

HOMOGENEOUS ANR -SPACES AND ALEXANDROFF MANIFOLDS

V. VALOV

ABSTRACT. We specify a result of Yokoi [18] by proving that if G is an abelian group and X is a homogeneous metric ANR compactum with $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$, then X is an (n, G) -bubble. This implies that any such space X has the following properties: $\check{H}^{n-1}(A; G) \neq 0$ for every closed separator A of X , and X is an Alexandroff manifold with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n-2$. We also prove that if X is a homogeneous metric continuum with $\check{H}^n(X; G) \neq 0$, then $\check{H}^{n-1}(C; G) \neq 0$ for any partition C of X such that $\dim_G C \leq n-1$. The last provides a partial answer to a question of Kallipoliti and Papasoglu [8].

1. INTRODUCTION

In this paper we establish some properties of homogeneous metric compacta. One of the main problems concerning homogeneous compacta is the Bing-Borsuk [2] question whether any closed separator of an n -dimensional homogeneous metric ANR -space is cyclic in dimension $n-1$. Yokoi's result [18, Theorem 3.3] provides a partial answer to this question. Our first result is a clarification of [18, Theorem 3.3], we omit the requirement G to be a principal ideal domain.

Theorem 1.1. *Let X be a homogeneous metric ANR -continuum with cohomological dimension $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$, where G is an abelian group. Then X is an (n, G) -bubble.*

Following Yokoi [18], a compactum X is called an (n, G) -bubble if $\check{H}^n(X; G) \neq 0$ and $\check{H}^n(A; G) = 0$ for every closed proper set $A \subset X$. This is a reformulation of the notion of an n -bubble introduced by Kuperberg [13] and Choi [4], see also Karimov-Repovš [9] for the stronger notion of an \check{H}^n -bubble.

Date: September 25, 2018.

2000 Mathematics Subject Classification. Primary 55M10, 55M15; Secondary 54F45, 54C55.

Key words and phrases. absolute neighborhood retracts, cohomological dimension, cohomology groups homogeneous compacta.

The author was partially supported by NSERC Grant 261914-08.

Corollary 1.2. *Let X be a homogeneous metric ANR-continuum with $\check{H}^n(X; G) \neq 0$ and $\dim_G X = n$. Then*

- (i) X is a strong V_G^n -continuum;
- (ii) X is an Alexandroff manifold with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n - 2$;
- (iii) If A is a closed separator of X , then $\check{H}^{n-1}(A; G) \neq 0$.

Item (ii) from Corollary 1.2 was proved in [11] under the additional requirement that G is a principal ideal domain. Here, $\check{H}^n(X; G)$ denotes the reduced n -th Čech cohomology group of X with coefficients from G . We say that a set $A \subset X$ is massive if A has a non-empty interior in X .

Recall that a space X is an *Alexandroff manifold with respect to a class \mathcal{C}* , see [10] and [16], if for every two disjoint, closed massive sets $X_0, X_1 \subset X$ there exists an open cover ω of X such that there is no partition P in X between X_0 and X_1 admitting an ω -map onto a space $Y \in \mathcal{C}$. This definition is inspired by the Alexandroff's notion of V^n -continua [1] which is obtained when \mathcal{C} is the class of all compacta whose covering dimension is $\leq n - 2$.

A compactum X is said to be a V_G^n -continuum [15] if for every two open, disjoint subsets U_1, U_2 of X there exists an open cover ω of $X_0 = X \setminus (U_1 \cup U_2)$ such that any partition P in X between U_1 and U_2 does not admit an ω -map g of P onto a space Y with $g^*: \check{H}^{n-1}(Y; G) \rightarrow \check{H}^{n-1}(P; G)$ being trivial. If, in addition, there exists also an element $e \in \check{H}^{n-1}(X_0; G)$ such that for any partition P between U_1 and U_2 and any ω -map g of P onto a space Y we have $0 \neq i_P^*(e) \in g^*(\check{H}^{n-1}(Y; G))$, where i_P is the embedding $P \hookrightarrow X_0$, X is called a *strong V_G^n -continuum*. For example, every (n, G) -bubble is a strong V_G^n -continuum, see [11].

It follows directly from the above definitions that V_G^n -continua are Alexandroff manifolds with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n - 2$. The converse is not true, for example the Menger n -dimensional compactum is a V^n -continuum, but it is not a V_G^n -continuum for any group G , see [11].

Homogeneous metric compacta (not necessary ANR) are also interesting class of spaces. Krupski [12] has shown that any such an n -dimensional space is a Cantor n -manifold. One of the ingredients of Krupski' proof is the classical result established by Hurewicz-Menger [6] and Tumarkin [17] that any n -dimensional compactum contains an n -dimensional Cantor n -manifold. Kuz'minov [14] provided a cohomological counterpart of this fact about V^n -continua (see [10] for more general results). Concerning V_G^n -continua we have the following statement which provides a positive answer of Question 4.3 from [11]:

Theorem 1.3. *Any compactum X with $\check{H}^n(X; G) \neq 0$ contains a strong V_G^n -continuum.*

Theorem 1.3 could be also compared with the cohomological version of Kuperberg's result [13, Theorem 5.5] that any n -dimensional n -cyclic metric compactum X for which $\check{H}^n(X; \mathbb{Z})$ is finitely generated (in particular, any n -dimensional n -cyclic ANR) contains an n -bubble.

The condition $\check{H}^n(X; G) \neq 0$ in the above theorem is essential. For example, let X be the square \mathbb{I}^2 and suppose X contains a (strong) $V_{\mathbb{Z}}^2$ -continuum K , where \mathbb{Z} is the group of all integers. Then $\dim K = 2$, so K contains a non-empty interior U in X . Now, take a segment \mathbb{I} joining two opposite sides of \mathbb{I}^2 and intersecting U . Obviously $\mathbb{I} \cap K$ is a partition of K . Since $\check{H}^{n-1}(P; G) \neq 0$ for every partition P of a V_G^n -continuum, $\check{H}^1(\mathbb{I} \cap K; \mathbb{Z}) \neq 0$. On the other hand, because $\mathbb{I} \cap K$ is an one-dimensional subset of \mathbb{I} , $\check{H}^1(\mathbb{I} \cap K; \mathbb{Z}) = 0$.

Kuperberg [13] asked whether any n -dimensional metric compactum contains an $(n-1)$ -bubble. This question is still open, but the following corollary provides a result in this direction.

Corollary 1.4. *Any compactum X with $\dim_G X = n$ contains a strong V_G^{n-1} -continuum.*

For finite-dimensional metric compacta and V_G^n -continua Theorem 1.3 and Corollary 1.4 were established in [15].

Proposition 1.5. *Let X be a homogeneous metric continuum with $\check{H}^n(X; G) \neq 0$. Then for any partition C of X there exists an open cover ω of X such that C does not admit any ω -map $g: C \rightarrow Y$ onto a space of dimension $\dim_G Y \leq n - 1$ with $g^* : \check{H}^{n-1}(Y; G) \rightarrow \check{H}^{n-1}(C; G)$ being a trivial homomorphism.*

Let us note that the n -dimensional universal Menger compactum μ^n , which is a homogeneous continuum cyclic in dimension n , contains a separator C such that $\dim C = n$ and $\check{H}^{n-1}(C; G) = 0$, see [11, Corollary 2.6]. Therefore, the restriction $\dim_G Y \leq n - 1$ in Proposition 1.5 and the condition $\dim_G P \leq n - 1$ in next corollary are essential.

Corollary 1.6. *Let X be a homogeneous metric continuum such that $\check{H}^n(X; G) \neq 0$. Then $\check{H}^{n-1}(P; G) \neq 0$ for every partition P of X with $\dim_G P \leq n - 1$.*

Kallipoliti and Papasoglu [8] have shown that every 2-dimensional locally connected, simply connected homogeneous metric continuum can not be separated by an arc, and asked if the simple connectedness can be dropped from this result. Corollary 1.6 provides a partial answer to the Kallipoliti-Papasoglu question.

2. COHOMOLOGICAL CARRIERS

In this section we consider cohomological carriers of non-trivial elements of $\check{H}^n(X; G)$ and establish some properties of them. We fix an abelian group G , an integer n and a metric compactum X with $\check{H}^n(X; G) \neq 0$. A closed non-empty set $A \subset X$ is said to be a *cohomological carrier* (shortly, a carrier) of a non-zero element $\alpha \in \check{H}^n(X; G)$ if $i_A^*(\alpha) \neq 0$ and $i_B^*(\alpha) = 0$ for every proper closed subset $B \subset A$, where i_A denotes the inclusion map $A \hookrightarrow X$.

Lemma 2.1. *For every non-zero element $\alpha \in \check{H}^n(X; G)$ there exists a carrier. Moreover, if A is a carrier of α , then $\check{H}^{n-1}(P; G) \neq 0$ for any closed partition P of A .*

Proof. The first part of Lemma 2.1 follows from Zorn's lemma and the continuity of Čech cohomology. For the second part, suppose A is a carrier of α and P a partition of A . Then there exist two closed proper subsets A_1 and A_2 of A such that $A = A_1 \cup A_2$ and $P = A_1 \cap A_2$. Consider the Mayer-Vietoris exact sequence

$$\check{H}^{n-1}(P; G) \rightarrow \check{H}^n(A; G) \rightarrow \check{H}^n(A_1; G) \oplus \check{H}^n(A_2; G).$$

For every $i = 1, 2$ let $\partial_i: \check{H}^n(A; G) \rightarrow \check{H}^n(A_i; G)$ be generated by the inclusion $A_i \hookrightarrow A$. Denote also by Δ and φ , respectively, the left and right homomorphism of the above sequence. Since each A_i is a proper subset of A we have $\varphi(\beta) = (\partial_1(\beta), \partial_2(\beta)) = 0$, where $\beta = i_A^*(\alpha)$. So, there exists $\gamma \in \check{H}^{n-1}(P; G)$ with $\Delta(\gamma) = \beta$. Because β is a non-trivial element of $\check{H}^n(A; G)$, so is γ . Hence, $\check{H}^{n-1}(P; G) \neq 0$. \square

Everywhere below, if $B \subset A$, then $i_{A,B}$ denotes the inclusion $B \hookrightarrow A$. The next lemma is an analogue of Lemma 4 from [4].

Lemma 2.2. *Let $A \subset X$ be a carrier of a non-trivial element $\alpha \in \check{H}^n(X; G)$ and B a closed subset of X . Then $A \subset B$ if and only if $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$, where $j_A = i_{A \cup B, A}$ and $j_B = i_{A \cup B, B}$ are the corresponding inclusions.*

Proof. Obviously $A \subset B$ implies $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$. Suppose that $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$, but B does not contain A . Then $A \cap B$ is a proper closed subset of A (possibly empty). The left homomorphism in the Mayer-Vietoris exact sequence

$$\check{H}^n(A \cup B; G) \rightarrow \check{H}^n(A; G) \oplus \check{H}^n(B; G) \rightarrow \check{H}^n(A \cap B; G).$$

is defined by (j_A^*, j_B^*) , while the right one i^* assigns to each $(\beta_1, \beta_2) \in$

$\check{H}^n(A; G) \oplus \check{H}^n(B; G)$ the difference $i_{A, A \cap B}^*(\beta_1) - i_{B, A \cap B}^*(\beta_2)$. Since $A \cap B$ is a proper subset of A , $i_{A \cap B}^*(\alpha) = 0$. Then $i^*((i_A^*(\alpha), 0)) = 0$. Consequently, there exists $\beta \in \check{H}^n(A \cup B; G)$ with $(j_A^*(\beta), j_B^*(\beta)) = (i_A^*(\alpha), 0)$. So, $\beta \in \text{Ker}(j_B^*)$ and, according to our assumption, $\beta \in \text{Ker}(j_A^*)$. The last relation contradicts $i_A^*(\alpha) \neq 0$. Therefore, $A \subset B$. \square

The next proposition is actually Theorem 5 from [4]. We provide a different proof of that theorem.

Proposition 2.3. *Let $A \subset X$ be a carrier for a non-trivial element of $\check{H}^n(X; G)$ and $f: X \rightarrow X$ a map homotopic to the identity on X . If $\dim_G X \leq n$, then $A \subset fA$.*

Proof. By [7], we can identify the cohomological group $\check{H}^n(A \cup fA; G)$ with the group $[A \cup fA, K(G, n)]$ of homotopy classes from $A \cup fA$ to $K(G, n)$, where $n > 0$ and $K(G, n)$ denotes an Eilenberg-MacLane complex. Similarly, $\check{H}^n(A; G)$ and $\check{H}^n(fA; G)$ are identified with the groups $[A, K(G, n)]$ and $[fA, K(G, n)]$, respectively.

By Lemma 2.2, it suffices to prove that if $\alpha \in \text{Ker}(j_{fA}^*)$ then $\alpha \in \text{Ker}(j_A^*)$. So, we fix $\alpha \in \check{H}^n(A \cup fA; G)$ with $j_{fA}^*(\alpha) = 0$. According to the above identifications, there exists a map $g: A \cup fA \rightarrow K(G, n)$ such that $\alpha = [g]$. Since $\dim_G X \leq n$, g can be extended to a map $\tilde{g}: X \rightarrow K(G, n)$. Because $j_{fA}^*(\alpha) = 0$, we can find a homotopy $H_1: fA \times [0, 1] \rightarrow K(G, n)$ with $H_1(x, 0) = g(x)$ and $H_1(x, 1) = *$ for all $x \in fA$, where $*$ is a point from $K(G, n)$. Then the homotopy $H_2: A \times [0, 1] \rightarrow K(G, n)$, $H_2(x, t) = H_1(f(x), t)$, connects the constant map $\kappa: A \hookrightarrow *$ and the map $h: A \rightarrow K(G, n)$ defined by $h(x) = g(f(x))$. Next, consider a homotopy $F: A \times [0, 1] \rightarrow X$ with $F(x, 0) = f(x)$ and $F(x, 1) = x$, and define $H: A \times [0, 1] \rightarrow K(G, n)$ by $H(x, t) = \tilde{g}(F(x, t))$. We have $H(x, 0) = g(f(x))$ and $H(x, 1) = g(x)$ for all $x \in A$. Hence, H is connecting the maps h and the restriction $g|_A$ of g over A . Finally, combining H and H_2 , we can produce a homotopy on A connecting the map $g|_A$ and the constant map κ . Hence, $j_A^*(\alpha) = [g|_A] = 0$. \square

Before proving the next property of carriers, we introduce some more notations. If ω is a finite open cover of a closed set $Z \subset X$, we denote by $|\omega|$ and p_ω , respectively, the nerve of ω and a map from Z onto $|\omega|$ generated by a partition of unity subordinated to ω . Furthermore, if $C \subset Z$ and $\omega(C) = \{W \cap C : W \in \omega\}$, then $p_{\omega(C)}: C \rightarrow |\omega(C)|$ is the restriction $p_\omega|_C$. Recall also that p_ω generates maps $p_\omega^*: \check{H}^k(|\omega|; G) \rightarrow \check{H}^k(Z; G)$ for $k \geq 0$. Moreover, if $q_\omega: Z \rightarrow |\omega|$ is a map generating

by (another) partition of unity subordinated to ω , then p_ω and q_ω are homotopic. So, $p_\omega^* = q_\omega^*$.

Proposition 2.4. *Let K be a carrier for a non-trivial element of $\alpha \in \check{H}^n(X; G)$. Then for any two open disjoint subsets U_1 and U_2 of K there exists an open cover ω of $K \setminus (U_1 \cap U_2)$ and an element $\eta \in \check{H}^{n-1}(|\omega|; G)$ such that $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) \neq 0$ for every partition C of K between U_1 and U_2 , where $i_{\omega(C)}$ is the inclusion $|\omega(C)| \hookrightarrow |\omega|$.*

Proof. Let $K_1 = K \setminus U_1$, $K_2 = K \setminus U_2$, C be a partition of K between U_1 and U_2 , and F_1, F_2 closed subsets of K such that: $F_1 \cap F_2 = C$, $F_1 \cup F_2 = K$, $F_1 \subset K_1$ and $F_2 \subset K_2$. Consider the commutative diagram whose rows are Mayer-Vietoris sequences:

$$\begin{array}{ccccc} \check{H}^{n-1}(K_1 \cap K_2; G) & \xrightarrow{\delta} & \check{H}^n(K; G) & \xrightarrow{j} & \check{H}^n(K_1; G) \oplus \check{H}^n(K_2; G) \\ \downarrow i^* & & \downarrow id & & \downarrow i_1^* \oplus i_2^* \\ \check{H}^{n-1}(C; G) & \xrightarrow{\delta_1} & \check{H}^n(K; G) & \xrightarrow{j_1} & \check{H}^n(F_1; G) \oplus \check{H}^n(F_2; G). \end{array}$$

Since $j(\beta) = 0$, where $\beta = i_K^*(\alpha)$, there exists a non-zero element $\gamma \in \check{H}^{n-1}(K_1 \cap K_2; G)$ with $\delta(\gamma) = \beta$. Consequently, we can find an open cover ω of $K_1 \cap K_2$ and a non-trivial element $\eta \in \check{H}^{n-1}(|\omega|; G)$ such that $p_\omega^*(\eta) = \gamma$. It follows from the commutativity of the above diagram that $i^*(\gamma) \neq 0$. Then the equality $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) = i^*(\gamma)$ completes the proof. \square

Proposition 2.5. *Every carrier for a non-trivial element of $\check{H}^n(X; G)$ is a strong V_G^n -continuum.*

Proof. Indeed, suppose U_1 and U_2 are open subsets of K having disjoint closures. Let ω be an open cover of $K_0 = K \setminus (U_1 \cup U_2)$ and $e \in \check{H}^{n-1}(|\omega|; G)$ a non-trivial element satisfying the hypotheses of Proposition 2.4. Assume C a partition of K between U_1 and U_2 admitting an ω -map g onto a space T . Thus, we can find a finite open cover τ of T such that $\nu = g^{-1}(\tau)$ is refining ω . Let $p_\nu: C \rightarrow |\nu|$ be a map onto the nerve of ν generated by a partition of unity subordinated to ν . Obviously, the function $V \in \tau \rightarrow g^{-1}(V) \in \nu$ provides a simplicial homeomorphism $g_\nu^\tau: |\tau| \rightarrow |\nu|$. Then the maps p_ν and $g_\tau = g_\nu^\tau \circ \pi_\tau \circ g$, where $\pi_\tau: T \rightarrow |\tau|$ is a map generated by a partition of unity subordinated to $|\tau|$, are homotopic. Hence, $p_\nu^* = g^* \circ \pi_\tau^* \circ (g_\nu^\tau)^*$.

On the other hand, since ν refines ω , we can find a map $\varphi_\nu: |\nu| \rightarrow |\omega(C)|$ such that $p_{\omega(C)}$ and $\varphi_\nu \circ p_\nu$ are homotopic. Therefore, $p_{\omega(C)}^* = p_\nu^* \circ \varphi_\nu^*$. According to Proposition 2.4, there exists $\eta \in \check{H}^{n-1}(|\omega|; G)$

with $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) \neq 0$. Since $i_C^*(p_\omega^*(\eta)) = p_{\omega(C)}^*(i_{\omega(C)}^*(\eta))$, $e = p_\omega^*(\eta)$ is a non-zero element of $\check{H}^{n-1}(K_0; G)$. Here $p_\omega: K_0 \rightarrow |\omega|$ is a map generated by a partition of unity subordinated to ω and $i_C: C \hookrightarrow K_0$ is the inclusion map. Moreover, both equalities $p_\nu^* = g^* \circ \pi_\tau^* \circ (g_\nu^\tau)^*$ and $p_{\omega(C)}^* = p_\nu^* \circ \varphi_\nu^*$ yield that $i_C^*(e)$ is a non-trivial element of $g^*(\check{H}^{n-1}(T; G))$. \square

3. PROOF OF THEOREM 1.1 AND COROLLARY 1.2

Proof of Theorem 1.1. Suppose G is an abelian group, X is a non-trivial homogeneous metric ANR-continuum with $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$. Since X is an ANR, $n \geq 1$ and there exists a positive ϵ such that any two ϵ -close maps from X into X are homotopic (we say that two maps $f_1, f_2: X \rightarrow X$ are ϵ -close if $\text{dist}(f_1(x), f_2(x)) < \epsilon$ for each $x \in X$).

It suffices to show that if A is a carrier for a non-trivial element $\alpha \in \check{H}^n(X; G)$, then $A = X$. Indeed, suppose there exists a proper subset $B \subset X$ with $\check{H}^n(B; G) \neq 0$, and choose a non-trivial element $\beta \in \check{H}^n(B; G)$. Since $\dim_G X = n$, there exists $\alpha \in \check{H}^n(X; G)$ with $\beta = i_B(\alpha)$. Because the carrier of α is X and B is a proper subset of X , $i_B(\alpha) = 0$, a contradiction.

Next, suppose $A \subset X$ is a carrier for a non-trivial $\alpha \in \check{H}^n(X; G)$ and A is a proper set. According to the Effros' theorem [5], there corresponds a positive number δ with the following property: whenever x and y are points from X and $\text{dist}(x, y) < \delta$, there is a homeomorphism $h: X \rightarrow X$ such that $h(x) = y$ and h is ϵ -close to the identity id_X on X . Because $A \neq X$, we can choose points $a \in A$ and $b \notin A$ with $\text{dist}(a, b) < \delta$. Consequently, there would be a homeomorphism $f: X \rightarrow X$ such that $f(a) = b$ and $\text{dist}(f, \text{id}_X) < \epsilon$. Obviously, $g = f^{-1}: (X, fA) \rightarrow (X, A)$ generates the isomorphism $g^*: \check{H}^n(X; G) \rightarrow \check{H}^n(X; G)$ and $B = f(A)$ is a carrier for the element $g^*(\alpha)$. Moreover, the homeomorphism g is also ϵ -close to id_X . Hence, g is homotopic to id_X . Applying Proposition 2.3 to the carrier B and the homeomorphism g , we obtain that $B \subset g(B) = A$ which contradicts $b \in B \setminus A$. Hence, A should be the whole space X . \square

Proof of Corollary 1.2. Item (i) follows from Proposition 2.5. Item (ii) follows from the simple observation that any V_G^n -continuum is an Alexandroff manifold with respect to the class D_G^{n-2} . Since X is a carrier for every non-trivial $\alpha \in \check{H}^n(X; G)$, item (iii) follows from Lemma 2.1. \square

4. V_G^n -CONTINUA

In this section we provide the proofs of Theorem 1.3, Corollary 1.4 and Propositions 1.5-1.7.

Proof of Theorem 1.3. Since $\check{H}^n(X; G) \neq 0$, X contains a carrier A of a non-trivial element of $\check{H}^n(X; G) \neq 0$. Then, according to Propositions 2.5, A is a strong V_G^n -continuum. \square

Proof of Corollary 1.4. This corollary follows directly from Theorem 1.3 because every compactum X with $\dim_G X = n$ contains a closed subset F such that $\check{H}^{n-1}(F; G) \neq 0$ (see, for example, [14]). \square

Proof of Proposition 1.5. Suppose there exists a partition C of X such that for every open cover ω of X , C admits an ω -map $g_\omega: C \rightarrow Y_\omega$ onto a space of dimension $\dim_G Y_\omega \leq n - 1$ with $g_\omega^*: \check{H}^{n-1}(Y_\omega; G) \rightarrow \check{H}^{n-1}(C; G)$ being a trivial homomorphism. Then, according to [3, Theorem 2.4], $\dim_G C \leq n - 1$. Obviously, the boundary B of C in X is also a partition of X and $\dim_G B \leq n - 1$. Moreover, we have the commutative diagram below, where $g_\omega|_B: B \rightarrow g_\omega(B)$ is the restriction of g_ω

$$\begin{array}{ccc} \check{H}^{n-1}(Y_\omega; G) & \xrightarrow{g_\omega^*} & \check{H}^{n-1}(C; G) \\ \downarrow i_{g(B)}^* & & \downarrow i_B^* \\ \check{H}^{n-1}(g_\omega(B); G) & \xrightarrow{(g_\omega|_B)^*} & \check{H}^{n-1}(B; G) \end{array}$$

Since $\dim_G Y_\omega \leq n - 1$, $i_{g(B)}^*$ is a surjection. This implies that $(g_\omega|_B)^*$ is the trivial homomorphism because so is g_ω^* . Therefore, considering B instead of C , we may assume that C does not have interior points in X . The above diagram also shows that for every closed subset $A \subset C$ and every ω the restriction $g_\omega|_A$ is an ω -map onto $g_\omega(A)$ such that $(g_\omega|_A)^*: \check{H}^{n-1}(g_\omega(A); G) \rightarrow \check{H}^{n-1}(A; G)$ is the trivial homomorphism.

By Theorem 1.3, there exists a strong V_G^n -continuum $K \subset X$. Since X is homogeneous, we may also assume that $K \cap C \neq \emptyset$. Observe that $z \in K \setminus C$ for some z . Indeed, the inclusion $K \subset C$ would imply that if P is a partition of K and γ any open cover of K , then P admits a γ -map h_γ onto a space T such that $(h_\gamma)^*: \check{H}^{n-1}(T_\gamma; G) \rightarrow \check{H}^{n-1}(P; G)$ is trivial. This would contradict the fact that K is a strong V_G^n -continuum. Let $X \setminus C = U \cup V$ and $z \in V$, where U and V are nonempty, open and disjoint sets in X . Then the Effros theorem [5] allows us to push K towards U by a small homeomorphism $h: X \rightarrow X$ so that the image $h(K)$ meets both U and V (see the proof of Lemma 2 from [12] for a similar application of Effros' theorem). Therefore, $S = h(K) \cap C$ is a partition of $h(K)$ such that for any ω the restriction $g_\omega|_S$

an ω -map generating a trivial homomorphism $(g_\omega|_S)^*$, a contradiction. \square

Proof of Corollary 1.6. It follows directly from Proposition 1.5. \square

Acknowledgments. The author would like to express his gratitude to K. Kawamura and K. Yokoi for providing some information. The author also thanks the referee for his/her valuable remarks and suggestions which improved the paper.

REFERENCES

- [1] P. S. Alexandroff, *Die Kontinua (V^p) - eine Verschärfung der Cantorschen Mannigfaltigkeiten*, Monatshefte für Math. **61** (1957), 67–76 (German).
- [2] R. H. Bing and K. Borsuk, *Some remarks concerning topological homogeneous spaces*, Ann. of Math. **81** (1965), no. 1, 100–111.
- [3] A. Chigogidze and V. Valov, *Extension dimension and refinable maps*, Acta Math. Hungar. **92** (2001), no. 3, 185–194.
- [4] J. Choi, *Properties of n -bubbles in n -dimensional compacta and the existence of $(n - 1)$ -bubbles in n -dimensional clC^n compacta*, Top. Proceed. **23** (1998), 101–120.
- [5] E. G. Effros, *Transformation groups and C^* -algebras*, Ann. of Math. **81** (1965), 38–55.
- [6] W. Hurewicz and K. Menger, *Dimension und Zusammenhangsstufe*, Math. Ann. **100** (1928), 618–633 (German).
- [7] P. Huber, *Homotopical cohomology and Čech cohomology*, Math. Annalen **144** (1961), 73–76.
- [8] M. Kallipoliti and P. Papasoglu, *Simply connected homogeneous continua are not separated by arcs*, Top. Appl. **154** (2007), 3039–3047.
- [9] U. Karimov and D. Repovš, *On \tilde{H}^n -bubbles in n -dimensional compacta*, Colloq. Math. **75** (1998), 39–51.
- [10] A. Karassev, P. Krupski, V. Todorov and V. Valov, *Generalized Cantor manifolds and homogeneity*, Houston J. Math. **38** (2012), no. 2, 583–609.
- [11] A. Karassev, V. Todorov and V. Valov, *Alexandroff manifolds and homogeneous continua*, <http://dx.doi.org/10.4153/CMB-2013-010-8>.
- [12] P. Krupski, *Homogeneity and Cantor manifolds*, Proc. Amer. Math. Soc. **109** (1990), 1135–1142.
- [13] W. Kuperberg, *On certain homological properties of finite-dimensional compacta. Carriers, minimal carriers and bubbles*, Fund. Math. **83**, (1973), 7–23.
- [14] V. Kuz'minov, *On V^n continua*, Dokl. Akad. Nauk SSSR **139** (1961), 24–27 (in Russian).
- [15] S. Stefanov, *A cohomological analogue of V^n -continua and a theorem of Mazurkiewicz*, Serdica **12** (1986), no. 1, 88–94 (in Russian).
- [16] V. Todorov and V. Valov, *Generalized Cantor manifolds and indecomposable continua*, Questions and Answers in Gen. Topology, **30** (2012), 93–102.
- [17] L. A. Tumarkin, *Sur la structure dimensionnelle des ensembles fermés*, C.R. Acad. Paris **186** (1928), 420–422.
- [18] K. Yokoi, *Bubbly continua and homogeneity*, Houston J. Math. **29** (2003), no. 2, 337–343.

DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS, NIPISSING UNIVERSITY, 100 COLLEGE DRIVE, P.O. BOX 5002, NORTH BAY, ON, P1B 8L7, CANADA

E-mail address: `veskov@nipissingu.ca`