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Large spin expansion of the wrapping correction to Freyhult-Rej-Zieme twist operators

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ABSTRACT: Twist operators in the closed $\mathfrak{sl}(2)$ sector of planar $\mathcal{N} = 4$ SYM are characterized by their spin. The explicit dependence of anomalous dimensions on this important parameter is a source of interesting information. Wrapping corrections are a non trivial part of the calculation and are under control in the framework of thermodynamical Bethe Ansatz valid for the full theory and thoroughly checked in that sector. The extension to more general twist operators beyond $\mathfrak{sl}(2)$ has been recently accomplished for the so-called 3-gluon operators that are a special case of the generalized twist operators introduced by Freyhult, Rej and Zieme. Such operators are dual to spinning strings configurations with two spins S_1, S_2 in AdS_5 and charge in S^5. We compute the expansion of the weak-coupling leading order wrapping correction in the gauge theory limit dual to large S_1 and fixed S_2. We present a simple algorithm for the calculation and provide explicit results illustrating the general structure of the expansion.

Contents

1. Introduction

The computation of finite size corrections to states/operators in AdS/CFT correspondence is an important technical issue. Recently, in the integrable planar limit, this problem has been solved in full generality by means of the mirror thermodynamic Bethe Ansatz developed for the $AdS_5 \times S^5$ superstring in [1]. Formerly, the associated Y-system had been proposed in [2] based on symmetry arguments and educated guesses about the analyticity and asymptotic properties of the Y-functions. The predicted finite size corrections have been deeply tested in [3], mainly in the closed $\mathfrak{sl}(2)$ subsector. The relevant operators are represented by the insertion of n covariant derivatives $\mathscr{D}$ into the protected half-BPS state $\text{Tr}\mathcal{Z}^L$ ($\mathcal{Z}$ being one of the three complex scalars of $\mathcal{N} = 4$ SYM)

$$\mathbb{O}^{\mathcal{Z}}_{n,L} = \sum_{s_1,\dots,s_L} c_{s_1,\dots,s_L} \,\text{Tr}\,(\mathscr{D}^{s_1}\mathcal{Z}\cdots\mathscr{D}^{s_L}\mathcal{Z})\,, \quad \text{with} \quad n = s_1 + \cdots + s_L\,. \tag{1.1}$$

Their anomalous dimensions can be obtained from a non-compact, length-L $\mathfrak{sl}(2)$ invariant integrable spin chain with n excitations. The interaction range between scattering magnons increases with the perturbative order. As soon as it exceeds the length of the spin chain and *wraps* around it, the S-matrix picture fails and no asymptotic region can be defined any longer. For length L operators this effect is delayed by superconformal invariance and starts at order g^{2L+4}. In this regime, wrapping corrections cannot be obtained within the asymptotic Bethe Ansatz and require the full use of the thermodynamical Bethe Ansatz framework. The most accurate available calculations are at five-loop order for the special length $L = 2$ [4] and at six-loop order for $L = 3$ [5]. In these cases, the minimal anomalous dimension of $\mathbb{O}^{\mathcal{Z}}_{n,L}$ can be obtained in closed form as a function of the number of excitations n.

The availability of n as a control parameter is a remarkable fact since it opens the door to very interesting cross checks of the calculations. For instance, at large n, it is found that a generalized Gribov-Lipatov reciprocity (see [6] and the recent review [7]) holds predicting half of the large n expansion in terms of the other half. Also, in the twist-2 case, the analytic continuation in the spin parameter n allows to test the predictions of the BFKL equation [8] governing the poles around unphysical negative values of n.

Apart from these important tests, the computation of wrapping corrections as functions of (or series expansions in) the parameter n is also very useful in order to predict general features. For instance, a recurring theme in AdS/CFT is the assumption that wrapping corrections are somewhat suppressed at large n [1]. This permits, in first approximation, to neglect them. A nice example where such an approximation is needed is the computation of Maldacena *et al.* of the two loop expressions for polygonal Wilson loops expectation values [9] . It is based on an operator product expansion where the spectrum of excitations of the flux tube stretching between two null Wilson lines can also be viewed as the spectrum of excitations around the infinite spin limit of finite twist operators in the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM , or the GKP [10] string. Integrability and AdS/CFT correspondence effectively help in computing such spectrum and wrapping corrections are assumed to be negligible. Such a statement is safe for the GKP background, but is only a conjecture (although reasonable) for the excitations over the GKP string.

Thus, generally speaking, it is clearly important to increase our knowledge of the structure of wrapping corrections to twist operators beyond the simple $\mathfrak{sl}(2)$ sector [2]. Such an extension has been recently presented in [11] where we studied wrapping corrections to operators whose multi-loop asymptotic contributions had been computed in [12]. We shall refer to these operators as 3-gluon operators [3]. Indeed, in [11] we computed the leading order wrapping correction to the lowest anomalous dimension of such operators in closed form as a function of n.

Actually, 3-gluon operators are related by superconformal invariance to a special case of a larger family studied in [13] which we shall dub Freyhult-Rej-Zieme (FRZ) twist operators. For the length 3 case we are interested in, they take the following (schematic)

[1] In all known cases they scale like $\frac{1}{n^2}$ with possible enhancement factors growing like powers of $\log n$.

[2] Here, the loose term *twist operator* refers to gauge invariant composite operators built with a fixed number of elementary fields and an increasing number of covariant derivatives acting on them.

[3] At one-loop they have the same form as $\mathfrak{sl}(2)$ operators, with the scalar $\mathcal{Z}$ being replaced by a physical gauge field component.

form

$$\mathbb{O}^{\mathrm{FRZ}}_{n,m} = \mathrm{Tr}\,(\mathscr{D}^{n+m}\bar{\mathscr{D}}^{m}\mathcal{Z}^{3}) + \cdots\,, \tag{1.2}$$

where dots denote a linear combination of similar terms with the covariant derivatives spread over the scalar fields. These operators reduce to length 3 operators in the $\mathfrak{sl}(2)$ subsector for $m = 0$. For $m = 1$ we get descendants of twist–2 operators. For $m = 2$ we get the 3-gluon operators. At strong coupling, the FRZ operators are duals to minimal energy spinning strings configurations with two spins S_1 and S_2 in AdS_5 and charge J in S^5 given by

$$S_1 = n + m - \frac{1}{2}\,, \qquad S_2 = m - \frac{1}{2}\,, \qquad J = L = 3. \tag{1.3}$$

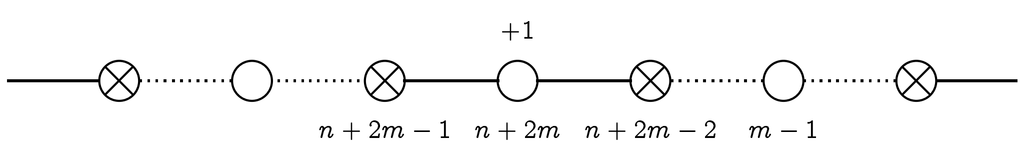
The main result of [13] is the large n expansion of the asymptotic minimal anomalous dimension of $\mathbb{O}^{\mathrm{FRZ}}_{n,m}$ for fixed ratio n/m or fixed m. The expansion is obtained at all orders in the coupling and including the leading term $\sim \log n$ as well as the subleading asymptotically constant correction $\sim n^0$. These two contributions are expected to be free of wrapping corrections. In this paper, we consider precisely the leading order wrapping correction which appears at four loops. We provide an algorithm to compute its large n expansion for fixed m and present explicit results for $m = 2, 3, 4$. As we mentioned, for $m = 2$ we have to match the 3-gluon result obtained in [11]. The expansions for the other two values are new. In full generality, we prove the $\frac{\log n}{n^2}$ scaling behaviour at large n thus confirming the assumption in [13]. Notice that for the considered states with $m > 2$ no asymptotic closed form of the anomalous dimension is known beyond one-loop.

The plan of the paper is the following. In Sec. (2), we summarize the one-loop solution of the Bethe Ansatz equations for FRZ operators. Sec. (3) presents the necessary Y-system formulae for the efficient computation of the leading order wrapping correction. The algorithm for the derivation of its large spin expansion is described and tested in Sec. (4). Our results are summarized in Sec. (5) .

2. One loop solution of the FRZ operators

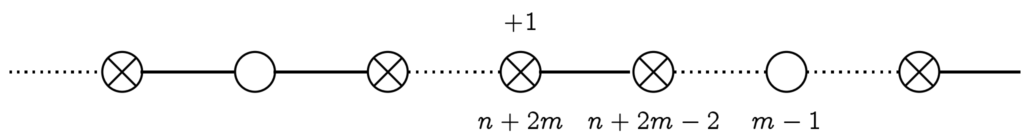
In this section, we review the one-loop solution of the FRZ states and give explicit information on the Baxter polynomials entering the wrapping calculation. In particular, we shall provide the explicit form of the Baxter polynomials whose degree is independent on the spin n.

The excitation pattern of the operators in Eq. (1.2) has the following form in the higher Dynkin diagram of $\mathfrak{psu}(2,2|4)$ in the $\mathfrak{su}(2)$ grading



(2.1)

By dualizing the diagram on node 1, 3 we arrive at the simpler configuration



(2.2)

The Bethe equations in this grading (2.2) are [4]

$$\left(\frac{u_{4,k}^{+}}{u_{4,k}^{-}}\right)^3 = \frac{Q_5^-}{Q_5^+}\bigg|_{u_{4,k}}, \qquad 1 = \frac{Q_6^+}{Q_6^-}\bigg|_{u_{5,k}} \frac{Q_4^-}{Q_4^+}\bigg|_{u_{5,k}}, \qquad -1 = \frac{Q_6^{++}}{Q_6^{--}}\bigg|_{u_{6,k}} \frac{Q_5^-}{Q_5^+}\bigg|_{u_{6,k}}. \tag{2.5}$$

The solution to the system (2.5) is explicitly written out in Appendix A where we provide the expressions of the polynomials $Q_{4,5,6}$. Since the degree of $Q_{4,5}$ is dependent on n, the computation of the large n limit is definitely non - trivial.

Although the definition of Q_6 is rather complicated, it is a polynomial of order $m-1$ whose coefficients depend on n. We can reconstruct them explicitly for general values of n at least for the first values of m. We do this for $m = 2, 3, \dots, 8$. The polynomials are

$$\begin{aligned}
Q_6^{m=2} &= u, \\
Q_6^{m=3} &= 4(n+3)(n+8)u^2 + 32 + 11n + n^2, \\
Q_6^{m=4} &= (132 + 4n(n+14))\, u^3 + (213 + 4n(n+14))\, u, \\
Q_6^{m=5} &= 16(n+3)(n+5)(n+12)(n+14)u^4 \\
&\quad +8(n+5)(n+12)(402 + 5n(n+17))u^2 \\
&\quad +47256 + 3n(n+17)(434 + 3n(n+17)), \\
Q_6^{m=6} &= 16(n+3)(n+5)(n+15)(n+17)u^5 \\
&\quad +(681000 + 40n(n+20)(377 + 2n(n+20)))u^3 \\
&\quad +(670425 + 8n(n+20)(1633 + 8n(n+20)))u, \\
Q_6^{m=7} &= 64(n+3)(n+5)(n+7)(n+16)(n+18)(n+20)u^6 \\
&\quad +80(n+5)(n+7)(n+16)(n+18)(7n(n+23) + 1068)u^4 \\
&\quad +4(n+7)(n+16)(n(n+23)(259n(n+23) + 71250) + 4936680)u^2 \\
&\quad +45(n(n+23)(n(n+23)(5n(n+23) + 1958) + 255720) + 11140992), \\
Q_6^{m=8} &= 64(n+3)(n+5)(n+7)(n+19)(n+21)(n+23)u^7
\end{aligned} \tag{2.6}$$

[4]Shifted quantities are defined as

$$F^{\overbrace{\pm\cdots\pm}^{a}}(u) = F^{[\pm a]}(u) = F\left(u \pm i\,\frac{a}{2}\right). \tag{2.3}$$

Q_ℓ's are the Baxter polynomials vanishing on the ℓ-th node roots

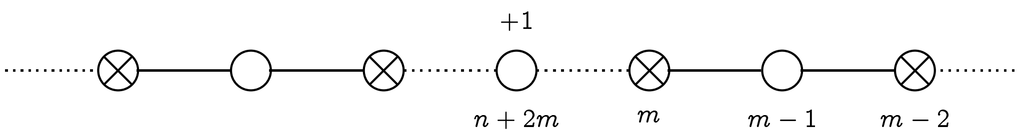
$$Q_\ell(u) = \prod_{i=1}^{K_\ell} (u - u_{\ell,i}), \tag{2.4}$$

K_ℓ being the number of excitations on the ℓ-th node.

$$+112(n+5)(n+7)(n+19)(n+21)(8n(n+26)+1581)u^5$$
$$+196(n+7)(n+19)(4n(n+26)(4n(n+26)+1431)+516495)u^3$$
$$+3(4n(n+26)(16n(n+26)(12n(n+26)+6085)+16477937)+3724800415)u.$$

These results will be useful in the following since they are explicit in n and can be used to extract large n contributions.

In order to efficiently evaluate wrapping corrections it is convenient to dualize the diagram (2.2) at nodes 5 and 7. We get a configuration where the number of roots at nodes 5, 6, 7 does not depend anymore on n. More precisely, we get



(2.7)

This is the direct extension of eq. (3.13) in [11]. An important difference are the $m-2$ roots appearing on node 7.

The one–loop Bethe equations are now

$$-\left(\frac{u_{4,k}^{+}}{u_{4,k}^{-}}\right)^3=\frac{Q_4^{--}}{Q_4^{++}}\bigg|_{u_{4,k}}\frac{\widetilde{Q}_5^{+}}{\widetilde{Q}_5^{-}}\bigg|_{u_{4,k}},\quad 1=\frac{Q_6^{+}}{Q_6^{-}}\bigg|_{u_{5,k}}\frac{Q_4^{-}}{Q_4^{+}}\bigg|_{\widetilde{u}_{5,k}},\quad 1=\frac{\widetilde{Q}_5^{-}}{\widetilde{Q}_5^{+}}\bigg|_{u_{6,k}},\quad 1=\frac{Q_6^{-}}{Q_6^{+}}\bigg|_{\widetilde{u}_{7,k}}, \tag{2.8}$$

where the dual Baxter functions $\widetilde{Q}_5$, $\widetilde{Q}_7$ are defined by

$$\widetilde{Q}_5 Q_5 = Q_4^{+}Q_6^{-}-Q_4^{-}Q_6^{+},\qquad\qquad \widetilde{Q}_7=Q_6^{+}-Q_6^{-}. \tag{2.9}$$

They are polynomials of order m and $m-2$ respectively. Explicitly, for $m=2,3,\ldots,8$ they read

$$\widetilde{Q}_5^{m=2}=(n+3)(n+5)u^2+\frac{1}{4}(n+4)^2,$$
$$\widetilde{Q}_5^{m=3}=\frac{1}{2}\Big((n+3)(n+8)u^3+(n+5)(n+6)u\Big),$$
$$\widetilde{Q}_5^{m=4}=\frac{1}{16}\Big(+16(n+3)(n+5)(n+9)(n+11)u^4+8(n+5)(n+9)(246+5n(n+14)u^2$$
$$+9(n+6)^2(n+8)^2\Big),$$
$$\widetilde{Q}_5^{m=5}=\frac{1}{4}\Big((n+3)(n+5)(n+12)(n+14)u^5+5(n+5)(n+12)(74+n(n+17))u^3$$
$$+4(n+7)(n+8)(n+9)(n+10)u),$$
$$\widetilde{Q}_5^{m=6}=\frac{1}{64}\Big(64(n+3)(n+5)(n+7)(n+13)(n+15)(n+17)u^6$$
$$+80(n+5)(n+7)(n+13)(n+15)(732+7n(n+20))u^4$$
$$+4(n+7)(n+13)(2548800+n(n+20)(51384+259n(n+20)))u^2$$
$$+225(n+8)^2(n+10)^2(n+12)^2\Big),$$

$$\begin{aligned}
\tilde{Q}_5^{m=7} = \frac{1}{8}\Big(&(n+3)(n+5)(n+7)(n+16)(n+18)(n+20)u^7\\
&+14(n+5)(n+7)(n+16)(n+18)(n(n+23)+141)u^5\\
&+7(n+7)(n+16)(n(n+23)(7n(n+23)+1860)+123660)u^3\\
&+36(n+9)(n+10)(n+11)(n+12)(n+13)(n+14)u\,, \qquad (2.10)\\
\tilde{Q}_5^{m=8} = \frac{1}{256}\Big(&256(n+3)(n+5)(n+7)(n+9)(n+17)(n+19)(n+21)(n+23)u^8\\
&+1792(n+5)(n+7)(n+9)(n+17)(n+19)(n+21)(3n(n+26)+550)u^6\\
&+224(n+7)(n+9)(n+17)(n+19)(n(n+26)(141n(n+26)+48544)\\
&\qquad\qquad +4185720)u^4\\
&+16(n+9)(n+17)(n(n+26)(n(n+26)(3229n(n+26)+1613162)\\
&\qquad\qquad +268631440)+14910974400)u^2\\
&+11025(n+10)^2(n+12)^2(n+14)^2(n+16)^2\Big)\,.
\end{aligned}$$

For $\tilde{Q}_7$ we have

$$\begin{aligned}
\tilde{Q}_7^{m=2} &= 1,\\
\tilde{Q}_7^{m=3} &= u,\\
\tilde{Q}_7^{m=4} &= n^2+4(n+3)(n+11)u^2+14n+60,\\
\tilde{Q}_7^{m=5} &= (n+3)(n+14)u^3+(n(n+17)+90)u,\\
\tilde{Q}_7^{m=6} &= 16(n+3)(n+5)(n+15)(n+17)u^4\\
&\quad +40(n+5)(n+15)(n(n+20)+126)u^2\\
&\quad +3n(n+20)(3n(n+20)+628)+100800,\\
\tilde{Q}_7^{m=7} &= (n+3)(n+5)(n+18)(n+20)u^5 \qquad (2.11)\\
&\quad +5(n+5)(n+18)(n(n+23)+168)u^3\\
&\quad +4(n(n+23)(n(n+23)+285)+20790)u,\\
\tilde{Q}_7^{m=8} &= 64(n+3)(n+5)(n+7)(n+19)(n+21)(n+23)u^6\\
&\quad +560(n+5)(n+7)(n+19)(n+21)(n(n+26)+216)u^4\\
&\quad +28(n+7)(n+19)(n(n+26)(37n(n+26)+13788)+1315440)u^2\\
&\quad +45(n(n+26)(n(n+26)(5n(n+26)+2564)+439712)+25276160).
\end{aligned}$$

3. Explicit formulae for the leading order wrapping correction

The Y-system is a set of functional equations for the functions $Y_{a,s}(u)$ defined on the fat-hook diagram associated with $\mathfrak{psu}(2,2|4)$ which is a suitable (a,s) grid described in details in [2, 14]. The anomalous dimension of a generic state is given by the TBA formula

$$E = \underbrace{\sum_i \epsilon_1(u_{4,i})}_{\text{asymptotic}} + \underbrace{\sum_{a\geq 1}\int_{\mathbb{R}} \frac{du}{2\pi i}\frac{\partial \epsilon_a^\star}{\partial u}\,\log(1+Y_{a,0}^\star(u))}_{\text{wrapping W}}\,. \qquad (3.1)$$

In this formula, the dispersion relation is

$$\epsilon_a(u) = a + \frac{2\,i\,g}{x^{[a]}} - \frac{2\,i\,g}{x^{[-a]}}, \tag{3.2}$$

and the star means evaluation in the mirror kinematics [5]. The first term in E is the sum of asymptotic one-magnon energies and is the so-called asymptotic contribution to the anomalous dimension. The second term is the wrapping correction. The Bethe roots $\{u_{4,i}\}$ are fixed by the exact Bethe equations (in physical kinematics) $Y_{1,0}(u_4) = -1$. Any solution of the Y-system can be written in terms of a solution of the Hirota integrable discrete equation. For large L, (or small g) it can be shown that the Hirota equation splits in two $\mathfrak{su}(2|2)_{\mathrm{L,R}}$ wings. One can have a simultaneous finite large L limit on both wings after a suitable gauge transformation of the Hirota solution. Thus, we have

$$Y_{a,0}(u) \simeq \left(\frac{x^{[-a]}}{x^{[+a]}}\right)^L \frac{\Phi^{[-a]}}{\Phi^{[+a]}}\,T^{\mathrm{L}}_{a,1}\,T^{\mathrm{R}}_{a,1}, \tag{3.5}$$

where Φ is an arbitrary function and $T^{\mathrm{L,R}}_{a,1}$ are transfer matrices of the antisymmetric rectangular representations of $\mathfrak{su}(2|2)_{\mathrm{L,R}}$. They are given explicitly by the generating functional

$$\begin{aligned}\sum_{a=0}^{\infty}(-1)^a\,T^{[1-a]}_{a,1}\,\overline{\mathscr{D}}^a &= \left(1-\frac{Q_3^+}{Q_3^-}\,\overline{\mathscr{D}}\right)^{-1}\left(1-\frac{Q_3^+}{Q_3^-}\frac{Q_2^{--}}{Q_2}\frac{R^{(+)-}}{R^{(-)-}}\,\overline{\mathscr{D}}\right) \\ &\times\left(1-\frac{Q_2^{++}}{Q_2}\frac{Q_1^-}{Q_1^+}\frac{R^{(+)-}}{R^{(-)-}}\,\overline{\mathscr{D}}\right)\left(1-\frac{Q_1^-}{Q_1^+}\frac{B^{(+)+}}{B^{(-)+}}\frac{R^{(+)-}}{R^{(-)-}}\overline{\mathscr{D}}\right)^{-1},\end{aligned} \tag{3.6}$$

where $\overline{\mathscr{D}} = e^{-i\partial_u}$ and

$$R^{(\pm)} = \prod_{i=1}^{K_4}\frac{x(u)-x^{\mp}_{4,i}}{(x^{\mp}_{4,i})^{1/2}}, \qquad B^{(\pm)} = \prod_{i=1}^{K_4}\frac{\frac{1}{x(u)}-x^{\mp}_{4,i}}{(x^{\mp}_{4,i})^{1/2}}. \tag{3.7}$$

The function Φ has been determined in [2] and reads

$$\frac{\Phi^-}{\Phi^+} = \sigma^2\,\frac{B^{(+)+}\,R^{(-)-}}{B^{(-)-}\,R^{(+)+}}\,\frac{B^+_{1,\mathrm{L}}\,B^-_{3,\mathrm{L}}}{B^-_{1,\mathrm{L}}\,B^+_{3,\mathrm{L}}}\,\frac{B^+_{1,\mathrm{R}}\,B^-_{3,\mathrm{R}}}{B^-_{1,\mathrm{R}}\,B^+_{3,\mathrm{R}}}, \tag{3.8}$$

where σ is the dressing phase . At weak coupling, evaluating the various terms at leading order in the mirror dynamics, the wrapping correction (second term in the r.h.s. of (3.1)) is simply given by the expression

$$W = -\frac{1}{\pi}\sum_{a=1}^{\infty}\int_{\mathbb{R}} du\,Y^{\star}_{a,0}. \tag{3.9}$$

[5]We recall that the physical and mirror branches of the Zhukowsky relation

$$x + \frac{1}{x} = \frac{u}{g}, \tag{3.3}$$

are

$$x_{\mathrm{ph}}(u) = \frac{1}{2}\left(\frac{u}{g}+\sqrt{\frac{u}{g}-2}\sqrt{\frac{u}{g}+2}\right), \qquad x_{\mathrm{mir}}(u) = \frac{1}{2}\left(\frac{u}{g}+i\sqrt{4-\frac{u^2}{g^2}}\right). \tag{3.4}$$

3.1 Explicit formulae for the computation of $Y^\star_{a,0}$

In the following, we shall need a compact efficient formula for the evaluation of $Y^\star_{a,0}$. According to (3.5), we need the contribution from the dispersion (ratio of $x^\pm$), the fusion of scalar factors (Φ terms), and the $\mathfrak{su}(2|2)$ transfer matrices. After a straightforward computation we obtain:

Transfer matrices

Using the relations, valid at leading order in the coupling constant

$$\begin{aligned} \frac{R^{(+)}}{R^{(-)}} &= \frac{Q_4^{[+]}}{Q_4^{[-]}}\left(1 + g^2\frac{ic}{u} + \mathcal{O}(g^4)\right), \\ \frac{B^{(+)}}{B^{(-)}} &= \left(1 - g^2\frac{ic}{u} + \mathcal{O}(g^4)\right), \\ c &= \sum_j \frac{1}{u^+_{4,j}u^-_{4,j}} = i\left(\log\left(Q_4\right)\right)'\Big|_{u=-\frac{i}{2}}^{u=+\frac{i}{2}}. \end{aligned} \tag{3.10}$$

we get the following expression for the transfer matrices $T^*_{a,1}$ in mirror dynamics:

$$T^*_{a,1} = (-1)^{a+1}\frac{Q_5^{[a]}Q_7^{[-a]}}{Q_4^{[1-a]}}\sum_{\substack{k=1-a\\ \Delta k=2}}^{a-1}\frac{Q_4^{[k]}}{Q_6^{[k]}}\left(\frac{Q_6^{[k+2]}-Q_6^{[k]}}{Q_5^{[k+1]}Q_7^{[k+1]}} + \frac{Q_6^{[k-2]}-Q_6^{[k]}}{Q_5^{[k-1]}Q_7^{[k-1]}}\right) + \mathcal{O}(g^2). \tag{3.11}$$

We remark that this expression is valid for **any distributions of roots** on Dynkin diagrams like the one of the right wing of picture (2.7). So, for example, taking the expression for $\widetilde{Q}_{5,m=2}$, $Q_{6,m=2}$, $\widetilde{Q}_{7,m=2}$ from equations (2.10), (2.6) and (2.11), we get back to eq. (4.22) of [11]. If Q_6 is trivial, i.e. $Q_6 = 1$, formula (3.11) shows that the transfer matrix is $\mathcal{O}(g^2)$.

To compute the wrapping corrections we can use formula (3.11) for the transfer matrices of the right wing of the diagram (2.7), while for the $\mathcal{O}(g^2)$ left wing we use

$$T^{*,L}_{a,1} = icg^2\frac{(-1)^{a+1}}{Q_4^{[1-a]}}\sum_{\substack{k=-a\\ \Delta k=2}}^{a}\frac{Q_4^{[-1-k]}-Q_4^{[1-k]}}{u-i\frac{k}{2}}\Bigg|_{Q_4^{[-1-a]},Q_4^{[-1-a]}\to 0} + \mathcal{O}(g^4). \tag{3.12}$$

Dispersion relation

This is the universal factor

$$\left(\frac{4g^2}{a^2+4u^2}\right)^3. \tag{3.13}$$

Fusion scalar factor

From the relation

$$\frac{\Phi^-}{\Phi^+} = \sigma^2\,\frac{B^{(+)+}R^{(-)-}}{B^{(-)-}R^{(+)+}}\,\frac{B_1^+B_3^-}{B_1^-B_3^+}\,\frac{B_7^+B_5^-}{B_7^-B_5^+}, \tag{3.14}$$

the following formula follows

$$\Phi_a^* = \left[Q_4^+(0)\right]^2 \frac{Q_4^{[1-a]}}{Q_4^{[-1-a]}Q_4^{[a-1]}Q_4^{[a+1]}} \frac{Q_5^{[-a]}}{Q_5(0)} \frac{Q_7(0)}{Q_7^{[-a]}}. \tag{3.15}$$

This formula is valid for even Q_4, Q_5 and Q_7, i.e. for even values of n and m. For m odd, the ratio Q_7/Q_5 is indeterminate at $u = 0$, but has a smooth limit for $u \to 0$.

4. Large n expansion: The algorithm

The wrapping correction can be computed by summing the residues of the Y_a–functions at $u = \frac{ia}{2}$. The precise relation is

$$W = -\frac{1}{\pi}\sum_{a=1}^{\infty}\int_{\mathbb{R}} du\, Y_{a,0}^{\star} = -2i\sum_{a=1}^{\infty}\operatorname*{Res}_{u=i\frac{a}{2}} Y_{a,0}^{\star}. \tag{4.1}$$

The physical reason of this property, that we explicitly checked for all the cases we are interested in, is presumably the same as in the Konishi case discussed in [15]. The pole at $u = \frac{ia}{2}$ is of kinematical origin and does not depend on the scattering matrix. Instead, other poles are determined by the dynamics and correspond to μ terms in the Lüscher approach to wrapping corrections. It is expected that such terms are absent in the weakly coupled limit [16].

Since we are interested in the large spin limit of W, we can attempt to exchange this limit with the sum over the intermediate virtual states in the r.h.s of Eq. (4.1). This possibility is supported by the fact that the large n structure perfectly matches the exact result in all known cases in the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM theory, in its β-deformed version and in ABJM theory as shown in [17].

In practice, one evaluates the above residue at fixed $a = 1, 2, \ldots$ without assigning n and then taking the limit over it in two steps: The dependence on n, in fact, comes from the polynomials Q_4, its derivatives (which are written in terms of the basic hypergeometric function $F_{n,m}$ defined in (A.2)) and from the explicit n-dependent coefficients of the other Baxter polynomials. At this point one can use the Baxter equation to shift the argument of $F_{n,m}$ to some minimal value and take the large n limit on the coefficients. This gives a first expansion containing various derivatives of the logarithm of $F_{n,m}$ which in turn can be systematically computed as explained in Appendix B or by means of the method explained in [18]. The outcome of this procedure are sequences of rational numbers being the a-dependent coefficients of the large n expansion of $\mathrm{Res}_{u=i\frac{a}{2}}\, Y_{a,0}^{\star}$. These sequences turn out to be rather simple rational functions which are easily identified and summed over a.

In the following, we first apply this strategy to the case $m = 2$ reproducing the known results for 3-gluon operators. Then, we move to unexplored cases $m > 2$ for which we provide new asymptotic expansions for the wrapping correction.

4.1 $m = 2$, checking 3-gluon operators

Let $L(u)$ be the logarithm of the basic hypergeometric function

$$L(u) = \log F_{n,m=2}(u), \tag{4.2}$$

and R_a be the residue

$$g^8\, R_a(n) = \operatorname*{Res}_{u=\frac{ia}{2}} Y^{\star}_{a,0}(u). \tag{4.3}$$

We find the following explicit results for the first residues expanded at first order for large $n \equiv 1/\epsilon$

$$\begin{aligned} R_1 &= \frac{160}{81}\epsilon^2 \left(L'\left(\frac{i}{2}\right) - 5i\right)\left(76 + 9L''\left(\frac{i}{2}\right)\right) + O\left(\epsilon^3\right), \\ R_{a\geq 2} &= 32\epsilon^2 \left(L'\left(\frac{i}{2}\right) - 5i\right)\left(f_1\left(a\right) + f_2\left(a\right) L''\left(\frac{i}{2}\right)\right) + O\left(\epsilon^3\right). \end{aligned} \tag{4.4}$$

Notice that the whole dependence on the spin n is inside the derivatives of $L(u)$ evaluated at special points. Instead, the dependence on the label of intermediate virtual states a is in the coefficient functions $f_{1,2}(a)$. In principle these functions could be very non trivial. In our case, we find that they are rather simple rational functions precisely as in other cases analyzed in [17]. In particular, we find

$$\begin{aligned} f_1(a) &= \frac{\left(192a^{10} - 960a^9 + 2640a^8 - 4800a^7 + 4916a^6 - 1980a^5 - 405a^4 + 430a^3 + 12a^2 - 45a + 9\right)}{(a-1)^3\, a^3\, (2a-3)^3(2a-1)^3(2a+1)^3}, \\ f_2(a) &= \frac{1}{(a-1)\, a\, (2a-3)(2a-1)(2a+1)}. \end{aligned} \tag{4.5}$$

The derivatives $L^{(n)}\left(\frac{i}{2}\right)$ are computed in Appendix A. Summing over a and in terms of $\overline{n} \equiv \frac{1}{2}\, e^{\gamma_E}\, n$ (γ_E is the Euler number), we find

$$\sum_{a=1}^{\infty} R_a = -\frac{256\, i}{3}(3\zeta_3 - 1)\, \frac{\log \overline{n} + 1}{n^2} + \mathcal{O}\left(\frac{\log \bar{n}}{n^3}\right).$$

This result is in perfect agreement with the large n expansion of the results of [11]. This provides the validity of the computational method we are using. We now apply this same method to cases with $m > 2$.

4.2 $m = 3$

As in the previous case, let $L(u)$ be the logarithm of the basic hypergeometric function

$$L(u) = \log F_{n,m=3}(u), \tag{4.6}$$

and R_a be the residue

$$g^8\, R_a(n) = \operatorname*{Res}_{u=\frac{ia}{2}} Y^{\star}_{a,0}(u). \tag{4.7}$$

We find the following explicit results for the first three residues expanded at large $n \equiv 1/\epsilon$

$$
\begin{aligned}
R_1 &= \frac{9}{2}\epsilon^2 \left(2L'(i) - 11i\right) \left(6 + L''(i)\right) - \frac{99}{2}\epsilon^3 \left(\left(2L'(i) - 11i\right) \left(6 + L''(i)\right)\right) + \\
&+ \frac{3}{8}\epsilon^4 \left(-11624 i L''(i) + 12590 L'(i) + 2096 L'(i) L''(i) - 69821 i\right) + O\left(\epsilon^5\right), \qquad (4.8)\\
R_2 &= \frac{1}{36}\epsilon^2 \left(2L'(i) - 11i\right) \left(191 + 18L''(i)\right) - \frac{11}{36}\epsilon^3 \left(\left(2L'(i) - 11i\right) \left(191 + 18L''(i)\right)\right) + \\
&+ \frac{1}{216}\epsilon^4 \left(-84816 i L''(i) + 177886 L'(i) + 15264 L'(i) L''(i) - 987541 i\right) + O\left(\epsilon^5\right), \\
R_3 &= -\frac{35}{288}\epsilon^2 \left(2L'(i) - 11i\right) + \frac{385}{288}\epsilon^3 \left(2L'(i) - 11i\right) + \\
&+ \frac{1}{576}\epsilon^4 \left(-8448 i L''(i) - 1758 L'(i) + 1536 L'(i) L''(i) + 10229 i\right) + O\left(\epsilon^5\right).
\end{aligned}
$$

For $a \geq 4$, we find instead

$$
\begin{aligned}
R_{a\geq 4} &= \left(\epsilon^2 - 11\,\epsilon^3\right) \left[L'(i) - \frac{11 i}{2}\right] f_1(a) + \qquad (4.9)\\
&+ \epsilon^4 [L'(i)\, f_2(a) + f_3(a)] + \mathcal{O}(\epsilon^5), \qquad (4.10)
\end{aligned}
$$

where again we can match the functions f's with rational functions

$$
\begin{aligned}
f_1(a) &= -\frac{12(2a-1)\left(3a^2 - 3a - 4\right)}{(a-2)^2(a-1)^3 a^3 (a+1)^2}, \qquad (4.11)\\
f_2(a) &= -\frac{12\left(20a^8 + 426a^7 - 4399a^6 + 12288a^5 - 6156a^4 - 19036a^3 + 17131a^2 + 8538a - 6108\right)}{(a-3)^2(a-2)^3(a-1)^3 a^3 (a+1)^3}, \\
f_3(a) &= \frac{6i\left(220a^8 + 4734a^7 - 48797a^6 + 136256a^5 - 68196a^4 - 211220a^3 + 190033a^2 + 94734a - 67764\right)}{(a-3)^2(a-2)^3(a-1)^3 a^3 (a+1)^3}.
\end{aligned}
$$

The derivatives $L^{(n)}(i)$ are computed in Appendix A. Summing over a, we find the final result (setting now $\overline{n} \equiv e^{\gamma_E}\, n$)

$$
\begin{aligned}
&\sum_{a=1}^{\infty} R_a = -\frac{2\,i}{3}(2\log\overline{n} + 1)(-3 + 5\pi^2 + 36\zeta_3)\,\frac{1}{n^2} + \qquad (4.12)\\
&\frac{44\,i}{3}\,\log\overline{n}\,(-3 + 5\pi^2 + 36\zeta_3)\,\frac{1}{n^3} + \\
&\left[-\frac{4\,i}{9}(-615 + 1289\pi^2 + 9108\zeta_3)\,\log\overline{n} + \frac{i}{9}(-1527 + 2017\pi^2 + 14868\zeta_3)\right]\frac{1}{n^4} + \ldots
\end{aligned}
$$

A comparison with a numerical estimate of the (imaginary part of the) sum of the residues is shown in the following table

n	estimate	(LO	NLO	NNLO)	full expansion	diff %
10	-1.857411	$(-4.038730$	3.785374	$-2.548066)$	-2.801422	51%
30	-0.438781	$(-0.594614$	0.193683	$-0.045355)$	-0.446286	1.7%
50	-0.197270	$(-0.238478$	0.047207	$-0.006716)$	-0.197986	0.36%

(4.13)

4.3 $m = 4$

Again, let $L(u)$ be the logarithm of the basic hypergeometric function

$$L(u) = \log F_{n,m=4}(u), \tag{4.14}$$

and R_a be the residue

$$g^8\, R_a(n) = \operatorname*{Res}_{u=\frac{ia}{2}} Y^\star_{a,0}(u). \tag{4.15}$$

We find the following explicit results for the first two residues expanded at large $n \equiv 1/\epsilon$

$$\begin{aligned} R_1 = \; & \epsilon^2 \left(-\frac{9632}{225} i L''\left(\frac{i}{2}\right) + \frac{942928 L'\left(\frac{i}{2}\right)}{16875} + \frac{448}{75} L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) - \frac{20272952 i}{50625} \right) + \\ & + \epsilon^3 \left(\frac{134848}{225} i L''\left(\frac{i}{2}\right) - \frac{13200992 L'\left(\frac{i}{2}\right)}{16875} - \frac{6272}{75} L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) + \frac{283821328 i}{50625} \right) + \\ & + \epsilon^4 \left(-\frac{7420192 i L''\left(\frac{i}{2}\right)}{1125} + \frac{28672}{75} L'\left(\frac{i}{2}\right)^3 - \frac{917504}{225} i L'\left(\frac{i}{2}\right)^2 - \frac{328511984 L'\left(\frac{i}{2}\right)}{84375} + \right. \\ & \left. + \frac{288448}{375} L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) - \frac{12897148504 i}{253125} \right) + \mathcal{O}\left(\epsilon^5\right), \end{aligned} \tag{4.16}$$

$$\begin{aligned} R_2 = \; & \epsilon^2 \left(-\frac{688}{105} i L''\left(\frac{i}{2}\right) + \frac{4510928 L'\left(\frac{i}{2}\right)}{385875} + \frac{32}{35} L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) - \frac{96984952 i}{1157625} \right) + \\ & + \epsilon^3 \left(\frac{1376}{15} i L''\left(\frac{i}{2}\right) - \frac{9021856 L'\left(\frac{i}{2}\right)}{55125} - \frac{64}{5} L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) + \frac{193969904 i}{165375} \right) + \\ & + \epsilon^4 \left(-\frac{48512752 i L''\left(\frac{i}{2}\right)}{55125} + \frac{2048}{35} L'\left(\frac{i}{2}\right)^3 - \frac{65536}{105} i L'\left(\frac{i}{2}\right)^2 - \frac{10476726928 L'\left(\frac{i}{2}\right)}{28940625} + \right. \\ & \left. + \frac{1831328 L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right)}{18375} - \frac{872054313448 i}{86821875} \right) + \mathcal{O}\left(\epsilon^5\right). \end{aligned} \tag{4.17}$$

For $a \geq 3$, we find the same general structure

$$\begin{aligned} R_{a\geq 3} = \; & \epsilon^2 \left(f_{2,0}(a) + f_{2,1}(a)\, L'\left(\frac{i}{2}\right) + f_{2,2}(a)\, L''\left(\frac{i}{2}\right) + f_{2,12}(a)\, L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) \right) + \\ & + \epsilon^3 \left(f_{3,0}(a) + f_{3,1}(a)\, L'\left(\frac{i}{2}\right) + f_{3,2}(a)\, L''\left(\frac{i}{2}\right) + f_{3,12}(a)\, L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) \right) + \\ & + \epsilon^4 \left(f_{4,0}(a) + f_{4,1}(a)\, L'\left(\frac{i}{2}\right) + f_{4,11}(a)\, L'\left(\frac{i}{2}\right)^2 + f_{4,111}(a)\, L'\left(\frac{i}{2}\right)^3 + \right. \\ & \left. + f_{4,2}(a)\, L''\left(\frac{i}{2}\right) + f_{4,12}(a)\, L'\left(\frac{i}{2}\right) L''\left(\frac{i}{2}\right) \right) + \ldots, \end{aligned}$$

where

$$
\begin{aligned}
f_{2,0}(a) \;=\; & \frac{344\,i}{(a-1)^3a^3(2a-5)^3(2a-3)^3(2a-1)^3(2a+1)^3(2a+3)^3}\times \\
& \Big(15360\,a^{14} - 107520\,a^{13} + 302848\,a^{12} - 419328\,a^{11} - 61824\,a^{10} + \\
& 1590400\,a^{9} - 2375328\,a^{8} - 260736\,a^{7} + 3289132\,a^{6} - 2275140\,a^{5} - \\
& 26229\,a^{4} + 382662\,a^{3} - 20277\,a^{2} - 34020\,a + 8100\Big)\,,
\end{aligned}
\tag{4.18}
$$

$$
f_{2,1}(a) \;=\; \frac{6\,i}{43}\;f_{2,0}(a), \tag{4.19}
$$

$$
f_{2,2}(a) \;=\; \frac{1376\,i}{(a-1)a(2a-5)(2a-3)(2a-1)(2a+1)(2a+3)}, \tag{4.20}
$$

$$
f_{2,12}(a) \;=\; \frac{6\,i}{43}\;f_{2,2}(a), \tag{4.21}
$$

$$
f_{3,0}(a) \;=\; -14\;f_{2,0}(a), \qquad f_{3,1}(a) = \frac{6\,i}{43}\;f_{3,0}(a), \tag{4.22}
$$

$$
f_{3,2}(a) \;=\; -14\;f_{2,2}(a), \qquad f_{3,1,2}(a) = \frac{6\,i}{43}\;f_{3,2}(a), \tag{4.23}
$$

$$
\begin{aligned}
f_{4,0}(a) \;=\; & \frac{8i}{(a-2)^3(a-1)^3a^3(2a-7)^3(2a-5)^3(2a-3)^3(2a-1)^3(2a+1)^4(2a+3)^4}\times \\
& \Big(105676800\,a^{23} + 193789952\,a^{22} - 30633238528\,a^{21} + 374751035392\,a^{20} - \\
& 2338797209600\,a^{19} + 9042207928320\,a^{18} - 21933773253120\,a^{17} + \\
& 25092446111744\,a^{16} + 32261701882496\,a^{15} - 181183264537856\,a^{14} + \\
& 259510085417632\,a^{13} + 62844522906624\,a^{12} - 666497213030424\,a^{11} + \\
& 681006561227024\,a^{10} + 220531002799826\,a^{9} - 879621870442904\,a^{8} + \\
& 464871876325207\,a^{7} + 160461344177928\,a^{6} - 197687865980727\,a^{5} \\
& +10634069596050\,a^{4} + 27492240914352\,a^{3} - 3490888295808\,a^{2} - \\
& 1670217479280\,a + 429309266400)\,,
\end{aligned}
\tag{4.24}
$$

$$
\begin{aligned}
f_{4,1}(a) \;=\; & -\frac{16}{(a-2)^3(a-1)^3a^3(2a-7)^3(2a-5)^3(2a-3)^3(2a-1)^3(2a+1)^4(2a+3)^4}\times \\
& \Big(7372800\,a^{23} - 655032320\,a^{22} + 12236677120\,a^{21} - \\
& 103330938880\,a^{20} + 446158370816\,a^{19} - 770280591360\,a^{18} - \\
& 1416142109184\,a^{17} + 9605107320832\,a^{16} - 15447524759936\,a^{15} - \\
& 8228184007936\,a^{14} + 60190240489376\,a^{13} - 59275890588672\,a^{12} - \\
& 47305035162840\,a^{11} + 130190626486672\,a^{10} - 49532653784990\,a^{9} - \\
& 75968166506584\,a^{8} + 70367565767567\,a^{7} + 3329446699848\,a^{6} -
\end{aligned}
$$

$$21692142654831\,a^5 + 3791318445090\,a^4 + 2416824867888\,a^3 - 531904185216\,a^2 - 96835480560\,a + 24890392800\Big), \tag{4.25}$$

$$f_{4,11}(a) = \frac{131072\,i}{(a-1)a(2a-5)(2a-3)(2a-1)(2a+1)(2a+3)}, \tag{4.26}$$

$$f_{4,111}(a) = -\frac{12288}{(a-1)a(2a-5)(2a-3)(2a-1)(2a+1)(2a+3)}, \tag{4.27}$$

$$f_{4,2}(a) = \frac{32i\,(1720\,a^5 + 56864\,a^4 - 203410\,a^3 - 163652\,a^2 + 521499\,a + 270414)}{(a-2)(a-1)a(2a-7)(2a-5)(2a-3)(2a-1)(2a+1)^2(2a+3)^2}, \tag{4.28}$$

$$f_{4,12}(a) = -\frac{192\,(40\,a^5 + 1120\,a^4 - 4022\,a^3 - 3148\,a^2 + 10129\,a + 5226)}{(a-2)(a-1)a(2a-7)(2a-5)(2a-3)(2a-1)(2a+1)^2(2a+3)^2}. \tag{4.29}$$

The derivatives $L^{(n)}\left(\frac{i}{2}\right)$ are computed in Appendix A. Summing over a, we find the final result ($\overline{n} \equiv \frac{1}{2}\,e^{\gamma_E}\,n$)

$$\begin{aligned}\sum_{a=1}^{\infty} R_a = &-\frac{512\,i}{1215}\,(3\log\overline{n}+4)\,(-32+81\zeta_3)\,\frac{1}{n^2} + \\ &\frac{3584\,i}{1215}\,(6\log\overline{n}+5)\,(-32+81\zeta_3)\,\frac{1}{n^3} + \\ &\frac{512\,i}{8505}\,(135(1971\zeta_3-760)\log\overline{n} + 143289\zeta_3 - 53248)\,\frac{1}{n^4}\end{aligned} \tag{4.30}$$

Again, we can present a numerical table showing the accuracy of the computed asymptotic expansion

n	estimate	(LO	NLO	NNLO)	full expansion	diff %
10	-1.201460	$(-2.908787$	3.493848	$-3.576129)$	-2.991068	149%
30	-0.293765	$(-0.424071$	0.176476	$-0.061888)$	-0.309484	5.4%
50	-0.134246	$(-0.169551$	0.042847	$-0.009090)$	-0.135794	1.2%

(4.31)

5. Summary and a reciprocity conjecture

In summary, our results for the large spin expansion of the leading order wrapping correction at $m = 2, 3, 4$ are (we set here $\bar{n} = e^{\gamma_E}\,n$ for all m)

$$\begin{aligned}g^{-8}\,W_{n,m=2} = &-\frac{512}{3}\,(3\,\zeta_3-1)\,\frac{3\,\log\frac{\bar{n}}{2}+1}{n^2} + \frac{2048}{3}\,(3\,\zeta_3-1)\,\frac{2\,\log\frac{\bar{n}}{2}+1}{n^3} + \\ &-\frac{1536}{5}\,(77\,\zeta_3-24)\,\frac{\log\frac{\bar{n}}{2}}{n^4} + \dots,\end{aligned} \tag{5.1}$$

$$\begin{aligned}g^{-8}\,W_{n,m=3} = &-\frac{4}{3}\,(36\,\zeta_3+5\pi^2-3)\,\frac{2\,\log\bar{n}+1}{n^2} + \frac{88}{3}\,(36\,\zeta_3+5\pi^2-3)\,\frac{\log\bar{n}}{n^3} + \\ &-\frac{2}{9\,n^4}\Big[4\,(9108\,\zeta_3+1289\pi^2-615)\,\log\bar{n} - 14868\,\zeta_3 - 2017\pi^2 + 1527\Big] + \dots,\end{aligned} \tag{5.2}$$

$$g^{-8}\,W_{n,m=4} = -\frac{1024}{1215}\,(81\,\zeta_3-32)\,\frac{3\,\log\frac{\bar{n}}{2}+4}{n^2} + \frac{7168}{1215}\,(81\,\zeta_3-32)\,\frac{6\,\log\frac{\bar{n}}{2}+5}{n^3} + \tag{5.3}$$

$$-\frac{1024}{8505\,n^4}\left[138240\,(1971\,\zeta_3-760)\,\log\frac{\bar{n}}{2}+143289\,\zeta_3-53248\right]+\dots.$$

Following the general idea of [6] (see for instance the review [7] for its many tests in AdS/CFT), we are led to rewrite the above large n expansions in terms of the quantity

$$\mathcal{J}_m^2=n\,(n+a_m). \tag{5.4}$$

The possible vanishing of odd terms $1/\mathcal{J}^{2k+1}$ is linked to the Gribov-Lipatov reciprocity and allows to interpret $\mathcal{J}$ as the Casimir of a suitable additional symmetry of anomalous dimensions. From previous experience, it can be expected such reciprocity relations to hold not only for the full anomalous dimension, but also separately for the leading order wrapping correction. It turns out that the coefficients of the two odd terms $1/\mathcal{J}^3$ and $\log\mathcal{J}/\mathcal{J}^3$ indeed vanish for the choice

$$a_{2,3,4}=8,\ 11,\ 14. \tag{5.5}$$

This is not completely trivial since we have one parameters and two structures. It is tempting to conjecture the simple relation $a_m = 3\,m+2$ and to claim that reciprocity in the above sense holds for the full anomalous dimension as well. This remark could help in the task of finding a closed expression for the asymptotic anomalous dimensions which is currently unavailable beyond one loop. For completeness, we report the expansion of wrapping in terms of $\mathcal{J}_m=n\,(n+3\,m+2)$

$$\begin{aligned}
g^{-8}\,W_{n,m=2} &= -\frac{256}{3\mathcal{J}_2^2}\,(3\,\zeta_3-1)\left(\log\frac{\bar{\mathcal{J}}_2{}^2}{4}+2\right)+ \\
&\quad+\frac{256}{15\mathcal{J}_2^4}\left[(267\,\zeta_3-104)\,\log\frac{\bar{\mathcal{J}}_2{}^2}{4}+480\,\zeta_3-160\right],
\end{aligned} \tag{5.6}$$

$$\begin{aligned}
g^{-8}\,W_{n,m=3} &= -\frac{4}{3\mathcal{J}_3^2}\,(36\,\zeta_3+5\pi^2-3)\left(\log\bar{\mathcal{J}}_3{}^2+1\right)+ \\
&\quad+\frac{4}{9\mathcal{J}_3^4}\left[2\,(1980\,\zeta_3+263\pi^2-237)\,\log\bar{\mathcal{J}}_3{}^2+900\,\zeta_3+101\pi^2-219\right],
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
g^{-8}\,W_{n,m=4} &= -\frac{512}{1215\mathcal{J}_4^2}\,(81\,\zeta_3-32)\left(3\,\log\frac{\bar{\mathcal{J}}_4{}^2}{4}+8\right)+ \\
&\quad+\frac{512}{8505\mathcal{J}_4^4}\left[(67311\,\zeta_3-29112)\,\log\frac{\bar{\mathcal{J}}_4^2}{4}+102384\,\zeta_3-47168\right].
\end{aligned} \tag{5.8}$$

6. Conclusions

In this paper we have applied a simple algorithm to derive the large spin expansion of the leading order wrapping correction to a class of twist operators introduced by S. Zieme, A. Rej and L. Freyhult in [13]. Our analysis extends previous work on simple $\mathfrak{sl}(2)$-like rank one classes of states in β-deformed or ABJM theories. We could easily obtain accurate asymptotic expansions for various special cases. This analytic results can be used to claim the correct scaling behaviour of the wrapping correction, but also to explore other interesting properties like reciprocity constraint. In principle, our analysis could be helpful in a possible attempt to derive the currently unavailable explicit expression of the asymptotic anomalous dimension beyond one-loop.

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A. One-loop explicit Baxter polynomials

The general solutions to the 1-loop Bethe equations (2.5) are given by the following Baxter polynomials [13]

$$
\begin{aligned}
Q_4(u) &= \sum_{k=0}^{m}(-1)^k\binom{m}{k}\prod_{j=1}^{k}\left(u-i\frac{2j-1}{2}\right)^3\prod_{j=1}^{m-k}\left(u+i\frac{2j-1}{2}\right)^3 F_{n,m}\left(u+i\frac{m-2k}{2}\right),\\
Q_5(u) &= \sum_{k=0}^{m-1}(-1)^k\binom{m-1}{k}\prod_{j=1}^{k}(u-ij)^3\prod_{j=1}^{m-1-k}(u+ij)^3 F_{n,m}\left(u+i\frac{m-1-2k}{2}\right),\\
Q_6(u) &= \prod_{k=0}^{m-2} f_k\left(u+i\frac{k}{2}\right)\\
&+\sum_{r=1}^{m-1}(-1)^r\sum_{j_1=0}^{m-2}\sum_{j_2=0}^{j_1-1}\cdots\sum_{j_r=0}^{j_{r-1}-1}\prod_{s=1}^{r}\tilde{f}_{j_s}\left(u+i\frac{j_s-2(r-s)}{2}\right)\prod_{k=0}^{j_r-1} f_k\left(u+i\frac{k}{2}\right)\\
&\times\prod_{s=2}^{r}\prod_{k=j_s+1}^{j_{s-1}-1} f_k\left(u+i\frac{k-2(r-s+1)}{2}\right)\prod_{k=j_1+1}^{m-2} f_k\left(u+i\frac{k-2r}{2}\right),
\end{aligned}
\tag{A.1}
$$

where the hypergeometric function

$$
F_{n,m}(u) = {}_4F_3\left(\left.\begin{matrix}-\frac{n}{2} & \frac{n}{2}+1+\frac{3m}{2} & \frac{1}{2}+i\,u & \frac{1}{2}-i\,u\\ & 1+\frac{m}{2} & 1+\frac{m}{2} & 1+\frac{m}{2}\end{matrix}\right|1\right). \tag{A.2}
$$

obeys the Baxter equation

$$
\begin{aligned}
&\left(u-i\frac{m+1}{2}\right)^3 F_{n,m}(u-i)+\left(u+i\frac{m+1}{2}\right)^3 F_{n,m}(u+i) = t_3\,(u)\,F_{n,m}(u),\\
&t_3\,(u) = 2u^3-\left(n^2-n+3(m+1)n+\frac{3}{2}(m+1)^2\right)\,u.
\end{aligned}
\tag{A.3}
$$

In the formula for Q_6, we defined

$$
f_l\,(u) = -\frac{P_l\left(u-\frac{i}{2}\right)}{P_{l+1}\,(u)} \qquad\qquad \tilde{f}_l\,(u) = -\frac{P_l\left(u+\frac{i}{2}\right)}{P_{l+1}\,(u)}, \tag{A.4}
$$

and

$$
P_l(u) = \sum_{k=0}^{m-1-l}(-1)^k\binom{m-1-l}{k}\prod_{j=1}^{k}\left(u-i\frac{2j+l}{2}\right)^3\times \tag{A.5}
$$

$$\times \prod_{j=1}^{m-1-k-l} \left(u + i\frac{2j+l}{2}\right)^3 F_{n,m}\left(u + i\frac{m-1-l-2k}{2}\right).$$

B. Expansion of various hypergeometric functions

Let

$$L_{n,m}(u) = \log F_{n,m}(u). \tag{B.1}$$

We can easily obtain closed expressions for the specialized derivatives

$$L_{n,m}^{(k)}\left(\frac{m+1}{2}i\right). \tag{B.2}$$

In particular, the first two derivatives for $m = 2, 3, 4$ are

$$L'_{n,2}(3i/2) = -\frac{i\left(4(n+4)S_1\left(\frac{n}{2}+1\right) - 5n - 16\right)}{2(n+4)}, \tag{B.3}$$

$$L''_{n,2}(3i/2) = \frac{n(n+8)}{4(n+4)^2}, \tag{B.4}$$

$$L'_{n,3}(2i) = -iS_1\left(\frac{n}{2}+2\right) - iS_1\left(\frac{n}{2}+4\right) - \frac{1}{2}i(4\log(2) - 9), \tag{B.5}$$

$$L''_{n,3}(2i) = S_2\left(\frac{n}{2}+2\right) - S_2\left(\frac{n}{2}+4\right) + \frac{1}{12}\left(4\pi^2 - 37\right), \tag{B.6}$$

$$L'_{n,4}(5i/2) = -iS_1\left(\frac{n}{2}+2\right) - iS_1\left(\frac{n}{2}+4\right) + \frac{43i}{12}, \tag{B.7}$$

$$L''_{n,4}(5i/2) = \frac{n\left(25n^3 + 700n^2 + 6148n + 17472\right)}{144\left(n^2 + 14n + 48\right)^2}. \tag{B.8}$$

We can now use the Baxter equation to shift the arguments and move them to $i/2$ for even m of i for odd m. Expanding at large n, we find

$$\bar{n} = \frac{1}{2}e^{\gamma_E} n, \tag{B.9}$$
$$L'_{n,2}(i/2) = (3i - 2i\log(\bar{n})) - \frac{8i}{n} + \frac{\frac{62i}{3} - 8i\log(\bar{n})}{n^2} + \frac{64i\log(\bar{n}) - 112i}{n^3} + \ldots,$$
$$L''_{n,2}(i/2) = -3 + \frac{32\log^2(\bar{n}) - 16\log(\bar{n}) + 4}{n^2} + \frac{-256\log^2(\bar{n}) + 384\log(\bar{n}) - 96}{n^3} + \ldots,$$

$$\bar{n} = e^{\gamma_E} n, \tag{B.10}$$
$$L'_{n,3}(i) = \left(\frac{9i}{2} - 2i\log(\bar{n})\right) - \frac{11i}{n} + \frac{217i}{6n^2} - \frac{176i}{n^3} + \ldots,$$
$$L''_{n,3}(i) = \left(\frac{\pi^2}{3} - \frac{15}{4}\right) - \frac{2}{n^2} + \frac{22}{n^3} + \ldots,$$

$$\bar{n} = \frac{1}{2}e^{\gamma_E}\, n, \tag{B.11}$$

$$L'_{n,4}(i/2) = \left(\frac{9i}{2} - 2i\log(\bar{n})\right) - \frac{14i}{n} + \frac{\frac{446i}{3} - 64i\log(\bar{n})}{n^2} + \frac{896i\log(\bar{n}) - 2072i}{n^3} + \ldots,$$

$$L''_{n,4}(i/2) = -\frac{15}{4} + \frac{256\log^2(\bar{n}) - 704\log(\bar{n}) + 536}{n^2} + \frac{-3584\log^2(\bar{n}) + 13440\log(\bar{n}) - 12432}{n^3} + \ldots.$$

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