Factorization of integrals defining the two-loop β-function for the general renormalizable $N = 1$ SYM theory, regularized by the higher covariant derivatives, into integrals of double total derivatives

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Abstract

The integrals defining the two-loop β-function for the general renormalizable $N = 1$ supersymmetric Yang–Mills theory, regularized by higher covariant derivatives, are investigated. It is shown that they are given by integrals of double total derivatives. These integrals are not equal to zero due to appearing of δ-functions. These δ-functions allow to reduce the two-loop integrals to one-loop integrals, which can be easily calculated. The result agrees with the exact NSVZ β-function and calculations made by different methods.

1 Introduction.

Quantum correction in supersymmetric theories were studied for a long time. For example, the β-function for $N = 1$ supersymmetric Yang–Mills theories was calculated in one- [1], two- [2], three- [3, 4], and four-loop [5] approximations. All these calculation were made with the dimensional reduction [6] in the $\overline{MS}$-scheme [7], because the dimensional regularization [8] breaks the supersymmetry. However, it is well known [9] that the dimensional reduction is inconsistent. Although ways allowing to overcome the corresponding problems are discussed in the literature [10], removing of inconsistencies leads to the loss of the supersymmetry in higher orders [11, 12]. In particular [11, 13], it was shown that obtaining a three-loop β-function by different methods (from different vertexes) leads to different results for the $N = 2$ supersymmetric Yang–Mills theory. As a consequence, the dimensional reduction scheme breaks the supersymmetry in higher loops. (In [11] it was argued that this also takes place for the $N = 1$ and $N = 4$ supersymmetric Yang–Mills theories in the three-loop approximation, but recent calculation [13] showed that in the three-loop approximation this is true only for the $N = 2$ theory.

For the $N = 4$ supersymmetric Yang–Mills theory the dimensional reduction does not also break the supersymmetry in the four-loop approximation [14].)

Other regularizations are also applied to calculations of quantum corrections. For example, in Ref. [15] the two-loop β-function of $N = 1$ supersymmetric electrodynamics (and also the β-functions of the scalar and spinor electrodynamics) was calculated using a method based on the operator product expansion. The two-loop β-function for $N = 1$ supersymmetric Yang–Mills theory was calculated in Ref. [16] with the differential renormalization [17]. Some calculations were made with the higher covariant derivative regularization, proposed in [18], and generalized to the supersymmetric case in [19] (another variant was proposed in [20]). The higher covariant derivative regularization is an invariant regularization and does not break the supersymmetry [19, 20, 21]. However, it was not frequently applied to concrete calculations, because it is very difficult to calculate the corresponding loop integrals analytically. For example, the one-loop β-function of the (non-supersymmetric) Yang-Mills theory was first calculated only in [22]. Taking into account correction made in subsequent papers [23] the result coincided with the well-known one, obtained with the dimensional regularization [24]. It is possible to prove that in the one-loop approximation the results obtained with the higher derivative regularization always agree with the results obtained with the dimensional regularization [25]. Some calculations in the one-loop and two-loop approximations were made for various theories [26, 27] with a variant of the higher covariant derivative regularization proposed in [28]. The structure of the corresponding integrals was discussed in Ref. [27].

Calculations of quantum corrections in supersymmetric theories with the higher derivative regularization show that the β-function is given by integrals of total derivatives. This was first noted in [29], where all integrals defining the three-loop β-function of $N = 1$ supersymmetric electrodynamics were calculated using integration by parts. This feature was also found in Ref. [30], where the factorization of integrands into total derivatives is explained using a special technique, based on the covariant Feynman rules in the background field method [31, 32]. A proof of the factorization for $N = 1$ SQED by a different method [33, 34] is made in [35]. This factorization allows natural explaining the origin of the exact NSVZ β-function [36], because one of the loop integrals can be calculated explicitly. As a consequence, say, in $N = 1$ SQED integrals defining the β-function in the n-th loop are reduced to integrals defining the anomalous dimension in the $(n-1)$-th loop [37]. It is important to note that with the higher derivative regularization in order to obtain the NSVZ β-function one should not make a special redefinition of the coupling constant [35], which is needed if the calculations are made with the dimensional reduction [4, 38].

For the general renormalizable $N = 1$ supersymmetric Yang–Mills theory, regularized by the higher covariant derivatives, the two-loop β-function have been calculated in [39]. Similar results were obtained with two different versions of the higher derivative regularization in [40, 41]. In these papers it was also verified that all integrals defining the β-function are integrals of total derivatives, and this feature does not depend on a particular choice of the regularizing term. However, in Ref. [30] it was argued that the integrals defining the β-function are integrals of double total derivatives. For $N = 1$ SQED, regularized by higher derivatives, this was also proved by a different method in [35]. In the present paper we demonstrate that for a general renormalizable $N = 1$ supersymmetric Yang–Mills theory, regularized by the higher covariant derivatives, two-loop integrals for the β-function can be also written as integrals of double total derivatives.

The paper is organized as follows:

In Sec. 2 we introduce the notation and recall basic information about the higher covariant derivative regularization. The integrals defining the β-function for the considered theory are rewritten as integrals of double total derivatives in Sec. 3. The result is briefly discussed in the Conclusion.

2 $N=1$ supersymmetric Yang–Mills theory and the higher covariant derivative regularization

In this paper we consider a general renormalizable $N=1$ supersymmetric Yang–Mills theory. In the massless case it is described by the action [42, 43][1]

$$S=\frac{1}{2e^2}\mathrm{Re}\,\mathrm{tr}\int d^4x\,d^2\theta\,W_a C^{ab} W_b+\frac{1}{4}\int d^4x\,d^4\theta\,(\phi^*)^i(e^{2V})_i{}^j\phi_j+\\+\Big(\frac{1}{6}\int d^4x\,d^2\theta\,\lambda^{ijk}\phi_i\phi_j\phi_k+\mathrm{h.c.}\Big),\qquad(1)$$

where ϕ_i are chiral matter superfields in a representation R, which is in general reducible. V is a real scalar gauge superfield. The superfield W_a is a supersymmetric gauge field stress tensor, which is defined by

$$W_a=\frac{1}{8}\bar D^2(e^{-2V}D_a e^{2V}).\qquad(2)$$

In our notation D_a and $\bar D_a$ are the right and left supersymmetric covariant derivatives respectively, $V=eV^AT^A$, and the generators of the fundamental representation are normalized by the condition

$$\mathrm{tr}(t^At^B)=\frac{1}{2}\delta^{AB}.\qquad(3)$$

Because action (1) should be invariant under the gauge transformations, the coefficient λ^{ijk} satisfies the condition

$$(T^A)_m{}^i\lambda^{mjk}+(T^A)_m{}^j\lambda^{imk}+(T^A)_m{}^k\lambda^{ijm}=0.\qquad(4)$$

It is convenient to calculate quantum corrections using the background field method [42]. We make the substitution

$$e^{2V}\to e^{2V'}\equiv e^{\mathbf{\Omega}^+}e^{2V}e^{\mathbf{\Omega}}\qquad(5)$$

in action (1), where $\mathbf{\Omega}$ is a background superfield. Then the theory is invariant under the background gauge transformations

$$\phi\to e^{i\Lambda}\phi;\quad V\to e^{iK}Ve^{-iK};\quad e^{\mathbf{\Omega}}\to e^{iK}e^{\mathbf{\Omega}}e^{-i\Lambda};\quad e^{\mathbf{\Omega}^+}\to e^{i\Lambda^+}e^{\mathbf{\Omega}^+}e^{-iK},\qquad(6)$$

[1] In our notation $\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$; $\theta^a\equiv\theta_bC^{ba}$; θ_a and $\bar\theta_a$ denote the right and left components of θ, respectively.

where K is an arbitrary real superfield, and Λ is a background-chiral superfield. This invariance allows to set $\mathbf{\Omega} = \mathbf{\Omega}^+ = \mathbf{V}$. We choose a regularization and a gauge fixing so that invariance (6) is unbroken. A gauge is fixed by adding

$$S_{\rm gf} = -\frac{1}{32e^2}\,\mathrm{tr}\int d^4x\, d^4\theta \left(V\boldsymbol{D}^2\bar{\boldsymbol{D}}^2V + V\bar{\boldsymbol{D}}^2\boldsymbol{D}^2V\right) \qquad (7)$$

to the action. The actions for the corresponding Faddeev–Popov and Nielsen–Kallosh ghosts are

$$\begin{aligned} S_{\rm FP} = &\frac{1}{2e^2}\mathrm{tr}\int d^4x\, d^4\theta\, (e^{\mathbf{\Omega}}\tilde{c}e^{-\mathbf{\Omega}} + e^{-\mathbf{\Omega}^+}\tilde{c}^+e^{\mathbf{\Omega}^+})\Big(V_{\rm Ad}(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}} + e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+})\\ &+V_{\rm Ad}\mathrm{cth}\, V_{\rm Ad}(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}} - e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+})\Big);\\ S_{\rm NK} = &\frac{1}{2e^2}\mathrm{tr}\int d^4x\, d^4\theta\, b^+e^{\mathbf{\Omega}^+}e^{\mathbf{\Omega}}b\, e^{-\mathbf{\Omega}}e^{-\mathbf{\Omega}^+}, \end{aligned} \qquad (8)$$

where

$$f(V_{\rm Ad})c = f(0)c + \frac{1}{1!}f'(0)[V,c] + \frac{1}{2!}f''(0)[V,[V,c]] + \ldots \qquad (9)$$

In order to introduce the regularization it is necessary to add terms with the higher covariant derivatives to the action. There are different possibilities for choosing such terms. For example, in [39] the following terms were added:

$$\begin{aligned} S_\Lambda = &\frac{1}{2e^2}\mathrm{tr}\,\mathrm{Re}\int d^4x\, d^4\theta\, V\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{8}\int d^4x\, d^4\theta\, \Bigg((\phi^*)^i\Big[e^{\mathbf{\Omega}^+}e^{2V}\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}e^{\mathbf{\Omega}}\Big]_i{}^j\phi_j +\\ &+(\phi^*)^i\Big[e^{\mathbf{\Omega}^+}\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}e^{2V}e^{\mathbf{\Omega}}\Big]_i{}^j\phi_j\Bigg), \end{aligned} \qquad (10)$$

where $\boldsymbol{D}_\alpha$ is the background covariant derivative and we assume that $m < n$. Below we call this choice "variant 1". It is important that the higher covariant derivative term is also introduced for the matter superfields, because the considered theory contains a nontrivial superpotential.

A simpler variant of the regularization is obtained if terms with the higher covariant derivatives are chosen in the form ("variant 2")[40]

$$S_\Lambda = \frac{1}{2e^2}\mathrm{tr}\,\mathrm{Re}\int d^4x\, d^4\theta\, V\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{4}\int d^4x\, d^4\theta\, (\phi^*)^i\Big[e^{\mathbf{\Omega}^+}\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}e^{\mathbf{\Omega}}\Big]_i{}^j\phi_j. \qquad (11)$$

where m and n are arbitrary positive integers.

In both cases the regularized theory is evidently invariant under the background gauge transformations. However, the higher derivative terms considered here break BRST-invariance of the action, and it is necessary to use a special subtraction scheme, which restore the Slavnov–Taylor identities in each order of the perturbation theory [44]. For the supersymmetric case such a scheme was constructed in Ref. [45].

After adding S_Λ divergences remain only in the one-loop approximation [46]. In order to regularize them, it is necessary to introduce into the generating functional the Pauli–Villars determinants

$$\prod_I \Big(\int D\phi_I^* D\phi_I e^{iS_I}\Big)^{-c_I} \prod_i \Big(\int Dc_i^+ Dc_i D\tilde c_i^+ D\tilde c_i Db_i^+ Db_i e^{iS_i}\Big)^{-c_i}, \qquad (12)$$

where S_I and S_i are the actions for the Pauli–Villars fields corresponding to ϕ and ghosts, respectively. For variant 1 (if S_Λ is given by Eq. (10)), the Pauli–Villars action can be chosen as [25]

$$S_I = \frac{1}{8}\int d^4x\, d^4\theta\, \Bigg((\phi_I^*)^i \Big[e^{\mathbf{\Omega}^+} e^{2V}\Big(1+\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}\Big)e^{\mathbf{\Omega}}\Big]_i{}^j(\phi_I)_j + (\phi_I^*)^i\Big[e^{\mathbf{\Omega}^+}\Big(1+\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}\Big)\times$$
$$\times e^{2V}e^{\mathbf{\Omega}}\Big]_i{}^j(\phi_I)_j\Bigg) + \Big(\frac{1}{4}\int d^4x\, d^2\theta\, M_I^{ij}(\phi_I)_i(\phi_I)_j + \mbox{h.c.}\Big). \qquad (13)$$

For variant 2 the Pauli–Villars action is

$$S_I = \frac{1}{4}\int d^4x\, d^4\theta\, (\phi_I^*)^i \Big[e^{\mathbf{\Omega}^+}\Big(1+\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}\Big)e^{\mathbf{\Omega}}\Big]_i{}^j(\phi_I)_j + \Big(\frac{1}{4}\int d^4x\, d^2\theta\, M_I^{ij}(\phi_I)_i(\phi_I)_j + \mbox{h.c.}\Big). \qquad (14)$$

The mass terms for the ghost Pauli–Villars fields are

$$\frac{1}{2e^2}\mbox{tr}\int d^4x\, d^2\theta\, \Big(m_b b^2 + 2m_c\tilde c c\Big) + \mbox{h.c.} \qquad (15)$$

The masses of all Pauli–Villars fields are proportional to the parameter Λ:

$$M_I^{ij} = a_I^{ij}\Lambda; \qquad m_i = a_i\Lambda, \qquad (16)$$

where a-s are numerical constants. As a consequence, Λ is the only dimensionful parameter of the regularized theory. We assume that the mass term does not break the gauge invariance. Also we will choose the masses so that

$$M_I^{ij}(M_I^*)_{jk} = M_I^2\delta_k^i. \qquad (17)$$

The coefficients c_I and c_i satisfy the conditions

$$\sum_I c_I = 1; \qquad \sum_I c_I M_I^2 = 0; \qquad \sum_i c_i = 1; \qquad \sum_i c_i m_i^2 = 0. \qquad (18)$$

The generating functional for connected Green functions and the effective action are defined by the standard way.

3 Two-loop β-function

Let us write terms in the effective action corresponding to the renormalized two-point Green function of the gauge superfield in the form

$$\Gamma^{(2)}_V = -\frac{1}{8\pi}\mathrm{tr}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\, d^{-1}(\alpha,\lambda,\mu/p), \tag{19}$$

where α is a renormalized coupling constant. In this paper we investigate the expression

$$\frac{d}{d\ln\Lambda}\left(d^{-1}(\alpha_0,\lambda_0,\Lambda/p) - \alpha_0^{-1}\right)\Big|_{p=0} = -\frac{d\alpha_0^{-1}}{d\ln\Lambda} = \frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} \tag{20}$$

in the two-loop approximation. After the calculation of the supergraphs the two-loop β-function can be presented in the form:

$$\beta_2(\alpha,\lambda) = \alpha^2 C_2(I_{\mathrm{FP}} + I_{\mathrm{NK}}) + \alpha^2 T(R) I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r} C(R)_i{}^j C(R)_j{}^i I_2 +$$
$$+\alpha^3 T(R) C_2 I_3 + \alpha^2 C(R)_i{}^j \frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r} I_4, \tag{21}$$

where the following notation is used:

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA}. \tag{22}$$

Here

$$I = I(0) - \sum_I c_I I(M_I) \quad \text{for} \quad I_0, I_2, I_3;$$
$$I = I(0) - \sum_i c_i I(m_i) \quad \text{for} \quad I_{\mathrm{NK}}, I_{\mathrm{FP}}, \tag{23}$$

and the integrals $I_0(M)$, I_1, $I_2(M)$, $I_3(M)$ and I_4 can be found in Ref. [39] for variant 1 and in Ref. [40] for variant 2. In Refs. [39, 40] these integrals are written as integrals of total derivatives. However, they are actually the integrals of double total derivatives. Note that in this paper the notation is different from Ref. [35], where

$$\int \frac{d^4q}{(2\pi)^4}\frac{\partial}{\partial q^\mu} \equiv \int\limits_{S_\infty} \frac{dS_\mu}{(2\pi)^4} \tag{24}$$

corresponds to $\mathrm{Tr}[x_\mu, \ldots]$. Here we use the ordinary notation

$$\int \frac{d^4q}{(2\pi)^4}\frac{\partial}{\partial q^\mu} \equiv \int\limits_{\partial} \frac{dS_\mu}{(2\pi)^4} = \int\limits_{S_\infty} \frac{dS_\mu}{(2\pi)^4} - \text{integrals of } \delta\text{-singularities}, \tag{25}$$

and ∂ denotes a boundary of a region where the integrand is regular.

The result for the integrals defining the β-function for the variant 1 can be written as follows (a two-loop contribution of the Faddeev–Popov ghosts is 0, exactly as in [31]):

$$I_{\rm NK}(m) = \frac{1}{2} I_{\rm FP}(m) = \pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big\{ \frac{1}{q^2} \ln\Big(q^2 + m^2\Big) \Big\}; \qquad (26)$$

$$I_0(M) = -\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big\{ \frac{1}{q^2} \ln\Big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2\Big) \Big\}; \qquad (27)$$

$$I_1 = -12\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial k^\mu} \frac{\partial}{\partial k_\mu} \Big\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n}) q^2(1+q^{2n}/\Lambda^{2n}) (q+k)^2}$$
$$\times \frac{1}{(1+(q+k)^{2n}/\Lambda^{2n})} \Big\}; \qquad (28)$$

$$I_2(M) = 2\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big\{ \frac{(2+(q+k)^{2m}/\Lambda^{2m} + q^{2m}/\Lambda^{2m})^2}{k^2(1+k^{2n}/\Lambda^{2n})}$$
$$\times \frac{(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{\Big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2\Big)\Big((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2 + M^2\Big)} \Big\}; \qquad (29)$$

$$I_3(M) = 2\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial k_\mu} \Big\{ \frac{(2+k^{2m}/\Lambda^{2m} + q^{2m}/\Lambda^{2m})^2}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})}$$
$$\times \frac{(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}{\Big(k^2(1+k^{2m}/\Lambda^{2m})^2 + M^2\Big)\Big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2\Big)} \Big\}; \qquad (30)$$

$$I_4 = -8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big\{ \frac{1}{k^2(1+k^{2m}/\Lambda^{2m}) q^2(1+q^{2m}/\Lambda^{2m}) (q+k)^2}$$
$$\times \frac{1}{(1+(q+k)^{2m}/\Lambda^{2m})} \Big\}. \qquad (31)$$

These integrals are not equal to 0 because

$$\int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big(\frac{f(q^2)}{q^2} \Big) = \lim_{\varepsilon\to 0} \int\limits_{S_\varepsilon} \frac{dS_\mu}{(2\pi)^4} \frac{(-2) q^\mu f(q^2)}{q^4} = \frac{1}{4\pi^2} f(0) \qquad (32)$$

for a nonsingular function $f(q^2)$ which rapidly decreases at the infinity. As a consequence,

$$I_{\rm NK} = -\frac{1}{4\pi} \frac{d}{d\ln\Lambda} \Big(\sum_i c_i \ln m_i^2 \Big) = -\frac{1}{2\pi};$$
$$I_0 = \frac{1}{4\pi} \frac{d}{d\ln\Lambda} \Big(\sum_I c_I \ln M_I^2 \Big) = \frac{1}{2\pi};$$
$$I_1 = -6 \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \Big[\frac{1}{q^4(1+q^{2n}/\Lambda^{2n})^2} \Big] = -\frac{3}{4\pi^2};$$

$$I_2 = \int \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \left[\frac{(2+k^{2m}/\Lambda^{2m})^2}{k^4(1+k^{2n}/\Lambda^{2n})(1+k^{2m}/\Lambda^{2m})}\right] = \frac{1}{2\pi^2};$$
$$I_3 = \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \left[\frac{2}{q^4} - \sum_I c_I \frac{2(1+q^{2m}/\Lambda^{2m})^4}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M_I^2)^2}\right] = \frac{1}{4\pi^2};$$
$$I_4 = -4\int \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \left[\frac{1}{k^4(1+k^{2m}/\Lambda^{2m})^2}\right] = -\frac{1}{2\pi^2}. \qquad (33)$$

(The Pauli–Villars fields nontrivially contribute only to integrals $I_{\rm NK}$, $I_{\rm FP}$, I_0 and I_3, where they cancel the one-loop (sub)divergence.)

For variant 2 the integrals $I_{\rm FP}$, $I_{\rm NK}$, I_0, I_1, and I_4 are the same. However, the integrals I_2 and I_3 are different:

$$I_2(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Bigg\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})}$$
$$\times \frac{(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{\Big(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2\Big)\Big((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2+M^2\Big)} \Bigg\}; \qquad (34)$$

$$I_3(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial k_\mu} \Bigg\{ \frac{1}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})}$$
$$\times \frac{(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}{\Big(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2\Big)\Big(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2\Big)} \Bigg\}. \qquad (35)$$

As a consequence,

$$I_2 = \int \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \left[\frac{4}{k^4(1+k^{2n}/\Lambda^{2n})(1+k^{2m}/\Lambda^{2m})}\right] = \frac{1}{2\pi^2}; \qquad (36)$$
$$I_3 = \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \left[\frac{2}{q^4(1+q^{2m}/\Lambda^{2m})^2} - \sum_I c_I \frac{2(1+q^{2m}/\Lambda^{2m})^2}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M_I^2)^2}\right] = \frac{1}{4\pi^2}.$$

Therefore, for both variants of the regularization the two-loop β-function is given by

$$\beta(\alpha,\lambda) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(-3C_2^2 + T(R)C_2 + \frac{2}{r}C(R)_i{}^j C(R)_j{}^i\Big) -$$
$$-\frac{\alpha^2 C(R)_i{}^j \lambda^*_{jkl}\lambda^{ikl}}{8\pi^3 r} + \ldots \qquad (37)$$

and agrees with the exact NSVZ β-function [36, 37]

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha,\lambda)/r\Big]}{2\pi(1-C_2\alpha/2\pi)}. \qquad (38)$$

Up to notation, this result is in agreement with the results of calculations made with the dimensional reduction in [2].

4 Conclusion

With the higher covariant derivative regularization all integrals defining the two-loop β-function of the general renormalizable $N=1$ supersymmetric Yang–Mills theory are integrals of total derivatives. In this paper using two different versions of the higher covariant derivative regularization we show that they are not only integrals of total derivatives, but also integrals of double total derivatives. Due to the identity

$$\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\frac{1}{q^2} = -4\pi^2\delta^4(q) \tag{39}$$

these integrals do not vanish. Calculating them one obtain the exact NSVZ β-function. Possibly, this situation also takes place in all orders of the perturbation theory.

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