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On Black Holes in Massive Gravity

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Abstract

In massive gravity the so-far-found black hole solutions on Minkowski space happen to convert horizons into a certain type of singularities. Here we explore whether these singularities can be avoided if space-time is not asymptotically Minkowskian. We find an exact analytic black hole (BH) solution which evades the above problem by a transition at large scales to self-induced de Sitter (dS) space-time, with the curvature scale set by the graviton mass. This solution is similar to the ones discovered by Koyama, Niz and Tasinato, and by Nieuwenhuizen, but differs in detail. The solution demonstrates that in massive GR, in the Schwarzschild coordinate system, a BH metric has to be accompanied by the Stückelberg fields with nontrivial backgrounds to prevent the horizons to convert into the singularities. We also find an analogous solution for a Reissner-Nordström BH on dS space. A limitation of our approach, is that we find the solutions only for specific values of the two free parameters of the theory, for which both the vector and scalar fluctuations loose their kinetic terms, however, we hope our solutions represent a broader class with better behaved perturbations.

1 Introduction and Summary

According to the representation theory of the Poincaré group in 4D, a massive spin-2 state has to have five degrees of freedom; these can be thought of as the helicity-0, ± 1, ± 2 states. A good Lagrangian for the massive spin-2 has to be able to describe these states. The Fierz-Pauli mass term [1] is the only ghost- and tachyon-free term at the quadratic order that describes the above 5 states [2]. However, in the zero mass limit it does not recover the linearized Einstein's gravity, since the helicity-0 mode couples to the trace of the matter stress-tensor with strength equal to that of the helicity-2; this is called the vDVZ discontinuity [3]. If true, it would rule out massive gravity on the grounds of solar system observations. However, Vainshtein [4] argued that the troublesome longitudinal mode is suppressed at measurable distances by nonlinear effects, making the nonlinear theory compatible with current empirical data [5]. On the other hand, in a broad class of models, the same nonlinear terms that are responsible for the above-mentioned suppression, give rise to an instability known as the Boulware-Deser (BD) ghost [6]. This ghost appears as a 6th degree of freedom in the theory, and even though it is infinitely heavy on the Minkowski background, it becomes sufficiently light on locally nontrivial backgrounds, thus invalidating the theory [7, 8, 9, 10].

More recently, however, using the effective field theory formalism of [7], it has been shown in Ref. [11] that there exists a two parameter family of nonlinear generalization of the linear Fierz-Pauli theory, that is free of the BD ghost order-by order and to all orders, at least in the decoupling limit.

Most importantly, it was shown in Ref. [12] that the absence of the BD ghost in the decoupling limit is such a powerful requirement that it leads to the resummation of the entire infinite series of the terms in the effective Lagrangian. As a result, a candidate theory of massive General Relativity free of BD ghost, was proposed [12].

Using the Hamiltonian analysis in the unitary gauge it was shown that for a certain choice of the free parameters of the theory, and in the 4th order in nonlinearities, the Hamiltonian constraint that forbids the BD ghost is maintained in the theory of [12]. Note that the quartic order is special, since the lapse necessarily enters nonlinearly in all massive theories precisely in this order [8], and it may appear that the hamiltonian constraint should necessarily be lost then. In spite of this, the constraint is maintained in a subtle way for special theories, as was shown for a toy model in [11], and shown in the 4th order for massive GR in [12].

The existence of the Hamiltonian constraint to all orders in the unitary gauge, and for generic values of the parameters, was shown in Ref. [13], using the method of dealing with the lapse and shift proposed in [11, 12]. Moreover, Ref. [13] has also argued for a secondary constraint, that follows from the conservation of the Hamiltonian constraint [1]. Very recently, the existence of the secondary constraint was explicitly confirmed in Ref. [15].

The absence of the BD ghost among the local fluctuations of the theory of [12]

[1] The argument of the existence of the secondary constraint was challenged in [14].

in a generic gauge has been shown using the Stückelberg decomposition [16], as well as the helicity decompositions [17] to quartic orders in nonlinearities (in the latter two references, previous misconceptions in the literature claiming the presence of the BD ghost were also clarified). Motivated by the above developments, in the present work we will proceed to study certain subtle properties black holes (BH) in the theory of [12].

In the unitary gauge Lagrangian of the theory the object $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, is the gravitational analog of the Proca field of massive electrodynamics, describing all the five modes of the graviton. The diffeomorphism invariance can be restored by introducing the four scalar fields ϕ^a (the Stückelberg fields) [18, 7, 19], and replacing the Minkowski metric by the covariant tensor $\partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$

$$g_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} + H_{\mu\nu}, \tag{1}$$

where $H_{\mu\nu}$ denotes the covariantized metric perturbation, and $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$. The existence of the 4 Stückelberg scalars ϕ^a in this theory leads to the existence of new invariants in addition to the ones usually encountered in GR (Ricci scalar, Ricci tensor square, Riemann tensor square, etc); one new basic invariant is $I^{ab} = g^{\mu\nu}\partial_\mu \phi^a \partial_\nu \phi^b$. Note that the unitary gauge is set by the condition $\phi^a = x^\mu \delta^a_\mu$. In this gauge, $I^{ab} = g^{\mu\nu}\delta^a_\mu \delta^b_\nu$. Hence, any inverse metric that has divergence (even those which are innocuous in GR) would exhibit a singularity in the invariant I^{ab}. Is this singularity of any significance? The singularity in the above invariant does not necessarily affect the geodesic motion of external observers – the geodesic equation is identical to that of GR, and due to its covariance, one could remove from it what would have been a coordinate singularity in $g^{\mu\nu}$ in GR. However, one would expect the singularities in $I^{ab} = g^{\mu\nu}\delta^a_\mu \delta^b_\nu$ to be a problem for fluctuations around classical solutions exhibiting it. Since $g^{\mu\nu}$ could change signs on either side of the singularity, this could lead to emergence of ghosts and/or tachyons in the fluctuations around a given classical solution. In what follows, we will take a conservative point of view and will only accept solutions that have non-singular I^{ab}. These arguments, in a somewhat different form, have already been emphasized recently by Deffayet and Jacobson [20].

The above arguments give rise to the following seeming puzzle. On the one hand, according to the Vainshtein mechanism [4], spherically symmetric solutions of massive gravity should approximate those of GR better and better, as we increase the mass of the source and come closer to it. This would imply that the metric of a BH near its horizon should very much be similar to that of GR. On the other hand, the conventional Schwarzschild metric – if it were the solution of massive gravity in unitary gauge – would be singular according to the arguments above.

We reiterate this central point in more general terms: In order for a metric to qualify as a valid description of a BH configuration, the physical singularities must be absent at the horizon. Then, in the unitary gauge of massive gravity the

Schwarzschild-like metric

$$ds^2 = -(1-f)dt^2 + \frac{dr^2}{1-f} + r^2 d\Omega^2, \quad \text{with e.g.} \quad f = r_g/r, \tag{2}$$

cannot be a legitimate BH solution of the theory. The same applies to the metric of de Sitter (dS) space in the static coordinates for which $f = m^2 r^2$.

Recently, interesting BH solutions of massive gravity have been found in Refs. [21, 22, 23] (for other interesting solutions, which will not be discussed here, see, [24]- [32]). Following Koyama, Niz and Tasinato (KNT) [21], one can start in the unitary gauge, and consider a most general stationary spherically symmetric metric. Then, using the method developed by KNT, very interesting full non-linear solutions for stars and black holes with Minkowskian asymptotics were found by Gruzinov and Mirbabayi in [23]. These solutions do exhibit the Vainshtein mechanism, and therefore are potentially viable classical solutions for stars and other compact objects in massive gravity (although their stability still remains to be studied). Nevertheless, it is not clear, as emphasized in [23], whether these are appropriate solutions for BHs. Even in the best case solution, when all the GR invariants are finite, the invariant $g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}$ diverges, [23]. As noted above, this divergence does not affect the geodesic motion of any external observer, however, we expect it to be a problem for fluctuations.

Could there be any solution that avoids the above issue? The answer is positive, and the resolution is in the identification of the unitary gauge to the coordinate system in which the black hole has no horizon (the Kruskal-Szekeres, Eddington-Finkelstein, or Gullstrand-Painlevé systems come to our mind). The most convenient one for our purposes is the Gullstrand-Painlevé (GP) system, in which the metric has the following form

$$ds^2 = -dt^2 + (dr \pm \sqrt{f}dt)^2 + r^2 d\Omega^2, \tag{3}$$

and is free of horizon singularities. It corresponds to the frame of an in-falling observer and covers half the whole space (for either choice of sign).

Furthermore, if one has the metric (3) as a solution in unitary gauge, then the coordinate transformation to the metric (2) will lead us to a background with $\phi^a \neq x^a$. This means that if the configuration is described by the metric (2), the presence of a halo of helicity ± 1 and/or 0 fields around the BH is unavoidable. We will show in the present work that massive gravity in unitary gauge admits BH solutions precisely of this type.

Interestingly, the dS-Schwarzschild solution found in [21, 33] do happen to satisfy our conservative criterion of non-singularity. However, the solution that we present here is not among the ones of [21, 33].

One more point worth emphasizing is that the BH solutions of [23] do exhibit the "helicity-0 hair" (e.g., produce an extra scalar force), while the ones found in [21, 33], and in the present work do not. The status of the "no-hair" theorems in

GR with the galileon field (which should capture some properties of the helicity-0 of massive gravity) will be discussed in Ref. [34].

A limitation of our work is that we only manage to find these exact analytic solutions for a specific choice of the two free parameters of massive gravity. Such a choice is peculiar since on the obtained background, as we will show, the kinetic terms for both the vector and scalar fluctuations vanish in the decoupling limit.

This fact would imply infinitely strong interactions for these modes (unless these modes happen to be nondynamical to all orders, e.g., due to the specific choice of the coefficients of the theory). Because of this issue, we would like to regard the solutions obtained here as just examples demonstrating how non-singular solutions should emerge. We also hope that our solutions are representative of a broader class of solutions which may have better behaved fluctuations.

In this regard, there seems to be a few directions in which the studies of massive gravity BH's can be extended. First, one could look at the metric in the unitary gauge which would be some generalization of the Kruskal-Szekeres form. Second, one can extend the massive theory of [12] by adding more degrees of freedom to the existing 5 helicity states of massive graviton. In fact, two consistent extensions have already been discussed so far: (I) adding one real scalar field that makes the graviton mass dynamical [28]; (II) adding one massless tensor field with two degrees of freedom [35] that makes the internal space metric of the Stückelberg field dynamical (bigravity). In the latter case cosmological solutions were found recently in [31] and [32], while BH's were studied in [36].

The work is organized as follows. Section 2 gives a brief review of the theory of massive gravity [12]. In section 3 we find an exact Schwarzschild-de Sitter solution, and in section 4 an exact Reissner-Nordström-de Sitter, solution which have nonsingular I^{ab}. These solution are similar to those discovered by Koyama, Niz and Tasinato, and by Th. Nieuwenhuizen, but differ in detail: our dS solution has no ghost even though the vector field is present (compare to [33]). Moreover, on the obtained solution the singularities in the invariant I^{ab} are absent (compare to [22]). In the Appendix A we give another exact Schwarzschild solution that asymptotes to a conformaly rescaled Minkowski space, and briefly mention its peculiarities. In the Appendix B we discuss fluctuations on the selfaccelerated solution of section 3.

2 The Theory

A massive graviton is described by the Lagrangian density of [12] specified below

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\sqrt{-g}\left(R + m^2\mathcal{U}(g,\phi^a)\right), \tag{4}$$

where $\mathcal{U}$ is the potential for the graviton that depends on two free parameters $\alpha_{3,4}$

$$\mathcal{U}(g,\phi^a) = \left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right), \tag{5}$$

and the individual terms in the potential are defined as follows:

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]\,, \tag{6}$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\,, \tag{7}$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\,, \tag{8}$$

where $\mathcal{K}^\mu_\nu(g, \phi^a) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$; rectangular brackets denote traces, $[\mathcal{K}] \equiv \mathrm{Tr}(\mathcal{K}) = \mathcal{K}^\mu_\mu$. The above potential is unique – no further polynomial terms can be added to the action without introducing the BD ghost.

The tensor $H_{\mu\nu}$ represents the covariantized metric perturbation, as discussed in the introduction, which reduces to the $h_{\mu\nu}$ in unitary gauge. While in a gauge unfixed theory we have

$$H_{\mu\nu} = g_{\mu\nu} - \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}. \tag{9}$$

Moreover, $\mathcal{U}$ is constructed in such a way that the theory admits the Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \phi^a = x^\mu\delta^a_\mu. \tag{10}$$

Hence, it is natural to split ϕ's as the background plus the 'pion' contribution $\phi^a = x^a - \pi^a$, and as it was already mentioned in the introduction, the unitary gauge is defined by the condition $\pi^a = 0$. In the non-unitary gauge, on the other hand, it proves to be useful to adopt the following decomposition

$$\pi^a = \frac{mA^a + \partial^a\pi}{\Lambda^3}, \tag{11}$$

where A^μ describes in the decoupling limit the helicity ± 1, while π is the longitudinal mode of the graviton (in the decoupling limit [7], $M_{\rm pl} \to \infty$ and $m \to 0$, while $\Lambda^3 \equiv M_{\rm pl}m^2$ is held fixed). This limit captures the approximation in which the energy scale is much greater than the graviton mass scale, $E \gg m$.

For convenience, in what follows, we define the coefficients α and β which are related to those of (5) by $\alpha_3 = -(-\alpha + 1)/3$ and $\alpha_4 = -\beta/2 + (-\alpha + 1)/12$. For generic values of the parameters α and β the theory exhibits the Vainshtein mechanism, as show in the decoupling limit [11], and beyond [21, 26]. As was emphasized in [11], for one special choice, $\alpha = \beta = 0$, the nonlinear interactions vanish in the decoupling limit with fixed Λ, leaving the theory weakly coupled (i.e., no Vainshtein mechanism) in this limit. For this particular choice of the coefficients the action of massive gravity with the potential (6)-(8) (which can be rewritten in terms of just $[K]$ and tuned to it cosmological constant [25], referred as a minimal model in Ref. [25]), was shown not to exhibit the Vainshtein mechanism also away from the decoupling limit [21].

3 A Black Hole on de Sitter

In this section we present the Schwarzschild-de Sitter solution in the theory of massive gravity described above. The obtained solution is free of singularities (except from the conventional one appearing in GR).

For convenience we choose unitary gauge for the metric. In this gauge the symmetric tensor $g_{\mu\nu}$ is an observable describing all the five degrees of freedom of a massive graviton. The equations of motion in empty space read as follows

$$G_{\mu\nu} + m^2 X_{\mu\nu} = 0\,, \tag{12}$$

where $X_{\mu\nu}$ is the effective energy-momentum tensor due to the graviton mass,

$$X_{\mu\nu} = -\frac{1}{2}\Big[\mathcal{K}g_{\mu\nu} - \mathcal{K}_{\mu\nu} + \alpha\left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}\mathcal{K}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right) \tag{13}$$
$$+6\beta\left(\mathcal{K}^3_{\mu\nu} - \mathcal{K}\mathcal{K}^2_{\mu\nu} + \frac{1}{2}\mathcal{K}_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right) - \frac{1}{6}g_{\mu\nu}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)\right)\Big]\,.$$

Using the Bianchi identities, from (12) we obtain the following constraint on the metric

$$m^2\nabla^\mu X_{\mu\nu} = 0, \tag{14}$$

where ∇^μ denotes the covariant derivative.

In order to obtain the expression for $X_{\mu\nu}$ we make use of the fact that the Lagrangian is written as the trace of the polynomial of the matrix $\mathcal{K}^\mu_\nu$. Thus, following the method by Koyama, Niz and Tasinato [21], we choose the basis which diagonalizes the expression appearing under the square root in the definition of $\mathcal{K}^\mu_\nu$ [One should bear in mind that this is not a coordinate transformation, but rather a trick to simplify the procedure of getting the equations of motion]. As a result, the potential becomes a function of the components of the inverse metric, rather than the combination of square roots of matrices. Having done this, one is free to vary the action with respect to the inverse metric components to obtain explicit expression for (12). Since these expressions are quite cumbersome we will not give them here.

Below, we concentrate on one particular family of the ghost-free theory of massive gravity in which there is the following relations between the two free coefficients:

$$\beta = -\frac{\alpha^2}{6}\,. \tag{15}$$

That this choice of the coefficients is special was first shown by Th. Nieuwenhuizen [22] (see also [23]). In particular, it was shown in [22] that for this choice the equation (14) is automatically satisfied for a certain diagonal (in spherical coordinates) and time-independent metrics. It is interesting, however, that the above property persists for a more general class of non-diagonal spherically-symmetric metrics written as follows:

$$ds^2 = -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + w^2r^2d\Omega^2, \tag{16}$$

where w is a constant, while $A(r)$, $B(r)$ and $C(r)$ are arbitrary functions.

In subsection 3.1 we find an exact de Sitter solution to (12), and in subsection 3.2 we find an exact BH solution on the obtained dS background. Note that the dS background is entirely due to the graviton mass.

3.1 The de Sitter Solution

We note that we would find an exact dS solution if we required that

$$m^2 X_{\mu\nu} = \lambda g_{\mu\nu}, \tag{17}$$

where λ is some constant. The solution of the equations (12) that also satisfies (17) with a positive but otherwise arbitrary α is given by

$$ds^2 = -\kappa^2 dt^2 + \left(\frac{\alpha}{\alpha+1}dr \pm \kappa\sqrt{\frac{2}{3\alpha}}\frac{\alpha}{(\alpha+1)}mrdt\right)^2 + \frac{\alpha^2}{(\alpha+1)^2}r^2d\Omega^2. \tag{18}$$

Here, κ is a positive integration constant. It is straightforward to check that for (18) we have $X_{\mu\nu} = (2/\alpha)\, g_{\mu\nu}$, leading to the expression for the Ricci scalar

$$R = \frac{8}{\alpha}m^2, \tag{19}$$

as expected. Hence, this is a dS space with curvature scale set by the graviton mass and one free parameter α. One could imagine that $m \sim (0.1-1)H_0$, and $\alpha \sim (0.01-1)$, in which case the obtained dS solution (if stable) could describe dark energy.

Up to a rescaling of the coordinates, the expression (18) looks exactly like the de Sitter solution of GR written in the Gullstrand–Painlevé frame. Either $\pm$ solution covers half of dS space. One can rotate the obtained solution to the static coordinate system at the expense of nonzero Stückelberg fields. This will be done in the next subsection. In either form, the solution has no additional singularities.

3.2 Schwarzschild-de Sitter Background

Having the solution of the previous subsection worked out, it is straightforward to show that the system of equations (12) admits the following exact solution

$$ds^2 = -\kappa^2dt^2 + \left(\tilde{\alpha}dr \pm \kappa\sqrt{\frac{r_g}{\tilde{\alpha}r} + \frac{2\tilde{\alpha}^2}{3\alpha}m^2r^2}dt\right)^2 + \tilde{\alpha}^2r^2d\Omega^2\,, \tag{20}$$

where $\tilde{\alpha} \equiv \alpha/(\alpha+1)$, and as before, κ is an integration constant. In order to bring this solution to a more familiar form let us perform the following rescaling

$$\begin{aligned} r &\to \frac{\alpha+1}{\alpha} r, \\ dt &\to \frac{1}{\kappa} dt. \end{aligned} \tag{21}$$

The resulting metric reads

$$ds^2 = -dt^2 + \left(dr \pm \sqrt{\frac{r_g}{r} + \frac{2}{3\alpha} m^2 r^2} dt \right)^2 + r^2 d\Omega^2. \tag{22}$$

This is the Schwarzschild-de Sitter solution in the GP coordinates.

However, the above rescaling takes us away from the unitary gauge

$$\phi^0 = t \to t - \left(1 - \frac{1}{\kappa}\right) t, \tag{23}$$

$$\phi^r = r \to r + \frac{1}{\alpha} r. \tag{24}$$

In terms of the 'pions', $\pi^\mu \equiv x^\mu - \phi^\mu$, which can be decomposed as $\pi^\mu = (mA^\mu + \partial^\mu \pi)/\Lambda^3$, we have

$$\pi = \frac{\Lambda^3}{2} \left[-\left(1 - \frac{1}{\kappa}\right) t^2 - \frac{1}{\alpha} r^2 \right], \tag{25}$$

$$A^\mu = 0. \tag{26}$$

The fields in (26) correspond to the canonically normalized fields carrying the helicity eigenstates in the decoupling limit.

Let us now rewrite our solution into a more familiar coordinate system. The metric can be transformed to a static slicing by means of the following coordinate transformation

$$dt \to dt + f'(r) dr, \tag{27}$$

with $f'(r) \equiv -g_{01}/g_{00}$ given by

$$f'(r) = \pm \frac{\sqrt{\frac{r_g}{r} + \frac{2}{3\alpha} m^2 r^2}}{1 - \frac{r_g}{r} - \frac{2}{3\alpha} m^2 r^2}. \tag{28}$$

The resulting expression for the metric reads as follows:

$$ds^2 = -\left(1 - \frac{r_g}{r} - \frac{2}{3\alpha} m^2 r^2\right) dt^2 + \frac{dr^2}{1 - \frac{r_g}{r} - \frac{2}{3\alpha} m^2 r^2} + r^2 d\Omega^2. \tag{29}$$

This is nothing but the metric of the Schwarzschild-de Sitter solution of GR in the static coordinates. However, this metric should be accompanied by a nontrivial backgrounds for the Stückelberg fields. Indeed, it is evident that (27) gives rise to the shift $\delta\phi^0 = f(r)$. In turn, this gives rise to a background for the 'vector mode'

$$A^0 = -\frac{\Lambda^3}{\kappa m} f(r), \tag{30}$$
$$A^i = 0. \tag{31}$$

This particular field assignment has been chosen according to scaling in the decoupling limit. Namely, $f(r)$ vanishes in the decoupling limit linearly in m hence it was ascribed to the "vector mode".

We would like to make two important comments in the remainder of this section. The first one concerns the integration constant κ. Although, all the invariants of GR are independent of κ, the new invariant that is characteristic of massive gravity

$$I^{ab} \equiv g^{\mu\nu}\partial_\mu \phi^a \partial_\nu \phi^b, \tag{32}$$

does depend on this integration constant; in the unitary gauge I^{ab} is just the inverse of the GP metric (18) which reads as follows

$$\begin{pmatrix} -\frac{1}{\kappa^2} & \pm\frac{1}{\kappa}\frac{\alpha+1}{\alpha}\sqrt{\frac{\alpha+1}{\alpha}\frac{r_g}{r} + \frac{2}{3\alpha}\frac{\alpha^2}{(\alpha+1)^2}m^2r^2} & 0 \\ \pm\frac{1}{\kappa}\frac{\alpha+1}{\alpha}\sqrt{\frac{\alpha+1}{\alpha}\frac{r_g}{r} + \frac{2}{3\alpha}\frac{\alpha^2}{(\alpha+1)^2}m^2r^2} & \left(\frac{\alpha+1}{\alpha}\right)^2\left(1 - \frac{\alpha+1}{\alpha}\frac{r_g}{r} - \frac{2}{3\alpha}\frac{\alpha^2}{(\alpha+1)^2}m^2r^2\right) & 0 \\ 0 & 0 & \left(\frac{\alpha+1}{\alpha}\right)^2\Omega^{-1}_{2\times 2} \end{pmatrix}.$$

Thus, the backgrounds with different values of κ correspond to distinct superselection sectors labeled by the values of I^{ab}.

The second comment concerns the issue of small fluctuations on top of this solution. One may worry that the scalar perturbations on this background are infinitely strongly coupled in the light of the results of [24]. In the latter work it was found that, for the parameters chosen as in (15), the de Sitter background has infinitely strongly coupled fluctuations in the decoupling limit. However, we should point out that the self-accelerated background discussed in this section is different from that studied in [24]. This distinction is manifest in (31) by the presence of the background for A_0, which vanishes in the case of [24].

Still, one could argue that it is unnecessary to perform the transformation of variables (27) responsible for this difference, and limit oneself to the rescaling of the coordinates

$$r \to \frac{\alpha+1}{\alpha} r\,, \quad t \to \frac{1}{\kappa} t\,, \tag{33}$$

which clearly does not give rise to the vector background. As a result, the 'pion' configuration will become similar to that of [24], while the metric itself will be quite different, namely[2]

$$ds^2 = -(1 - m^2r^2)dt^2 + 2mrdtdr + dr^2 + r^2d\Omega^2. \tag{34}$$

Now, if we were to take this metric as the one in which the decoupling limit should be taken, then we would find that the gauge freedom that is left in this limit

$$g_{\mu\nu} \to g_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}, \tag{35}$$

would not be enough for bringing (34) to the form of de Sitter space in either conformal or static slicing. Furthermore, the canonically normalized (34) diverges in the decoupling limit, in such a way that this divergence can be isolated only in the vector mode. If so, then no conclusion can be drawn about the perturbations around our background based on the results of [24]. This, on the other hand, does not necessarily imply that the fluctuations are fine. As we show in the Appendix B the vector and scalar fluctuations may be infinitely strongly coupled.

3.3 From Gullstrand-Painlevé to Kruskal-Szekeres

In this section we ask the question whether the BH solution of the GP form could be analytically continued to cover the other half of the space-time as well. This can be done by going to the Kruskal-Szekeres (KS) coordinates and analyzing the Stückelberg fields.

Let us start addressing this point by considering the following background

$$ds^2 = -dt_{GP}^2 + \left(dr + \sqrt{\frac{r_g}{r}}dt_{GP}\right)^2 + r^2d\Omega^2, \qquad \phi^a = x^a. \tag{36}$$

First we rewrite the metric in static slicing by performing the following change of the time variable

$$\phi^0 = t_{GP} = t + 2r_g\sqrt{\frac{r}{r_g}} + r_g \ln\left(\frac{\sqrt{\frac{r}{r_g}} - 1}{\sqrt{\frac{r}{r_g}} + 1}\right). \tag{37}$$

As a result the metric takes on the Schwarzschild form. In order to go to KS coordinates we use reparametrizations identical to the one used in GR

$$\left(\frac{r}{r_g} - 1\right)e^{r/r_g} = X^2 - T^2, \tag{38}$$

$$t = r_g \ln\left(\frac{X+T}{X-T}\right). \tag{39}$$

For the analysis of the ϕ's in KS coordinates we make the 'near the horizon' approximation $r/r_g \to 1$, since this is the the region of our interest. In this limit

[2] For simplicity we set $r_g = 0$ and drop the numerical factors.

the above coordinate transformation simplifies to (near the horizons $T = \pm X$, with signs corresponding to the black- and white-hole respectively)

$$\left(\frac{r}{r_g} - 1\right) = \frac{1}{e}(X^2 - T^2), \tag{40}$$

$$t = r_g \ln\left(\frac{X+T}{X-T}\right). \tag{41}$$

As a result the ϕ's take the following form (using the fact that $T^2 - X^2$ is small)

$$\phi^0 = 2r_g \ln(X+T) + r_g(\ln(1/4) + 1), \tag{42}$$

$$\phi^r = r_g\left(1 + \frac{1}{e}(X^2 - T^2)\right). \tag{43}$$

Notice that ϕ^0 is singular on the horizon of the white hole (while being regular on the black hole horizon). The metric in these coordinates is given by

$$ds^2 = 4r_g^3 \frac{e^{-r/r_g}}{r}(-dT^2 + dX^2) + r^2 d\Omega^2 \tag{44}$$

The invariant $I^{ab} = g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b$ on the above background is singular at $X = -T$, corresponding to the horizon of the white hole.

If one takes the original GP metric (36) to describe the white hole instead of the black hole (this is achieved by flipping the relative sign of the expressions in parentheses) then after analytical continuation to KS coordinates the singularity will appear on the black hole horizon rather than on the one of the white hole. The generalization of this arguments for the case of the dS is straightforward.

4 Reissner-Nordström solution on de Sitter

The ghost-free theory of massive gravity with $\beta = -\alpha^2/6$, upon its coupling to the Maxwell's theory of electromagnetism, possesses the following Reissner-Nordström solution on dS space

$$ds^2 = -dt^2 + \left(\tilde{\alpha} dr \pm \sqrt{\frac{r_g}{\tilde{\alpha} r} + \frac{2\tilde{\alpha}^2}{3\alpha} m^2 r^2 - \frac{\tilde{Q}^2}{\tilde{\alpha}^4 r^2}}\, dt\right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2, \tag{45}$$

with $\tilde{\alpha} \equiv \alpha/(\alpha+1)$ and the electromagnetic field given by

$$E = \frac{\tilde{Q}}{r^2} \qquad \text{and} \qquad B = 0. \tag{46}$$

In order to normalize the radial coordinate appropriately and to rewrite the solution in the static slicing, one needs to perform the rescaling

$$r \to \frac{r}{\tilde{\alpha}}, \tag{47}$$

supplemented with the following transformation of time

$$dt \to dt + f'(r)dr, \quad \text{with} \quad f'(r) \equiv -\frac{g_{01}}{g_{00}} = \pm \frac{\sqrt{\frac{r_g}{r} + \frac{2}{3\alpha}m^2r^2 - \frac{\tilde{Q}^2}{\tilde{\alpha}^2r^2}}}{1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2r^2 + \frac{\tilde{Q}^2}{\tilde{\alpha}^2r^2}}. \tag{48}$$

As a result the metric takes the familiar form

$$ds^2 = -(1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2r^2 + \frac{\tilde{Q}^2}{\tilde{\alpha}^2r^2})dt^2 + \frac{dr^2}{1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2r^2 + \frac{\tilde{Q}^2}{\tilde{\alpha}^2r^2}} + r^2d\Omega^2, \tag{49}$$

while the Stückelberg fields become

$$\phi^0 = t + f(r), \tag{50}$$

$$\phi^r = r + \frac{1}{\alpha}r. \tag{51}$$

In this reference frame the electromagnetic field is

$$E = \frac{\tilde{Q}}{\tilde{\alpha}r^2} \qquad \text{and} \qquad B = 0. \tag{52}$$

And, for obvious reasons the actual charge should be defined by $Q \equiv \tilde{Q}/\tilde{\alpha}$.

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A Schwarzschild-like Solution

In this appendix we concentrate on a different choice of the parameters

$$\beta = -\frac{\alpha^2}{8}, \tag{I}$$

for which some exact solutions can also be obtained. In particular, we show that there exists an exact non-singular and asymptotically flat BH solution. We choose the unitary gauge and find that X_{tt} and X_{tr} from eq. (12) vanish identically on the ansatz (16), for $w = \alpha/(\alpha + 2)$. Suggesting, that there exist fluctuations which are infinitely strongly coupled.

Then, the solution to the full set of the equations of motion takes the following form

$$
\begin{aligned}
ds^2 = & -\frac{(\alpha+2)^3\alpha^2}{(\alpha+2)^5+\alpha^5\delta}\left(1-\frac{r_g(\alpha+2)}{r\alpha}\right)dt^2 \\
& \pm \frac{2\alpha(\alpha+2)}{(\alpha+2)^5+\alpha^5\delta}\sqrt{\frac{r_g^2(\alpha+2)^6}{r^2}+\frac{r_g\alpha^6\delta}{r}}dtdr \\
& + \frac{\alpha^2}{(\alpha+2)^2}\left(1+\frac{r_g(\alpha+2)^6}{\alpha r((\alpha+2)^5+\alpha^5\delta)}\right)dr^2+\frac{\alpha^2 r^2}{(\alpha+2)^2}d\Omega^2\,,
\end{aligned}
\tag{II}
$$

where δ and r_g are positive integration constants. The transformation to the Schwarzschild coordinates is carried out in a way similar to the previous section

$$
\begin{aligned}
r &\to \frac{\alpha+2}{\alpha}r, \\
dt &\to \zeta\left(dt+f'(r)dr\right), \quad \text{with} \quad \zeta^2 \equiv \frac{(\alpha+2)^2}{\alpha^2}+\frac{\alpha^3}{(\alpha+2)^3}\delta.
\end{aligned}
\tag{III}
$$

As a result the metric takes the conventional form

$$
ds^2 = -\left(1-\frac{r_g}{r}\right)dt^2+\frac{dr^2}{1-\frac{r_g}{r}}+r^2d\Omega^2, \tag{IV}
$$

however, this should be accompanied by the 'pion' configuration

$$
\pi = \frac{\Lambda^3}{2}\left[-\left(1-\zeta\right)t^2-\frac{2}{\alpha}r^2\right], \tag{V}
$$

$$
A^0 = -\zeta\frac{\Lambda^3}{m}f(r), \tag{VI}
$$

$$
A^i = 0. \tag{VII}
$$

It is interesting that there exists a choice of parameters for which one gets the background metric identical to that of GR, supplemented with the 'pion' fields listed above. All the invariants are regular on this solution (away from the singularity in the center). It should be pointed out that (II) is the only background among those given in [21] with vanishing cosmological constant. This solution, however, has infinitely strongly coupled fluctuations; this and related issues will be discussed in a forthcoming paper [38].

B Fluctuations

In this section we study the fluctuations on the backgrounds of section 3. The analysis is done in the decoupling limit [7]

$$
m \to 0, \qquad M_{\rm pl} \to \infty, \quad \text{with} \quad \Lambda^3 \equiv M_{\rm pl}m^2 - \text{fixed}. \tag{VIII}
$$

In this limit the Lagrangian can be decomposed into tow pieces. The first describes the dynamics of the helicity-0, ± 2 modes and their interactions with each other. While the second one accounts for the helicity-± 1 modes and their nonlinear couplings to the helicity-0 degree of freedom.

The scalar-tensor Lagrangian, with the condition $\beta = -\alpha^2/6$, is given by [11]

$$\mathcal{L}_{ST} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \left(X^{(1)}_{\mu\nu} - \frac{\alpha}{\Lambda^3} X^{(2)}_{\mu\nu} - \frac{\alpha^2}{6\Lambda^3} X^{(3)}_{\mu\nu} \right). \tag{IX}$$

Here, the first term represents the linearized Einstein-Hilbert Lagrangian, while X's are defined in terms of the longitudinal mode as follows

$$\begin{aligned} X^{(1)}_{\mu\nu} &= -\frac{1}{2} \epsilon_{\mu\alpha\rho\sigma} \epsilon_{\nu\beta\rho\sigma} \Pi_{\alpha\beta}, \\ X^{(2)}_{\mu\nu} &= \frac{1}{2} \epsilon_{\mu\alpha\rho\gamma} \epsilon_{\nu\beta\sigma\gamma} \Pi_{\alpha\beta} \Pi_{\rho\sigma}, \\ X^{(3)}_{\mu\nu} &= \epsilon_{\mu\alpha\rho\gamma} \epsilon_{\nu\beta\sigma\delta} \Pi_{\alpha\beta} \Pi_{\rho\sigma} \Pi_{\gamma\delta}, \end{aligned}$$

with $\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi$ and all the repeated indices contructed by the flat space metric.

The scalar-vector Lagrangian, on the other hand, contains an infinite number of terms and schematically is given by

$$\mathcal{L}_{SV} = -\frac{1}{4} F^2_{\mu\nu} + \sum_{n=1}^{\infty} \partial A \partial A \left(\frac{\partial\partial\pi}{\Lambda^3} \right)^n, \tag{X}$$

where $F_{\mu\nu}$ denotes the field strength of the helicity-1 mode, A_μ, and the whole Lagrangian is invariant under the $U(1)$ gauge transformation $\delta A_\mu = \partial_\mu \alpha$. Remarkably, this expression has been recently resummed for the spherically symmetric ansatz [33], making the analysis of the vector fluctuations possible.

After expanding $\mathcal{L}_{ST}$, and the resummed version of $\mathcal{L}_{SV}$ (eq. (C.5) of Ref. [33] which is too lengthy to be reproduced here) to the second order in perturbations around the backgrounds of section 3, we find that the kinetic terms for both helicity-0 and helicity-± 1 fields vanish identically, on the dS as well as the Schwarzschild-dS space. This does imply that these modes are infinitely strongly coupled in this limit (unless of course they are rendered nondynamical to all orders by some symmetry or constraint). Whether this problem can be remedied by going beyond the decoupling limit, and/or by invoking kinetic terms due to quantum loops (which will be generated as long as they're not prohibited by symmetries), remains to be seen.

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