A Novel Scheme for Downlink Opportunistic Interference Alignment

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Abstract—In this paper we propose a downlink codebook-based opportunistic interference alignment (OIA) in a three-cell MIMO system. A codebook composed of multiple transmit vector sets is utilized to improve the multiuser selection diversity. The sum rate increases as the size of the codebook grows. In addition, during the user selection, effective channel gain and alignment metric are combined to generate a novel criterion, which improves the system performance, especially at low SNR. Furthermore, a threshold-based feedback approach is introduced to reduce the feedback load in the proposed scheme. Both the analytical results and simulations show that the proposed scheme provides significant improvement in terms of sum rates with no feedback load growth and slight increase of complexity.

I. INTRODUCTION

With the exponential growth in mobile data traffic, interference has been one of the major challenges in wireless communication. Interference alignment (IA) [1] is a technique recently introduced to improve the performance of interference networks. Unfortunately, extensive channel station information and a large amount of computation is required to achieve the optimal DoFs [2], which makes IA too complicated to be implemented in practice. Motivated by opportunistic beamforming (OBF) [3], OIA schemes are developed in [4]–[9], which only require limited feedback and modest computational complexity. Though OIA takes advantage of multiuser diversity via opportunistic user equipment (UE) scheduling, [5] proves that the number of required UEs grows with an exponential scale in order to achieve an optimal DoF. In practical systems, the number of UEs is usually limited, so the improvement of sum rate performance via OIA is not obvious. On the other hand, UEs are selected from the perspective of interference reduction in OIA, while the selection is done from the point of view of channel gains in Maximum SNR (MAX-SNR) scheduling. OIA outperforms MAX-SNR in an interference limited environment while MAX-SNR provides better performance in a noise limited environment. Neither of them have a wide SNR range of application.

In this paper, we propose a downlink codebook-based opportunistic interference alignment (COIA) scheme. Compared with the conventional OIA schemes,three improvements are made:

(i) A codebook composed of multiple transmit beamforming vector sets for three base stations (BSs) is utilized to bring more selection diversity. More specifically, besides the UE scheduling, BSs select a transmit beamforming vector set from the codebook to enhance system sum rate. Consequently, fewer UEs are needed in COIA to achieve the same sum rate compared with the conventional OIA. Particularly, with the theoretical analysis of the expectation of the alignment metric value, we can pre-calculate the required numbers of candidate UEs with various codebook sizes for the same sum rate as that of the conventional OIA. Note that code-book based uplink OIA schemes have recently been proposed in [10], [11]. Our downlink COIA is completely different from them because the codebook in our scheme is utilized to exploit the selection diversity, while the codebook in [10], [11] is used to reduce the feedforward load. We propose our downlink COIA scheme to improve the sum rate performance, while they focus on the UE and feedback bit scaling law with their uplink COIA schemes.

(ii) An effective UE selection metric adaptively balancing the noise and interference power is introduced to overcome the shortcoming of OIA schemes at low SNR. With the above two improvements, COIA achieves better sum rate performance than MAX-SNR scheduling and the conventional OIA the same number of candidate UEs.

(iii) When the codebook size is very large, the feedback load becomes unacceptable of our previous COIA scheme in [12]. In this paper, a threshold-based feedback scheme, which has never been discussed in OIA to the best of our knowledge, is explored to reduce the system feedback load in our COIA. We address the relationship between the threshold value and feedback load by an explicit expression, so that the feedback load of the proposed scheme can be adjusted to the same as that of the conventional OIA by setting an appropriate threshold.

Throughout the paper, we describe matrices and vectors by bold upper and lower case letters. $\mathbf{A}^H$, $\lambda_{\max}(\mathbf{A})$, $\mathbf{v}_{\max}(\mathbf{A})$, $\|\mathbf{A}\|$ and $\mathbf{A}^{-1}$ denote the conjugate transpose, the largest eigenvalue, the eigenvector corresponding to the largest eigenvalue, L_2-norm and the inverse of matrix $\mathbf{A}$, respectively.

II. SYSTEM MODEL

We consider a 3-cell MIMO downlink system with a single BS and K UEs in each cell. Both the BS and the UEs are each equipped with two antennas. In the i-th cell, $i = 1, 2, 3$, the BS sends a data stream to a scheduled UE with a normalized transmit beamforming vector $\mathbf{w}_i$, where $\|\mathbf{w}_i\| = 1$. For convenience we denote the k-th UE in the i-th cell as UE $[k, i]$, where $1 \le k \le K, k \in \mathbb{N}$ and $i = 1, 2, 3$. Quasi-static channels between BSs and UEs are assumed. The received signal at UE $[k, i]$ is

$$\mathbf{y}_{k_i} = \sqrt{P_S}\mathbf{H}_{k_i,i}\mathbf{w}_i x_i + \sqrt{P_I} \sum_{j=1,j\neq i}^{3} \mathbf{H}_{k_i,j}\mathbf{w}_j x_j + \mathbf{n}_{k_i}, \quad (1)$$

where $\mathbf{H}_{k_i,j} \in \mathbb{C}^{2\times 2}$ is the channel matrix from the BS in the j-th cell to UE $[k, i]$. Elements of $\mathbf{H}_{k_i,j}$ are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance. x_j is the signal transmitted by the j-th BS with a transmit power constraint $\mathbb{E}[|x_j|^2] = 1$. $\mathbf{n}_{k_i} \in \mathbb{C}^{2\times 1} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I})$ is the additive complex Gaussian noise at UE $[k, i]$. P_S stands for the received data power and P_I is the received average interference power from each interfering BS. Denoting the receive beamforming vector of UE $[k, i]$ by $\mathbf{v}_{k_i} \in \mathbb{C}^{2\times 1}$, the received signal after receive beamforming is

$$\mathbf{v}_{k_i}^H\mathbf{y}_{k_i} = \sqrt{P_S}\mathbf{v}_{k_i}^H\mathbf{H}_{k_i,i}\mathbf{w}_i x_i + \sqrt{P_I} \sum_{j=1,j\neq i}^{3} \mathbf{v}_{k_i}^H\mathbf{H}_{k_i,j}\mathbf{w}_j x_j + \mathbf{v}_{k_i}^H\mathbf{n}_{k_i}. \quad (2)$$

We also assume there exist low-rate but reliable and delay-free backhaul links between each UE with its relevant BS as well as among the BSs.

Based on (2), the signal-to-interference-plus-noise ratio (SINR) of the data stream of UE $[k, i]$ is given by

$$\text{SINR}_{k_i} = \frac{P_S|\mathbf{v}_{k_i}^H\mathbf{H}_{k_i,i}\mathbf{w}_i|^2}{\sigma_n^2 + P_I\sum_{j=1,j\neq i}^{3}|\mathbf{v}_{k_i}^H\mathbf{H}_{k_i,j}\mathbf{w}_j|^2}. \quad (3)$$

III. CONVENTIONAL OPPORTUNISTIC USER SELECTION SCHEMES

In both OBF and conventional OIA, the transmit beamforming vectors $\mathbf{w}$ are all generated randomly, while the UE selection criteria are significantly different. We discuss three opportunistic UE selection schemes (i.e., MAX-SINR, MAX-SNR and the conventional OIA) in this section.

A. MAX-SINR

MAX-SINR has been shown to be an optimal opportunistic UE selection scheme in the sense of sum rate so far [5]. The receive beamforming vector of UE $[k, i]$ is $\mathbf{v}_{k_i}^{\text{MAX-SINR}} = \mathbf{v}_{\max}(\mathbf{A}_{k_i}^{-1}\mathbf{B}_{k_i})$ to maximize SINR_{k_i}, where $\mathbf{A}_{k_i} = \sigma_n^2\mathbf{I} + P_I\sum_{j=1,j\neq i}^{3}\mathbf{H}_{k_i,j}\mathbf{w}_j\mathbf{w}_j^H\mathbf{H}_{k_i,j}^H$ and $\mathbf{B}_{k_i} = P_S\mathbf{H}_{k_i,i}\mathbf{w}_i\mathbf{w}_i^H\mathbf{H}_{k_i,i}^H$. The corresponding SINR is

$$\text{SINR}_{k_i} = \lambda_{\max}(\mathbf{A}_{k_i}^{-1}\mathbf{B}_{k_i}).$$

The UE with the largest SINR is selected, i.e.,

$$k_i^{\text{MAX-SINR}} = \underset{1\le k_i \le K}{\arg\max} \text{SINR}_{k_i}.$$

B. MAX-SNR

In MAX-SNR, the receive beamforming vector of UE $[k, i]$ is designed as $\mathbf{v}_{k_i}^{\text{MAX-SNR}} = \frac{\mathbf{H}_{k_i,i}\mathbf{w}_i}{\|\mathbf{H}_{k_i,i}\mathbf{w}_i\|}$ to maximize the SNR. The corresponding SNR is $\text{SNR}_{k_i} = \frac{P_S\|\mathbf{H}_{k_i,i}\mathbf{w}_i\|^2}{\sigma_n^2}$. In homogeneous network, each UE calculates its effective channel gain

$$\beta_{k_i} = \|\mathbf{H}_{k_i,i}\mathbf{w}_i\|^2 \quad (4)$$

and informs the corresponding BS. The BS selects the UE with the largest SNR, i.e.,

$$k_i^{\text{MAX-SNR}} = \underset{1\le k_i \le K}{\arg\max} \beta_{k_i}. \quad (5)$$

C. Conventional OIA

In OIA [5], the UE whose interference signals are most aligned with each other is selected. The alignment of interfering signals is measured by their chordal distance. The metric value of UE $[k, i]$ is

$$\gamma_{k_i} = \frac{\|\mathbf{w}_{i'}^H\mathbf{H}_{k_i,i'}^H\mathbf{H}_{k_i,i''}\mathbf{w}_{i''}\|^2}{\|\mathbf{H}_{k_i,i'}\mathbf{w}_{i'}\|^2 \cdot \|\mathbf{H}_{k_i,i''}\mathbf{w}_{i''}\|^2}, \quad (6)$$

where i' is the i-th element of vector $[2, 3, 1]$, and i'' is the i-th element of vector $[3, 1, 2]$. Each UE sends the value back to the relevant BS. The preferred UE in the ith cell is

$$k_i^{\text{OIA}} = \underset{1\le k_i \le K}{\arg\max} \gamma_{k_i}. \quad (7)$$

IV. NOVEL CODEBOOK-BASED OIA SCHEME

In this section, we propose an OIA scheme with a codebook of transmit beamforming vector sets. A novel selection criterion adaptive to noise and interference power as well as a threshold-based feedback are further developed to enhance the sum rate performance and control the feedback load of the proposed scheme.

A. Codebook-Based OIA

In codebook-based downlink OIA, BSs choose transmit beamforming vectors from multiple vectors in a codebook every time slot. The codebook composed of transmit beamforming vector sets is denoted by $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_S\}$, where $\mathbf{c}_s$ is the concatenation of the s-th set of random unit-norm transmit beamforming vectors, i.e., $\mathbf{c}_s = [\mathbf{w}_{1,s}^H, \mathbf{w}_{2,s}^H, \mathbf{w}_{3,s}^H]^H \in \mathbb{C}^{6\times 1}$, and S is the size of the codebook. All the UEs and BSs know the codebook $\mathcal{C}$.

The UE selection and data transmission in COIA is shown as follows:

Step 1: Each BS broadcasts pilots for channel estimation. Every UE obtains channel estimations $\hat{\mathbf{H}}_{k_i,i}$ and $\hat{\mathbf{H}}_{k_i,j}$.

Step 2: Using the estimated channel information, each UE calculates S alignment metric values for S transmit beamforming vector sets in $\mathcal{C}$. The alignment metric value of UE $[k, i]$ for the s-th transmit beamforming vector set is

$$\gamma_{k_i,s} = \frac{\|\mathbf{w}_{i'',s}^H \hat{\mathbf{H}}_{k_i,i'}^H \hat{\mathbf{H}}_{k_i,i''} \mathbf{w}_{i'',s}\|^2}{\|\hat{\mathbf{H}}_{k_i,i'} \mathbf{w}_{i',s}\|^2 \cdot \|\hat{\mathbf{H}}_{k_i,i''} \mathbf{w}_{i'',s}\|^2}. \tag{8}$$

Each UE feeds the analog metric values back to the BS in its own cell.

Step 3: BSs exchange the analog feedback, then select the preferred transmit beamforming vector set as well as the corresponding served UEs. Regarding a specific transmit beamforming vector set $\mathbf{c}_s$, we first find the UE with the largest alignment metric value in the i-th cell and the corresponding metric value, denoted by

$$\bar{k}_{i,s} = \arg\max_{1 \le k_i \le K} \gamma_{k_i,s}, \quad \bar{\gamma}_{i,s} = \max_{1 \le k_i \le K} \gamma_{k_i,s}. \tag{9}$$

After that, we calculate the average of the largest alignment metric values of three cells for $\mathbf{c}_s$, which is given by

$$\bar{\gamma}_s = \frac{1}{3} \sum_{i=1}^{3} \bar{\gamma}_{i,s}. \tag{10}$$

The preferred transmit beamforming vector set is then selected among all sets in the codebook as

$$s^* = \arg\max_{1 \le s \le S} \bar{\gamma}_s, \tag{11}$$

which means we choose the codeword to maximize the average of the largest alignment metric values of three cells. Once s^* is determined, the selected transmit beamforming vector for the i-th transmitter is $\mathbf{w}_{i,s^*}$ and the preferred UE being served in the i-th cell is $\bar{k}_{i,s^*}$.

Step 4: Each BS serves the selected UE with the preferred transmit beamforming vector.

B. Analysis of Codebook-Based OIA

We provide a theoretical analysis of the codebook-based OIA. The expectation of the alignment metric value of the selected UE increases as S grows. In other words, compared with the conventional OIA ($S = 1$), the interfering signals of the selected UE are aligned more and more closely when the codebook size increases in COIA.

The expectation of $\bar{\gamma}_{s^*}$ (i.e., the average of alignment metric value of the selected UE $\bar{k}_{i,s^*}$), is approximately given by

$$\mathbb{E}[\bar{\gamma}_{s^*}]$$
$$= \frac{S}{(\mathbb{B}(a,b))^S} \cdot P(a,b,S) \cdot \mathbb{B}(aS + k_1 + \cdots + k_{S-1} + 1, b), \tag{12}$$

where $\mathbb{B}(\cdot)$ is the beta function, $a = \frac{3K(K+2)}{K+1}$, $b = \frac{3(K+2)}{K+1}$, and

$$P(a,b,S)$$
$$= \sum_{k_1=0}^{\infty} \cdots \sum_{k_{S-1}=0}^{\infty} \frac{(1-b)_{k_1} \cdots (1-b)_{k_{S-1}}}{(a+k_1) \cdots (a+k_{S-1}) k_1! \cdots k_{S-1}!}. \tag{13}$$

See Appendix for the derivation of (12).

TABLE I
$\mathbb{E}[\bar{\gamma}_{s^*}]$ OF VARIOUS K AND S.

	$S=1$	$S=2$	$S=3$	$S=4$
$K=10$	0.9091	0.9351	0.9461	0.9529
$K=15$	0.9375	0.9559	0.9635	0.9680
$K=20$	0.9524	0.9666	0.9725	0.9759

Given number of UEs K and codebook size S, we can get the expectation efficiently because $\mathbb{B}(\cdot)$ can be calculated directly in MATLAB. TABLE I shows $\mathbb{E}[\bar{\gamma}_{s^*}]$ for various K and S. It can be seen that the expectation increases with the growth of S for the same K.

Since the average rate of the selected UE increases as the expectation of its alignment metric value grows [5], with the help of (12), we can get the number of UEs K with variable codebook sizes S for the same expectation, i.e., the same sum rate performance. For example, when $K = 20, S = 1$, $\mathbb{E}[\bar{\gamma}_{s^*}] = 0.9524$. Then letting $S = 4$ and setting the left hand side value of (12) as 0.9524, we can get the required number of UEs $K = 10$ for the same performance. Only half of UEs are needed in COIA with $S = 4$ codebook compared with the conventional OIA.

C. Hybrid Criterion in COIA

The effective channel gain in MAX-SNR of UE $[k, i]$ with the s-th transmit beamforming vector is defined as

$$\beta_{k_i,s} = \|\hat{\mathbf{H}}_{k_i,i} \mathbf{w}_{i,s}\|^2. \tag{14}$$

We introduce a hybrid criterion with (8) and (14), which is given by

$$\alpha_{k_i,s} = [0, (1-\theta)]^+ \cdot \gamma_{k_i,s} + \theta \cdot \beta_{k_i,s}, \tag{15}$$

where $\theta = \frac{P_S/P_I}{P_S/\sigma_n^2} = \frac{\sigma_n^2}{P_I}$ and $[x, 0]^+ = \max(x, 0)$. The BSs select the transmit beamforming vector set and UEs in the same way as that mentioned in Part. IV-A, except replacing $\gamma_{k_i,s}$ with $\alpha_{k_i,s}$ in (9). We can see that when the power of interference is smaller than that of noise, i.e., the system is at low SNR, the hybrid metric value only depends on the effective channel gain. With the increase of interference power, the proportion of the effective channel gain decreases and the effect of OIA UE selection is enhanced. At very high SNR, the hybrid metric value is almost equal to the OIA metric value. With the proposed hybrid criterion adaptive to noise and interference power, the COIA scheme achieves better sum rate performance in both low and high SNR regions.

D. Threshold-Based Feedback in COIA

In the OIA scheme proposed in [5], every UE feeds back an alignment metric value to the corresponding BS, which we refer to as full feedback. K values are needed to complete a UE selection in each cell. In COIA, if full feedback is adopted, the amount of feedback will be $K \cdot S$ due to the utilization of the S size transmit beamforming vector codebook. The feedback load becomes unacceptable when S is large. Here we propose a threshold-based feedback technique to reduce the

feedback needs (by more than 75%) while preserving the sum rate performance in COIA. Similar techniques are introduced in [13], [14]. However, they take only signal and noise into consideration and ignore interference, which degrades their performance in multi-cell systems.

In the proposed threshold-based feedback scheme, each UE compares its selection metric value to a predefined threshold T and decides locally whether it sends feedback to the BS, only those who fall above T are allowed to be fed back. BSs make selections with the feedback. If no feedback is received by all three BSs, transmit beamforming vector set and UE in each cell is selected randomly.

Choosing a proper threshold is critical. We first characterize the statistics of the alignment metric γ and the effective channel gain β in terms of cumulative distributive function (CDF) and probability density function (PDF). As (6) shows, the alignment metric γ is related to the chordal distance between two vectors. Using the results of [15],the CDF of γ, denoted by $F_\gamma(x)$ is given by

$$F_\gamma(x) = P(\gamma \leq x) = x, \quad 0 \leq x \leq 1. \quad (16)$$

The PDF of γ is

$$f_\gamma(x) = 1, \quad 0 \leq x \leq 1. \quad (17)$$

The effective channel gain β is defined as (4). With a certain unit-norm vector $\mathbf{w}$, β has a central chi-square distribution. The CDF of β is given by

$$F_\beta(y) = P(\beta \leq y) = 1 - e^{-y}(1 + y), \quad y \geq 0. \quad (18)$$

The PDF of β is

$$f_\beta(y) = ye^{-y}, \quad y \geq 0. \quad (19)$$

The normalized average feedback load $\bar{F}$ is defined as the ratio of the average load per selection to the total amount of full feedback (KS) in each cell. Apparently, with a threshold T, we have

$$\bar{F}^{\text{OIA}}(T) = 1 - F_\gamma(T) \quad (20)$$

and

$$\bar{F}^{\text{MAX-SNR}}(T) = 1 - F_\beta(T). \quad (21)$$

For a given feedback load requirement $\bar{F}$ (e.g., 1/4), we can get the threshold T^{OIA} and $T^{\text{MAX-SNR}}$ with (20) and (21), respectively.

In COIA, the metric value α is given by (15). With (17) and (19), when $0 < \theta < 1$, the CDF of α is given by (we omit the derivation due to space limitations)

$$F_\alpha(z) = \begin{cases} \dfrac{1}{1-\theta} - \dfrac{e^{-\frac{z}{\theta}}(\theta + z)}{(1-\theta)\theta}, & 0 \leq z < 1-\theta \\ \dfrac{e^{-\frac{z}{\theta}}\left(\theta + z - e^{\frac{1-\theta}{\theta}}(2\theta - 1 + z)\right)}{(1-\theta)\theta}, & z \geq 1-\theta \end{cases}. \quad (22)$$

When $\theta \geq 1$, $\alpha = \theta \cdot \beta$, the CDF of α is given by

$$F_\alpha(z) = 1 - e^{-\frac{z}{\theta}}(1 + \frac{z}{\theta}), \quad z \geq 0. \quad (23)$$

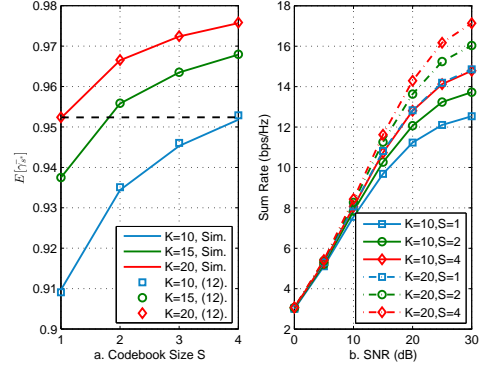Fig. 1. Expectations and sum rates of full feedback codebook-based OIA of various S and K. $P_S = P_I$.

The normalized average feedback load $\bar{F}$ is of COIA is

$$\bar{F}^{\text{COIA}}(T) = 1 - F_\alpha(T). \quad (24)$$

We can choose a proper threshold T^{COIA} with (24). It should be remarked that T^{COIA} is related to θ, i.e., T^{COIA} is adaptive to noise and interference power, because we consider both signal channel quality and interference condition in COIA. It is different form T^{OIA} and $T^{\text{MAX-SNR}}$ as (20) and (21) are only functions of T.

E. Complexity Analysis

We analysis the computational complexity in the UE selection step of each UE in COIA briefly. Only complex multiplication is considered for simplicity. Assume the number of receive antennas is N. For every vector set in the codebook, the effective channel gain consumes $\mathcal{O}(N)$ computation, and operations of $\mathcal{O}(N)$ are needed to get the alignment metric. So $2S \cdot \mathcal{O}(N)$ computation is required for each UE in COIA. Just like MAX-SNR and the conventional OIA, the computational complexity of COIA is $\mathcal{O}(N)$. Note that we do not take the computation of channel estimation into account here because it is necessary for receive beamforming regardless of UE selection schemes.

V. NUMERICAL SIMULATIONS

In this section, we simulate the performance of the proposed COIA scheme. Preferred UErs are selected with different schemes, then the selected UE k_i^* executes MAX-SINR receive beamforming. It should be mentioned that we focus on the UE selection scheme while the receive beamforming vectors design after UE selection is not studied in depth. The sum rate is obtained according to the equation

$$R = \sum_{i=1}^{3} \log_2(1 + \text{SINR}_{k_i^*}), \quad (25)$$

where $\text{SINR}_{k_i^*}$ can be obtained by (3). Perfect channel estimation is assumed at all the UEs.

Fig. 1. a shows the expectation of the alignment metric value of the selected UEs in the codebook-based OIA. It is clear that the expectation increases with the increase of
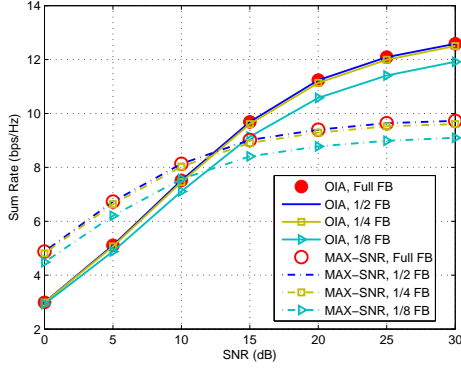
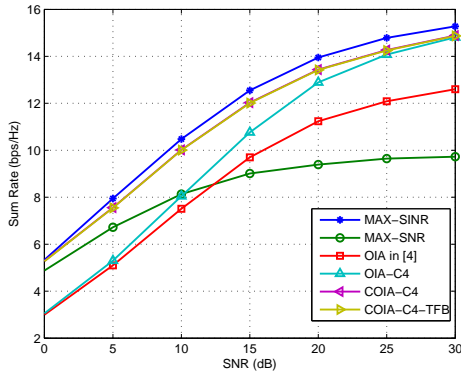Fig. 2. Sum rates of threshold-based feedback OIA and MAX-SNR. $K = 10, P_S = P_I$.

Fig. 3. Sum rates of various opportunistic UE selection schemes. $K = 10, P_S = P_I$.

codebook size S for the same number of UEs K. Further, the configuration $K = 10, S = 4$ has almost the same expectation as $K = 20, s = 1$ has. Fig. 1. b shows the sum rates of full feedback codebook-based OIA. The sum rates increase with the growth of K and S, especially at high SNR. Only $K = 10$ UEs are needed in $S = 4$ codebook-based OIA to achieve almost the same sum rate performance as $K = 20$ UEs in the conventional OIA (i.e., $S = 1$), which is consistent with the analytical result in Part. IV-B.

Fig. 2 shows the sum rates of threshold-based feedback OIA and MAX-SNR with various feedback load requirment $\bar{F}$. In OIA, the threshold value T^{OIA} is 0.5, 0.75 and 0.875 when $\bar{F}(T^{\text{OIA}})$ is 1/2, 1/4 and 1/8, respectively. In MAX-SNR, the threshold value $T^{\text{MAX-SNR}}$ is 1.6785, 2.6925 and 3.6070 when $\bar{F}(T^{\text{MAX-SNR}})$ is 1/2, 1/4 and 1/8, respectively. The sum rate loss is negligible with 1/2 and 1/4 feedback load in the threshold-based feedback scheme. It means that a large reduction of the feedback is possible while preserving most of the sum rate performance.

In Fig. 3, the sum rates of various schemes are shown with $K = 10$ UEs in each cell and $P_S = P_I$. The word "C4" in legends means the codebook size $S = 4$. The threshold-based feedback scheme is marked as "TFB". For a fair comparison, we choose $\bar{F}^{\text{COIA}} = 1/S = 1/4$ and calculate T^{COIA} according

to (22), (23) and (24). MAX-SNR scheme outperforms the conventional OIA [5] and even codebook-based OIA in the low SNR region but gets significant performance degradation at high SNR. The proposed COIA with $S = 4$ codebook approaches better sum rate performance than MAX-SNR and the conventional OIA in all range of SNR. In COIA, the sum rate performance of threshold-based feedback is almost the same as that of full feedback, which means the proposed COIA with threshold-based feedback outperforms MAX-SNR and the conventional OIA with the same feedback load.

VI. CONCLUSIONS

In this paper, we have proposed a codebook-based opportunistic interference alignment with a hybrid selection criterion and threshold-based feedback in a three-cell MIMO downlink system. A codebook composed of multiple transmit vector sets is utilized to improve the multiuser selection diversity. Effective channel gain and alignment metric are combined to generate a novel metric for a wide SNR range of application. A threshold is employed to reduce the feedback load in COIA. Both the analytical results and simulations indicate that the proposed COIA scheme provides higher sum rates in wide SNR region than the conventional OIA scheme with the same feedback load. In the future, we will focus on the COIA scheme with multiple data streams for each UE.

APPENDIX

Defined in (9), it can be proved easily that $\bar{\gamma}_{i,s} \sim \text{Beta}(K, 1)$ in a similar way to [5], where $\text{Beta}(\cdot)$ is the beta distribution. We omit the proof due to space limitations.

Lemma 1. ([16]): Let $S = \sum_{i=i}^{k} X_i$ where X_i are i.i.d. random variables of $Beta(\alpha, \beta)$. The distribution of S can be approximated by:

$$Beta(e, f); e = Ff, f = \frac{F}{\sigma^2(1+F)^3} \qquad (26)$$

where $E = \sum \mathbb{E}[X_i]$, $F = \frac{E}{1-E}$, and $\sigma^2 = \sum Var(X_i)$.

As $\bar{\gamma}_{i,s}, i = 1, 2, 3$ are i.i.d. beta-distributed random variables, using Lemma 1, we can consider $\bar{\gamma}_s$ defined by (10) as a new beta-distributed random variable, i.e., $\bar{\gamma}_s \sim \text{Beta}(a, b)$, where $a = \frac{3K(K+2)}{K+1}$, $b = \frac{3(K+2)}{K+1}$.

Let $x = \bar{\gamma}_{s*}$ for convenience, the explicit expression of the expectation of the maximum of i.i.d. beta-distributed random variables $\bar{\gamma}_1, \ldots, \bar{\gamma}_S$ is

$$\mathbb{E}[\bar{\gamma}_{s*}] = \mathbb{E}[\max_{1 \le s \le S} \bar{\gamma}_s] = \mathbb{E}[x]$$
$$= \int_0^1 x \cdot f_X(x)dx = \int_0^1 x \cdot Sf(x)(F(x))^{S-1}dx \qquad (27)$$
$$= \int_0^1 x \cdot \frac{Sx^{a-1}(1-b)^{b-1}}{\mathbb{B}(a,b)}(\mathbb{I}_x(a,b))^{S-1}dx,$$

where $\mathbb{B}(\cdot)$ is the beta function and $\mathbb{I}_x(\cdot)$ is the regularized incomplete beta function. Using the series expansion

$$\mathbb{I}_x(a,b) = \frac{x^a}{\mathbb{B}(a,b)}\sum_{k=0}^{\infty}\frac{(1-b)_k x^k}{(a+k)k!},$$

(27) can be expressed as

$$\mathbb{E}[\bar{\gamma}_{s*}]$$

$$= \frac{S}{(\mathbb{B}(a,b))^S} \int_0^1 x^a (1-x)^{b-1} \left(x^a \sum_{k=0}^{\infty} \frac{(1-b)_k x^k}{(a+k)k!} \right)^{S-1}$$

$$= \frac{S}{(\mathbb{B}(a,b))^S}$$

$$\times \sum_{k_1=0}^{\infty} \cdots \sum_{k_{S-1}=0}^{\infty} \frac{(1-b)_{k_1} \cdots (1-b)_{k_{S-1}}}{(a+k_1) \cdots (a+k_{S-1})k_1! \cdots k_{S-1}!}$$

$$\times \int_0^1 x^{aS+k_1+\cdots+k_{S-1}} (1-x)^{b-1} dx$$

$$= \frac{S}{(\mathbb{B}(a,b))^S}$$

$$\times \sum_{k_1=0}^{\infty} \cdots \sum_{k_{S-1}=0}^{\infty} \frac{(1-b)_{k_1} \cdots (1-b)_{k_{S-1}}}{(a+k_1) \cdots (a+k_{S-1})k_1! \cdots k_{S-1}!}$$

$$\times \mathbb{B}(aS+k_1+\cdots+k_{S-1}+1, b),$$

where $(\cdot)_k$ is the Pochhammer symbol defined as $(x)_k = x(x+1)\cdots(x+k-1)$.

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