Gauged Kaluza-Klein AdS Pseudo-supergravity

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ABSTRACT

We obtain the pseudo-supergravity extension of the D-dimensional Kaluza-Klein theory, which is the circle reduction of pure gravity in $D+1$ dimensions. The fermionic partners are pseudo-gravitino and pseudo-dilatino. The full Lagrangian is invariant under the pseudo-supersymmetric transformation, up to quadratic order in fermion fields. We find that the theory possesses a $U(1)$ global symmetry that can be gauged so that all the fermions are charged under the Kaluza-Klein vector. The gauging process generates a scalar potential that has a maximum, leading to the AdS vacuum. Whist the highest dimension for gauged AdS supergravity is seven, our gauged AdS pseudo-supergravities can exist in arbitrary dimensions.

1 Introduction

There are usually two criteria for a successful construction of supergravities. The first is that the degrees of freedom of the bosonic and fermionic fields should match. This condition itself does not provide much restriction on the construction. The second, which is much more non-trivial, is that the bosonic part of the Lagrangian should admit consistent Killing spinor equations, whose projected integrability conditions give rise to the full set of the bosonic equations of motion. Let us consider eleven-dimensional supergravity [1] as an example. The on-shell degrees of freedom of the graviton and 3-form gauge potential match with those of gravitino. However, in addition to the Einstein-Hilbert action and the kinetic term for the 3-form potential, eleven-dimensional supergravity requires a Chern-Simons term for the 3-form with a specific coupling. This term does not affect the total degrees of freedom, but is essential for the consistency of the projected integrability condition for the Killing spinor equations [2, 3, 4].

Recently, it was discovered that the low-energy effective action of the bosonic string, which is an intrinsically non-supersymmetric theory, admits consistent Killing spinor equations [5]. The results were extended to include Yang-Mills fields and α' order corrections [6], as well as the conformal anomaly term [7]. Based on these results, the pseudo-supergravity extension of the bosonic string was constructed in [8]. Pseudo-supersymmetric partners, namely the pseudo-gravitino and pseudo-dilatino, are introduced. The full Lagrangian is invariant under the pseudo-supersymmetric transformation, up to the quadratic fermion order. The pseudo-supersymmetric theory can be extended by coupling it to a Yang-Mills pseudo-supermultiplet. This also allows one to construct "α' corrections" involving quadratic curvature terms. An exponential dilaton potential term, associated with the conformal anomaly for a bosonic string outside its critical dimension, can also be pseudo-supersymmetrized. Of course, in $D = 10$, where the degrees of freedom for the bosons and fermions match, the theory may become fully supersymmetric after adding quartic fermion terms. The full ten-dimensional $\mathcal{N} = 1$ supergravity with Yang-Mills supermultiplets were given in [9, 10, 11]. However, when α' correction terms are involved, the supersymmetry was proved only at the quadratic fermion order [10, 11].

Killing spinor equations for the D-dimensional Kaluza-Klein theory that is the circle reduction of pure gravity in $D+1$ dimensions were obtained in [5]. It was shown in [7] that the Killing spinors can be charged under the Kaluza-Klein vector. This charging process

generates a scalar potential, yielding the full bosonic Lagrangian

$$e^{-1}\mathcal{L} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{a\phi}F_{(2)}^2 + g^2(D-1)\Big((D-3)e^{-\sqrt{\frac{2}{(D-1)(D-2)}}\,\phi} + e^{\frac{\sqrt{2}\,(D-3)}{\sqrt{(D-1)(D-2)}}\,\phi}\Big), \quad (1)$$

where $e = \sqrt{-g}$, $F_{(2)} = dA_{(1)}$ is the 2-form field strength for the Kaluza-Klein vector $A_{(1)}$, and

$$a = \sqrt{\frac{2(D-1)}{D-2}}. \qquad (2)$$

In this paper, we obtain the gauged pseudo-supergravity extension for this bosonic Lagrangian.

The paper is organized as follows. In section 2, we construct ungauged Kaluza-Klein pseudo-supergravity that is pseudo-supersymmetrization of the Kaluza-Klein theory. In section 3, we pseudo-supersymmetrizing the scalar/gravity system as a warmup exercise, since as we explained earlier, the gauging process will generate a scalar potential. In section 4, we combine the results of sections 2 and 3, and obtain ungauged Kaluza-Klein pseudo-supergravity with a scalar potential. We show that the scalar potential has a single exponential term which has no fixed point. The theory possesses $U(1) \sim SO(2)$ global symmetry that rotates the fermion fields. In section 5, we gauge the theory by letting all the fermions charged under the Kaluza-Klein vector. This effectively turns the $U(1)$ symmetry to become a local one. The gauging process extends the previous scalar potential to involve two exponential terms. The new scalar potential has a maximum, implying that the AdS spacetime is its vacuum solution. We conclude the paper in section 6. We summarize our results in the appendix. Since we consider pseudo-supergravities in all dimensions, we also present the fermion conventions in diverse dimensions in the appendix.

2 Kaluza-Klein pseudo-supergravity

The D-dimensional Kaluza-Klein theory is the S^1 reduction of pure gravity in $D+1$ dimensions. The Lagrangian is given by (1) with $g = 0$, namely

$$e^{-1}\mathcal{L}_{\mathrm{KK},B} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{a\phi}F_{(2)}^2. \qquad (3)$$

The consistent Killing spinor equations for this were obtained in [5]. Here we examine whether the theory can be pseudo-supersymmetrized. We introduce pseudo-gravitino and dilatino fields (ψ_μ^i, λ^i), and propose that the fermionic extension is given by

$$\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{KK},F} = {} & s^{ij}\Big[\tfrac{1}{2}\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho}D_\nu\psi_\rho^j + \tfrac{1}{2}\bar{\lambda}^i\slashed{D}\lambda^j + e_1\,\bar{\psi}_\mu^i\Gamma^\nu\Gamma^\mu\lambda^j\partial_\nu\phi\Big] \\
& + t^{ij}\Big[e_2\,\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho\sigma}\psi_\sigma^j + e_3\,\bar{\psi}^{i\nu}\psi^{j\rho} + e_4\,\bar{\psi}_\mu^i\Gamma^{\nu\rho}\Gamma^\mu\lambda^j + e_5\,\bar{\lambda}^i\Gamma^{\nu\rho}\lambda^j\Big]e^{\frac{1}{2}a\phi}F_{\nu\rho},
\end{aligned} \qquad (4)$$

3

where the constant coefficients $e_1, \ldots, e_5$ are to be determined. Note that this is very different from the previous supergravity construction, which is typically on a specific dimension with specific type of fermion fields. Here we construct the theory in a generic dimension. It is thus necessary to summarize the properties of fermions in general dimensions. We adopt exactly the same convention given in [8], which follows the convention of [12]. We present the convention in Table 1 at the end of the appendix, and refer readers to [8, 12] for details. In addition to the Γ-matrix symmetries and spinor representations in diverse dimensions, we also present the s^{ij} and t^{ij} (and also u^{ij} that will appear in later constructions) in Table 1. The indices i, j takes two values, 1 and 2. From Table 1, we note that s^{ij} and t^{ij} can be either Kronecker δ^{ij} or ε^{ij}, where $\varepsilon^{ij} = -\varepsilon^{ji}$ with $\varepsilon^{12} = 1$. In dimensions where $s^{ij} = \delta^{ij}$, the fermions are two copies of Majorana; when $s^{ij} = \varepsilon^{ij}$, they are of a single symplectic Majorana. When $s^{ij} = t^{ij}$, which occurs only for $\beta = +1$, the fermions in (4) can be reduced to either a single copy of Majorana or symplectic Majorana, depending on the dimensions.

Having presented the convention for the fermions in general dimensions, we now give the ansatz for the pseudo-supersymmetric transformation rules

$$
\begin{aligned}
\delta\psi_\mu^i &= D_\mu \epsilon^i + \tfrac{\mathrm{i}}{8(D-2)} c_1 t^{ij} s^{kj} \left(\Gamma_\mu \Gamma^{\nu\rho} - 2(D-2)\delta_\mu^\nu \Gamma^\rho \right) e^{\frac{1}{2}a\phi} F_{\nu\rho} \epsilon^k , \\
\delta\lambda^i &= c_2 \left[\Gamma^\mu \partial_\mu \phi \epsilon^i + \tfrac{1}{4} c_3\, a\, t^{ij} s^{kj}\, \Gamma^{\mu\nu} e^{\frac{1}{2}a\phi} F_{\mu\nu} \epsilon^k \right], \\
\delta e_\mu^a &= \tfrac{1}{4} s^{ij}\, \bar\psi_\mu^i \Gamma^a \epsilon^j , \qquad \text{so} \qquad \delta g_{\mu\nu} = \tfrac{1}{2} s^{ij}\, \bar\psi_{(\mu}^i \Gamma_{\nu)} \epsilon^j , \\
\delta\phi &= c_4\, s^{ij}\, \bar\lambda^i \epsilon^j , \\
\delta A_\mu &= e^{-\frac{1}{2}a\phi} t^{ij} \left[c_5\, \bar\psi_\mu^i \epsilon^j + c_6\, \bar\lambda^i \Gamma_\mu \epsilon^j \right],
\end{aligned}
\tag{5}
$$

where the coefficients $c_1, \ldots, c_6$ are to be determined. Note that the pseudo-supersymmetric transformation rules for the fermionic fields (ψ_μ^i, λ^i) are inspired by the Killing spinor equations obtained in [7] for single copy of fermions.

We now require that the full Lagrangian

$$
\mathcal{L}_{\mathrm{KK}} = \mathcal{L}_{\mathrm{KK},B} + \mathcal{L}_{\mathrm{KK},F}
\tag{6}
$$

be invariant by the pseudo-supersymmetric transformation (5), up to quadratic order in fermion fields. We find that this fixes the coefficients in the ansatze completely, given by

$$
\begin{aligned}
e_1 &= \frac{\mathrm{i}}{2\sqrt{2\beta}}, \quad e_2 = \frac{\mathrm{i}\sqrt{\beta}}{16}, \quad e_3 = \frac{\mathrm{i}\sqrt{\beta}}{8}, \quad e_4 = \frac{a}{8\sqrt{2\beta}}, \quad e_5 = -\frac{\mathrm{i}\,D\sqrt{\beta}}{16(D-2)}, \\
c_1 &= c_3 = \sqrt{\beta}, \quad c_2 = \frac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}, \quad c_4 = -\frac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}, \quad c_5 = -\frac{\mathrm{i}\sqrt{\beta}}{4}, \quad c_6 = \frac{\beta\,a}{4\sqrt{2}}.
\end{aligned}
\tag{7}
$$

4

3 Pseudo-supersymmetrizing scalar/gravity

As discussed in the introduction, our ultimate goal of this paper is to construct gauged Kaluza-Klein pseudo-supergravity. The gauging process will generate a scalar potential. In this section, as a warmup exercise, we consider the pseudo-supersymmetrization of the scalar/gravity system. The bosonic Lagrangian is given by

$$e^{-1}\mathcal{L}_{\text{scalar},B} = R - \tfrac{1}{2}(\partial\phi)^2 - V(\phi) \,. \tag{8}$$

The potential V can be expressed in terms of a superpotential W, *via*

$$V = W'^2 - \frac{D-1}{2(D-2)}W^2 \,, \tag{9}$$

where a prime denotes a derivative with respect to ϕ. Killing spinor equations for this system in the context of domain wall solution were given in [13]. We propose the ansatz for the fermionic extension

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{scalar},F} &= s^{ij}\Big[\tfrac{1}{2}\bar{\psi}^i_\mu\Gamma^{\mu\nu\rho}D_\nu\psi^j_\rho + \tfrac{1}{2}\bar{\lambda}^i\slashed{D}\lambda^j + e_1\,\bar{\psi}^i_\mu\Gamma^\nu\Gamma^\mu\lambda^j\partial_\nu\phi\Big] \\
&\quad + u^{ij}\Big[e_6\,\bar{\psi}^i_\mu\Gamma^{\mu\nu}\psi^j_\nu\,W + e_7\,\bar{\psi}^{i\mu}\Gamma^\mu\lambda^j\,W' + \bar{\lambda}^i\lambda^j\,(e_8W'' + e_9W)\Big] \,.
\end{aligned}
\tag{10}
$$

Note that e_1 is given by (7) and $e_6, \dots, e_9$ are to be determined. The ansatz for the pseudo-supersymmetric transformation rules is given by

$$
\begin{aligned}
\delta\psi^i_\mu &= D_\mu\epsilon^i + \tfrac{1}{2\sqrt{2}\,(D-2)}b_1\,u^{ij}s^{kj}\,W\Gamma_\mu\epsilon^k \,, \\
\delta\lambda^i &= c_2\Big[\Gamma^\mu\partial_\mu\phi\epsilon^i - \sqrt{2}\,b_2\,u^{ij}s^{kj}\,W'\epsilon^k\Big] \,, \\
\delta e^a_\mu &= \tfrac{1}{4}s^{ij}\,\bar{\psi}^i_\mu\Gamma^a\epsilon^j \,, \\
\delta\phi &= c_4\,s^{ij}\,\bar{\lambda}^i\epsilon^j \,.
\end{aligned}
\tag{11}
$$

The coefficients c_2 and c_4 are given by (7) and coefficients b_1 and b_2 are to be determined. It is worth mentioning that s^{ij} and u^{ij}, which can be found in Table 1 in the appendix, can be the same only for cases with $\beta = -1$. We find that the requirement for pseudo-supersymmetry implies that

$$e_6 = -\frac{\mathrm{i}\sqrt{\beta}}{4\sqrt{2}} \,, \quad e_7 = -\frac{1}{2} \,, \quad e_8 = \frac{\mathrm{i}\sqrt{\beta}}{\sqrt{2}} \,, \quad e_9 = -\frac{\mathrm{i}\sqrt{\beta}}{4\sqrt{2}} \,, \quad b_1 = b_2 = \mathrm{i}\sqrt{\beta} \,. \tag{12}$$

4 Kaluza-Klein pseudo-supergravity with a scalar potential

We now combine the results of section 2 and section 3 together. Keep in mind that in section 3, the superpotential W can be an arbitrary function of ϕ. We shall examine the

restriction on W in Kaluza-Klein pseudo-supergravity. The full Lagrangian is given by

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{KK,pot}} = {}& R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{a\phi}F_{(2)}^2 - V \\
& + s^{ij}\Big[\tfrac{1}{2}\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho}D_\nu\psi_\rho^j + \tfrac{1}{2}\bar{\lambda}^i\slashed{D}\lambda^j + e_1\,\bar{\psi}_\mu^i\Gamma^\nu\Gamma^{j\mu}\lambda\partial_\nu\phi\Big] \\
& + t^{ij}\Big[e_2\,\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho\sigma}\psi_\sigma^j + e_3\,\bar{\psi}^{i\nu}\psi^{j\rho} + e_4\,\bar{\psi}_\mu^i\Gamma^{\nu\rho}\Gamma^\mu\lambda^j + e_5\,\bar{\lambda}^i\Gamma^{\nu\rho}\lambda^j\Big]e^{\frac{1}{2}a\phi}F_{\nu\rho} \\
& + u^{ij}\Big[e_6\,\bar{\psi}_\mu^i\Gamma^{\mu\nu}\psi_\nu^j\,W + e_7\,\bar{\psi}^{i\mu}\Gamma^\mu\lambda^j\,W' + \bar{\lambda}^i\lambda^j\,(e_8W'' + e_9W)\Big].
\end{aligned} \tag{13}
$$

The pseudo-supersymmetric transformation rules are given by

$$
\begin{aligned}
\delta\psi_\mu^i &= D_\mu\epsilon^i + \tfrac{\mathrm{i}}{8(D-2)}c_1 t^{ij}s^{kj}\Big(\Gamma_\mu\Gamma^{\nu\rho} - 2(D-2)\delta_\mu^\nu\Gamma^\rho\Big)e^{\frac{1}{2}a\phi}F_{\nu\rho}\epsilon^k \\
&\quad + \tfrac{1}{2\sqrt{2}\,(D-2)}b_1\,u^{ij}s^{kj}\,W\Gamma_\mu\epsilon^k\,, \\
\delta\lambda^i &= c_2\Big[\Gamma^\mu\partial_\mu\phi\epsilon^i + \tfrac{\mathrm{i}}{4}c_3\,a\,t^{ij}s^{kj}\,\Gamma^{\mu\nu}e^{\frac{1}{2}a\phi}F_{\mu\nu}\epsilon^k - \sqrt{2}\,b_2\,u^{ij}s^{kj}\,W'\epsilon^k\Big]\,, \\
\delta e_\mu^a &= \tfrac{1}{4}s^{ij}\,\bar{\psi}_\mu^i\Gamma^a\epsilon^j\,, \\
\delta\phi &= c_4\,s^{ij}\,\bar{\lambda}^i\epsilon^j\,, \\
\delta A_\mu &= e^{-\frac{1}{2}a\phi}\,t^{ij}\Big[c_5\,\bar{\psi}_\mu^i\epsilon^j + c_6\,\bar{\lambda}^i\Gamma_\mu\epsilon^j\Big]\,,
\end{aligned} \tag{14}
$$

We find that the variation of the Lagrangian, up to quadratic order in fermions, yields

$$
\begin{aligned}
\delta\mathcal{L} &= -\frac{a\sqrt{\beta}}{8}\gamma\Big(W'' + \frac{\sqrt{2}}{\sqrt{(D-1)(D-2)}}W' - \frac{D-3}{2(D-2)}W\Big)\bar{\lambda}^i\Gamma^{\mu\nu}\epsilon^j\varepsilon^{ij}e^{\frac{1}{2}a\phi}F_{\mu\nu} \\
&\quad -\frac{1}{8\sqrt{2}}\gamma\Big(a\,W' - \frac{D-3}{D-2}W\Big)e^{\frac{1}{2}a\phi}\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho}\epsilon^j\varepsilon^{ij}e^{\frac{1}{2}a\phi}F_{\nu\rho}\,,
\end{aligned} \tag{15}
$$

where $\gamma = \pm 1$ is given by (34) in the appendix. The vanishing of the above variation requires that

$$
W = m\,e^{\frac{D-3}{\sqrt{2(D-1)(D-2)}}\phi} \qquad \Longrightarrow \qquad V = -\frac{2m^2}{D-1}e^{\frac{2(D-3)}{\sqrt{2(D-1)(D-2)}}\phi}\,, \tag{16}
$$

where m is a free parameter. Thus we see that in Kaluza-Klein pseudo-supergravity, the scalar potential cannot be arbitrary, but a specific single exponential structure. Note that (s, t, u) cannot be all the same in this case. In dimensions where Majorana spinors are allowed, two copies are needed with ε^{ij} and δ^{ij} bilinear structures. The global symmetry is $U(1)$, which is a subgroup of $SU(2)$. In dimensions where symplectic Majorana is necessary, there is also additional δ^{ij} bilinear structure. The global symmetry is broken down from $Sp(2)$ to $U(1)$. In the next section, we consider gauging the $U(1)$ global symmetry.

5 Gauged Kaluza-Klein pseudo-supergravity

In the previous sections, we consider Kaluza-Klein pseudo-supergravities where the fermions are all neutral under the Kaluza-Klein vector $A_{(1)}$. In this section, we gauge the theory by

considering that all the fermions are charged under $A_{(1)}$. As we shall see presently, this turns the global symmetry of the $U(1)$ rotation of the fermions into a local symmetry. The charged covariant derivative on fermions is given by

$$D_\mu \xi^i \to \mathcal{D}(A)\xi^i = D_\mu \xi^i + b\, A_\mu\, s^{li} u^{lk} s^{mk} t^{mj} \xi^j = D_\mu \xi^i - \beta\gamma\, b\, A_\mu\, \varepsilon^{ij} \xi^j\,, \tag{17}$$

where $\gamma = \pm 1$ is given by (34), and the charge parameter b is a constant to be determined. Note that here ξ^i represents both ψ_μ^i and λ^i. The full Lagrangian for gauged pseudo-supergravity is now given by

$$\mathcal{L}_{\mathrm{KK,gauged}} = \mathcal{L}_{\mathrm{KK,pot}}(D \to \mathcal{D}(A))\,. \tag{18}$$

The pseudo-supersymmetric transformation rules also take the same form as (14), but with the covariant derivative D on the spinors replaced by $\mathcal{D}(A)$.

We find that the variation of the Lagrangian $\mathcal{L}_{\mathrm{KK,gauged}}$ leads to the following

$$
\begin{aligned}
\delta\mathcal{L} = & -\frac{a\sqrt{\beta}}{8}\gamma\Big(W'' + \frac{\sqrt{2}}{\sqrt{(D-1)(D-2)}}W' - \frac{D-3}{2(D-2)}W\Big)e^{\frac{1}{2}a\phi}\bar{\lambda}^i\Gamma^{\mu\nu}\epsilon^j\varepsilon^{ij}F_{\mu\nu} \\
& -\frac{1}{8\sqrt{2}}\gamma\Big((a\,W' - \frac{D-3}{D-2}W)e^{\frac{1}{2}a\phi} + \tfrac{1}{2}\beta\, b\Big)\bar{\psi}_\mu^i\Gamma^{\mu\nu\rho}\epsilon^j\varepsilon^{ij}F_{\nu\rho}\,.
\end{aligned} \tag{19}
$$

Thus we have

$$W = \frac{g}{\sqrt{2}}\Big((D-3)e^{-\frac{D-1}{\sqrt{2(D-1)(D-2)}}\phi} + (D-1)e^{\frac{D-3}{\sqrt{2(D-1)(D-2)}}\phi}\Big)\,. \tag{20}$$

The corresponding scalar potential is

$$V = -g^2(D-1)\Big((D-3)e^{-\sqrt{\frac{2}{(D-1)(D-2)}}\phi} + e^{\frac{\sqrt{2}\,(D-3)}{\sqrt{(D-1)(D-2)}}\phi}\Big)\,. \tag{21}$$

Note that the potential has a maximum and we have chosen the parameters so that it occurs at $\phi = 0$, with $V(0) = -(D-1)(D-2)g^2$. The charging parameter b is given by

$$b = -\frac{\beta}{4}(D-3)g\,. \tag{22}$$

Note that in $D = 3$, appropriate scaling limit should be performed to obtain non-vanishing results. Under the local gauge transformation,

$$A_{(1)} \to A_{(1)} + d\Lambda\,, \tag{23}$$

the fermions transform as follows

$$\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} \to \exp\left[\theta\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}\,, \tag{24}$$

or equivalently

$$\xi^1 + i\,\xi^2 \rightarrow e^{-i\theta}(\xi^1 + i\,\xi^2)\,, \tag{25}$$

where

$$\theta = \frac{\gamma}{4}(D-3)g\Lambda\,. \tag{26}$$

Note that the $U(1) \sim SO(2)$ rotation (24) leaves both the structures δ^{ij} and ε^{ij} invariant, and hence the Lagrangian is invariant under the gauge symmetry. By gauging, we see that the global symmetry of the $U(1)$ rotation of the fermions of the ungauged theory becomes the local symmetry, with the original constant θ now relating to the gauge parameter Λ by (26).

Thus we obtain the pseudo-supersymmetrization of the bosonic Lagrangian (1). For $D \geq 4$, the theory cannot be fully supersymmetrized. However, it can be embedded in maximally gauged supergravities in $D = 4, 5, 6$ and 7. The embedding in $D = 4, 5$ and 7 can be understood as follows. The gauged group of maximally gauged supergravities in these dimensions are $SO(8)$, $SO(6)$ and $SO(5)$ respectively, which admit $U(1)^N$ truncations with $N = 4, 3, 2$. The Kaluza-Klein vector in our theory is the one of the N vectors of the truncated theories. (See, for example, [14].) The $D = 6$ example can be embedded [15] in six-dimensional gauged supergravity [16] with a vector multiplet, which has an origin [17, 15] in massive type IIA supergravity [18].

6 Conclusions

In this paper, we construct gauged Kaluza-Klein AdS pseudo-supergravity in diverse dimensions. By pseudo-supergravity, we mean that the full Lagrangian is invariant under pseudo-supersymmetric transformation rules up to quadratic fermion order. We start with pseudo-supersymmetrizing the D-dimensional Kaluza-Klein theory that is the S^1 reduction of pure gravity in $D + 1$ dimensions. We then consider pseudo-supersymmetrizing the scalar/gravity system. Combining the two results, we obtain the pseudo-supersymmetric Kaluza-Klein theory with a single-exponential scalar potential. By requiring that the fermions are all charged under the Kaluza-Klein vector, we obtain gauged Kaluza-Klein pseudo-supergravity. In dimensions where there can be Majorana spinors, two copies are needed, with bilinear structures of either δ^{ij} or ε^{ij}, which has a $U(1)$ global symmetry. In dimensions where there can be symplectic-Majorana, the global symmetry is also $U(1)$, which is a subgroup of $Sp(2)$. The effect of the gauging is that the $U(1)$ symmetry becomes

a local one, associated with the Kaluza-Klein vector. The scalar potential now involves two exponential terms and it has a maximum, giving rise to the AdS spacetime as its vacuum solution.

The success of our construction, together with the previous example of pseudo-super-symmetrizing of the bosonic string [8], suggests that when a bosonic system admits consistent Killing spinor equations, it can always be pseudo-supersymmetrized. In dimensions when the fermion and boson degrees of freedom happen to match, full supersymmetry may be realized.

The highest dimension in gauged supergravities with AdS vacua is $D = 7$. In our gauged pseudo-supergravities, it can be arbitrary. Solutions of charged rotating [19] and static black holes [7] were previously obtained. These solutions provide interesting higher dimensional backgrounds to test the AdS/CFT correspondence. The pseudo-supersymmetry that our theories possess makes them superior to an ad hoc concocted AdS theory. The existence of consistent Killing spinor equations implies that we can in principle derive the complete set of solutions that preserves pseudo-supersymmetry. These solutions are as good as BPS solutions in gauged supergravities in testing the AdS/CFT correspondence.

Finally, we would like to emphasize that the possible bosonic theories that can be pseudo-supersymmetrized are rather limited. It was shown that Einstein gravity coupled to an n-form field strength cannot in general pseudo-supersymmetrized, unless it happens to be part of a supergravity theory [4]. The known non-trivial examples of pseudo-supergravities constructed so far, besides our examples here, are the pseudo-supergravity extensions of the bosonic string [8]. It is of great interest to investigate whether there exists a classification scheme of pseudo-supergravities, as in the case of supergravities.

Acknowledgement

A Summary of the results

The field content of the gauged Kaluza-Klein AdS pseudo-supergravity we have constructed consists of the metric, the dilaton ϕ and the Kaluza-Klein vector $A_{(1)}$, together with the pseudo-supersymmetric partners, pseudo-gravitino and pseudo-dilatino (ψ^i_μ, λ^i). The total on-shell degrees of freedom of bosonic fields are $\frac{1}{2}(D+1)(D-2)$; the ones of the fermionic fields are $(D-2)2^{[\frac{D}{2}]}$. Thus the theory does not have real supersymmetry, except for $D=3$; the ungauged $D=3$ theory is simply the circle reduction of $D=4$, $\mathcal{N}=1$ Poincaré supergravity. The full Lagrangian in general dimensions is given by

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{KK,gauged}} = {} & R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{a\phi}F^2_{(2)} - V \\
& + s^{ij}\Big[\tfrac{1}{2}\bar\psi^i_\mu\Gamma^{\mu\nu\rho}\mathcal{D}_\nu(A)\psi^j_\rho + \tfrac{1}{2}\bar\lambda^i\slashed{\mathcal{D}}(A)\lambda^j + \tfrac{\mathrm{i}}{2\sqrt{2\beta}}\,\bar\psi^i_\mu\Gamma^\nu\Gamma^\mu\lambda^j\partial_\nu\phi\Big] \\
& + t^{ij}\Big[\tfrac{\mathrm{i}\sqrt{\beta}}{16}\,\bar\psi^i_\mu\Gamma^{\mu\nu\rho\sigma}\psi^j_\sigma - \tfrac{\mathrm{i}\sqrt{\beta}}{8}\,\bar\psi^{i\nu}\psi^{j\rho} + \tfrac{a}{8\sqrt{2}}\,\bar\psi^i_\mu\Gamma^{\nu\rho}\Gamma^\mu\lambda^j - \tfrac{\mathrm{i}D\sqrt{\beta}}{16(D-2)}\,\bar\lambda^i\Gamma^{\nu\rho}\lambda^j\Big]e^{\frac{1}{2}a\phi}F_{\nu\rho} \\
& + u^{ij}\Big[-\tfrac{\mathrm{i}\sqrt{\beta}}{4\sqrt{2}}\,\bar\psi^i_\mu\Gamma^{\mu\nu}\psi^j_\nu\,W - \tfrac{1}{2}\,\bar\psi^i_\mu\Gamma^\mu\lambda^j\,W' + \mathrm{i}\sqrt{\tfrac{\beta}{2}}\,\bar\lambda^i\lambda^j\,(W''-\tfrac{1}{4}W)\Big] .
\end{aligned}
\tag{27}
$$

where $F_{(2)} = dA_{(1)}$, $a^2 = 2(D-1)/(D-2)$ and the charged covariant derivative is given by

$$
\mathcal{D}(A)\xi^i = D_\mu\xi^i - \beta\gamma\, b\, A_\mu\, \varepsilon^{ij}\xi^j ,
\tag{28}
$$

for any fermion ξ^i, where $\gamma = \pm 1$, given by (34). The superpotential W and the potential V are given by (20) and (21) respectively. The quantities (s^{ij}, t^{ij}, u^{ij}) take either δ^{ij} or ε^{ij}, satisfying the following identities

$$
s^{ij}s^{ik} = \delta^{jk} , \qquad s^{jk}t^{jl}s^{lm} = t^{km} , \qquad s^{jk}t^{jl} = t^{kj}s^{lj} , \qquad s^{kj}t^{jl} = t^{kj}s^{jl} .
\tag{29}
$$

Note that in the above, s and t can interchange, and each can interchange with u and the identities still hold.

The pseudo-supersymmetric transformation rules for all the involved fields are given by

$$
\begin{aligned}
\delta\psi^i_\mu &= \Big[\mathcal{D}_\mu\epsilon^i + \tfrac{\mathrm{i}\sqrt{\beta}}{8(D-2)}\,t^{ij}s^{kj}\Big(\Gamma_\mu\Gamma^{\nu\rho} - 2(D-2)\delta^\nu_\mu\Gamma^\rho\Big)e^{\frac{1}{2}a\phi}F_{\nu\rho}\epsilon^k \\
&\qquad\qquad + \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}(D-2)}\,u^{ij}s^{kj}\,W\Gamma_\mu\epsilon^k\Big] , \\
\delta\lambda^i &= \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}\Big[\Gamma^\mu\partial_\mu\phi\,\epsilon^i + \tfrac{\mathrm{i}\sqrt{\beta}}{4}a\,t^{ij}s^{kj}\,e^{\frac{1}{2}a\phi}F_{\mu\nu}\,\Gamma^{\mu\nu}\epsilon^k - \mathrm{i}\sqrt{2\beta}\,u^{ij}s^{kj}\,W'\epsilon^k\Big] , \\
\delta e^a_\mu &= \tfrac{1}{4}s^{ij}\,\bar\psi^i_\mu\Gamma^a\epsilon^j , \qquad \text{so} \qquad \delta g_{\mu\nu} = \tfrac{1}{2}s^{ij}\bar\psi^i_{(\mu}\Gamma_{\nu)}\epsilon^j , \\
\delta\phi &= -\tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}\,s^{ij}\,\bar\lambda^i\epsilon^j , \\
\delta A_\mu &= e^{-\frac{1}{2}a\phi}\,t^{ij}\Big[-\tfrac{\mathrm{i}\sqrt{\beta}}{4}\,\bar\psi^i_\mu\epsilon^j + \tfrac{a\beta}{4\sqrt{2}}\,\bar\lambda^i\Gamma_\mu\epsilon^j\Big] .
\end{aligned}
\tag{30}
$$

To verify that the Lagrangian is indeed invariant under the transformation rules, up to the quadratic fermion order, it is useful to derive first the following projected integrability

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conditions, given by

$$s^{ji}\Gamma^{\mu\nu\rho}\mathcal{D}_\nu\delta\psi^j_\rho$$

$$
\begin{aligned}
=\ & \tfrac{1}{2}\Gamma^\nu\left[R^\mu{}_\nu - \tfrac{1}{2}\partial^\mu\phi\,\partial_\nu\phi - \tfrac{1}{2}F^{2\mu}{}_\nu - \tfrac{1}{2}\delta^\mu{}_\nu\left(R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}F^2 - V\right)\right]\epsilon^i \\
& + \tfrac{\mathrm{i}\sqrt{\beta}}{8}s^{ji}t^{jk}s^{mk}\left(2g^{\mu_1\nu}g^{\mu\mu_2} - \Gamma^{\mu\nu\mu_1\mu_2}\right)e^{\frac{1}{2}a\phi}F_{\mu_1\mu_2}\delta\psi^m_\nu + \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}s^{ji}u^{jk}s^{mk}\Gamma^{\mu\nu}W\delta\psi^m_\nu \\
& - \tfrac{\mathrm{i}}{2\sqrt{2\beta}}s^{ji}\nabla_\nu\phi\,\Gamma^\nu\Gamma^\mu\delta\lambda^j - \tfrac{\sqrt{2}}{16}s^{ji}t^{jk}s^{mk}a\,e^{\frac{1}{2}a\phi}F_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2}\Gamma^\mu\delta\lambda^m \\
& + \tfrac{1}{2}s^{ji}u^{jl}s^{kl}W'\Gamma^\mu\delta\lambda^l + \tfrac{\mathrm{i}\sqrt{\beta}}{8}s^{ji}t^{jk}\left(2e^{-\frac{1}{2}a\phi}\nabla_\nu(e^{a\phi}F^{\nu\mu}) - e^{\frac{1}{2}a\phi}\Gamma^{\mu\nu\mu_1\mu_2}\nabla_\nu F_{\mu_1\mu_2}\right)\epsilon^k \\
& - \tfrac{1}{8\sqrt{2}}\gamma\varepsilon^{ij}\left[(\tfrac{D-3}{D-2}W - a\,W')e^{\frac{1}{2}a\phi} - \tfrac{\beta}{2}b\right]\Gamma^{\mu\nu\rho}F_{\nu\rho}\epsilon^j\,,
\end{aligned}
\tag{31}
$$

and

$$s^{ji}\Gamma^\mu\mathcal{D}_\mu\delta\lambda^j$$

$$
\begin{aligned}
=\ & \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}s^{ji}\,\Gamma^\mu\Gamma^\nu\partial_\nu\phi\,\delta\psi^j_\mu - \tfrac{\beta\,a}{8\sqrt{2}}s^{ji}t^{jk}s^{lk}e^{\frac{1}{2}a\phi}F_{\mu_1\mu_2}\Gamma^\mu\Gamma^{\mu_1\mu_2}\delta\psi^l_\mu + \tfrac{1}{2}\beta s^{ji}u^{jk}s^{lk}\Gamma^\mu W'\delta\psi^l_\mu \\
& + \tfrac{\mathrm{i}D\sqrt{\beta}}{8(D-2)}s^{ji}t^{jk}s^{lk}e^{\frac{1}{2}a\phi}F_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2}\delta\lambda^l + \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}s^{ji}u^{jk}s^{lk}W\delta\lambda^l \\
& - \mathrm{i}\sqrt{2\beta}\,s^{ji}u^{jk}s^{lk}W''\delta\lambda^l + \tfrac{\mathrm{i}\sqrt{\beta}}{2\sqrt{2}}\left(\nabla^2\phi - \tfrac{a}{4}e^{a\phi}F^2 - V'\right)\epsilon^i \\
& - \tfrac{\beta\,a}{8\sqrt{2}}s^{ji}t^{jk}\left[e^{\frac{1}{2}a\phi}\Gamma^{\mu\mu_1\mu_2}\nabla_\mu F_{\mu_1\mu_2} + 2e^{-\frac{1}{2}a\phi}\Gamma^{\mu_2}\nabla^\mu\left(e^{a\phi}F_{\mu\mu_2}\right)\right]\epsilon^k \\
& - \tfrac{\mathrm{i}\sqrt{\beta}a}{8}\gamma\varepsilon^{ij}e^{\frac{1}{2}a\phi}F_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2}\left[W'' + \tfrac{2}{(D-2)a}W' - \tfrac{(D-3)}{2(D-2)}W\right]\epsilon^m\,.
\end{aligned}
\tag{32}
$$

Note that for Killing spinors, we have $\delta\psi^i_\mu = 0$ and $\delta\lambda^i = 0$. Substituting these into the above equations and we find that what remain are precisely the full set of bosonic equations of motion attached to various Γ-matrix structures.

It is worth pointing out that since $s^{ij} = t^{ij}$ occurs only for $\beta = +1$ and $s^{ij} = u^{ij}$ only for $\beta = -1$, the quantities (s, t, u) cannot be all the same in any dimensions. In dimensions where Majorana spinors are allowed, two copies are necessary with both δ^{ij} and ε^{ij} bilinear structures. When symplectic Majorana is available, the symplectic structure is broken down to include δ^{ij} structure as well.

Finally we present the Γ-matrix and fermion conventions. We adopt exactly the same convention given in [8], which follows the convention of [12]. We present the convention in Table 1. In addition to the Γ-matrix symmetries and spinor representations in diverse dimensions, we also present the s^{ij}, t^{ij} and u^{ij} that appear in the construction. From Table 1, we can derive the following important identities

$$s^{ji}t^{jk}s^{mk}t^{ml} = \beta\,\delta^{il}\,, \qquad s^{ji}u^{jk}s^{mk}u^{ml} = -\beta\,\delta^{il}\,, \qquad s^{ji}u^{jk}s^{lk}t^{lm} = \gamma\beta\,\varepsilon^{im}\,, \tag{33}$$

where

$$\gamma = \begin{cases} +1\,, & \text{if } t^{ij} = \delta^{ij}\,, \\ -1\,, & \text{if } t^{ij} = \varepsilon^{ij}\,. \end{cases} \tag{34}$$

D mod 8	$C\Gamma^{(0)}$	$C\Gamma^{(1)}$	Spinor	β	s^{ij}	t^{ij}	u^{ij}
0	S	S	M	$+1$	δ^{ij}	δ^{ij}	ε^{ij}
	S	A	S-M	-1	ε^{ij}	δ^{ij}	ε^{ij}
1	S	S	M	$+1$	δ^{ij}	δ^{ij}	ε^{ij}
2	S	S	M	$+1$	δ^{ij}	δ^{ij}	ε^{ij}
	A	S	M	-1	δ^{ij}	ε^{ij}	δ^{ij}
3	A	S	M	-1	δ^{ij}	ε^{ij}	δ^{ij}
4	A	S	M	-1	δ^{ij}	ε^{ij}	δ^{ij}
	A	A	S-M	$+1$	ε^{ij}	ε^{ij}	δ^{ij}
5	A	A	S-M	$+1$	ε^{ij}	ε^{ij}	δ^{ij}
6	A	A	S-M	$+1$	ε^{ij}	ε^{ij}	δ^{ij}
	S	A	S-M	-1	ε^{ij}	δ^{ij}	ε^{ij}
7	S	A	S-M	-1	ε^{ij}	δ^{ij}	ε^{ij}

Table 1: Γ-matrix symmetries, spinor representations and (s, t, u) in diverse dimensions.

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