### LSZ reduction formula in many-dimensional theory with space-space noncommutativity

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An analogue of the Lehmann—Symanzik—Zimmermann reduction formula is obtained for the case of noncommutative space-space theory. Some consequences of the reduction formula and Haag's theorem are discussed.

Keywords: LSZ reduction formula; noncommutative theory; axiomatic quantum field theory.

PACS numbers: 11.10.-z, 11.10.Cd

### 1 Introduction

117312, Russia.

It is well known that in conventional quantum field theory the LSZ reduction formula allows effective calculation of the scattering amplitudes from the Green functions [1]. In the present paper we analyze the applicability of this formula in the framework of noncommutative quantum field theory (NC-QFT). We will consider the case of a neutral scalar field in many-dimensional theory with space-space noncommutativity, so that the temporal variable commutes with the spatial ones. Let us consider the general case of SO(1, d)-invariant theory with d + 1commutative coordinates (including time) and an arbitrary even number lof noncommutative ones. The commutation relations between l noncommutative coordinates have the form

$$\left[\hat{x}^{i}, \, \hat{x}^{j}\right] = i\theta^{ij}, \quad i, \, j = 1, \, \dots, \, l, \tag{1}$$

where  $\theta^{ij}$  — real antisymmetric  $l \times l$  matrix. As we said, the rest (d + 1) variables commute with each other and all  $\hat{x}^{j}$  from (1).

In order to formulate the theory in commutative space-time, we use the Weyl ordered symbol [2, 3]  $\varphi(x)$  of the noncommutative field operator  $\Phi(\hat{x})$ :

$$\varphi(x) = \frac{1}{(2\pi)^l} \int d^l k \int \operatorname{Tr} e^{ik(x-\hat{x})} \Phi(\hat{x}), \qquad (2)$$

and the corresponding multiplication law  $\varphi_1 \star \varphi_2$  between the two symbols in the Weyl–Moyal–Groenewold form:

$$(\varphi_1 \star \varphi_2)(x) = \left[ e^{\frac{i}{2}\theta^{\mu\nu}\partial'_{\mu}\partial''_{\nu}}\varphi_1(x')\varphi_2(x'') \right]_{x'=x''=x}.$$
(3)

Relation (3) admits further generalization: for the symbols (fields) taken at different points one can define twisted tensor product [3, 4]

$$\varphi(x_1) \star \ldots \star \varphi(x_n) = \prod_{a < b} \exp\left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x_a^{\mu}} \frac{\partial}{\partial x_b^{\nu}}\right) \varphi(x_1) \ldots \varphi(x_n),$$

$$a, b = 1, 2, \ldots n.$$
(4)

Thus the algebra of field operators is deformed, and it is not clear whether one can apply the standard LSZ formula for the noncommutative fields or not.

## 2 Commutation Relations for Creation and Annihilation Operators in NCQFT

As in conventional field theory, a free real scalar field in NCQFT admits a normal mode expansion:

$$\varphi(x) = \varphi^{+}(x) + \varphi^{-}(x),$$

$$\varphi^{\pm}(x) = \frac{1}{(2\pi)^{(d+l)/2}} \int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}} e^{\pm ikx} a^{\pm}(\vec{k}) \Big|_{k^{0} = \omega(\vec{k})},$$
(5)

where  $\omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2}$ ,  $\vec{k}^2 = \vec{k_c}^2 + \vec{k}_{nc}^2$ ,  $\vec{k_c}$ — commutative part of the (d+l)-dimensional vector  $\vec{k}$ ,  $\vec{k}_{nc}$ — noncommutative part of the same vector.

Let us obtain commutation relations for the creation and annihilation operators  $a^{\pm}$  directly from the assumption that the canonical quantization of a real scalar field in NCQFT is defined by the relations

$$\begin{aligned} \left[\varphi(x) \star \partial_0 \varphi(y)\right]|_{x^0 = y^0} &= i\delta(\vec{x} - \vec{y}), \\ \left[\varphi(x) \star \varphi(y)\right]|_{x^0 = y^0} &= 0, \quad \left[\partial_0 \varphi(x) \star \partial_0 \varphi(y)\right]|_{x^0 = y^0} = 0. \end{aligned}$$
(6)

Performing an inverse Fourier transform one can get the expression for  $a^{\pm}$  from (5):

$$a^{\pm}(\vec{k}) = \frac{1}{(2\pi)^{(d+l)/2}} \int d\vec{x} \, e^{\mp ikx} \left[ \sqrt{\frac{k_0}{2}} \varphi(x) \mp \frac{i}{\sqrt{2k_0}} \partial_0 \varphi(x) \right] \Big|_{k_0 = \omega(\vec{k})}.$$
 (7)

Let us take the operator product  $a^{-}(\vec{k})a^{+}(\vec{q})$  and multiply it by  $e^{\frac{i}{2}\theta^{\mu\nu}k_{\mu}q_{\nu}} = e^{\frac{i}{2}\theta^{\mu\nu}ik_{\mu}(-i)q_{\nu}}$ . Expanding the phase factor in a series, we get

$$e^{\frac{i}{2}\theta^{\mu\nu}ik_{\mu}(-i)q_{\nu}} a^{-}(\vec{k})a^{+}(\vec{q}) = \sum_{n=0}^{\infty} \frac{(i/2)^{n}}{n!} (\theta^{\mu\nu}ik_{\mu}(-i)q_{\nu})^{n} a^{-}(\vec{k})a^{+}(\vec{q}) =$$

$$= \frac{1}{(2\pi)^{(d+l)}} \int \int d\vec{x} d\vec{y} \sum_{n=0}^{\infty} \frac{(i/2)^{n}}{n!} (\theta^{\mu\nu}ik_{\mu}(-i)q_{\nu})^{n} e^{ikx} e^{-iqy} \times \qquad (8)$$

$$\times \left(\sqrt{\frac{k_{0}}{2}}\varphi(x) + \frac{i}{\sqrt{2k_{0}}}\partial_{0}\varphi(x)\right) \left(\sqrt{\frac{q_{0}}{2}}\varphi(y) - \frac{i}{\sqrt{2q_{0}}}\partial_{0}\varphi(y)\right).$$

Next, let us replace momenta k and q with the derivatives, using the relation

$$(ik_{\mu}(-i)q_{\nu})^n e^{ikx} e^{-iqy} = (\partial_{\mu}\partial_{\nu})^n e^{ikx} e^{-iqy}, \qquad (9)$$

and perform integration by parts, so that the derivatives act on the field  $\varphi$  in each term of the series. Thus we obtain \*-product of the field operators:

$$e^{\frac{i}{2}\theta^{\mu\nu}k_{\mu}q_{\nu}}a^{-}(\vec{k})a^{+}(\vec{q}) = \frac{1}{(2\pi)^{(d+l)}}\int\int d\vec{x}\,d\vec{y}\,e^{ikx}\,e^{-iqy}\times$$

$$\times\left(\sqrt{\frac{k_{0}}{2}}\varphi(x) + \frac{i}{\sqrt{2k_{0}}}\partial_{0}\varphi(x)\right) \star\left(\sqrt{\frac{q_{0}}{2}}\varphi(y) - \frac{i}{\sqrt{2q_{0}}}\partial_{0}\varphi(y)\right)\Big|_{k_{0}=\omega(\vec{k}),\,q_{0}=\omega(\vec{q})}.$$
(10)

Similarly,

$$e^{\frac{i}{2}\theta^{\mu\nu}q_{\mu}k_{\nu}}a^{+}(\vec{q})a^{-}(\vec{k}) = \frac{1}{(2\pi)^{(d+l)}}\int\int d\vec{x}\,d\vec{y}\,e^{ikx}\,e^{-iqy}\times$$

$$\times \left(\sqrt{\frac{q_{0}}{2}}\varphi(y) - \frac{i}{\sqrt{2q_{0}}}\partial_{0}\varphi(y)\right) \star \left(\sqrt{\frac{k_{0}}{2}}\varphi(x) + \frac{i}{\sqrt{2k_{0}}}\partial_{0}\varphi(x)\right)\Big|_{k_{0}=\omega(\vec{k}),\,q_{0}=\omega(\vec{q})}.$$
(11)

Now, taking the fields  $\varphi(x)$  and  $\varphi(y)$  at equal moments of time  $x^0 = y^0$  and subtracting (11) from (10), with the use of (6) we obtain:

$$e^{\frac{i}{2}\theta^{\mu\nu}k_{\mu}q_{\nu}}a^{-}(\vec{k})a^{+}(\vec{q}) - e^{\frac{i}{2}\theta^{\mu\nu}q_{\mu}k_{\nu}}a^{+}(\vec{q})a^{-}(\vec{k}) = \delta(\vec{k} - \vec{q}), \quad (12)$$

or, in a more convenient form:

$$a^{-}(\vec{k})a^{+}(\vec{q}) = e^{-i\theta^{\mu\nu}k_{\mu}q_{\nu}} a^{+}(\vec{q})a^{-}(\vec{k}) + e^{-\frac{i}{2}\theta^{\mu\nu}k_{\mu}q_{\nu}} \delta(\vec{k} - \vec{q}).$$
(13)

In the same way we get

$$a^{\pm}(\vec{k})a^{\pm}(\vec{q}) = e^{i\theta^{\mu\nu}k_{\mu}q_{\nu}} a^{\pm}(\vec{q})a^{\pm}(\vec{k}).$$
(14)

Commutation relations (13) and (14) are equivalent to the ones obtained in [5] from general group-theoretical considerations involving the twisted Poincaré symmetry.

# 3 Analogue of the LSZ reduction formula for spacespace NCQFT

In [4] it was proposed that the expression for the noncommutative Wightman functions has the following form:

$$W_{\star}(x_1,\ldots,x_n) = \langle 0|\varphi(x_1)\star\ldots\star\varphi(x_n)|0\rangle, \qquad (15)$$

where  $\star$ -product of fields taken at independent points is given by (4).

In accordance with (15) we suppose that the noncommutative Green functions are

$$G_{\star}(x_1, \ldots, x_n) = \langle 0 | T(\varphi(x_1) \star \ldots \star \varphi(x_n)) | 0 \rangle, \tag{16}$$

where we defined time-ordered  $\star$ -product of fields as straightforward generalization of the usual *T*-product:

$$T(\varphi_1(x_1) \star \ldots \star \varphi_n(x_n)) = \varphi_{\sigma_1}(x_{\sigma_1}) \star \ldots \star \varphi_{\sigma_n}(x_{\sigma_n}),$$
  
$$x_{\sigma_1}^0 > x_{\sigma_2}^0 > \ldots > x_{\sigma_n}^0.$$
 (17)

Below we extend the classical proof of the LSZ formula [1, 6, 7] to the case of space-space NCQFT.

Let us single out the variable  $x_1$  and consider the expression

$$\lim_{p_1^0 \to \omega(\vec{p}_1)} (p_1^2 - m^2) \int dx_1^0 d\vec{x}_1 \, e^{-ip_1 x_1} \langle 0 | T(\varphi(x_1) \star \dots \star \varphi(x_n)) | 0 \rangle.$$
(18)

Dividing integration over  $dx_1^0$  into three parts

$$\int (\dots) dx_1^0 = \int_{-\infty}^{-\tau} (\dots) dx_1^0 + \int_{-\tau}^{\tau} (\dots) dx_1^0 + \int_{\tau}^{+\infty} (\dots) dx_1^0, \qquad (19)$$

we denote the summands as  $I_1(\tau)$ ,  $I_2(\tau)$ , and  $I_3(\tau)$  respectively.

Using expression  $(p_1^2 - m^2)e^{-ip_1x_1} = (\Box_1 - m^2)e^{-ip_1x_1}$  and performing integration by parts, we get:

$$I_{1}(\tau) = \int d\vec{x}_{1} e^{i\vec{p}_{1}\vec{x}_{1} + i\omega(\vec{p}_{1})\tau} \langle 0|T(\varphi(x_{2}) \star \dots \star \varphi(x_{n})) \star$$
  
 
$$\star (i\omega(\vec{p}_{1}) - \frac{\partial}{\partial\tau})\varphi(-\tau, \vec{x}_{1})|0\rangle - \int_{-\infty}^{-\tau} dx_{1}^{0} \int d\vec{x}_{1} e^{i\vec{p}_{1}\vec{x}_{1} - i\omega(\vec{p}_{1})x_{1}^{0}} \times \qquad (20)$$
  
 
$$\times \langle 0|T(\varphi(x_{2}) \star \dots \star \varphi(x_{n})) \star (\Box_{1} - m^{2})\varphi(x_{1})|0\rangle.$$

Here  $\Box_1 \equiv \frac{\partial^2}{\partial (x_1^1)^2} + \frac{\partial^2}{\partial (x_1^2)^2} + \frac{\partial^2}{\partial (x_1^3)^2} - \frac{\partial^2}{\partial (x_1^0)^2}$ , and  $\tau$  is taken sufficiently large so that the permutation of  $\varphi(x_1)$  to the last position on the right is possible.

Next, we use the Fourier-expression for  $\varphi(-\tau, \vec{x}_1)$ :

$$\varphi(-\tau, \vec{x}_1) = \frac{1}{(2\pi)^{(d+l)/2}} \int dk_0 \, d\vec{k} \, e^{-ik_0\tau} \, e^{-i\vec{k}\vec{x}_1} \tilde{\varphi}(k). \tag{21}$$

All derivatives in the \*-product will act on the factor  $e^{-i\vec{k}\vec{x}_1}$  in the Fourier expansion of  $\varphi(x_1)$ . Therefore, additional factor  $N(k_{nc})$  will appear. Note that  $N(k_{nc})$  depends only on the noncommutative part of  $\vec{k}$ .

Let us also take into account the asymptotic representation for the field  $\varphi$ :

$$\lim_{t \to -\infty} \int dk_0 \, e^{it(k_0 - \omega(\vec{k}))} \tilde{\varphi}(k) = \frac{1}{\sqrt{2\omega(\vec{k})}} a_{in}^+(\vec{k}). \tag{22}$$

Taking the limit  $\tau \to \infty$ , we obtain:

$$I_{1} = \lim_{\tau \to \infty} I_{1}(\tau) = i(2\pi)^{(d+l)/2} \int dk_{0} (k_{0} + \omega(\vec{p}_{1})) \delta(k_{0} - \omega(\vec{p}_{1})) \times \\ \times \langle 0|T(\varphi(x_{2}) \star \dots \star \varphi(x_{n})) N(p_{1,nc}) \frac{a_{in}^{+}(\vec{p}_{1})}{\sqrt{2\omega(\vec{p}_{1})}} |0\rangle =$$
(23)
$$= i(2\pi)^{(d+l)/2} \sqrt{2\omega(\vec{p}_{1})} N(p_{1,nc}) \langle 0|T(\varphi(x_{2}) \star \dots \star \varphi(x_{n})) a_{in}^{+}(\vec{p}_{1}) |0\rangle.$$

In this limit the second term of the expression (20) is equal to null.

Similar calculations for  $I_3(\tau)$  will give:

$$I_{3} = i(2\pi)^{(d+l)/2} \sqrt{2\omega(\vec{p}_{1})} N(p_{1,nc}) \langle 0 | a_{out}^{+}(\vec{p}_{1}) T(\varphi(x_{2}) \star \ldots \star \varphi(x_{n})) | 0 \rangle = 0.$$
(24)

As to the second summand in (19),  $I_2(\tau)$  can be presented as

$$\int_{-\infty}^{\infty} dx_1^0 e^{-ip_1^0 x_1^0} \chi(x_1^0, \tau) F(x_1),$$

$$\chi(x_1^0, \tau) = \begin{cases} 1, & |x_1^0| \le \tau; \\ 0, & |x_1^0| > \tau. \end{cases}$$
(25)

The integrand contains a generalized function with compact support, so its Fourier-transform is a smooth function and doesn't have a pole. For this reason

$$\lim_{p_1^0 \to \omega(\vec{p}_1)} (p_1^2 - m^2) \int_{-\tau}^{\tau} dx_1^0 \int d\vec{x}_1 \, e^{-ip_1 x_1} \langle 0 | T(\varphi(x_1) \star \ldots \star \varphi(x_n)) | 0 \rangle = 0.$$
(26)

Following similar limiting procedure over  $x_1$  and  $x_2$  consecutively, we obtain

$$\lim_{p_{2}^{0} \to \omega(\vec{p}_{2})} \lim_{p_{1}^{0} \to \omega(\vec{p}_{1})} (p_{2}^{2} - m^{2}) (p_{1}^{2} - m^{2}) \int dx_{1} \int dx_{2} e^{-ip_{2}x_{2} - ip_{1}x_{1}} \times \\ \times \langle 0|T(\varphi(x_{1}) \star \dots \star \varphi(x_{n}))|0\rangle = \left[i(2\pi)^{(d+l)/2}\right]^{2} \sqrt{2\omega(\vec{p}_{1})} \sqrt{2\omega(\vec{p}_{2})} \times$$
(27)
$$\times N(p_{2,nc}) N(p_{1,nc}) \langle 0|T(\varphi(x_{3}) \star \dots \star \varphi(x_{n}))a_{in}^{+}(\vec{p}_{2})a_{in}^{+}(\vec{p}_{1})|0\rangle.$$

Now let us replace the secong procedure (over  $x_2$ ) with the one corresponding to transition to the bottom sheet of the mass hyperboloid, that is  $\lim_{p_2^0 \to -\omega(\vec{p}_2)} (p_2^2 - m^2).$  Making use of the asymptotic representation

$$\lim_{t \to \pm \infty} \int dk_0 \, e^{it(k_0 + \omega(\vec{k}))} \tilde{\varphi}(k) = \frac{1}{\sqrt{2\omega(\vec{k})}} a^-_{in(out)}(-\vec{k}), \tag{28}$$

we obtain:

$$\lim_{p_{2}^{0} \to -\omega(\vec{p}_{2})} \lim_{p_{1}^{0} \to \omega(\vec{p}_{1})} (p_{2}^{2} - m^{2}) (p_{1}^{2} - m^{2}) \int dx_{1} \int dx_{2} e^{-ip_{2}x_{2} - ip_{1}x_{1}} \times \\ \times \langle 0|T(\varphi(x_{1}) \star \dots \star \varphi(x_{n}))|0\rangle = \left(i(2\pi)^{(d+l)/2}\right)^{2} \sqrt{2\omega(\vec{p}_{1})} \sqrt{2\omega(\vec{p}_{2})} \times \\ \times [N(p_{2,nc})N(p_{1,nc})\langle 0|a_{out}^{-}(-\vec{p}_{2})T(\varphi(x_{3}) \star \dots \star \varphi(x_{n}))a_{in}^{+}(\vec{p}_{1})|0\rangle - \\ -\tilde{N}(p_{2,nc})\tilde{N}(p_{1,nc})\langle 0|T(\varphi(x_{3}) \star \dots \star \varphi(x_{n}))a_{in}^{-}(-\vec{p}_{2})a_{in}^{+}(\vec{p}_{1})|0\rangle].$$

$$(29)$$

The first term here is the contribution of  $I_3(\tau)$ , which is not equal to zero in this limit. Let us make the substitution  $\vec{p_2} \rightarrow -\vec{p_2}$ . We consider the scattering processes in which  $\vec{p_1}$  — incoming momentum,  $\vec{p_2}$  — outcoming momentum, and  $\vec{p_1} \neq \vec{p_2}$ . In accordance with (13) we can commute  $a_{in}^-(\vec{p_2})$ and  $a_{in}^+(\vec{p_1})$  in the second term of (29) so that  $a_{in}^-(\vec{p_2})$  can act on the vacuum state and give null.

We can repeat the above-mentioned procedure n times — until nothing is left under the time-ordered  $\star$ -product. Now the additional factor  $N(p_{1,nc}) \times$  ... ×  $N(p_{n,nc})$  can be expressed in explicit form: each derivative  $\partial_{\mu}$  in (4) should be replaced with  $ip_{\mu}$ , and we have:

$$N(p_{1,nc}) \times \ldots \times N(p_{n,nc}) = \exp\left[-\frac{i}{2}\theta_{\mu\nu}\sum_{a < b} p_a^{\mu}p_b^{\nu}\right]\Big|_{-p_{out}}, \qquad (30)$$
$$a, b = 1, \ldots, n,$$

where  $|_{-p_{out}}$  means that outcoming momenta should be taken with the minus sign (as the result of the substitution  $\vec{p} \rightarrow -\vec{p}$  we made earlier).

The final expression for the scattering amplitude:

$$\langle 0|a_{out}^{-}(\vec{p}_{1}) \dots a_{out}^{-}(\vec{p}_{k}) a_{in}^{+}(\vec{p}_{k+1}) \dots a_{in}^{+}(\vec{p}_{n})|0\rangle = \left[\frac{1}{i(2\pi)^{(d+l)/2}}\right]^{n} \times \\ \times \exp\left[\frac{i}{2} \theta_{\mu\nu} \sum_{a < b} p_{a}^{\mu} p_{b}^{\nu}\right] \bigg|_{-p_{out}} \prod_{j=1}^{n} \frac{p_{j}^{2} - m^{2}}{\sqrt{2\omega(\vec{p}_{j})}} G_{\star}(-p_{1}, \dots, -p_{k}, p_{n}, \dots, p_{k+1})$$

$$(31)$$

where  $G_{\star}(p_1, \ldots, p_n)$  — Fourier transform of the noncommutative Green function:

$$G_{\star}(p_1, \dots, p_n) = \int dx_1 \dots dx_n \exp\left[-i\sum_{j=1}^n p_j x_j\right] G_{\star}(x_1, \dots, x_n).$$
 (32)

Relation (31) is a noncommutative analogue of the LSZ reduction formula. This result corresponds to the one obtained in [8], authors of which didn't use the  $\star$ -product between the fields taken at different points and considered the Green functions with the usual time-ordered product of noncommutative fields. The difference between the two results is the additional phase-factor (30) due to the chosen form of the Green function (16).

#### 4 Consequences

Now we can extend to NCQFT the considerations that were originally proposed in [9] for the case of commutative theory. Suppose that we have two noncommutative SO(1, d)-invariant theories on Hilbert spaces  $\mathcal{H}_1$  and  $\mathcal{H}_2$  respectively, related by a unitary transformation. Let  $\varphi_1$  and  $\varphi_2$  be two irreducible sets of field operators defined in  $\mathcal{H}_1$  and  $\mathcal{H}_2$ . Let  $\langle p'_1, \ldots, p'_n | p_1, \ldots, p_m \rangle_i$ , i = 1, 2 be inelastic scattering amplitudes of the process  $m \to n$  for the fields  $\varphi_1$  and  $\varphi_2$  respectively. In accordance with the reduction formula (31)

$$< p'_{1}, \dots, p'_{n} | p_{1}, \dots, p_{m} >_{i} \sim$$

$$\sim \int dx_{1} \dots dx_{n+m} \exp\{i (-p_{1} x_{1} - \dots - p_{m} x_{m} + p'_{1} x_{m+1} + \dots + p'_{n} x_{n+m})\} \times$$

$$\times \prod_{j=1}^{n+m} (\Box_{j} - m^{2}) \langle 0 | T(\varphi_{i} (x_{1}) \star \dots \star \varphi_{i} (x_{n+m})) | 0 \rangle,$$

$$i = 1, 2.$$
(33)

Let us also take into account the results obtained for the generalized Haag's theorem in the context of noncommutative theory [10, 11]. Namely, it was shown that in two SO(1, d)-invariant theories, related by a unitary transformation, the two-, three, ..., d + 1-point Wightman functions coincide:

$$\langle 0|\varphi_1(x_1) \star \ldots \star \varphi_1(x_s)|0\rangle = \langle 0|\varphi_2(x_1) \star \ldots \star \varphi_2(x_s)|0\rangle,$$
  
$$2 \leqslant s \leqslant d+1.$$
(34)

From (33) and (34) it follows that the amplitudes  $\langle p'_1, \ldots, p'_n | p_1, \ldots, p_m \rangle_1$ and  $\langle p'_1, \ldots, p'_n | p_1, \ldots, p_m \rangle_2$  coincide in the two theories if

$$m+n \leqslant d+1. \tag{35}$$

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