Holographic Meson Spectra in the Dense Medium with Chiral Condensate

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Abstract

We study two $1/N_c$ effects on the meson spectra by using the AdS/CFT correspondence where the $1/N_c$ corrections from the chiral condensate and the quark density are controlled by the gravitational backreaction of the massive scalar field and $U(1)$ gauge field respectively. The dual geometries with zero and nonzero current quark masses are obtained numerically. We discuss meson spectra and binding energy of heavy quarkonium with the subleading corrections in the hard wall model.

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Contents

1 Introduction

Holographic QCD provides descriptions of the strongly interacting regime of the gauge theory based on the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1, 2, 3]. The properties of the gauge theory including confinement, chiral symmetry breaking, supersymmetry reduction and glueballs are discussed in [4, 5, 6, 7, 8, 9, 10]. QCD-like models are proposed in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Baryons and chemical potential are considered in [11, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

One of the simplest bottom-up approach is the hard wall model [13, 14] where the confinement is realized by introducing an infrared cut-off in the AdS spacetime. The predictions of the hard wall model match measured values of meson masses within 10% error.

AdS geometry is dual to the large N_c limit of $U(N_c)$ gauge theory on the boundary, which is conformal. Pure AdS geometry can be considered as a UV fixed point of the holographic QCD. If we consider a subleading correction having a typical scale, the conformal symmetry should be broken. As a result, the dual geometry should be modified to include the gravitational backreaction of the bulk field dual to the subleading correction, which may improve the hard wall model.

There are two important holographic subleading corrections. One is a correction from the hadronic medium and the other is a correction from the chiral condensate. The subleading correction from the hadronic medium is the gravitational backreaction of local $U(1)$ gauge fields. The Reissner-Nordström AdS black hole (RN AdS BH) and thermal charged AdS (tcAdS), which is the zero mass limit of the RN AdS BH, are proposed as the corresponding geometries for deconfinement and confinement phases respectively [28, 31, 32]. The Hawking-Page transition between the geometries, and the meson spectra and the binding energy of heavy quarkonium on the tcAdS are discussed in [28, 31, 33, 34]. The subleading correction from the chiral condensate is the gravitational backreaction of a massive scalar field [35, 36, 37, 38]. In [35] the contribution of the scalar field to the Hawking-Page transition is investigated. In [36, 37], holographic models capturing the gravitational backreaction of the scalar field are constructed. In [38], the dual geometries with zero and nonzero quark masses are obtained by numerically solving the equations of motion of the bulk action including the scalar field. The light meson spectra and the binding energy of heavy quarkonium are discussed on both backgrounds.

In this paper we study subleading corrections from the chiral condensate and the hadronic medium as an extension of [31, 33, 34, 38]. We numerically solve the equations of motion of the bulk action including a massive scalar field and $U(1)$ gauge fields to obtain the dual geometries with zero and nonzero current quark masses. In QCD phenomenology, it is known that the chiral condensate gets reduced in the hadronic medium [39]. We constrain the dual geometries with a model-independent relation between the chiral condensate and the quark density. We calculate meson masses and binding energy of heavy quarkonium on the geometrical backgrounds. By comparing the results with [31, 33, 34, 38], we discuss the effects of the chiral condensate and the quark density.

Organization of this paper is as follows. In section 2, the bulk action and the equations of motion of it for the dual geometries are discussed. In section 3 and 4, the equations of motion of the bulk action are solved with zero and nonzero current quark masses. The meson masses and the binding energy of heavy quarkonium are obtained on the asymptotic AdS geometries. In section 5, we summarize the results.

2 Asymptotic AdS background

The gravity action in the bulk is

$$
S = \int d^{5}x \sqrt{-G} \left\{ \frac{1}{2\kappa^{2}} (\mathcal{R} - 2\Lambda) - \text{Tr} \left[|D\Phi|^{2} + m^{2} |\Phi|^{2} + \frac{1}{4g^{2}} \left(F_{MN}^{(L)} F^{(L)MN} + F_{MN}^{(R)} F^{(R)MN} \right) \right] \right\}, \quad (2.1)
$$

where $m^2 = -\frac{3}{R^2}$ and $\Lambda = -\frac{6}{R^2}$. We follow the convention that $g^2 = \frac{12\pi^2 R}{N_c}$ $\frac{2\pi^2 R}{N_c}$ and $\kappa^2 = \frac{\pi^2 R^3}{4N_c^2}$ $\frac{7^2 R^6}{4N_c^2}$ [13]. We consider the case of $N_f = 2$ and $N_c = 3$. The superscripts (L) and (R) are of $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry with $F_{MN}^{(L,R)} = \partial_M A_N^{(L,R)} - \partial_N A_M^{(L,R)} - i \left[A_M^{(L,R)}, A_N^{(L,R)} \right]$. The covariant derivative of the complex scalar field is defined by $D_M \Phi = \partial_M \Phi - i A_M^{(L)} \Phi + i \Phi A_M^{(R)}$. We set

$$
\Phi(z) = \frac{1}{2\sqrt{N_f}} \phi(z) \mathbf{1}_f e^{i2\pi^a(z)T^a},\tag{2.2}
$$

where the modulus of the complex scalar field is regarded as a background field giving the gravitational backreaction while $\pi^a(z)$ corresponding to the scalar meson is regarded as fluctuations.

The background geometries of our interest are obtained from a gravity action with the massive scalar field of the Lagrangian (2.1) and $U(1)$ gauge interaction. The corresponding action is

$$
S = \int d^5 x \sqrt{-G} \Big\{ \frac{1}{2\kappa^2} (\mathcal{R} - 2\Lambda) - \frac{1}{4} \Big[\left(\partial_M \phi \right)^2 + m^2 \phi^2 \Big] - \frac{1}{4g^2} F_{MN} F^{MN} \Big\},\tag{2.3}
$$

where $\phi(z)$ is the scalar field in (2.2). We choose an ansatz for the asymptotic AdS metric in the Fefferman-Graham coordinate as

$$
ds^{2} = \frac{R^{2}}{z^{2}} \left[-F(z)dt^{2} + G(z)dx^{2} + dz^{2} \right],
$$
\n(2.4)

where R is the AdS radius. We take $R = 1$. For the AdS black hole without the chiral condensate and the quark density, the components of the metric (2.4) are given by

$$
F(z) = \frac{(1 - Mz^4)^2}{1 + Mz^4},
$$

\n
$$
G(z) = 1 + Mz^4,
$$
\n(2.5)

where M is the black hole mass, which is asymptotically the AdS space. In the AdS spacetime, the modulus of the scalar field $\phi(z)$ is

$$
\phi(z) = m_q z + \sigma z^3,\tag{2.6}
$$

where m_q and $\sigma = \langle \bar{q}q \rangle$ are related to the current quark mass and the chiral condensate of QCD. The $U(1)$ gauge fields in (2.3) are $A_0 = A_0(z)$ whereas $A_M = 0$ for $M = 1, 2, 3, z$. In the RN AdS BH or tcAdS, the time component vector field is

$$
A_0(z) = \mu - Qz^2,
$$
\n(2.7)

where μ and Q are related to the chemical potential and the quark number density.

Now, we generalize the metric (2.4) including the effects of the chiral condensate and the quark density. The Einstein equations and the equations of motion for $\phi(z)$ and $A_0(z)$ are

$$
0 = \frac{1}{4} \left\{ -\frac{2z^2 \kappa^2 A_0'^2}{g^2} + \frac{F}{z^2 G} \left[\kappa^2 G (3\phi^2 - z^2 \phi'^2) - 6z (-3G' + zG'') \right] \right\},
$$

\n
$$
0 = \frac{1}{4} \left\{ \kappa^2 G \left(-\frac{3\phi^2}{z^2} + \phi'^2 - \frac{2z^2 A_0'^2}{g^2 F} \right) + G' \left(-\frac{12}{z} + \frac{2F'}{F} \right) - \frac{G'^2}{G} - G \left(\frac{F'^2}{F^2} + \frac{6F'}{zF} - \frac{2F''}{F} \right) + 4G'' \right\},
$$

\n
$$
0 = \frac{1}{4} \left\{ \kappa^2 \left(-\frac{3\phi^2}{z^2} - \phi'^2 + \frac{2z^2 A_0'^2}{g^2 F} \right) + \frac{3G'}{G} \left(-\frac{6}{z} + \frac{G'}{G} \right) + \frac{3F'}{F} \left(-\frac{2}{z} + \frac{G'}{G} \right) \right\},
$$

\n
$$
0 = \frac{G^{1/2}}{2z^2 F^{3/2}} \left\{ A_0' \left[zGF' + F(2G - 3zG') \right] - 2zFG A_0'' \right\},
$$

\n
$$
0 = 3\phi - 3z\phi' + \frac{1}{2}z^2\phi' \frac{F'}{F} + \frac{3}{2}z^2\phi' \frac{G'}{G} + z^2\phi''.
$$

\n(2.8)

We solve the equations to obtain the geometries with zero and nonzero quark masses, and calculate the meson masses and the binding energy of heavy quarkonium on each geometry.

3 Mesons with a zero m_q

3.1 Light meson spectra

We solve (2.8) with a zero quark mass, $m_q = 0$, which corresponds to the chiral limit showing the spontaneous symmetry breaking effect caused by the chiral condensation. In the UV limit, the scalar field $\phi(z)$ and the vector field $A_0(z)$ should be (2.6) and (2.7) respectively. We also use the fact that in the absence of the chiral condensate and the quark density, the solution should be (2.5). The perturbative solutions to (2.8) near the boundary, up to $\mathcal{O}(z^{12})$ are

$$
F(z) = 1 - 3Mz^{4} + \frac{20Q^{2}\kappa^{2} - 3g^{2}\kappa^{2}\sigma^{2}}{36g^{2}}z^{6} + 4M^{2}z^{8} + \frac{M\kappa^{2}(-104Q^{2} + 3g^{2}\sigma^{2})}{60g^{2}}z^{10},
$$

\n
$$
G(z) = 1 + Mz^{4} + \frac{-4Q^{2}\kappa^{2} - 3g^{2}\kappa^{2}\sigma^{2}}{36g^{2}}z^{6} + \frac{M\kappa^{2}(8Q^{2} - 3g^{2}\sigma^{2})}{180g^{2}}z^{10},
$$

\n
$$
\phi(z) = \sigma z^{3} + \frac{\kappa^{2}\sigma(-2Q^{2} + 3g^{2}\sigma^{2})}{48g^{2}}z^{9} + \frac{3M^{2}\sigma}{10}z^{11},
$$

\n
$$
A_{0}(z) = \mu - Qz^{2} + MQz^{6} + \frac{-16Q^{3}\kappa^{2} - 3Qg^{2}\kappa^{2}\sigma^{2}}{144g^{2}}z^{8} - M^{2}Qz^{10}.
$$
\n(3.1)

We set $M = 0$, as we are interested in the meson spectra in the confining phase. From the above perturbative solutions, we can easily find the full numerical solutions depending on parameters, σ and Q . Note that if we concentrate on the canonical ensemble μ is not important for investigating the meson spectra.

The perturbation of the massive scalar field and the bulk gauge fields describe the pion, vector meson and axial-vector meson respectively. We transform perturbed gauge fields $a_M^{(L,R)}$ of $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry to vector and axial-vector fields:

$$
v_M = \frac{1}{2} (a_M^{(L)} + a_M^{(R)}), \quad a_M = \frac{1}{2} (a_M^{(L)} - a_M^{(R)}).
$$
\n(3.2)

The action describing the mesons is

$$
\Delta S = \int d^5 x \sqrt{-G} \left[-\frac{1}{2} (\phi \partial_M \pi - a_M \phi) (\phi \partial^M \pi - a^M \phi) - \frac{1}{4g^2} (f_{MN}^{(V)} f^{MN(V)} + f_{MN}^{(A)} f^{MN(A)}) \right], \quad (3.3)
$$

where the Φ is as defined in (2.2) and $f_{MN}^{(V)}$ and $f_{MN}^{(A)}$ are the field strengths of the gauge fields v_M and a_M . The Lagrangian is in the gauge where $v_z = 0$ and $a_z = 0$. The axial vector can be decomposed into a transverse component and a longitudinal component:

$$
a_{\mu} = \bar{a}_{\mu} + \partial_{\mu} \chi. \tag{3.4}
$$

We set $\bar{a}_0 = 0$ as the Lorentz boost symmetry is not manifest. The equations of motion for v_i , a_i , π and χ are

$$
\partial_z \left(\frac{1}{z} F^{\frac{1}{2}} G^{\frac{1}{2}} \partial_z v_i \right) + \frac{1}{z} F^{-\frac{1}{2}} G^{\frac{1}{2}} m_V^2 v_i = 0,
$$

\n
$$
\partial_z \left(\frac{1}{z} F^{\frac{1}{2}} G^{\frac{1}{2}} \partial_z \bar{a}_i \right) - \frac{1}{z^3} F^{\frac{1}{2}} G^{\frac{1}{2}} \left(g^2 \phi^2 - \frac{z^2}{F} m_A^2 \right) \bar{a}_i = 0,
$$

\n
$$
\partial_z \left(\frac{1}{z} F^{-\frac{1}{2}} G^{\frac{3}{2}} \partial_z \chi \right) + g^2 \phi^2 \frac{1}{z^3} F^{-\frac{1}{2}} G^{\frac{3}{2}} (\pi - \chi) = 0,
$$

\n
$$
m_\pi^2 \partial_z \chi - g^2 \frac{F}{z^2} \phi^2 \partial_z \pi = 0,
$$
\n(3.5)

where m_V , m_A and m_{π} are the masses of vector meson, axial-vector meson and pion respectively. We solve (3.5) to obtain the meson masses in the background geometry numerically obtained from $(3.1).$

The dual geometry corresponding to the ground state contains the chiral condensate as well as the quark density. The effects of the quark density and the chiral condensate on the meson spectra are discussed separately in [33, 38]. As the quark density increases, the ρ -meson mass and the a_1 -meson mass increase [33]. As the chiral condensate increases, the ρ -meson mass decreases whereas the a_1 -meson mass increases [38]. Thus it is worth investigating the effect of both $1/N_c$ corrections to the meson spectra and which of them is more dominant. In phenomenology, the chiral condensate and the quark density are not independent. One of the known relations between them is

$$
\frac{\sigma}{\sigma_0} = 1 - 0.35 \frac{\rho_B}{\rho_0},\tag{3.6}
$$

where ρ_B is the baryon number density and related to the quark number density with Q/N_c . The relation is considered to be model-independent, on the assumption that the pion-nucleon sigma term $\Sigma_{\pi N} \simeq 45$ MeV [39]. The quark density reduces the chiral condensate and it leads to the modification of the medium requiring non-trivial boundary conditions in the gravity set-up. Instead of finding such boundary conditions, we impose the relation (3.6) on the geometry (3.1).

The quark density becomes a parameter. In our notation, $Q/Q_0 = \rho_B/\rho_0$ with $Q_0 = 1 \text{GeV}^3$ as a unit. We also choose

$$
\sigma_0 = (0.304 \text{GeV})^3, \quad z_{IR} = 1/(0.3227 \text{GeV}), \tag{3.7}
$$

of which the physical aspects are discussed in [38]. We impose Dirichlet and Neumann boundary conditions at $z = 0$ and $z = z_{IR}$ respectively. The pion decay constant f_{π} is

$$
f_{\pi}^{2} = -\frac{1}{g^{2}} \frac{\partial_{z} \bar{a}(0)}{z} \bigg|_{z=0},
$$
\n(3.8)

where $\bar{a}(0)$ is a solution of the second equation of (3.5) with $m_A = 0$, satisfying the boundary conditions $\bar{a}(0) = 1$ and $\partial_z \bar{a}(z_{IR}) = 0$ [13]. The result is shown in Table 1, Figure 1 and Figure 2.

The ρ -meson mass increases as the quark density increases. This is qualitatively consistent with the meson spectra depending on the chiral condensate [38] and the quark density [33]. The a_1 -meson mass and pion mass, however, decrease with the quark density. This is consistent with the mass spectra depending on the chiral condensate [38], but contrary to the spectra depending on the quark density [33]. It shows that the effect of the chiral condensate dominates the effect of the quark density.

The pion decay constant decreases with the quark density. From Gell-Mann–Oakes–Renner (GOR) relation

$$
f_{\pi}^2 m_{\pi}^2 = 2m_q \sigma,\tag{3.9}
$$

the deviation is $\Delta = f_{\pi}^2 m_{\pi}^2$ since $m_q = 0$. The GOR relation is satisfied up to 10^{-13}GeV^4 .

$Q(\text{GeV}^3)$	m_{ρ} (GeV)	m_{a_1} (GeV)	m_{π} (GeV)	f_{π} (GeV)	$\Delta(GeV^4)$
0.00	0.77581	1.22166	4.78271×10^{-6}	8.3073×10^{-2}	1.5786×10^{-13}
0.01	0.77583	1.21970	4.77344×10^{-6}	8.2946×10^{-2}	1.5677×10^{-13}
0.02	0.77591	1.21779	4.76435×10^{-6}	8.2818×10^{-2}	1.5569×10^{-13}
0.03	0.77603	1.21594	4.75546×10^{-6}	8.2691×10^{-2}	1.5463×10^{-13}
0.04	0.77621	1.21415	4.74675×10^{-6}	8.2565×10^{-2}	1.5360×10^{-13}
0.05	0.77643	1.21242	4.73825×10^{-6}	8.2439×10^{-2}	1.5258×10^{-13}
0.06	0.77671	1.21074	4.72994×10^{-6}	8.2313×10^{-2}	1.5158×10^{-13}
0.07	0.77703	1.20913	4.72182×10^{-6}	8.2188×10^{-2}	1.5060×10^{-13}
0.08	0.77740	1.20757	4.71390×10^{-6}	8.2063×10^{-2}	1.4964×10^{-13}
0.09	0.77782	1.20607	4.70618×10^{-6}	8.1939×10^{-2}	1.4870×10^{-13}
0.10	0.77829	1.20463	4.69867×10^{-6}	8.1815×10^{-2}	1.4778×10^{-13}

Table 1: Masses of ρ -meson, a_1 -meson and pion. f_π is the pion decay constant. Δ is the deviation from GOR relation. $m_q = 0$, $z_{IR} = 1/(0.3227 \text{GeV})$ and $\sigma_0 = (0.304 \text{GeV})^3$.

3.2 Binding energy of heavy quarkonium

We study the binding energy of heavy quarkonium in the confining phase where the quark density is (3.6) and the asymptotic geometry is (3.1) with $M = 0$. We observe breaking of a fundamental string which connects two heavy quarks. It describes dissociation of a bound state of heavy quarks into two heavy-light quark bound states. The action for a fundamental string is

$$
S = \frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{\det \partial_a X^M \partial_b X^N G_{MN}},
$$
\n(3.10)

Figure 1: Meson masses with $m_q = 0$. (a) ρ -meson (b) a_1 -meson (c)pion

Figure 2: (a)pion decay constant (b)deviation from GOR relation. $m_q = 0$.

where a, b are the string worldsheet indices and G_{MN} is the metric (2.4). We take $R = 1$ and $\alpha' = 1/2\pi$. We choose a gauge condition and an ansatz for the coordinate z as

$$
\tau = t, \ \sigma_1 = x^1 \equiv x \text{ and } z = z(x). \tag{3.11}
$$

The action becomes

$$
S = \int_0^T dt \int_{-r/2}^{r/2} dx \frac{1}{z^2} \sqrt{F(G + z'^2)}.
$$
 (3.12)

Notice that we use the Fefferman-Graham coordinate and F and G are solutions obtained by numerically solving (2.8) , whose asymptotic expansions at the boundary are given in (3.1) . The Hamiltonian density is

$$
\mathcal{H} = -\frac{1}{z^2} \frac{FG}{\sqrt{F(G + z'^2)}}.\tag{3.13}
$$

We define $z = z_0$ as the point where $\frac{\partial z}{\partial x}\Big|_{z=z_0} = 0$. The Hamiltonian density at $z = z_0$ becomes

$$
\mathcal{H}_0 = -\frac{1}{z_0^2} \sqrt{F_0 G_0},\tag{3.14}
$$

where F_0 and G_0 are the values of F and G at $z = z_0$. As Hamiltonian is conserved we can get the relation between r and z_0 from (3.13) and (3.14) as

$$
r = 2 \int_0^{z_0} dz \frac{z^2 \sqrt{F_0 G_0}}{\sqrt{z_0^4 F G^2 - z^4 F_0 G_0 G}}.
$$
\n(3.15)

The kinetic energy of two heavy quarks can be ignored since we are considering a static configuration. The potential energy is obtained from (3.12) and (3.15) as

$$
V = 2 \int_0^{z_0} dz \frac{z_0^2}{z^2} \frac{FG}{\sqrt{z_0^4 FG^2 - z^4 F_0 G_0 G}}.
$$
 (3.16)

The potential energy diverges since the heavy quarks have infinite masses at the boundary. We regularize the potential energy by subtracting the energy of two straight strings corresponding to two free heavy quarks. By choosing a gauge condition and an ansatz for the two strings as

$$
\tau = t, \ \sigma_1 = z \text{ and } x = \text{constant},\tag{3.17}
$$

the energy of two static straight strings is

$$
V_0 = 2 \int_0^{z_{IR}} \frac{1}{z^2} \sqrt{F}.
$$
\n(3.18)

The regularized binding energy of heavy quarkonium is

$$
E = 2\int_0^{z_0} dz \frac{z_0^2}{z^2} \frac{FG}{\sqrt{z_0^4 FG^2 - z^4 F_0 G_0 G}} - 2\int_0^{z_{IR}} \frac{\sqrt{F}}{z^2}.
$$
 (3.19)

The dissociation length increases with the quark density as shown in Table 2. It indicates that it takes more energy to produce a pair of heavy-light quark bound states as the quark density increases. By taking account of (3.6), this is consistent with the result of [38] that the dissociation length increases as the chiral condensate decreases.

Table 2: Dissociation length. $m_q = 0$, $z_{IR} = 1/(0.3227 \text{GeV})$ and $\sigma_0 = (0.304 \text{GeV})^3$.

4 Mesons with a nonzero m_q

4.1 Light meson spectra

We solve (2.8) with a nonzero quark mass, $m_q = 0.002383 \text{GeV}$ [38], which breaks the chiral symmetry explicitly. The asymptotic solutions are

$$
F(z) = 1 - \frac{1}{12} \kappa^2 m_q^2 z^2 + \frac{1}{144} (\kappa^4 m_q^4 - 18 \kappa^2 m_q \sigma - 3 \kappa^4 m_q^4 \log z) z^4
$$

+
$$
\frac{1}{31104} \Big(17280 \frac{\kappa^2 Q^2}{g^2} + 65 \kappa^6 m_q^6 - 396 \kappa^4 m_q^3 \sigma - 2592 \kappa^2 \sigma^2
$$

-
$$
- 66 \kappa^6 m_q^6 \log z - 864 \kappa^4 m_q^3 \sigma \log z - 72 \kappa^6 m_q^6 (\log z)^2 \Big) z^6 + \cdots
$$

$$
G(z) = 1 - \frac{1}{12} \kappa^2 m_q^2 z^2 + \frac{1}{144} (\kappa^4 m_q^4 - 18 \kappa^2 m_q \sigma - 3 \kappa^4 m_q^4 \log z) z^4
$$

+
$$
\frac{1}{31104} \Big(-3456 \frac{\kappa^2 Q^2}{g^2} + 65 \kappa^6 m_q^6 - 396 \kappa^4 m_q^3 \sigma - 2592 \kappa^2 \sigma^2
$$

-
$$
- 66 \kappa^6 m_q^6 \log z - 864 \kappa^4 m_q^3 \sigma \log z - 72 \kappa^6 m_q^6 (\log z)^2 \Big) z^6 + \cdots
$$

$$
\phi(z) = m_q z + \Big(\sigma + \frac{1}{6} \kappa^2 m_q^3 \log z \Big) z^3 + \frac{1}{576} (-13 \kappa^4 m_q^5 + 144 \kappa^2 m_q^2 \sigma + 24 \kappa^4 m_q^5 \log z) z^5
$$

+
$$
\frac{1}{62208} \Big(-1728 \kappa^2 \frac{m_q Q^2}{g^2} - 209 \kappa^6 m_q^7 + 1044 \kappa^4 m_q^4 \sigma + 10368 \kappa^2 m_q \sigma^2
$$

+
$$
174 \kappa^6 m_q^7 \log z + 3456 \kappa^4 m_q^4 \sigma \log z + 288 \kappa^6 m_q^7 (\log z)^2
$$

$$
A_0(z) = \mu - Qz^2 - \frac{1}{24}\kappa^2 m_q^2 Qz^4 + \frac{1}{864}(Q\kappa^4 m_q^4 - 36Q\kappa^2 m_q \sigma - 6Q\kappa^4 m_q^4 \log z)z^6
$$

$$
-\frac{1}{248832} \left(27648 \frac{Q^3 \kappa^2}{g^2} - 205Q\kappa^6 m_q^6 + 1872Q\kappa^4 m_q^3 \sigma + 5184Q\kappa^2 \sigma^2 + 312Q\kappa^6 m_q^6 \log z + 1728Q\kappa^4 m_q^3 \sigma \log z + 144Q\kappa^6 m_q^6 (\log z)^2\right)z^8 + \cdots
$$
 (4.1)

To investigate the meson masses, we solve (3.5) on the full numerical solutions obtained from (4.1) . We also use the relation (3.6) with Q_0 as a unit and the values of the chiral condensate and IR cutoff (3.7). The result is shown in Table 3, Figure 3 and Figure 4.

$Q(\text{GeV}^3)$	m_{ρ} (GeV)	m_{a_1} (GeV)	m_{π} (GeV)	f_{π} (GeV)	$\Delta(GeV^4)$
0.00	0.77580	1.23056	0.139617	8.4645×10^{-2}	5.7645×10^{-6}
0.01	0.77583	1.22860	0.139590	8.4519×10^{-2}	5.7631×10^{-6}
0.02	0.77590	1.22669	0.139568	8.4394×10^{-2}	5.7756×10^{-6}
0.03	0.77603	1.22484	0.139551	8.4269×10^{-2}	5.8017×10^{-6}
0.04	0.77620	1.22305	0.139541	8.4145×10^{-2}	5.8416×10^{-6}
0.05	0.77643	1.22132	0.139536	8.4021×10^{-2}	$5.\overline{8956 \times 10^{-6}}$
0.06	0.77670	1.21965	0.139538	8.3897×10^{-2}	$5.963\overline{4 \times 10^{-6}}$
0.07	0.77702	1.21803	0.139545	8.3774×10^{-2}	6.0454×10^{-6}
0.08	0.77740	1.21648	0.139559	8.3652×10^{-2}	6.1418×10^{-6}
0.09	0.77782	1.21498	0.139579	8.3530×10^{-2}	6.2523×10^{-6}
0.10	0.77829	1.21354	0.139606	8.3408×10^{-2}	6.3772×10^{-6}

Table 3: Masses of ρ -meson, a_1 -meson and pion. f_π is the pion decay constant. Δ is the deviation from GOR relation. $m_q = 0.002383 \text{GeV}, z_{IR} = 1/(0.3227 \text{GeV})$ and $\sigma_0 = (0.304 \text{GeV})^3$.

The ρ -meson mass increases whereas the a_1 -meson mass decreases as the quark density increases. The patterns are the same as the results with a zero quark mass as observed in Section 3.1. The a_1 -meson mass decreases with the quark density in both cases of zero and nonzero quark masses. By taking the relation (3.6) into account, the spectrum is consistent with the result of [38] that the a_1 -meson mass increases with the chiral condensate, but contrary to the result of [33] that the a_1 -meson mass increases with the quark density. It shows that the effect of the chiral condensate on the mass spectra dominates the effect of the quark density. A distinguishing feature is that the spectrum of pion exhibits a critical point in the case of a nonzero quark mass whereas it decreases monotonically with the quark density in the case of a zero quark mass as observed in Section 3.1. At the low quark density, the chiral condensate is high so that the quark mass is negligible. Thus the pion mass spectrum is similar to the case of a zero quark mass. At the high quark density, however, the chiral condensate is small. The effect of the quark mass becomes comparable with the chiral condensate near the critical point. Above the critical point, the effect

Figure 3: Meson masses with $m_q = 0.002383 \text{GeV}$. (a) ρ -meson (b) a_1 -meson (c)pion

Figure 4: (a)pion decay constant (b)deviation from GOR relation. $m_q = 0.002383 \text{GeV}$.

of the quark mass, which is constant, dominates the chiral condensate so that the quark density mainly contributes to the medium. The pion mass therefore increases with the quark density as it is observed in [33].

The pion decay constant decreases as the quark density increases. The deviation from the GOR relation (3.9) is $\Delta = f_{\pi}^2 m_{\pi}^2 - 2m_q \sigma$. The GOR relation is satisfied up to 10^{-6}GeV^4 .

4.2 Binding energy of heavy quarkonium

We study the binding energy of heavy quarkonium in the confining phase. We numerically solve the equations (2.8) with (4.1) as asymptotic expansions of the metric components at the boundary. The binding energy of heavy quarkonium is (3.19) and the dissociation occurs when $E = 2m_q$. The result is shown in Table 4.

The quark mass is small so that there is no big difference between the cases of zero and nonzero quark masses as shown in Table 2 and Table 4. The dissociation length with a nonzero quark mass is slightly longer. It implies that the quark mass shifts the binding energy a little bit higher.

$Q(\rm{GeV}^3)$	length with $m_q \neq 0$ (GeV
0.00	2.2382
0.01	2.2383
0.02	2.2385
0.03	2.2389
0.04	2.2394
0.05	2.2400
0.06	2.2407
0.07	2.2416
0.08	2.2426
0.09	2.2438
0.10	2.2450

Table 4: Dissociation length. $m_q = 0.002383 \text{GeV}, z_{IR} = 1/(0.3227 \text{GeV})$ and $\sigma_0 = (0.304 \text{GeV})^3$.

5 Discussion

We have studied subleading $1/N_c$ corrections from the chiral condensate and the quark density to the meson spectra and the binding energy of heavy quarkonium. We have considered the gravitational backreaction of a massive scalar field, which corresponds to the current quark mass and the chiral condensate, and the time component of $U(1)$ gauge fields, which corresponds to the chemical potential and the quark density. The geometries are numerically solved, separately with zero and nonzero current quark masses. The geometries are constrained by a model-independent relation in QCD phenomenology that the chiral condensate gets reduced linearly with the quark density. The meson masses and binding energy of heavy quarkonium are calculated on each geometry.

With a zero quark mass, the ρ -meson mass increases as the quark density increases. This is consistent with the results of [33, 38], where the meson spectra are studied with the quark density

and the chiral condensate as parameters separately. The a_1 -meson mass and pion mass decrease with the quark density. The results are consistent with the spectra with the chiral condensate, but contrary to the spectra with the quark density.

With a nonzero quark mass, the spectra of the ρ -meson and a_1 -meson present the same pattern as the spectra with a zero quark mass. The a_1 -meson mass decreases with the quark density in both cases of zero and nonzero quark masses. By taking account of the fact that the chiral condensate gets reduced with the quark density, this is consistent with the result of [38] that the a_1 -meson mass increases with the chiral condensate, but contrary to the result of [33] that a_1 meson mass increases with the quark density. It indicates that the effect of the chiral condensate on the mass spectra dominates the effect of the quark density. The mass spectrum of pion exhibits a critical point. Below the critical point, where the quark density is low, the chiral condensate is high so the quark mass is negligible. Thus the pion mass decreases with the quark density as it does with a zero quark mass. Above the critical point, where the quark density is high and the chiral condensate is low, the effect of the constant quark mass dominates the chiral condensate. As the quark density becomes the main contribution to the medium, the pion mass increases with the quark density as observed in [33].

The dissociation length of heavy quarkonium increases with the quark density for both cases of zero and nonzero quark masses. It shows that it requires more energy to produce a pair of heavy-light quark bound states as the quark density increases. This result is consistent with the dissociation length depending on the chiral condensate [38]. The dissociation length is slightly longer with a nonzero quark mass. It indicates that the quark mass shifts the binding energy higher.

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