mu}(B^{\mu\nu} \times \bar{B}_{1}) - D_{\mu}[(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) \times \bar{C}_{1}] + D_{\mu}[(D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) \times C_{1}] + \frac{1}{2} (H^{\nu\eta\sigma} \times B_{\eta\sigma}) + (B^{\mu\nu} \times \bar{B}_{\mu}) + (\bar{B}^{\nu} \times \phi) + (D^{\nu}\bar{\beta} \times \beta) + \bar{C}_{\mu} \times [(D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu}] - (\bar{C}^{\nu} \times \lambda) + (C^{\nu} \times \rho) + (D^{\nu}\beta \times \bar{\beta}) - C_{\mu} \times [(D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu}] = 0, D_{\mu}H^{\mu\nu\eta} - \frac{m}{2} \varepsilon^{\nu\eta\kappa\sigma} F_{\kappa\sigma} + (D^{\nu}\bar{B}^{\eta} - D^{\eta}\bar{B}^{\nu}) + i (F^{\nu\eta} \times \bar{B}_{1}) = 0, \bar{B}_{\mu} = -(1/2) (D_{\mu}\phi + D^{\nu}B_{\nu\mu}), \qquad D_{\mu}\bar{B}^{\mu} = 0.$$
(22)

The conserved current  $J^{\mu}_{(ab)}$  leads to the derivation of the generator  $Q_{ab} = \int d^3x J^0_{(ab)}$  of the anti-BRST symmetry transformations (16) as

$$Q_{ab} = \int d^{3}x \Big[ \frac{m}{2} \, \varepsilon^{0ijk} F_{ij} \cdot \bar{C}_{k} - \frac{1}{2} \, H^{0ij} \cdot \Big( D_{i}\bar{C}_{j} - D_{j}\bar{C}_{i} - \bar{C}_{1} \times F_{ij} \Big) \\ - \Big( D_{0}C_{i} - D_{i}C_{0} - C_{1} \times F_{0i} \Big) \cdot (D^{i}\bar{\beta}) - \bar{B}^{0} \cdot \rho - (D^{0}\bar{\beta}) \cdot \lambda \\ - \Big( D_{0}\bar{C}_{i} - D_{i}\bar{C}_{0} - \bar{C}_{1} \times F_{0i} \Big) \cdot \bar{B}^{i} \Big].$$
(23)

The above charge is conserved  $(\dot{Q}_{ab} = 0)$  and off-shell nilpotent of order two  $(i.e. Q_{ab}^2 = 0)$ . The latter property establishes the fermionic nature of  $Q_{ab}$ .

A few comments are in order. It can be readily seen that  $Q_{ab}$  generates the anti-BRST symmetry transformations for the field  $B_{0i}$  as  $s_{ab}B_{0i} = -i[B_{0i}, Q_{ab}] = -(D_0\bar{C}_i - D_i\bar{C}_0) + \bar{C}_1 \times F_{0i}$ . Furthermore, it is interesting to point out that  $Q_b Q_{ab} + Q_{ab} Q_b = 0$  if and only if the CF-type restrictions (18) are exploited for its proof. Finally, the conserved and nilpotent charge  $Q_{ab}$  is unable to generate the anti-BRST symmetry transformation for the *special* auxiliary field  $K_{\mu}$  (*i.e.*  $s_{ab}K_{\mu} = D_{\mu}\bar{C}_1 - \bar{C}_{\mu}$ ). This is a novel observation.

The most surprising thing is that the above specific transformation can not be derived even from the requirements of (i) the (anti-)BRST invariance of CF-type restrictions (18), (ii) the nilpotency property, and (iii) the anticommutativity property of  $s_{(a)b}$  (*i.e.*  $s_bs_{ab} + s_{ab}s_b = 0$ ). This is a new observation in the context of the application of BRST approach to topologically massive 4D non-Abelian theory (which is drastically different from the application of the same approach to its Abelian counterpart (see, *e.g.*, [6,7])).

## 5. Ghost symmetry, ghost charge and BRST algebra

The ghost part of the Lagrangian density of the theory

$$\mathcal{L}_{g} = \frac{1}{2} \left[ (D_{\mu}\bar{C}_{\nu} - D_{\nu}\bar{C}_{\mu}) - \bar{C}_{1} \times F_{\mu\nu} \right] \cdot \left[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \right] + D_{\mu}\bar{\beta} \cdot D^{\mu}\beta + \rho \cdot (D_{\mu}C^{\mu} - \lambda) + (D_{\mu}\bar{C}^{\mu} - \rho) \cdot \lambda,$$
(24)

remains invariant under the following scale transformations

$$C_{1} \to e^{+\Sigma} C_{1}, \quad \bar{C}_{1} \to e^{-\Sigma} \bar{C}_{1}, \quad C_{\mu} \to e^{+\Sigma} C_{\mu}, \quad \bar{C}_{\mu} \to e^{-\Sigma} \bar{C}_{\mu}, \beta \to e^{+2\Sigma} \beta, \quad \bar{\beta} \to e^{-2\Sigma} \bar{\beta}, \quad \rho \to e^{-\Sigma} \rho, \quad s_{g} \lambda = e^{+\Sigma} \lambda, (A_{\mu}, B_{\mu\nu}, B_{\mu}, \bar{B}_{\mu}, B_{1}, \bar{B}_{1}, \phi) \to (A_{\mu}, B_{\mu\nu}, B_{\mu}, \bar{B}_{\mu}, B_{1}, \bar{B}_{1}, \phi),$$
(25)

where  $\Sigma$  is a global scale parameter and numbers in the exponential denote the corresponding ghost number of the fields (*e.g.* the ghost field  $\beta$  has the ghost number equal to +2). It is elementary to check that the following infinitesimal version of the above scale symmetry transformations

$$s_g C_1 = \Sigma C_1, \quad s_g \bar{C}_1 = -\Sigma \bar{C}_1, \quad s_g C_\mu = \Sigma C_\mu, \quad s_g \bar{C}_\mu = -\Sigma \bar{C}_\mu,$$
  

$$s_g \beta = 2 \Sigma \beta, \quad s_g \bar{\beta} = -2 \Sigma \bar{\beta}, \quad s_g \rho = -\Sigma \rho, \quad s_g \lambda = \Sigma \lambda,$$
  

$$s_g [A_\mu, B_{\mu\nu}, B_\mu, \bar{B}_\mu, B_1, \bar{B}_1, \phi] = 0,$$
(26)

leads to the derivation of the conserved Noether current

$$J_{(g)}^{\mu} = 2\beta \cdot D^{\mu}\bar{\beta} - 2\bar{\beta} \cdot D^{\mu}\beta - C_{\nu} \cdot \left[ (D^{\mu}\bar{C}^{\nu} - D^{\nu}\bar{C}^{\mu}) - \bar{C}_{1} \times F^{\mu\nu} \right] - \bar{C}_{\nu} \cdot \left[ (D^{\mu}C^{\nu} - D^{\nu}C^{\mu}) - C_{1} \times F^{\mu\nu} \right] - C^{\mu} \cdot \rho - \bar{C}^{\mu} \cdot \lambda.$$
(27)

The conservation law  $\partial_{\mu} J_g^{\mu} = 0$  can be proven by exploiting the E-L equation of motion derived from the  $\mathcal{L}_B$  and  $\mathcal{L}_{\bar{B}}$  (cf. (13), (22)) for the ghost, antighost and other appropriate fields of the theory. The conserved ghost charge  $Q_g = \int d^3x J^0_{(g)}$ , derived from the Noether conserved current  $J^{\mu}_{(g)}$ , is as follows

$$Q_{g} = \int d^{3}x \Big[ 2\beta \cdot D^{0}\bar{\beta} - 2\bar{\beta} \cdot D^{0}\beta - C_{i} \cdot [(D^{0}\bar{C}^{i} - D^{i}\bar{C}^{0}) - \bar{C}_{1} \times F^{0i}] \\ - \bar{C}_{i} \cdot [(D^{0}C^{i} - D^{i}C^{0}) - C_{1} \times F^{0i}] - C^{0} \cdot \rho - \bar{C}^{0} \cdot \lambda \Big].$$
(28)

The above charge generates<sup>5</sup> the infinitesimal version of (26). Using the definition of a generator (e.g.  $s_bQ_b = -i\{Q_b, Q_b\}, s_bQ_g = -i[Q_g, Q_b], etc.$ ), it can be seen that the following standard BRST algebra emerges, namely;

$$Q_b^2 = 0, \quad Q_{ab}^2 = 0, \quad \{Q_b, \ Q_{ab}\} = Q_b \ Q_{ab} + Q_{ab} \ Q_b = 0,$$
  
$$i \ [Q_g, \ Q_b] = + \ Q_b, \qquad i \ [Q_g, \ Q_{ab}] = - \ Q_{ab}. \tag{29}$$

It should be pointed out that, for the proof of the anticommutativity of the  $Q_b$  and  $Q_{ab}$  (*i.e.*  $\{Q_b, Q_{ab}\} = 0$ ), we have to exploit the beauty and strength of the CF-type restrictions in (18). It is trivial to note, in passing, that the ghost number of  $Q_{(a)b}$  is  $(\mp 1)$ . As a consequence, the transformations, generated by  $Q_{(a)b}$ , decrease/increase the ghost number of the fields by one.

## 6. Conclusions

In our present investigation, we have concentrated on the "vector" gauge symmetry transformations of the 4D topologically massive non-Abelian gauge theory and exploited it in the context of BRST analysis. We have shown that *all* the fields of the present theory have proper (*i.e.* off-shell nilpotent and absolutely anticommuting<sup>6</sup>) (anti-)BRST transformations (cf. (11), (16)). All these transformations have been tapped in the derivation of the conserved, nilpotent and anticommuting (anti-)BRST charges  $Q_{(a)b}$  (cf. (14), (23)).

As it turns out, the generators  $Q_{(a)b}$  are able to produce all the nilpotent (anti-)BRST symmetry transformations for the basic fields of the theory. Such transformations for the auxiliary fields are, as usual, derived from the requirements of the nilpotency and anticommutativity of the (anti-)BRST symmetry transformations. The (anti-)BRST invariance of the CF-type restrictions also plays a key role in such an endeavor. In our present work there exists a very *special* auxiliary field, however. It transpires that the generators  $Q_{(a)b}$  and nilpotency as well as anticommutativity requirements are unable

<sup>&</sup>lt;sup>5</sup>It is obvious that  $Q_g$  does not generate the ghost transformation for  $\rho, \lambda, C_1$  and  $\bar{C}_1$  which are auxiliary fields. These transformations are derived from other considerations.

<sup>&</sup>lt;sup>6</sup>To prove the absolute anticommutativity  $(s_b s_{ab} + s_{ab} s_b = 0)$ , one has to invoke the CF-type restrictions (18) that emerge in the superfield approach to BRST formalism [10].

to produce the nilpotent (anti-)BRST transformations of  $K_{\mu}$  field. This is a new observation in the application of BRST formalism to our present theory.

It may be mentioned, at this stage, that we have exploited the "scalar" gauge symmetry transformation for BRST analysis in our earlier work [11] where we have found that the (anti-)BRST charges are *not* able to generate the (anti-)BRST symmetry transformation for  $B_{0i}$  and  $K_{\mu}$  fields. This should be contrasted with our present investigation where  $Q_{(a)b}$  are unable to produce the (anti-)BRST symmetry transformations for *only* the auxiliary field  $K_{\mu}$ . Thus, there is a key difference between the novel features that emerge from the BRST analysis of the "scalar" and "vector" gauge symmetries.

It is worthwhile to point out that the construction of a renormalizable, unitary and consistent 4D non-Abelian 2-form gauge theory is an outstanding problem which is yet to be resolved completely. The Freedmen-Townsend (FT) model [2] and Lahiri model [3,4] are two of the quite well-known models which have their own virtues and vices. In a recent paper [15], one of us has shown the novel features of the FT model within the framework of BRST formalism. The central goal of our studies [10,11,15], including the present one, is to understand these models [2-4] clearly and, if possible, propose a model which is free of all the drawbacks of the above models.

#### Acknowledgements

One of us (RK) is grateful to UGC, Govt. of India, for financial support. It is a great pleasure for both of us to acknowledge very useful communications with A. Lahiri on certain aspects of our present investigation. We are also thankful to our esteemed referee for some clarifying and enlightening remarks.

# Appendix

Here we provide a brief synopsis of the similarities between Lahiri model [3,4] and the celebrated Proca model within the framework of Stückelberg formalism (see, e.g. [16] for a review). It can be noticed that if we express the Lagrangian density (1) in terms of the redefined field  $\tilde{B}_{\mu\nu}$ , viz.;

$$\ddot{B}_{\mu\nu} = B_{\mu\nu} + (D_{\mu}K_{\nu} - D_{\nu}K_{\mu}), \tag{30}$$

the auxiliary field  $(K_{\mu})$  disappears from the curvature tensor  $H_{\mu\nu\eta}$ . Furthermore, the redefined topological mass term of the Lagrangian density (1):

$$\frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} \tilde{B}_{\mu\nu} \cdot F_{\eta\kappa} = \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa} + \partial_{\mu} \left[ \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} K_{\nu} \cdot F_{\eta\kappa} \right] - \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} K_{\nu} \cdot \left( D_{\mu}F_{\eta\kappa} \right),$$
(31)

remains invariant (modulo a total spacetime derivative term) because of the validity of the Bianchi identity  $(D_{\mu}F_{\nu\eta} + D_{\nu}F_{\eta\mu} + D_{\eta}F_{\mu\nu} = 0)$ . Thus, the auxiliary field  $K_{\mu}$  is completely eliminated from the whole theory. As a consequence, even though the theory respects the "scalar" (i.e. Yang-Mills) gauge symmetry transformations, it fails to respect the "vector" (i.e. tensor) gauge symmetry transformations. This observation is, however, true for all the theories where the Stückelberg trick is applied.

The above key observation should be contrasted with the Proca model. The Lagrangian density of the latter is as follows

$$\mathcal{L}_P = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu, \qquad (32)$$

where  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$  and *m* is the mass of the Abelian 1-form gauge field  $A_{\mu}$ . With the inclusion of the Stückelberg field  $\phi$  (by the application of the standard technique), the above Lagrangian density becomes [16]

$$\mathcal{L}_S = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \frac{1}{2} \partial_\mu \phi \,\partial^\mu \phi + m A_\mu \partial^\mu \phi. \tag{33}$$

It is well-known fact that the above Lagrangian density (33) remains invariant under the following gauge transformations  $(\delta_q)$ 

$$\delta_g A_\mu = \partial_\mu \Lambda(x), \qquad \qquad \delta_g \phi = -m \Lambda(x), \qquad (34)$$

where  $\Lambda(x)$  is a local infinitesimal transformation parameter. It can be checked that, if we incorporate the following redefinition:

$$\tilde{A}_{\mu} = A_{\mu} - \frac{1}{m} \,\partial_{\mu}\phi, \qquad (35)$$

in the above Lagrangian density, the Stückelberg field  $\phi$  is totally eliminated from  $\mathcal{L}_S$ . The ensuing theory, thus, does not respect the gauge transformation  $\delta_g A_\mu = \partial_\mu \Lambda$ . We conclude that the Lahiri model of the 4D non-Abelian 2form theory (with the auxiliary field  $K_\mu$ ) is exactly same, in structure, as the Proca theory with the Stückelberg field  $\phi$ . Thus, the auxiliary field  $K_\mu$ of the Lahiri model is the Stückelberg field in the true sense of the word.

In contrast to the above Stückelberg's tricks [16], the spontaneous symmetry breaking, though the Higgs mechanism, is yet another technique to generate a mass for the gauge field. For instance, one knows that in the polar decomposition of the scalar field, the Goldstone mode is eliminated due to the U(1) gauge transformation and the photon field acquires a mass in the *Abelian* Higgs model (see, e.g. [17] for details). The final theory, with the mass term for the photon, does not respect, however, the local U(1) gauge symmetry transformations for obvious reasons. We lay stress on the fact that the Stückelberg trick and the SSB of the gauge symmetries, through the Higgs mechanism, are entirely different techniques to generate the mass.

# References

- [1] T. J. Allen, M. J. Bowick, A. Lahiri, *Mod. Phys. Lett.* A 6, 559 (1991)
- [2] D. Z. Freedman, P. K. Townsend, Nucl. Phys. B 177, 282 (1981)
- [3] A. Lahiri, *Phys. Rev.* D **55**, 5045 (1997)
- [4] A Lahiri, *Phys. Rev.* D **63**, 105002 (2001)
- [5] E. Harikumar, A. Lahiri, M. Sivakumar, *Phys. Rev.* D 63, 105020 (2001)
- [6] R. P. Malik, Eur. Phys. J. C 60, 457 (2009), hep-th/0702039
- [7] S. Gupta, R. Kumar, R. P. Malik, *Eur. Phys. J.* C 70, 491 (2010)), arXiv:1003.3390 [hep-th]
- [8] L. Bonora, P. Pasti, M. Tonin, *Nuovo Cim.* A, **63**, 353 (1981)
- [9] L. Bonora, M. Tonin, *Phys. Lett.* B, **98**, 48 (1981)
- [10] S. Krishna, A. Shukla, R. P. Malik, arXiv:1008.2649 [hep-th]
- [11] R. Kumar, R. P. Malik, Euro. Phys. Lett. (EPL), 94, 11001 (2011), arXiv:1012.5195 [hep-th]
- [12] See, e.g., K. Nishijima, Czech. J. Phys. 46, 1 (1996)
- [13] See, e.g., S. Weinberg, The Quantum Theory of Fields: Modern Applications, Vol. 2 (Cambridge University Press, Cambridge, 1996)
- [14] L. Bonora, R. P. Malik, J. Phys. A: Math. Theor. 43, 375403 (2010), arXiv: 0911.4919 [hep-th]
- [15] R. P. Malik, arXiv:1106.3764 [hep-th]
- [16] See, e.g., for a modern review, H. Ruegg, M. R. Altaba, Int. J. Mod. Phys. A 19, 3265 (2004), arXiv: hep-th/0304245
- [17] See, e.g., W. E. Burcham, M. Jobes, Nuclear and Particle Physics, (John Wiely and Sons Inc, 1995)