

# Negative Refraction and Superconductivity

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## Abstract

We discuss exotic properties of charged hydrodynamical systems, in the broken superconducting phase, probed by electromagnetic waves. Motivated by general arguments from hydrodynamics, we observe that negative refraction, namely the propagation in opposite directions of the phase velocities and of the energy flux, is expected for low enough frequencies. We corroborate this general idea by analyzing a holographic superconductor in the AdS/CFT correspondence, where the response functions can be explicitly computed. We study the dual gravitational theory both in the probe and in the backreacted case. We find that, while in the first case the refractive index is positive at every frequency, in the second case there is negative refraction at low enough frequencies. This is in agreement with hydrodynamic considerations.

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## Introduction

In the last years there have been much progress in realizing artificial materials which exhibit exotic electromagnetic phenomena. These artificial materials, commonly called electromagnetic metamaterials, consist of periodic arrays of metallic rods [1] and split ring resonators [2]. Among others, one of the most fascinating properties is the possibility of negative refraction, i.e. the energy flux of the electromagnetic wave flows in the opposite direction with respect to the phase velocity. This has been theoretically suggested by Veselago in 1968 [3] and only recently experimentally realized [4,5]. The main difficulties in the realization of such systems is the necessity of strong magnetic response, and their high degree of electromagnetic dissipation.

In [6] it was argued that negative refraction is a common feature for transverse electromagnetic waves probing relativistic charged hydrodynamical systems at low enough frequencies. This idea was supported by an explicit computation of the permittivity and the permeability of a strongly coupled system that admits a dual gravitational description. This system is supposed to describe a four dimensional charged strongly coupled plasma. Moreover in [7] the same topic was investigated for a charged black hole in four dimensions, and in [8,9] for systems with D7 flavor branes. Especially this last development seems promising for more realistic applications to the Quark Gluon Plasma physics.

One may wonder about the electromagnetic response properties at low frequencies of other materials [10]. For example in the last years an interesting proposal of superconductors as NIR (negative index of refraction) materials or metamaterials have been

investigated (for review see [11] and references therein). It was shown that in the radio, microwave and low-terahertz frequency range the superconductors can behave as metamaterials and exhibit negative refraction. The interest in building NIR superconductors is triggered by the fact that they can reduce the dissipation (lossy) of the electromagnetic energy, thus evading the constraints on the performance of the usual metamaterials and providing a convenient setup for experimental investigations.

In this paper, motivated by the experimental developments, we investigate the refractive index of a class of superconductors in which the response functions are rather simple and their dependence on the transport coefficients is known. They are superconductors that at low frequencies and wave-vectors can be described by hydrodynamics. We observe that quite generically it is possible to find frequency ranges where these media behave as NIR materials, when opportune choices of the transport coefficients are made.

Then we verify this general hydrodynamic prediction in a class of superconductors in which the transport coefficients can be explicitly computed. The class of materials that we are describing are referred in the recent literature as holographic superconductors. They are associated to strongly coupled gauge theories dual to some weakly coupled gravitational system. The latter weakly coupled theory simplifies the computation of the correlation functions describing the linear response of the material to an external electromagnetic source.

The possibility of describing a superconductor in the context of the gauge/gravity duality was shown for the first time in [12–14]. According to this duality, the conformal field theory describing the quantum critical region is mapped to a gravitational system in one higher dimension whose asymptotic spacetime is AdS. The  $U(1)$  symmetry of the field theory is associated to a gauge field living in the bulk”. The phase transition in the gravity theory is set by a scalar field which breaks the  $U(1)$  symmetry below a critical temperature. This is the mechanism first explained in [12]. Strictly speaking, the field theory  $U(1)$  symmetry is a global one, so we should talk about superfluidity rather than superconductivity. Anyway, a weak gauging of the symmetry is consistent, and it opens up the way to the description of a superconductor in AdS/CFT. In particular, below the critical temperature the breakdown of the  $U(1)$  adds a pole in the conductivity [13, 14], thus modifying the electric permittivity. Many examples of this type have been studied, and we refer the reader to the reviews [15] and references therein.

A previous study of the refractive index in holographic superconductors was done in [16] where the authors observed that the refractive index is positive for every frequency. That result holds for a holographic superconductor in a specific limit, called the probe limit. This corresponds to the limit in which the metric background does not fluctuate, and the response functions are computed only from the fluctuation of the Maxwell field. When the background is fixed the correlation function of the transverse electromagnetic current, below the critical temperature, is dominated by the Goldstone mode.<sup>5</sup> Then the conductivity only has the pole due to the Goldstone boson which is not enough, at least at the leading order for low frequencies and wave-vectors, to obtain a negative refractive

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<sup>5</sup>This result can be also understood quite generally from the hydrodynamical equations.

index.

Allowing background fluctuations corresponds in the hydrodynamical description to also considering the stress-energy tensor conservation equations, which imply translational invariance in the field theory due to momentum conservation. In this case there is also a diffusive pole in the current correlator. At low frequencies this suggests the existence of negative refraction.<sup>6</sup>

In this paper we look for negative refractive index in backreacted holographic superconductors. We indeed find that if the backreaction is taken into account the refractive index becomes negative at low frequencies. This result corroborates the general claim that in systems described by hydrodynamics at finite temperature and finite charge density the refractive index becomes negative in the low frequencies regime, because the backreaction is proportional to the charge density.

The rest of the paper is organized as follows. In section 1 we review the computations of the linear response of media and the electromagnetic response properties, focusing on the negative refractive index. Then in section 2 we illustrate the general prediction on the refractive index of media described by hydrodynamics at finite temperature and charge density. In particular, we observe that negative refraction is expected for a superconductor if the system has a finite charge density and the diffusive contribution cannot be neglected with respect to the Goldstone mode typical of the superconducting broken phase. In section 3 we introduce the bulk setting for the study of a holographic superconductor. In section 4 we review the probe limit by showing even at the analytical level that in that case the refractive index is always positive. In section 5 we allow the backreaction of the matter fields on the metric. In this case we observe that increasing the backreaction the refractive index becomes negative at low frequencies. Finally we conclude.

## 1 Linear response functions and negative refraction

In this section we review the formalism of the linear response theory of a continuous medium to the EM field, and its application to the study of the negative refractive index.

The electrodynamics response of a medium is described by the electric and magnetic fields and inductions  $E, B, D, H$ . These fields are related by the constitutive equations (in Fourier space)

$$D_i(w) = \epsilon_{ij}(w)E_j(w) \quad , \quad B_i(w) = \mu_{ij}(w)H_j(w) \quad (1)$$

The response functions  $\epsilon$  and  $\mu$  are the permittivity and permeability of the medium. They are complex quantities and depend generically on the frequency of the electromagnetic wave.

In hydrodynamical systems, which the strong coupling media we are going to describe are a particular example of, non local effects can be relevant, and hence the response functions are in this case functions also of the wave vector  $k$ . This phenomenon is known

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<sup>6</sup>We expect that breaking the translational invariance with small impurities does not qualitatively alter this conclusion. However a more detailed study would be interesting.

as spatial dispersion. The magnetic and electric response of the system can be here just described in terms of the field  $D$ ,  $B$  and  $E$  [17], where

$$D_i(w, k) = \epsilon_{ij}(w, k)E_j(w, k) \quad (2)$$

If the medium is isotropic the permittivity  $\epsilon_{ij}$  is decomposed in a transverse and in a longitudinal part,  $\epsilon_T$  and  $\epsilon_L$ , with

$$\epsilon_{ij}(w, k) = \frac{k_i k_j}{k^2} \epsilon_L(w, k) + \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \epsilon_T(w, k) \quad (3)$$

The EM properties of the medium are analyzed by solving the dispersion relation, obtained by the Maxwell equations. Here we focus on the transverse propagation, which has the dispersion relation

$$\epsilon_T(w, k) = \frac{k^2}{w^2} = n^2 \quad (4)$$

where  $n$  is the refractive index of the medium. It is a complex quantity which, once we impose the dispersion relation, is a function only of the frequency. The real part of  $n$  is the usual refractive index, while the complex part encodes the dissipation.

If the spatial dispersion is small, we can expand the transverse permittivity as [18]

$$\epsilon_T(w, k) = \epsilon(w) + \frac{k^2}{w^2} \left( 1 - \frac{1}{\mu(w)} \right) + \dots \quad (5)$$

This expansion connects the *EBD* approach with the *EBDH* one.<sup>7</sup> Indeed the expression (5) has been arranged in such a way that if we impose the dispersion relation (4) we obtain the usual dispersion relation for an electromagnetic wave in the  $\epsilon - \mu$  approach, that is  $\epsilon\mu = n^2$ .

It has been shown [19] that in case of dissipative media described by complex  $\epsilon$  and  $\mu$  the sign of the refractive index coincide with the sign of the function

$$n_{DL}(w) = Re(\epsilon(w))|\mu(w)| + Re(\mu(w))|\epsilon(w)| \quad (6)$$

This index was originally derived for dissipative systems without spatial dispersion. It corresponds to the requirement that the orientation of the Poynting vector  $S$  is opposite to the orientation of the phase velocity which is  $Sign[Re[n]]$ . In the case of the propagation of a transverse wave the Poynting vector can be written as

$$S = Re\left(\frac{n}{\mu}\right) |E_T|^2 \quad (7)$$

Imposing  $Sgn(Re(n/\mu)) = -Sgn(Re(n))$  is equivalent to require that (6) is negative. Here, in the presence of spatial dispersion the definition of the Poynting vector is slightly modified. Anyway, as shown in [6], the relation (7) is still valid if the spatial dispersion is not

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<sup>7</sup>However it is important to observe that the function  $\mu(w)$  we introduce here is an effective magnetic permeability, that contains the magnetic response of the medium [17] plus some  $k^2$  corrections to the electric response. In the case of dispersive media it is the right quantity to study waves propagation [18].

too large and  $n_{DL}$  is still a measure of the sign of the refractive index. In the rest of the paper we will use (6) as an index to study at which frequencies the system at hand shows negative refraction.

The response functions  $\epsilon(w)$  and  $\mu(w)$  can be computed from the linear response theory applying an external field  $A_j$ . Indeed the resulting  $U(1)$  current  $J_i$  is proportional to  $A_j$  through the relation  $J_i = q^2 G_{ij} A_j$ , where  $q$  is the electric charge and  $G_{ij}$  is the retarded correlator of  $U(1)$  current.<sup>8</sup> For an isotropic medium, we decompose  $G$  in a similar fashion to (3). The transverse component  $G_T(w, k)$  is related to  $\epsilon_T$  by<sup>9</sup>

$$\epsilon_T(w, k) = 1 + \frac{4\pi}{w^2} q^2 G_T(w, k) \quad (8)$$

As anticipated above in the case of small spatial dispersion we can expand this expression at the second order in the wave-vector (5). Denoting  $G_T(w, k) = G_T^{(0)}(w) + k^2 G_T^{(2)}(w) + \mathcal{O}(k^4)$ , the permittivity and permeability read

$$\begin{aligned} \epsilon(w) &= 1 + \frac{4\pi}{w^2} q^2 G_T^{(0)}(w) \\ \mu(w) &= \frac{1}{1 - 4\pi q^2 G_T^{(2)}(w)} \end{aligned} \quad (9)$$

In particular the conductivity is defined as:  $\sigma(w) = -iq^2 G_T^{(0)}(w)/w$  and  $\epsilon(w) = 1 + 4\pi i\sigma(w)/w$ .

## 2 Hydrodynamics and negative refraction

Before doing any explicit computations it is worth noting that we can infer the generic behavior of the refractive index for the type of systems we are going to consider in the rest of the paper from hydrodynamic arguments. As explained in the previous section, to reach this goal we need to know the generic form of the retarded correlator of the transverse current. This goal is actually possible at least for the leading order in the low frequencies  $w$  and low wave vector  $k$  expansion. Indeed in the limit of low frequencies and long wave lengths a system can be typically described by hydrodynamic equations. These equations describe the long time dynamics of the effective macroscopic degrees of freedom of the system: conserved charge densities and phases of order parameters. The linearized hydrodynamic equations describe the response of the macroscopic degrees of freedom to the system due to the application of a small external field.

In this section we will quickly review the general argument in the specific case of the relativistic superconductor, the interested reader is referred to [10] for more details. To obtain the correlator of the transverse current  $J^T$  it is useful to study the dynamics of

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<sup>8</sup>In many cases the explicit  $q^2$  dependence in this relation is absorbed in the Green function. Here we prefer keeping it explicitly to stress that the EM field in AdS/CFT is introduced as an external field.

<sup>9</sup>Here we follow the opposite convention of [6] for the sign of the Green function to be consistent with the conventions common in the literature (es. [20]). This changes the sign of the  $q^2$  contribution in  $\epsilon_T$ .

the transverse momentum density  $\pi^T$ . As usual in the hydrodynamic regime [20, 21] the transverse part of the momentum density decouples from the other degrees of freedom and it satisfies the diffusive equation:

$$(\partial_t - \mathcal{D}\nabla^2)\pi^T = 0 \quad (10)$$

where  $\mathcal{D} = \frac{\eta}{(\epsilon + P - \rho_s \mu_p)}$ , and  $\eta$  is the shear viscosity,  $\epsilon$  the energy density,  $P$  the pressure,  $\rho_s$  the charge density of the superconducting part of the fluid and  $\mu_p$  the electrostatic chemical potential.

On the other hand linear response theory gives the non equilibrium momentum density of the system as a function of its retarded correlator computed in the equilibrium state as:

$$\pi^T(w, k) = \frac{G_{\pi^T}(w, k) - G_{\pi^T}(0, k)}{iw G_{\pi^T}(0, k)} \pi^T(t = 0, k) \quad (11)$$

where  $\pi^T(t = 0, k)$  is the Fourier transform of the initial value for the momentum density.

Linear hydrodynamics and linear response theory at long time and large scale are two different description for the same physics, and they must coincide [20, 21]. If we equate (11) with the solution of the initial value problem (10), taking the  $k \rightarrow 0$  limit for the correlation function and considering that  $G_{\pi^T}(0, 0) = (\epsilon + P - \rho_s \mu_p)$ , we obtain the classical expression for the momentum correlator:  $G_{\pi^T}(w, k) = \frac{\mathcal{D}k^2 G_{\pi^T}(0, 0)}{-iw + \mathcal{D}k^2}$ .

We can now compute the retarded correlator of the transverse current:  $G_{J^T}(w, k)$ . The induced transverse current  $J^T$  is the response of the system to an external electromagnetic field  $A_{ext}^\mu$ . In presence of an electromagnetic field the transverse velocity field of the system is no more proportional to  $\pi^T$  but to  $\pi^T - A^T \rho_t$ , due to the minimal coupling of the system to the vector potential, where  $\rho_t = \rho_n + \rho_s$  is the total charge density: the sum of the charge density of the normal fluid plus the charge density of the superfluid component. The transverse current is proportional to the transverse field velocity of the system [20–23] and in particular:

$$J^T = \rho_n v^T = \frac{\rho_n}{(\epsilon + P - \rho_s \mu_p)} \pi^T - \frac{\rho_n \rho_t}{(\epsilon + P - \rho_s \mu_p)} A^T \quad (12)$$

We then obtain  $G_{J^T}(w, k)$ , if we assume that all the electromagnetic fields in the system are external: namely  $A = A^{ext}$ . This hypothesis can be a bit strong and amounts to treating even the internally generated electromagnetic field as an external perturbation. We believe that it can be justified in the case we are studying in the main text, where the system itself is dominated by the non-abelian strong interactions and there is no dynamical electromagnetic field if we do not couple it with an external field.

Using equation (12), the fact that  $J^T = G_{J^T} A_{ext}^T$  and the explicit form for  $G_{\pi^T}(w, k)$ , we obtain:

$$G_{J^T}(w, k) = \frac{iw \frac{(\rho_t - \rho_s)^2}{(\epsilon + P - \rho_s \mu_p)}}{-iw + \frac{\eta}{(\epsilon + P - \rho_s \mu_p)} k^2} - \frac{\rho_s (\rho_t - \rho_s)}{(\epsilon + P - \rho_s \mu_p)} \equiv \frac{iw \mathcal{B}}{-iw + \mathcal{D}k^2} - \mathcal{C} \quad (13)$$

where  $\mathcal{B}$ ,  $\mathcal{C}$  and  $\mathcal{D}$  are real constants. In the limit of normal fluid  $\rho_s \rightarrow 0$ ,  $\rho_n \rightarrow \rho_t$ , the constant contribution  $\mathcal{C}$  to  $G_{JT}$  goes to zero, and (13) reduces to the correlation function discussed in [6], for a normal relativistic, translation invariant, isotropic system at finite charge density.

From the generic form of (13) we can obtain  $G^{(0)}(w)$  and  $G^{(2)}(w)$  and hence the generic form of  $\epsilon(w)$  and  $\mu(w)$ :

$$\epsilon(w) = 1 - \frac{4\pi q^2}{w^2} (\mathcal{B} + \mathcal{C}), \quad \mu(w) = \frac{1}{1 - 4\pi q^2 \frac{i\mathcal{B}\mathcal{D}}{w}} \quad (14)$$

and of the conductivity:

$$\sigma(w) = \frac{i}{w} q^2 (\mathcal{B} + \mathcal{C}) \quad (15)$$

from which it becomes clear that the  $\mathcal{C}$  is responsible for the infinite DC conductivity of the superconducting phase, while  $\mathcal{B}$  is responsible of the infinite DC conductivity of the normal phase, associated to the translation invariance of the system.

From the equations (14) one can verify that for low enough frequencies  $n_{DL}(w)$  is negative.

Hence we conclude that an isotropic and homogeneous superconductor at equilibrium, with relativistic invariance and at finite charge density, has a negative refractive index for low enough frequencies in the hydrodynamical regime.<sup>10</sup>

Differently from system in the normal phase, in the case of superconductors the imaginary pole in the conductivity has two different contributions; one is associated to the translational invariance and it comes from the charge density of the normal fluid  $\rho_n$ , and the other is associated to the superconducting phase of the system and it comes from  $\rho_s$ . Both terms contribute to make  $\text{Re}(\epsilon(w))$  negative at low frequencies. This fact is usually a necessary condition to obtain negative refraction. Anyway also the imaginary part of the permeability conspires to give negative refraction. This suggests that the diffusive pole is crucial in order to have a negative refractive index. Indeed, by varying the amount of backreaction, we will be able to explore the interplay between the two poles and to show that by raising the size of the backreaction the refractive index becomes negative at low frequency and within our approximation.

Moreover in [6], even if negative refraction was found, the system was shown to be highly dissipative. In superconductors the real part of the conductivity has a gap for low enough frequencies in which the conductivity is almost zero, this implies that the imaginary part of the electric permittivity is very small and hence the dissipative effects are smaller than for a normal fluid. Indeed, due to the presence of a physical pole in the imaginary part of the conductivity and a gap in the real part of the conductivity, the superconductors can be convenient systems with negative refraction and low dissipation, and they have been already used to obtain negative refractive metamaterials in [11].

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<sup>10</sup>Actually the conclusion is more general and can be extended to system which are not relativistic invariant and are not in the superconducting phase. See [10].



### 3 Holographic superconductors: the bulk setting

In this section we start the study of the basic setup for the analysis of the superconductivity of a medium through the techniques of the gauge gravity duality. The minimal ingredients to build a holographic theory of s-wave superconductivity are a bulk abelian gauge field  $A_\mu$  and a charged massive scalar field  $\phi$  coupled to gravity. We are then led to consider the following action [13,14]

$$S = \int d^5x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{12}{L^2} \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\phi|^2 - m^2 |\phi|^2 \right] \quad (16)$$

where the Newton constant  $G_5$  is related to the backreaction by  $\kappa^2 = 8\pi G_5$ ,

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ D_\mu \phi &= \nabla_\mu \phi - iq_s A_\mu \phi \end{aligned} \quad (17)$$

are the gauge field strength and the covariant derivative respectively,  $\nabla_\mu$  is the metric covariant derivative and  $q_s$  is the charge of the scalar field in five dimensions. Note that we used the freedom of rescaling the gauge field so that its kinetic term is fixed, and  $q_s$  is the charge of the scalar field in these units. By also defining the real current

$$J_\mu \equiv i (\phi^* D_\mu \phi - D_\mu \phi^* \phi) \quad (18)$$

the Einstein, Maxwell and scalar equations of motion resulting from (16) can be written as

$$G_{\mu\nu} - \frac{6}{L^2} g_{\mu\nu} = \kappa^2 T_{\mu\nu} \quad (19)$$

$$\nabla^\mu F_{\mu\nu} = q_s J_\nu \quad (20)$$

$$(D^\mu D_\mu - m^2) \phi = 0 \quad (21)$$

with  $G_{\mu\nu}$  the Einstein tensor and  $T_{\mu\nu}$  the stress-energy tensor.

Throughout this paper we only consider the plane-symmetric ansatz for the metric

$$ds^2 = -r^2 f(r) e^{2\nu(r)} dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\mathbf{x}^2 \quad (22)$$

where the condition  $f(r_h) = 0$  determines the radius  $r_h$  of the black hole. Moreover, the equation of motion for  $f$  sets  $f(r) \xrightarrow{r \rightarrow \infty} 1$ , i.e. the spacetime is asymptotically AdS: according to the AdS/CFT, the conformal symmetry is recovered in the UV limit of the dual field theory.

We take the following ansatz for the gauge and scalar fields

$$\begin{aligned} A_t &= h(r), & A_i &= 0 \quad i = r, x, y, z \\ \phi &= \phi(r) \end{aligned} \quad (23)$$

which corresponds to a homogeneous system. Because the scalar field is supposed to be charged under the bulk  $U(1)$  gauge symmetry, we wrote it as a complex field in (16). However, the  $r$  component of the Maxwell equations sets its phase to a constant, and because this constant does not affect the other equations, we set it zero without any loss of generality. Note that the phase of the scalar field cannot be always set to zero.

We find more convenient to change the radial coordinate to  $u = \frac{r_h^2}{r^2}$  so that it is a compact direction: the horizon is located at  $u = 1$  and the boundary at  $u = 0$ . In this coordinate system, the ansatz (22) becomes

$$ds^2 = -\frac{f(u)e^{2\nu(u)}}{u}dt^2 + \frac{r_h^2}{4u^2}\frac{du^2}{f(u)} + \frac{r_h^2}{u}d\mathbf{x}^2 \quad (24)$$

Unless otherwise specified, from now on we use the coordinate  $u$  and we set  $r_h = 1$ .

The Hawking temperature of the black hole is given by

$$T_H = \frac{f'(1)e^{\nu(1)}}{4\pi} \quad (25)$$

and is identified with the dual field theory temperature only if  $\nu(0) = 0$ .

## 4 The probe limit

In this section we review the refractive index of the holographic theory described above in the limit of  $\kappa = 0$ . Even if this analysis was already performed in [16] here we prefer to start from this simplified example to fix the notations which will become important for the backreacted case.

We start with the analysis of the equations of motion and explain the derivation of the Green functions in this limit. Then we study the numerical solutions and we observe that the system has a positive refractive index even at low frequencies. We conclude this section comparing the numerical results with the analytic computation.

### 4.1 The holographic setup

In the probe limit  $\kappa^2 = 0$  the Einstein equations decouple from the gauge-scalar sector and the analytic solution for the metric is the usual Schwarzschild-AdS black hole: The  $\nu$  function is constrained to vanish, while  $f(u) = 1 - u^2$ . We are left with a system of two ordinary differential equations for the background

$$\begin{aligned} h''(u) - \frac{q_s^2\phi(u)^2}{2u^2f(u)}h(u) &= 0 \\ \phi''(u) + \left(-\frac{1}{u} + \frac{f'(u)}{f(u)}\right)\phi'(u) + \frac{(-m^2f(u) + uq_s^2h(u)^2)}{4u^2f(u)^2}\phi(u) &= 0 \end{aligned} \quad (26)$$

Because they are non-linear, an analytic solution where both fields are nontrivial is very hard to find. However, mainly motivated by the corresponding numerical solution, an

analytic perturbative solution can be computed by assuming a small scalar field [24]. We will use this method later, when we will compare our numerical computation with the perturbative analytic solution to check the strength of our numerical algorithm.

The system of equations (26) has to be supplemented by boundary conditions for the solutions to be completely determined. At the horizon  $u = 1$  we require the gauge field to have a finite norm and the equations (26) to be regular there. This amounts to set

$$h(1) = 0 \quad \phi'(1) = \frac{m^2}{f'(1)}\phi(1) \quad (27)$$

At the boundary, the general asymptotic behavior of the fields is

$$h(u) = \mu_p - \rho_t u + \dots \quad \phi(u) = \phi^{(1)}u^{\Delta/2} + \phi^{(2)}u^{2-\Delta/2} + \dots \quad (28)$$

where  $\mu_p$  and  $\rho_t$  are the field theory chemical potential and total charge density, respectively, and  $\Delta = 2 \pm \sqrt{4 + m^2}$ . As long as  $\Delta \geq 1$  we can choose both signs for  $\Delta$  [25]; for concreteness, we will choose only the upper one in what follows. Thus, we have a normalizable solution only when

$$\phi^{(2)} = 0 \quad (29)$$

According to the AdS/CFT dictionary,  $\phi^{(2)}$  is interpreted as the expectation value  $\langle O \rangle$  of a dual field theory operator with conformal dimension  $\Delta$ , and  $\phi^{(1)}$  is the value of its source. The boundary condition (29) corresponds to the spontaneous breaking of a symmetry in the dual field theory: the charged operator  $O$  is acquiring a vacuum expectation value without being sourced by any field. Once (29) has been imposed, the other quantities are read from the general falloff (28).

In the probe limit the Hawking temperature is a constant

$$T_H = \frac{1}{4\pi} \quad (30)$$

and an analytic solution of (26) is

$$h(u) = \mu_p(1 - u) \quad (31)$$

with a vanishing scalar field. The equation (31) represents the normal phase of the dual field theory, i.e. the non-superconducting phase above the critical temperature.

The superconducting phase corresponds to a solution in which the scalar field assumes a nontrivial profile. This scalar field is identified with the condensate triggering the phase transition. By fixing a reference scale with the chemical potential  $\mu_p$  one can look at the evolution of  $\sqrt{\phi^{(2)}}/\mu_p$  as a function of  $T/\mu_p$ . Once the ratio  $T/\mu_p$  is lower than a certain critical value, the normal phase solution (31) produces an instability towards the formation of a hairy black hole. Indeed, one can see that the scalar effective mass in (26) is proportional to [12]

$$m_{eff}^2 = m^2 - \frac{q_s^2 h(u)^2}{f(u)} u = m^2 + q_s^2 g^{tt}(u) h(u)^2 \quad (32)$$

When the negative second term is such that  $m_{eff}^2$  falls below the BF bound [26],  $m_{eff}^2 < m_{BF}^2 = -4$ , for a range of the coordinate  $u$ , the instability occurs and the solution with the vanishing scalar field is no longer the vacuum of the theory.

Till now, our discussion has been quite general. From now on we specialize to the case  $m^2 = -3$  with  $\Delta = 3$ .

The next step is to compute the retarded correlator for the transverse current in the superconducting phase. To reach this goal we need to study the oscillations of the five dimensional gauge field  $\delta A_x(u, t, z) = a_x(u)e^{-i\omega t + ikz}$  transverse to  $k$ , in the superconducting five dimensional gravitational background. As in the standard approach we define the dimensionless frequency and wave-vector by normalizing  $\omega$  and  $k$  by the temperature and then we work with the dimensionless quantities  $\mathbf{w}$  and  $\mathbf{k}$ . Due to the isotropy of the system we chose the  $x$  component of the field, but the same is true for the  $y$  component, they are indeed both transverse to  $\mathbf{k}$ , that in this particular case is along the  $z$  direction.

Following the general holographic dictionary [27] we can then compute the  $G_{xx}^{(0)}(\mathbf{w})$  and  $G_{xx}^{(2)}(\mathbf{w})$  contributions to the current-current correlation function from holography and from that obtain the necessary information on the refractive index in the case of small spatial dispersion.

In the probe limit it suffices to allow fluctuations of the  $x$  component of the Maxwell field, because all the other equations decouple. The linearized equation for this fluctuation is

$$a_x''(u) + a_x'(u) \frac{f'(u)}{f(u)} + \frac{1}{2uf(u)} \left( \frac{\mathbf{w}^2}{2f(u)} - \frac{\mathbf{k}^2}{2} - \frac{q_s^2 \phi(u)^2}{u} \right) a_x(u) = 0 \quad (33)$$

Near the horizon we impose the fluctuation  $a_x$  to be proportional to  $(1-u)^{-\frac{i\mathbf{w}}{4}}$ , where the minus sign corresponds to the ingoing boundary condition. Near the boundary,  $u \rightarrow 0$ , the solution behaves as

$$a_x(u) = a_{x,0} + a_{x,1}u - a_{x,0}(\mathbf{w}^2 - \mathbf{k}^2)u \log(u) \quad (34)$$

where  $a_{x,0}$  and  $a_{x,1}$  are integration constants.

By using Lorentz invariance, and because we only need up to the  $\mathbf{k}^2$  coefficient of the Green function, we expand  $a_x(u, \mathbf{w}, \mathbf{k}) = a_x^{(0)}(u, \mathbf{w}) + \mathbf{k}^2 a_x^{(2)}(u, \mathbf{w})$ . Then, the near boundary solution (34) reads

$$a_x^{(0)} = a_{x,0}^{(0)} + a_{x,1}^{(0)}u - \mathbf{w}^2 a_{x,0}^{(0)}u \log(u) \quad (35)$$

$$a_x^{(2)} = a_{x,0}^{(2)} + a_{x,1}^{(2)}u + a_{x,0}^{(0)}u \log(u) \quad (36)$$

The Green function is then

$$G_{xx}^{(0)}(\mathbf{w}) = \frac{a_{x,1}^{(0)}}{a_{x,0}^{(0)}} + c \mathbf{w}^2 \quad , \quad G_{xx}^{(2)}(\mathbf{w}) = \frac{a_{x,1}^{(2)}}{a_{x,0}^{(2)}} \left( \frac{a_{x,1}^{(2)}}{a_{x,1}^{(0)}} - \frac{a_{x,0}^{(2)}}{a_{x,0}^{(0)}} \right) - c \quad (37)$$

where  $c$  is an arbitrary real constant fixed by requiring  $\epsilon(\mathbf{w} \rightarrow \infty) \rightarrow 1$ , i.e. the system at large frequencies behaves like the vacuum.

In [16] this system was numerically solved and it was observed that  $n_{DL} > 0$  in the whole frequency range. In the next section we will confirm this result when the effect of the backreaction are negligible. In the next subsection, we show that near  $T_c$  this result can be recovered by looking at the analytical computation of [24].

## 4.2 Connection with the analytical results

The numerical results that we discussed in the probe limit show that for every temperature below  $T_c$  the refractive index is a positive quantity. This is consistent with the observation that at  $T > T_c$  the  $U(1)$  symmetry is unbroken and the physics is described by [28]. The  $T \simeq T_c$  behavior can be explained even at analytical level. We refer to the system studied in [24] as the analytical example to understand the physics of the superconductor near  $T_c$ . The relevant details necessary to compute the Green function of an holographic superconductor at non zero wave-vector  $\mathbf{k}$  can be found in [24]. Here we show the correlator  $G_{xx}$  at higher orders referring to [24] for the details on the setup.

The analytical transverse Green function is studied for low values of the frequency  $\mathbf{w}$ , of the wave-vector  $\mathbf{k}$  and of the vev of the condensate, which we assume to be  $\xi \sim (1 - T/T_c)$ . These low values of the condensate are associated to temperatures near  $T_c$ .

The refractive index at  $T \simeq T_c$  is obtained by the study of the Green function in this regime. By computing some higher order correction with respect to [24] we obtain

$$G_T(\mathbf{w}, \mathbf{k}, \xi) = i\mathbf{w} - \frac{1}{2}\mathbf{w}^2 \log 2 - \frac{i\xi^2\mathbf{w}}{16} - \frac{\xi^2}{4} - \frac{i\pi^2\mathbf{w}\mathbf{k}^2}{32} + \frac{\xi^2\mathbf{k}^2(\pi^2 - 6i\pi \log 2 + \log^2 2)}{64} \quad (38)$$

The leading contribution to the electric permittivity at low frequencies is computed from (38) by applying (9) and it is

$$\epsilon(\mathbf{w}) = 1 + 4\pi q^2 \left( -\frac{\xi^2}{4} \frac{1}{\mathbf{w}^2} + \left(1 - \frac{\xi^2}{16}\right) \frac{i}{\mathbf{w}} - \frac{\log 2}{2} \right) \quad (39)$$

while the magnetic permeability does not have any pole in  $\mathbf{w}$  and it can be well approximated for small  $q$  by

$$\mu(\mathbf{w}) = 1 + \frac{\pi q^2}{8} (\xi^2 (\pi^2 - 6i\pi \log 2 + \log^2 2) - i\pi^2\mathbf{w}) \quad (40)$$

We conclude that the leading contribution to the index  $n_{DL}$  is

$$n_{DL}(\mathbf{w}) \simeq \begin{cases} 0 & \text{if } \mathbf{w} < \sqrt{\pi}q\xi \\ 2 \left(1 - \frac{\pi q^2 \xi^2}{\mathbf{w}^2}\right) & \text{otherwise} \end{cases} \quad (41)$$

For lower temperatures the approximation of small  $\xi$  does not hold, and the numerical computation is necessary. As observed in the last subsection even at lower temperatures the refractive index remains positive.

## 5 The backreacted case

In this section we study the refractive index of the holographic superconductor by taking into account the effects of the backreaction.

In this case the Goldstone mode in the superconducting phase introduces a gap in the real part [29] and a pole in the imaginary part of the electric conductivity  $\sigma$ , and they depend on the Newton constant, as first noted in [30, 31]. Moreover this usual pole of the superconductivity, related to the  $U(1)$  breaking, mixes with a new pole due to the translational invariance. Indeed, differently from the probe limit, where the background was fixed, and translational invariance was broken, here the fluctuation of the background are allowed. This restores the translational invariance and generates a pole in the Green function, because an external electric field uniformly accelerates the charges, that cannot relax due to momentum conservation. The EM properties of this pole were already studied in [6], and at low frequencies negative refraction was shown to be generic. For low  $\kappa$  the dominating pole is the superconducting one and the refractive index is positive, as in the probe limit, while for higher values of the backreaction the negative refraction is allowed as predicted from hydrodynamics.

### 5.1 Holography

In the backreacted case the Einstein equations do not decouple from the gauge-scalar sector anymore, because  $\kappa \neq 0$ . An analytic solution for the metric is a RN AdS black hole in the unbroken phase

$$f(u) = (1 - u^2) + \frac{2\kappa^2 h(1)^2 u^3}{3} \left(1 - \frac{1}{u}\right) \quad (42)$$

The analysis is completely analogous to that of section 4, so we will only sketch it here. The background system of equations is

$$\phi'(u)^2 + \frac{3\nu'(u)}{4u\kappa^2} + \frac{q_s^2 h(u)^2 \phi(u)^2}{4e^{2\nu(u)} u f(u)^2} = 0 \quad (43)$$

$$-\frac{m^2 \phi(u)}{4u^2 f(u)} + \frac{q_s^2 h(u)^2 \phi(u)}{4e^{2\nu(u)} u f(u)^2} - \frac{\phi'(u)}{u} + \frac{f'(u)\phi'(u)}{f(u)} + \nu'(u)\phi'(u) + \phi''(u) = 0 \quad (44)$$

$$h''(u) - h'(u)\nu'(u) - \frac{2q_s^2 h(u)\phi(u)^2}{4u^2 f(u)} = 0 \quad (45)$$

$$2(1 - f(u)) - \frac{m^2 \kappa^2 \phi(u)^2}{3} - \frac{q_s^2 u \kappa^2 h(u)^2 \phi(u)^2 + 2u^3 \kappa^2 h'(u)^2 f(u)}{3e^{2\nu(u)} f(u)} + \quad (46)$$

$$+ u f'(u) - \frac{4u^2 \kappa^2 f(u) \phi'(u)^2}{3} = 0$$

where the fields satisfy equations (27)-(29) and  $f(1) = 0, \nu(0) = 0$ . Once a solution is found, the field theory temperature is computed by (25).

The response of the system is obtained from the correlation function. As usual these are computed from the holographic perspective in terms of the boundary values of the fluctuations of the Maxwell field and of the metric. We observe that the only fluctuations necessary to obtain the transverse current correlation function  $G_{J_x, J_x} \equiv G_{xx}$  are  $a_x$  and the metric components  $g_{xt}$  and  $g_{xz}$ . Moreover we rise the index  $x$  by using the metric component  $g_0^{xx}$ , obtaining  $g_t^x = g_0^{xx} g_{xt} = u g_{xt}$  and  $g_z^x = g_0^{xx} g_{xz} = u g_{xz}$  to simplify the equations. The final set of equations for the fluctuations is

$$\mathbf{w} g_t^{t'x}(u) + e^{2\nu(u)} \mathbf{k} f(u) g_z^{t'x}(u) + u \kappa^2 \mathbf{w} h'(u) a_x(u) = 0 \quad (47)$$

$$\frac{\mathbf{w} (\mathbf{w} g_t^x(u) + \mathbf{k} g_z^x(u))}{4e^{2\nu} u f(u)} - \left( \frac{1}{u} - \frac{f'(u)}{f(u)} - \nu'(u) \right) g_z^{t'x}(u) + g_z^{t'x}(u) = 0 \quad (48)$$

$$\frac{\mathbf{k} (\mathbf{k} g_t^x(u) + \mathbf{w} g_z^x(u))}{4u f(u)} - \frac{q_s^2 \kappa^2 a_x(u) h(u) \phi(u)^2}{u f(u)} - 2u \kappa^2 a_x'(u) h'(u) + \left( \frac{1}{u} + \nu'(u) \right) g_t^{t'x}(u) - g_t^{t'x}(u) = 0 \quad (49)$$

$$\frac{(e^{-2\nu(u)} u \mathbf{w}^2 - f(u) (\mathbf{k}^2 u + 2 q_s^2 \phi(u)^2))}{4u^2 f(u)^2} a_x(u) + \frac{h'(u)}{e^{2\nu} f(u)} g_t^{t'x}(u) + \left( \frac{f'(u)}{f(u)} + \nu'(u) \right) a_x'(u) + a_x''(u) = 0 \quad (50)$$

where the equations (47), (48) and (49) are not independent. Once we derived the equations of motion we can follow the procedure explained in the probe limit case to compute the Green functions numerically. The difference is that in this case we have a set of coupled equations and more care has to be taken in the expansion. Moreover in this case the relevant contribution to the boundary action is

$$S_{Bd.} = \int \left( \frac{1}{q_s^2} a_x(u) (f(u) a_x'(u) - \rho g_t^x(u)) + \frac{1}{2u^2 \kappa^2} g_t^x(u) (u g_t^{t'x}(u) - g_t^x(u)) - \frac{f(u)}{2u^2 \kappa^2} g_t^z(u) (u g_z^{t'x}(u) - g_z^x(u)) \right)_{u=0}^{u=1} \quad (51)$$

The second term plays a crucial role in the backreacted case because it leads to the diffusive pole and to the appearance of the negative refraction in the low frequencies regime.

## 5.2 Prelude: numerical computation of the Green functions in the backreacted case

According to the prescription for computing the Minkowski space correlators [27], the Green functions are given by the second derivative of the on-shell boundary action with

respect to the boundary values of the dual gravity fluctuations. Numerically, this procedure is very difficult to implement. Thus, we will extract the Green functions in the following way.

Let us first note that, near the boundary  $u = \varepsilon$ , we have

$$\begin{aligned} g_t^{x'}(\varepsilon) &= \frac{\varepsilon}{A_{xtxt}} (G_{xtxt} g_t^x(0) - G_{xtxz} g_z^x(0) + G_{xtx} a_x(0)) \\ g_z^{x'}(\varepsilon) &= \frac{\varepsilon}{A_{xzzz}} (-G_{xtxz} g_t^x(0) + G_{xzzz} g_z^x(0) - G_{xzx} a_x(0)) \\ a_x'(\varepsilon) &= \frac{1}{A_{xx}} ((G_{xtx} - A_{xx}) g_t^x(0) - G_{xzx} g_z^x(0) + G_{xx} a_x(0)) \end{aligned} \quad (52)$$

where we discarded all the terms, including the logarithmic ones, which will be subtracted by the holographic renormalization procedure, and do not affect the present discussion. They will be taken into account by adding a constant to the Green functions.<sup>11</sup> The coefficients  $A_{ij}$  are read from the boundary action (51); for instance,  $A_{xtxt} = (2u\kappa^2)^{-1}$ . The equations (52) are a consequence of the on-shell boundary action (51). Given the above equations, we conclude that, for instance,

$$\frac{\partial^2 (g_z^{x'}(\varepsilon) g_t^x(0))}{\partial g_t^x(0)^2} = -\frac{G_{xtxz}}{A_{xzzz}} \quad (53)$$

We now consider the linearized set (47)-(50). Once we have solved the set (48)-(50), we still have to satisfy (47). We interpret it as a set of constraints on the different Green functions, in the following sense. If we multiply, for instance, the equation by  $g_t^x(u)$ , take the  $u \rightarrow 0$  limit, take the second derivative with respect to  $g_t^x(0)$  and use (53) and its analogous, we arrive at

$$\mathbf{w} G_{xtxt} + \mathbf{k} G_{xtxz} = 0 \quad (54)$$

where we used  $A_{xtxt} = -A_{xzzz}$ . Equation (54) is nothing than a Ward identity. A similar argument applies when we multiply by  $g_z^x(u)$  or  $a_x(u)$ .<sup>12</sup> Thus, we have a total of six equations, namely the set (48)-(50), plus the three constraints above, for six unknown Green functions (assuming the  $G_{ij} = G_{ji}$  symmetry). Because we are interested in the  $\mathbf{k}^0$  and  $\mathbf{k}^2$  coefficients of the Green functions, we expand both the functions  $g_t^x(u)$ ,  $g_z^x(u)$ ,  $a_x(u)$  and the correlators in powers of  $\mathbf{k}$ . Using again (47) and Lorentz invariance, it is easy to convince oneself that  $g_t^x(u)$  and  $a_x(u)$  only contain even powers of  $\mathbf{k}$ , while  $g_z^x(u)$  only contains odd powers of  $\mathbf{k}$ . Accordingly, the correlators which do not involve the coordinate  $z$ , together with  $G_{xzzz}$ , only contain even powers of the momentum, while the expansion of the other two correlators starts with a linear term in  $\mathbf{k}$ . The last statement is obvious if one looks at equation (54), or reminds the underlying Lorentz invariance. This procedure allows us to numerically compute the desired Green functions even when more than a

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<sup>11</sup>This is very usual in holographic renormalization. The arbitrary constant corresponds to an ambiguity in the choice of the holographic renormalization scale. We will fix this constant by the physical requirement that the real part of the electric permittivity approaches 1 at large frequencies.

<sup>12</sup>Note that when we multiply by  $a_x(u)$ , also the third term in (47) contributes.



fluctuation is involved, and we are not able to decouple the equations. To have further support of the validity of this procedure we checked it against some analytic results [28]. In the next subsection we will use it to show that our backreacted model exhibits negative refraction. Finally, note that, because in the probe limit the entire procedure reduces to the usual definition of the single-field prescription, it is perfectly consistent with previous results [16].

### 5.3 Numerical results

The following step consists in numerically solving the coupled set of equations of motion for  $a_x(u)$ ,  $g_x^t(u)$  and  $g_x^z(u)$  and extracting the Minkowski correlator [27] from the boundary action by using the procedure explained above. After we obtain the numerical behavior for the current correlator  $G_{xx}(\mathbf{w}, \mathbf{k})$  we can discuss the EM properties of the backreacted holographic superconductor. We start by showing the different results for the permittivity and the permeability at different values of  $\kappa$  and  $T/T_c$ . For the backreaction we choose  $\kappa^2 = \{10^{-10}, .1; .2; .3\}$  while for the ratio  $T/T_c$  we choose  $T/T_c = \{.45; .6; .75; .9\}$ . We plot the real and imaginary part of the electric permittivity and magnetic permeability.

We observe from the figures 1-4 that the real part of the permittivity is negative at low frequencies in all the cases. This result is expected both in the  $\kappa \rightarrow 0$  limit and in the fully backreacted case. Indeed in the first case the Goldstone mode dominates over the diffusive pole and this results coincides with the probe limit. In the case of a fully backreacted superconductor this contribution is corrected by the presence of the diffusive pole at zero wave-vector  $\mathbf{k}$ . The real part of the permeability is always positive and it is more peaked near the origin as far as the backreaction is raised. The imaginary part of the permittivity is always positive and has a pole at zero frequency. The imaginary part of the permeability is negative if the backreaction is very small.<sup>13</sup> When the backreaction is turned on, the diffusive pole of the green function introduces a pole in the imaginary part of  $G^{(2)}$  and thus in the imaginary part of the permeability. These results are perfectly consistent with the general prevision from hydrodynamics (14).

In figure 5 we plot the behavior of the index  $n_{DL}$  until  $\mathbf{w} = 2$ , while in figure 6 we zoom the plot to the frequency region where negative refraction is sizeable. When the backreaction is turned on, a region of negative refraction opens up at low frequencies. The size of the region increases with increasing the backreaction parameter. This confirms the fact that the negative refractive index is related to the diffusive pole in the current correlator and that the coefficient  $\mathcal{B}$  in (13) is related to  $\kappa^2$ . In figure 7 we plot the ratio  $\text{Re}(n)/\text{Im}(n)$ , which represents the amount of propagation with respect to the dissipation. Unfortunately this ratio is not large in the region of frequencies with negative refraction. Note that higher values of the backreaction make this ratio larger, improving the propagation. However we do not find in the holographic superconductor a net improvement with respect to the case studied in [6]. Moreover, as observed in [14] this

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<sup>13</sup>The permeability (9) is an effective permeability which is not simply related to any response function. However the fact that  $\text{Im}[\mu] < 0$  could indicate some problem in the  $\epsilon - \mu$  approach. This is a delicate issue that goes beyond the scope of this paper.

numerical procedure does not allow  $\kappa^2 > .3$ , even if this regime is physically meaningful. It would be interesting to confirm our predictions for the case of larger backreaction with  $T \simeq T_c$  by an analytical computation.

We conclude with a discussion about the validity of our results. In section 1 we used a long-wavelength expansion to relate the electrical permittivity and the magnetic permeability to the Green function. The validity of this expansion is verified *a posteriori* by the on shell relation  $|\mathbf{k}| = |n(\mathbf{w})|\mathbf{w} \ll 1$ . We explicitly verified that this inequality is satisfied in every plot that we have shown. As observed in [32] if this approximation breaks down it is a signal that the dispersion relation has not been correctly solved. A detailed analysis of this case was given for the hydrodynamical diffusive pole in [33].

A second constraint comes from the requirement of the validity of the perturbative expansion in the electrical charge  $q$ . Indeed in the hydrodynamical derivation we assumed that the external field  $A_{ext}$  is the dominating one. This analysis ignored the propagation of the induced internal field. By taking into account its contribution some constraints are imposed on the validity of the  $q^2$  expansion. This requirement enforces another lower bound on the frequency. We checked that there are regimes of parameters in which this bound does not affect the whole region of negative refraction. Anyway we leave deeper analysis for further studies.

A last observation is related to the fact that we consider a translation invariant system. We have checked that the refractive index is still negative even if we add a small amount of impurities that break the translation invariance. However this point requires further quantitative studies because it is crucially related to any concrete comparison to physical systems.

## Conclusions

In this paper we have observed that the two fluid description of superconductors allows the existence of negative refraction in the low frequencies regime. We reached this conclusion from the analysis of the general behavior of the correlation functions of the transverse current correlator in the hydrodynamical regime, and we corroborated this claim by analyzing an explicit example taken from high energy physics. This example deals with some powerful tools developed in the context of gauge/gravity duality. Indeed the correlation functions and the transport coefficients of a general class of superconductors can be exactly computed from the AdS/CFT dictionary.

On the gravity side our superconductor corresponds to an AdS/RN black hole coupled to a scalar field. In the broken phase this model is the dual of the EM symmetry breaking which leads usually to superconductivity. The EM properties of this system were already investigated in [16], where it was shown that the refractive index remains positive if the background metric is kept fixed. Here instead we studied the full backreacted example and we showed that, as expected from the hydrodynamic description, negative refraction is allowed.

Many interesting further lines of research are possible. A first important step should

be to set some numerical scale to the prediction of negative refraction in holographic superconductor, and try to make some connection with experimental physics.

Then it would be interesting to look at the behavior of the Josephson junction: two layered superconductor separated by another material (an insulator, a conductor or another superconductor). They are high- $T_c$  anisotropic superconductors and are promising metamaterials candidates because they are easier to fabricate and less dissipative than usual metamaterials [34]. From the holographic perspective a gravity dual description has been proposed in [35] and then developed and extended in [36–38].

Another interesting line of investigation would be the extension of our work to the  $AdS_4$  case. Indeed in the normal phase it was already shown that the negative refractive index is allowed [7], and a similar parallelism is expected in the broken phase.

Finally, it would be interesting to study the current correlation function for other systems in absence of the translation invariance pole and outside the hydrodynamic regime, to find if they can show negative refraction. The AdS/CFT is a natural tool to study this problem: it can indeed describe globally charged stable systems without the first contribution in (13). In this case, to claim something about the refractive index of the system, we cannot rely on the hydrodynamical approach and we need to compute higher order term in the  $\mathbf{w}, \mathbf{k}$  expansion.

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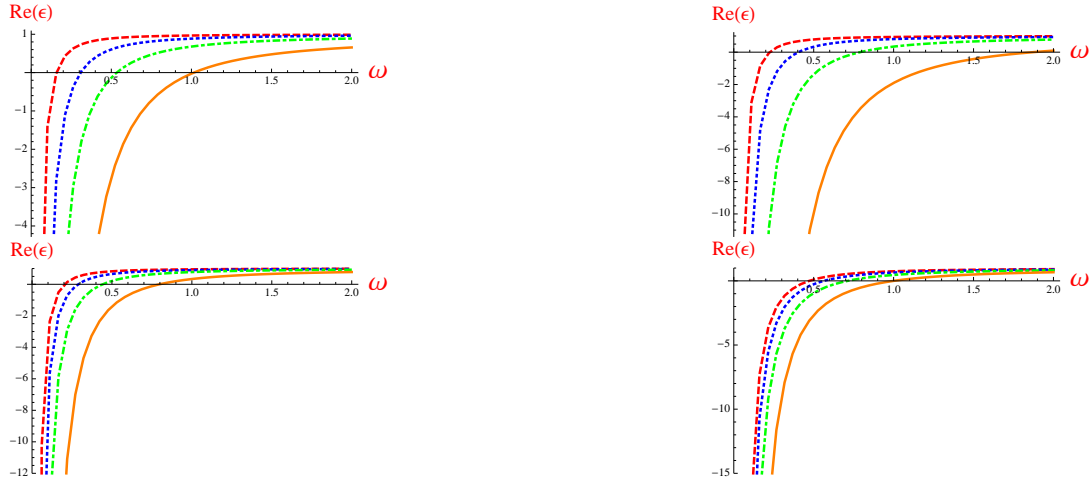


Figure 1: Real part of the permittivity as a function of  $\omega$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red) .75 (blue) .6 (green) and .45 (orange).

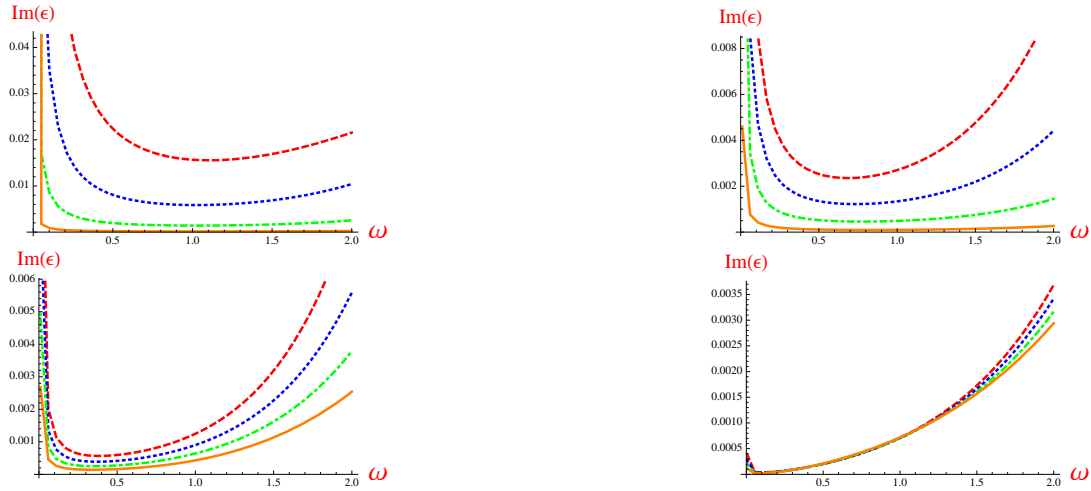


Figure 2: Imaginary part of the permittivity as a function of  $\omega$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red) .75 (blue) .6 (green) and .45 (orange).

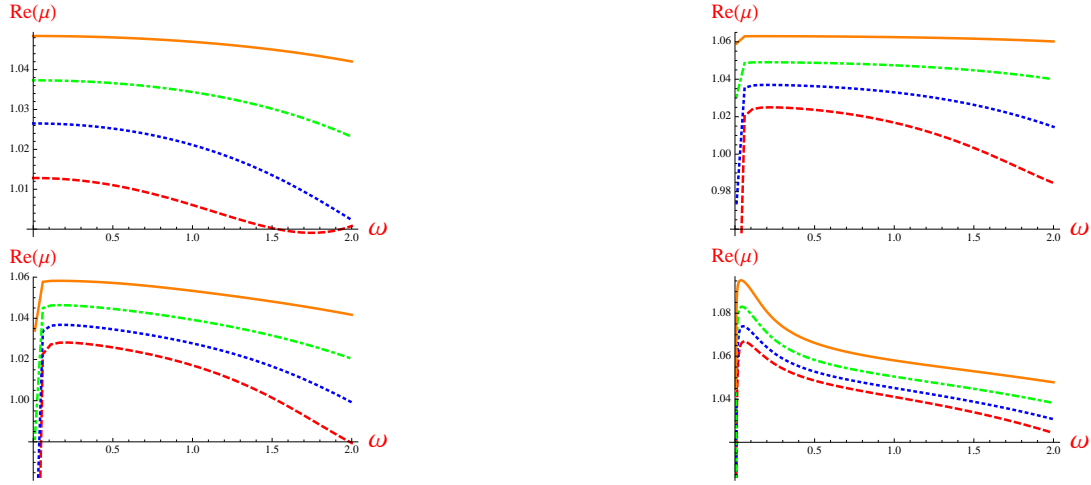


Figure 3: Real part of the permeability as a function of  $\omega$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red)  $.75$  (blue)  $.6$  (green) and  $.45$  (orange).

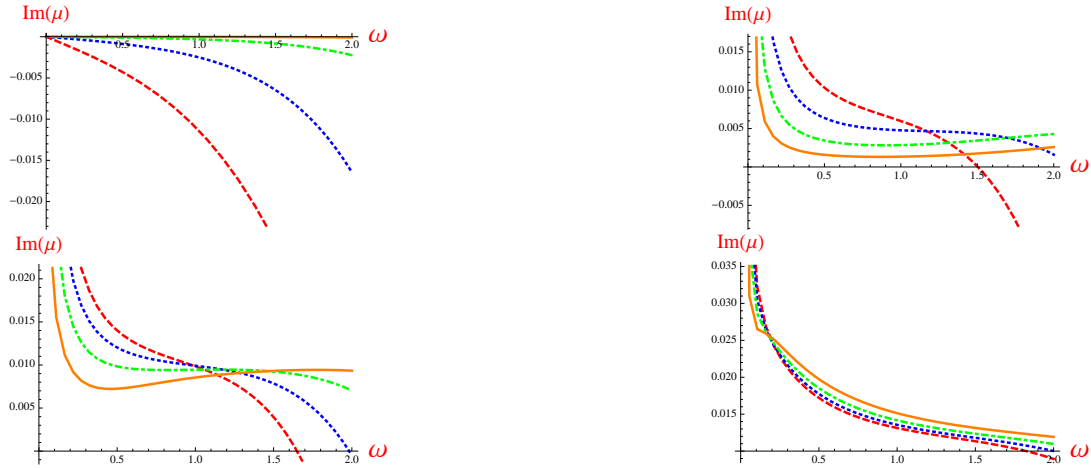


Figure 4: Imaginary part of the permeability as a function of  $\omega$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red)  $.75$  (blue)  $.6$  (green) and  $.45$  (orange).

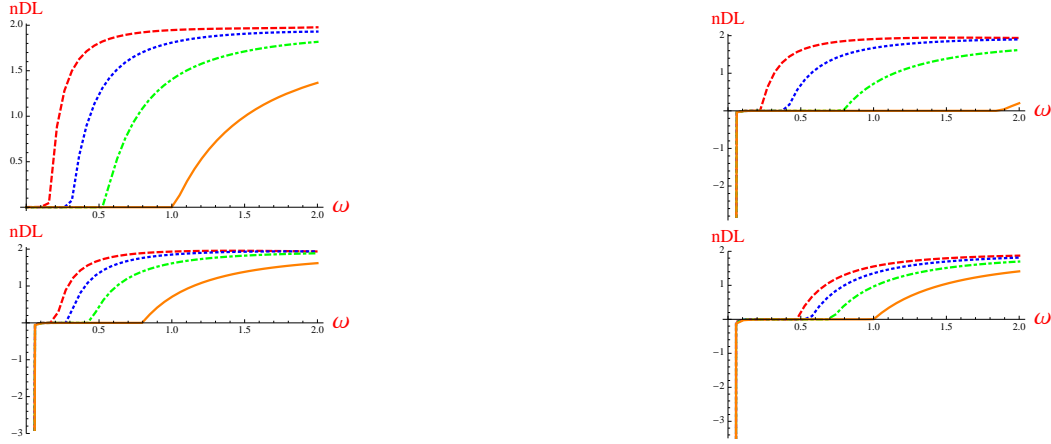


Figure 5: Index  $n_{DL}$  as a function of  $\omega$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red) .75 (blue) .6 (green) and .45 (orange).



Figure 6: Zoom of the index  $n_{DL}$  in the region of significant negative refraction (low  $\omega$ ) for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red) .75 (blue) .6 (green) and .45 (orange).

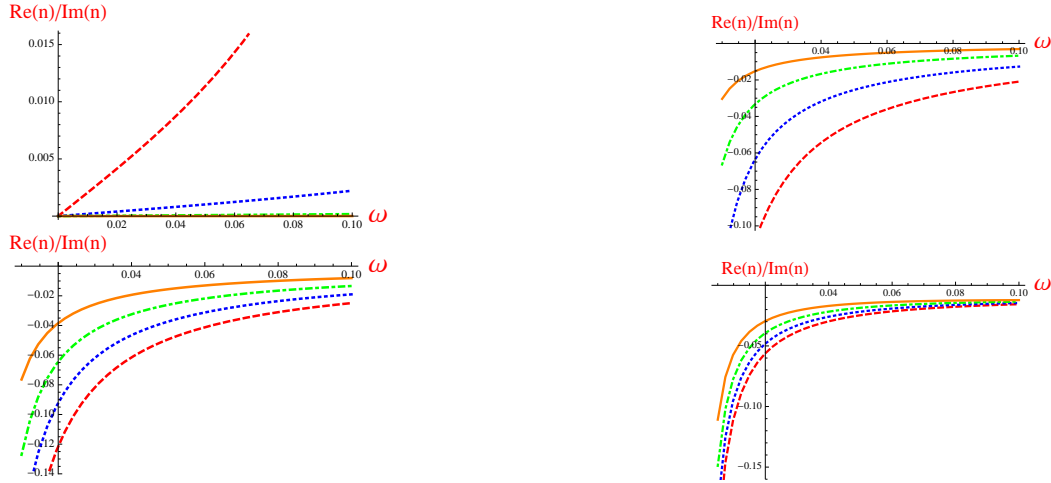


Figure 7: Ratio  $\text{Re}(n)/\text{Im}(n)$  for  $\kappa^2 = 10^{-10}, .1, .2, .3$  and for  $T/T_c = .9$  (red)  $.75$  (blue)  $.6$  (green) and  $.45$  (orange) in the low  $\mathbf{w}$  regime.

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