On the Number of Cycles in a Graph

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Abstract

In this paper, we obtain explicit formulae for the number of 7-cycles and the total number of paths of lengths 6 and 7 those contain a specific vertex v_i in a simple graph G, in terms of the adjacency matrix and with the help of combinatorics.
Keywords: *Adjacency Matrix, Cycle, Graph Theory, Path, Subgraph, Walk .*

1. Introduction

In a simple graph G, a walk is a sequence of vertices and edges of the form $v_0, e_1, v_1, ..., e_k, v_k$ such that the edge e_i has ends v_{i-1} and v_i. A walk is called closed if $v_0 = v_k$. If the vertices of a walk are distinct then the walk is called a path. A cycle is a non-trivial closed walk in which all the vertices are distinct except the end vertices.
It is known that if a graph G has adjacency matrix A=$[a_{ij}]$, then for k = 0, 1, ... , the ij-entry of A^k is the number of $v_i - v_j$ walks of length k in G. It is also known that tr(A^n) is the sum of the diagonal entries of A^n and d_i is the degree of the vertex v_i.
In 1971, Frank Harary and Bennet Manvel [4], gave formulae for the number of cycles of lengths 3 and 4 in simple graphs as given by the following theorems:

Theorem 1.1 [4] *If G is a simple graph with adjacency matrix A, then the number of* 3*-cycles in G is* $\frac{1}{6}$ $tr(A^3)$.
(*It is known that* $tr(A^3) = \sum_{i=1}^{n} a_{ii}^{(3)} = \sum_{j\neq i} a_{ij}^{(2)} a_{ij}$).

Theorem 1.2 [4] *If G is a simple graph with adjacency matrix A, then the number of* $4-$*cycles in G is* $\frac{1}{8}[\ tr(A^4) - 2q - 2\sum_{j\neq i} a_{ij}^{(2)}]$, *where q is the number of edges in G.*
(*It is obvious that the above formula is also equal to* $\frac{1}{8}\ [trA^4 - \ trA^2 - 2\sum_{j\neq i} a_{ij}^2]$)

Theorem 1.3 [4] *If G is a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*, then the number of* $5-$*cycles in G is* $\frac{1}{10}[tr(A^5) + 5\ tr(A^3) - 5\sum_{i=1}^{n} d_i a_{ii}^{(3)}]$

In 2003, V. C. Chang and H. L. Fu [5], found a formula for the number of 6−cycles in a simple graph which is stated below:

Theorem 1.4 [5] *If G is a simple graph with adjacency matrix A, then the number of* $6-$*cycles in G is* $\frac{1}{12}[tr(A^6) - 6\ tr(A^4) + 5tr(A^3) - 4tr(A^2) - 3\sum_{i,j=1}^{p} a_{ij}^{(3)} + 12\sum_{i,j=1}^{p} a_{ij}^{(2)} - 3\sum_{i=1}^{p} [a_{ii}^{(3)}]^2 + 9\sum_{j\neq i} a_{ij}^{(2)}(a_{ij}^{(2)}-1)a_{ij} - 6\sum_{j\neq i} a_{ij}^{(2)}(a_{ij}^{(2)}-1)(a_{ij}^{(2)}-2)$

$-2\sum_{i=1}^{p} a_{ii}^{(2)}(a_{ii}^{(2)}-1)(a_{ii}^{(2)}-2)]$, *where p is the number of vertices in G.*

Their proofs are based on the following fact:
The number of n-cycles (n= $3,4,5,6$) in a graph G is equal to $\frac{1}{2n}(\mathrm{tr}(\mathrm{A}^n) - x\ )$ where x is the number of closed walks of length n, which are not n-cycles.

In 1997, N. Alon, R. Yuster and U. Zwick [6], gave number of 7- cyclic graphs. They also gave some formulae for the number of cycles of lengths 5 , which contains a specific vertex v_i in a graph G.
In [6, 11, 12, 13, 15, 16, 18], we have also some bounds to estimate the total time complexity for finding or counting paths and cycles in a graph.

In our recent works [1, 2], we obtained some formulae to find the exact number of paths of lengths 3, 4 and 5, in a simple graph G, given below:

Theorem 1.5 [1] *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*. The number of paths of length* 3 *in G is* $\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$.

Theorem 1.6 [1] *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*. The number of paths of length* 4 *in G is* $\sum_{j\neq i}[a_{ij}^{(4)} - 2a_{ij}^{(2)}(d_j - a_{ij})] - \sum_{i=1}^{n}[(2d_i-1)a_{ii}^{(3)} + 6\begin{pmatrix} d_i \\ 3 \end{pmatrix}]$.

Theorem 1.7 [1] *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*. The number of paths of length* 3 *in G, each of which starts from a specific vertex* v_i *is* $\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$.

Theorem 1.8 [1] *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*. The number of paths of length* 4 *in G, each of which starts from a specific vertex* v_i *is* $\sum_{j\neq i}[a_{ij}^{(4)} - (d_i + d_j - 3a_{ij})a_{ij}^{(2)} - (a_{ii}^{(3)} + a_{jj}^{(3)} + 2\begin{pmatrix} d_j - 1 \\ 2 \end{pmatrix})a_{ij}]$.

Theorem 1.9 [2] *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*. The number of paths of length* 5 *in G is* $\sum_{j\neq i} a_{ij}^{(5)} - 2\sum_{j\neq i} a_{ij}^{(4)} + 2\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2) + 4\sum_{j\neq i} a_{ij}^{(2)} - 2\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1) - 4\sum_{j\neq i} a_{ij}^{(2)}\begin{pmatrix} d_i - a_{ij} - 1 \\ 2 \end{pmatrix}$ $+ 6\sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} - 2\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)} - 2\sum_{i=1}^{n} a_{ii}^{(3)}\begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix} - 2\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)})(d_i - 2) - \sum_{j\neq i} a_{ij} -$ *3 tr* A^4 *+ 6 tr* A^3 *+ 3 tr* A^2.

Theorem 1.10 [2] *If G is a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*, then the number of* $4-$*cycles each of which contains a specific vertex* v_i *of G is* $\frac{1}{2}\ [a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)}]$.

In [6] we can also see a formula for the number of 5-cycles each of which contains a specific vertex v_i but, their formula has some problem in coefficients. In [3] we gave the correct formula as considered bellow :

Theorem 1.11 [3] *If G is a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*, then the number of* $5-$*cycles each of which contains a specific vertex* v_i *of G is* $\frac{1}{2}\ [a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i - 2)a_{ii}^{(3)} - 2\sum_{j=1, j\neq i}^{n} a_{ij}^{(2)}a_{ij}(d_j - 2) - 2\sum_{j=1, j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$.

In this paper we give a formula to count the exact number of cycles of length 7 and the number of cycles of lengths 6 and 7 those contain a specific vertex v_i in a simple graph G, in terms of the adjacency matrix of G and with the helps of combinatorics.

2. Number of 7- Cycles :

In 1997, N. Alon, R. Yuster and U. Zwick [6], gave number of 7- cyclic graphs. The n- cyclic graph is a graph that contain a closed walk of length n and these walks are not necessarily a cycle. In this section we obtain a formula for the number of cycles of length 7 in a simple graph G with the helps of [6].

Method: To count N in the cases those considered below, we first count tr(A^7) for the graph of first configuration. This will give us the number of all closed walks of length 7 in the corresponding graph. But, some of these walks do not pass through all the edges and vertices of that configuration and to find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once. So, we delete the number of closed walks of length 7 those do not pass through all the edges and vertices. To find these kind of walks we also have to count tr(A^7) for all the subgraphs of the corresponding graph those can contain a closed walk of length 7.

Theorem 2.1 *: If G is a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*, then the number of* $7-$*cycles in G is* $\frac{1}{14}\ (tr(A^7) - x)$*, where x is equal to* $\sum_{i=1}^{11} F_i$ *in the cases those are considered below.*

Proof: The number of $7-$cycles of a graph G is equal to $\frac{1}{14}(tr(A^7) - x)$, where x is the number of closed walks of length 7 that are not 7-cycles. To find x, we have 11 cases as considered below; the cases are based on the configurations-(subgraphs) that generate all closed walks of length 7 that are not 7-cycles. In each case, N denote the number of closed walks of length 7 that are not 7-cycles in the corresponding subgraph, M denote the number of subgraphs of G of the same configuration and F_n, (n= 1, 2, ...) denote the total number of closed walks of length 7 that are not cycles in all possible subgraphs of G of the same configuration. However, in the cases with more that one figure (Cases 5, 6, 8, 9, 11), N, M and F_n are based on the first graph of the respective figures and P_1, P_2, ... denote the number of subgraphs of G which don't have the same configuration as the first graph but are counted in M. It is clear that F_n is equal to N× (M− P_1− P_2− ...). To find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once.

Case 1: For the configuration of Fig 1, N= 126, M= $\frac{1}{6}$tr(A^3) and F_1= 21 tr(A^3). (See theorem 1.1)

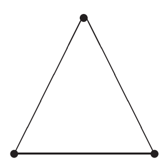


Fig 1

Case 2: For the configuration of Fig 2, N= 84, M= $\frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2)$ and F_2= $42\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2)$.

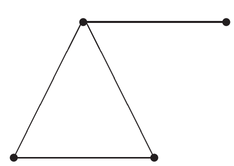


Fig 2

Case 3: For the configuration of Fig 3, N= 28, M= $\frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}\binom{d_i - 2}{2}$ and F_3= $14\sum_{i=1}^{n} a_{ii}^{(3)}\binom{d_i - 2}{2}$.

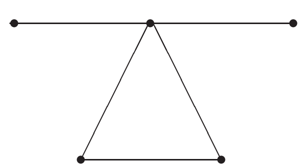


Fig 3

Case 4: For the configuration of Fig 4, N= 112, M= $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$ and F$_4$= $56\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$.

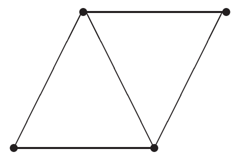
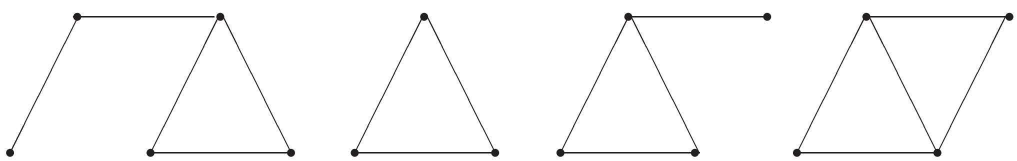


Fig 4

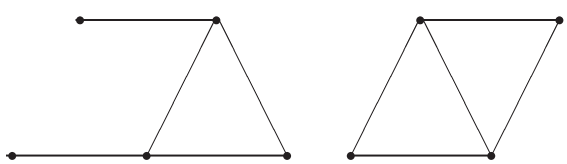
Case 5: For the configuration of Fig 5(a), N= 14, M= $\frac{1}{2}\sum_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)}$. Let P$_1$ denote the number of subgraphs of G that have the same configuration as the graph of Fig 5(b) and are counted in M. Thus P$_1$= $6\times(\frac{1}{126}$ F$_1$), where $\frac{1}{126}$ F$_1$ is the number of subgraphs of G that have the same configuration as the graph of Fig 5(b) and 6 is the number of times that this subgraph is counted in M. Let P$_2$ denote the number of subgraphs of G that have the same configuration as the graph of Fig 5(c) and are counted in M. Thus P$_2$= $2\times(\frac{1}{84}$ F$_2$), where $\frac{1}{84}$ F$_2$ is the number of subgraphs of G that have the same configuration as the graph of Fig 5(c) and 2 is the number of times that this subgraph is counted in M. Let P$_3$ denote the number of subgraphs of G that have the same configuration as the graph of Fig 5(d) and are counted in M. Thus P$_3$= $4\times(\frac{1}{112}$ F$_4$), where $\frac{1}{112}$ F$_4$ is the number of subgraphs of G that have the same configuration as the graph of Fig 5(d) and 4 is the number of times that this subgraph is counted in M. Consequently, F$_5$= $7\sum_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)} - \frac{2}{3}$ F$_1 - \frac{1}{3}$ F$_2 - \frac{1}{2}$ F$_4$.



(a) (b) (c) (d)

Fig 5

Case 6: For the configuration of Fig 6(a), N= 14, M= $\frac{1}{2}\sum_{j\neq i}a_{ij}^{(2)}a_{ij}(d_j-2)(d_i-2)$. Let P$_1$ denote the number of subgraphs of G that have the same configuration as the graph of Fig 6(b) and are counted in M. Thus P$_1$= $2\times(\frac{1}{112}$ F$_4$), where $\frac{1}{112}$ F$_4$ is the number of subgraphs of G that have the same configuration as the graph of Fig 6(b) and 2 is the number of times that this subgraph is counted in M. Consequently, F$_6$= $7\sum_{j\neq i}a_{ij}^{(2)}a_{ij}(d_j-2)(d_i-2) - \frac{1}{4}$ F$_4$.



(a) (b)

Fig 6

Case 7: For the configuration of Fig 7, N= 70, M= $\frac{1}{10}[\ \mathrm{tr}(A^5) + 5\ \mathrm{tr}(A^3) - 5\sum_{i=1}^{n}d_i a_{ii}^{(3)}]$ (See Theorem 1.3) and F$_7$= $7\mathrm{tr}(A^5) + 35\ \mathrm{tr}(A^3) - 35\sum_{i=1}^{n}d_i a_{ii}^{(3)}$.

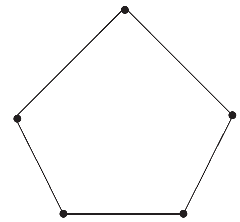
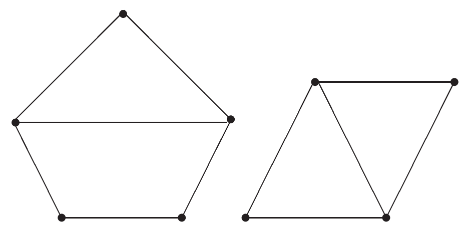


Fig 7

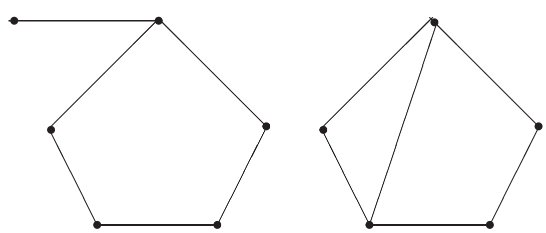
Case 8: For the configuration of Fig 8(a), N= 42, M= $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)})$ (See Theorem 1.5). Let P_1 denotes the number of subgraphs of G that have the same configuration as the graph of Fig 8(b) and are counted in M. Thus $P_1 = 4\times(\frac{1}{112}\ F_4)$, where $\frac{1}{112}\ F_4$ is the number of subgraphs of G that have the same configuration as the graph of Fig 8(b) and 4 is the number of times that this subgraph is counted in M. Consequently, $F_8 = 21\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - \frac{3}{2}\ F_4$.



(a) (b)

Fig 8

Case 9: For the configuration of Fig 9(a), N= 14, M= $\frac{1}{2}\ [\sum_{i=1}^{n}(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)})(d_i-2) - 2\sum_{j\neq i} a_{ij}^{(2)}a_{ij}(d_j - 2)(d_i-2)\ - 2\sum_{j\neq i} a_{ij}(d_i-2)(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$ (See Theorem 1.11). Let P_1 denote the number of subgraphs of G that have the same configuration as the graph of Fig 9(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}\ F_8)$, where $\frac{1}{42}\ F_8$ is the number subgraphs of G that have the same configuration as the graph of Fig 9(b) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_9 = 7\sum_{i=1}^{n}(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)})(d_i-2) - 14\sum_{j\neq i} a_{ij}^{(2)}a_{ij}(d_j-2)(d_i-2)\ - 14\sum_{j\neq i} a_{ij}(d_i-2)(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)}) - \frac{2}{3}\ F_8$.



(a) (b)

Fig 9

Case 10: For the configuration of Fig 10, N= 84, M= $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3}a_{ij}$ and $F_{10} = 42\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3}a_{ij}$.

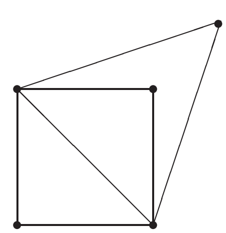
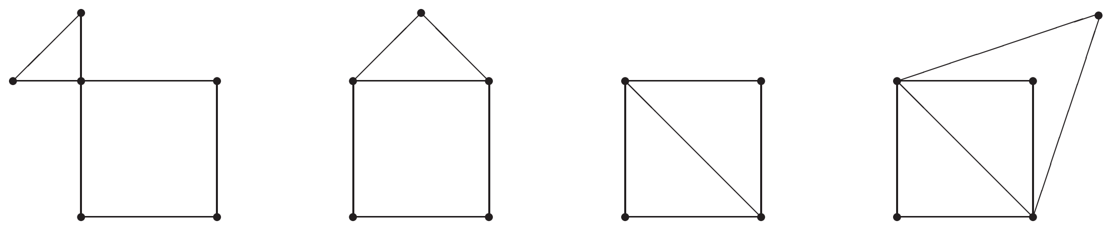


Fig 10

Case 11: For the configuration of Fig 11(a), N= 28, M= $\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ii}^{(3)}$. Let P_1 denote the number of subgraphs of G that have the same configuration as the graph of Fig 11(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}$ $F_8)$, where $\frac{1}{42}$ F_8 is the number of subgraphs of G that have the same configuration as the graph of Fig 11(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of subgraphs of G that have the same configuration as the graph of Fig 11(c) and are counted in M. Thus $P_2 = 6\times$ ($\frac{1}{112}$ F_4), where $\frac{1}{112}$ F_4 is the number of subgraphs of G that have the same configuration as the graph of Fig 11(c) and 6 is the number of times that this subgraph is counted in M. Let P_3 denote the number of subgraphs of G that have the same configuration as the graph of Fig 11(d) and are counted in M. Thus $P_3 = 6\times$ ($\frac{1}{84}$ F_{10}), where $\frac{1}{84}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 11(d) and 6 is the number of times that this subgraph is counted in M. Consequently, $F_{11} = 14\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ii}^{(3)} - \frac{4}{3} F_8 - \frac{3}{2} F_4 - 2\, F_{10}$.



(a) (b) (c) (d)

Fig 11

Now we add the values of F_n arising from the above cases and determine x. By putting the value of x in $\frac{1}{14}(\text{tr}(A^7)-x)$ we get the desired formula. □

Example 2.2 *In K_7, F_1= 4410, F_2= 35280, F_3= 17640, F_4= 23520, F_5= 17640, F_6= 17640, F_7= 17640, F_8= 52920, F_9= 35280, F_{10}= 17640, F_{11}= 35280 and trA^7= 279930. So x= 274890 and by Theorem 2.1, the number of cycles of length 7 in K_7 is $\frac{1}{14}(279930 - 274890) = 360$.*

3. Number of Cycles Passing the Vertex V_i :

In this section we give formulae to count the number of cycles of lengths 6 and 7, each of which contain a specific vertex v_i of the graph G.

Theorem 3.1 *If G is a simple graph with n vertices and the adjacency matrix $A = [a_{ij}]$, then the number of $6-$cycles each of which contains a specific vertex v_i of G is $\frac{1}{2}$ ($a_{ii}^{(6)} - x$), where x is equal to $\sum_{i=1}^{17} F_i$ in the cases those are considered below.*

Proof: The number of $6-$cycles each of which contain a specific vertex v_i of the graph G is equal to $\frac{1}{2}$ ($a_{ii}^{(6)} - x$), where x is the number of closed walks of length 6 form the vertex v_i to v_i that are not $6-$cycles. To find x, we have 17 cases as considered below; the cases are based on the configurations-(subgraphs) that generate $v_i - v_i$ walks of length 6 that are not cycles. In each case, N denote the number of walks of length 6 from v_i to v_i that are not cycles in the corresponding subgraph, M denote the number of subgraphs of G of the same configuration and F_n, (n= 1, 2, ...) denote the total number of $v_i - v_i$ walks of length 6 that are not cycles in all possible subgraphs of G of the same configuration. However, in the cases with more than one figure (Cases 11, 12, 13, 14, 15, 16, 17), N, M and F_n are based on the first graph of the respective figures and P_1, P_2,... denote the number of subgraphs of G which don't have the same configuration as the first graph but are counted in M. It is clear that F_n is equal to N × (M− P_1−P_2−...). To find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once.

Case 1: For the configuration of Fig 12, N= 1, M= $a_{ii}^{(2)}$ and $F_1 = a_{ii}^{(2)}$.

Fig 12

Case 2: For the configuration of Fig 13, N= 3, M=$\sum_{j\neq i} a_{ij}^{(2)}$ and F_2= $3\sum_{j\neq i} a_{ij}^{(2)}$.

Fig 13

Case 3: For the configuration of Fig 14, N= 6, M= $\binom{d_i}{2}$ and F_3= $6\binom{d_i}{2}$.

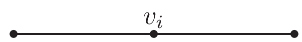


Fig 14

Case 4: For the configuration of Fig 15, N= 1, M=$\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$ and F_4=$\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$.
(See Theorem 1.7)

Fig 15

Case 5: For the configuration of Fig 16, N= 2, M=$\sum_{j\neq i} a_{ij}^{(2)}(d_i - a_{ij} - 1)$ and F_5= $2\sum_{j\neq i} a_{ij}^{(2)}(d_i - a_{ij} - 1)$.

Fig 16

Case 6: For the configuration of Fig 17, N= 2, M=$\sum_{j\neq i} a_{ij}\binom{d_j - 1}{2}$ and F_6= $2\sum_{j\neq i} a_{ij}\binom{d_j - 1}{2}$.

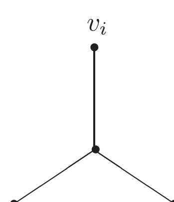


Fig 17

Case 7: For the configuration of Fig 18, N= 6, M= $\binom{d_i}{3}$ and F_7= $6\binom{d_i}{3}$.

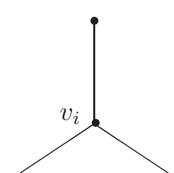


Fig 18

Case 8: For the configuration of Fig 19, N= 8, M= $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}$ and $F_8 = 4\sum_{j\neq i} a_{ij}^{(2)} a_{ij}$.

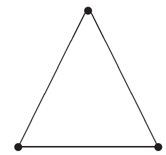


Fig 19

Case 9: For the configuration of Fig 20, N= 12, M= $\sum_{j\neq i} \binom{a_{ij}^{(2)}}{2}$ and $F_9 = 12\sum_{j\neq i} \binom{a_{ij}^{(2)}}{2}$.

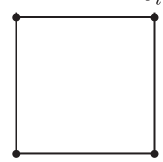


Fig 20

Case 10: For the configuration of Fig 21, N= 12, M= $\sum_{j\neq i} \binom{a_{ij}^{(2)}}{2} a_{ij}$ and $F_{10} = 12\sum_{j\neq i} \binom{a_{ij}^{(2)}}{2} a_{ij}$.

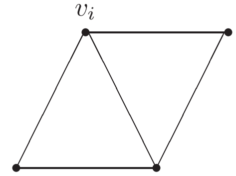
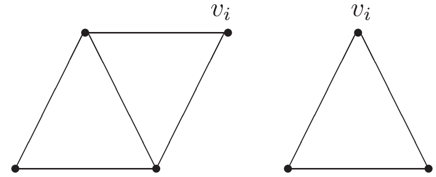


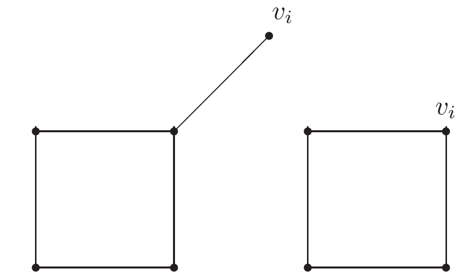
Fig 21

Case 11: For the configuration of Fig 22(a), N= 6, M= $\frac{1}{2} \sum_{k\neq j,\ j,k\neq i} a_{jk}^{(2)} a_{ij} a_{ik} a_{jk}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 22(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{8}\ F_8)$, where $\frac{1}{8}\ F_8$ is the number of subgraphs of G that have the same configuration as the graph of Fig 22(b) and this subgraph is counted only once in M. Consequently, $F_{11} = 3 \sum_{k\neq j,\ j,k\neq i} a_{jk}^{(2)} a_{ij} a_{ik} a_{jk} - \frac{3}{4}\ F_8$.



(a) Fig 22 (b)

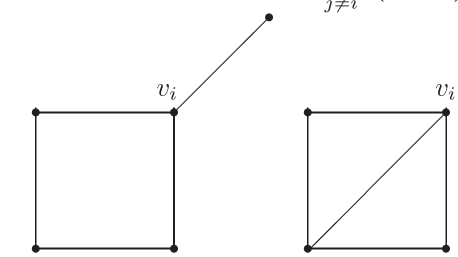
Case 12: For the configuration of Fig 23(a), N= 2, M= $\sum_{k\neq j,\ j,k\neq i}^{n} \binom{a_{jk}^{(2)}}{2} a_{ij}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 23(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{12}\ F_9)$, where $\frac{1}{12}\ F_9$ is the number of subgraphs of G that have the same configuration as the graph of Fig 23(b) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{12} = 2 \sum_{k\neq j,\ j,k\neq i}^{n} \binom{a_{jk}^{(2)}}{2} a_{ij} - \frac{1}{3}\ F_9$.



(a) (b)

Fig 23

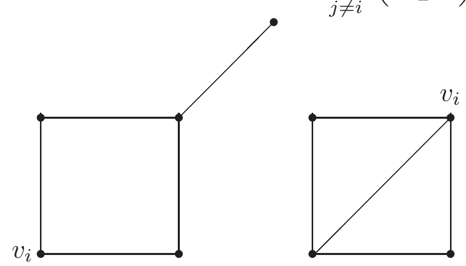
Case 13: For the configuration of Fig 24(a), N= 4, M= $\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 24(b) and are counted in M. Thus $P_1=1\times(\frac{1}{12}$ $F_{10})$, where $\frac{1}{12}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 24(b) and this subgraph is counted only once in M. Consequently, $F_{13}= 4\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-2)-\frac{1}{3}$ F_{10}.



(a) (b)

Fig 24

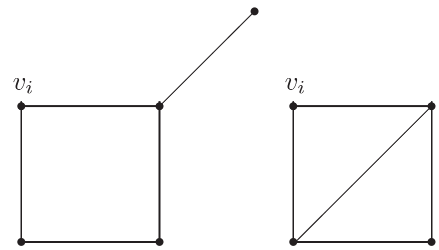
Case 14: For the configuration of Fig 25(a), N= 2, M= $\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_j-2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 25(b) and are counted in M. Thus $P_1=1\times(\frac{1}{12}$ $F_{10})$, where $\frac{1}{12}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 25(b) and this subgraph is counted only once in M. Consequently, $F_{14}= 2\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_j-2)-\frac{1}{6}$ F_{10}.



(a) (b)

Fig 25

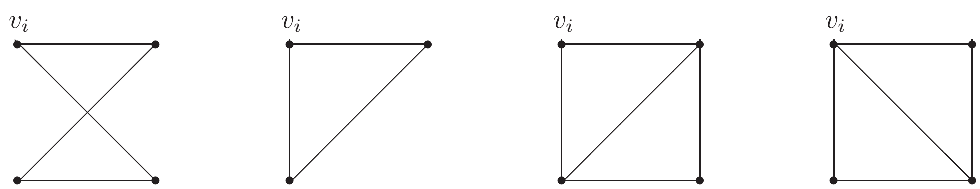
Case 15: For the configuration of Fig 26(a), N= 2, M=$\sum_{j\neq i}a_{ij}^{(2)}(d_j-a_{ij}-1)a_{ij}(d_j-2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 26(b) and are counted in M. Thus $P_1=2\times(\frac{1}{6}$ $F_{11})$, where $\frac{1}{6}$ F_{11} is the number of subgraphs of G that have the same configuration as the graph of Fig 26(b) and 2 is the number of times that this subgraph is counted in M. Consequently,
$F_{15}= 2\sum_{j\neq i}a_{ij}^{(2)}(d_j-a_{ij}-1)a_{ij}(d_j-2)-\frac{2}{3}$ F_{11}.



(a) (b)

Fig 26

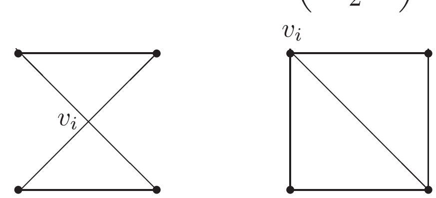
Case 16: For the configuration of Fig 27(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i} a_{jj}^{(3)} a_{ij}^{(2)} a_{ij}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 27(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{8}\ F_8)$, where $\frac{1}{8}\ F_8$ is the number of subgraphs of G that have the same configuration as the graph of Fig 27(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 27(c) and are counted in M. Thus $P_2 = 2 \times (\frac{1}{6}\ F_{11})$, where $\frac{1}{6}\ F_{11}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 27(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 27(d) and are counted in M. Thus $P_3 = 2 \times (\frac{1}{12}\ F_{10})$, where $\frac{1}{12}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 27(d) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{16} = 2\sum_{j\neq i} a_{jj}^{(3)} a_{ij}^{(2)} a_{ij} - F_8 - \frac{4}{3}\ F_{11} - \frac{2}{3}\ F_{10}$.



(a) (b) (c) (d)

Fig 27

Case 17: For the configuration of Fig 28(a), N= 8, M= $\binom{\frac{1}{2}a_{ii}^{(3)}}{2}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 28(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}\ F_{10})$, where $\frac{1}{12}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 28(b) and this subgraph is counted only once in M. Consequently, $F_{17} = 8\binom{\frac{1}{2}a_{ii}^{(3)}}{2} - \frac{2}{3}\ F_{10}$.



(a) (b)

Fig 28

Now we add the values of F_n arising from the above cases and determine x. Substituting the value of x in $\frac{1}{2}(a_{ii}^{(6)} - x)$ and simplifying, we get the number of 6−cycles each of which contains a specific vertex v_i of G. □

Example 3.2 *In the graph of Fig* 29 *we have,* $F_1 = 5$, $F_2 = 60$, $F_3 = 60$, $F_4 = 60$, $F_5 = 120$, $F_6 = 60$, $F_7 = 60$, $F_8 = 80$, $F_9 = 360$, $F_{10} = 360$, $F_{11} = 180$, $F_{12} = 120$, $F_{13} = 240$, $F_{14} = 120$, $F_{15} = 240$, $F_{16} = 240$, $F_{17} = 120$. *So, we have* $x = 2485$. *Consequently, by Theorem* 3.1, *the number of* 6−*cycles each of which contains the vertex* v_1 *in the graph of Fig* 29 *is* 60.

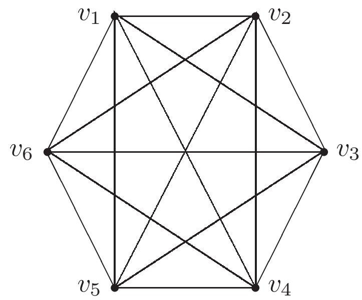


Fig 29

Theorem 3.3 *If G is a simple graph with n vertices and the adjacency matrix A= $[a_{ij}]$, then the number of 7−cycles each of which contains a specific vertex v_i of G is $\frac{1}{2}$ $(a_{ii}^{(7)} - x)$, where x is equal to $\sum_{i=1}^{30} F_i$ in the cases those are considered below.*

Proof: The number of 7−cycles each of which contains a specific vertex v_i of the graph G is equal to $\frac{1}{2}$ $(a_{ii}^{(7)} - x)$, where x is the number of closed walks of length 7 form the vertex v_i to v_i that are not 7−cycles. To find x, we have 30 cases as considered below; the cases are based on the configurations-(subgraphs) that generate $v_i - v_i$ walks of length 7 that are not cycles. In each case, N denote the number of walks of length 7 from v_i to v_i that are not cycles in the corresponding subgraph, M denote the number of subgraphs of G of the same configuration and F_n, (n= 1, 2, ...) denote the total number of $v_i - v_i$ walks of length 7 that are not cycles in all possible subgraphs of G of the same configuration. However, in the cases with more than one figure (Cases 9, 10, ..., 18, 20,..., 30), N, M and F_n are based on the first graph of the respective figures and P_1, P_2,... denote the number of subgraphs of G which don't have the same configuration as the first graph but are counted in M. It is clear that F_n is equal to N × (M− P_1−P_2−...). To find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once.

Case 1: For the configuration of Fig 30, N= 42, M= $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}$ and $F_1 = 21\sum_{j\neq i} a_{ij}^{(2)} a_{ij}$.

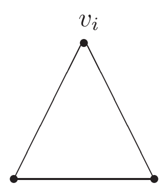


Fig 30

Case 2: For the configuration of Fig 31, N= 34, M= $\frac{1}{2}a_{ii}^{(3)}(d_i - 2)$ and F_2= $17a_{ii}^{(3)}(d_i - 2)$.

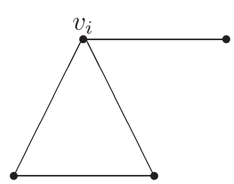


Fig 31

Case 3: For the configuration of Fig 32, N= 18, M=$\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j - 2)$ and F_3= $18\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j - 2)$.

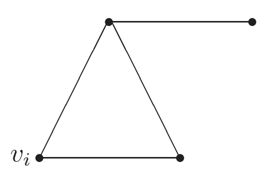


Fig 32

Case 4: For the configuration of Fig 33, N= 4, M=$\sum_{j\neq i}(\frac{1}{2}a_{jj}^{(3)}a_{ij} - a_{ij}^{(2)}a_{ij})(d_j - 3)$ and F_4= $4\sum_{j\neq i}(\frac{1}{2}a_{jj}^{(3)}a_{ij} - a_{ij}^{(2)}a_{ij})(d_j - 3)$.

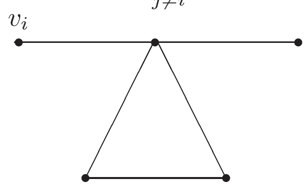


Fig 33

Case 5: For the configuration of Fig 34, N= 12, M= $\frac{1}{2}a_{ii}^{(3)}\binom{d_i - 2}{2}$ and F_5= $6a_{ii}^{(3)}\binom{d_i - 2}{2}$.

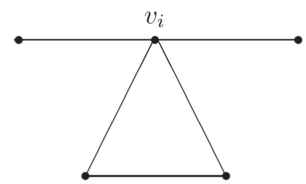


Fig 34

Case 6: For the configuration of Fig 35, N= 4, M=$\sum_{j\neq i}a_{ij}^{(2)}a_{ij}\binom{d_j - 2}{2}$ and F_6= $4\sum_{j\neq i}a_{ij}^{(2)}a_{ij}\binom{d_j - 2}{2}$.

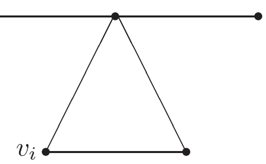


Fig 35

Case 7: For the configuration of Fig 36, N= 32, M=$\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$ and F_7= $32\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$.

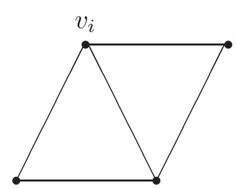


Fig 36

Case 8: For the configuration of Figure 37, N=24, M= $\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3}a_{ij}$, F_8= $24\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3}a_{ij}$.

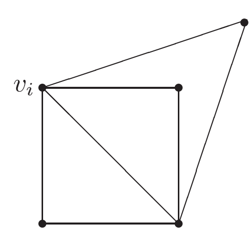


Fig 37

Case 9: For the configuration of Figure 38(a), N= 12, M= $\frac{1}{2}\sum_{k\neq j,\ j,k\neq i}\binom{a_{jk}^{(2)}}{2}a_{jk}a_{ij}a_{ik}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 38(b) and are counted in M.

Thus $P_1 = 1 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 38(b) and this subgraph is counted only once in M. Consequently, $F_9 = 6 \sum_{k\neq j,\ j,k\neq i} \begin{pmatrix} a^{(2)}_{jk} \\ 2 \end{pmatrix} a_{jk}a_{ij}a_{ik} - \frac{1}{2}\ F_{10}$.

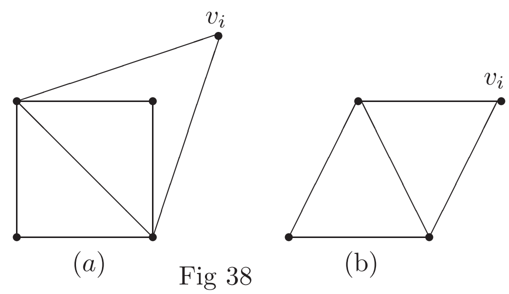


Fig 38

Case 10: For the configuration of Fig 39(a), N= 24, $M = \frac{1}{2} \sum_{k\neq j,\ j,k\neq i} a^{(2)}_{jk} a_{ij}a_{ik}a_{jk}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 39(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{42}\ F_1)$, where $\frac{1}{42}\ F_1$ is the number of subgraphs of G that have the same configuration as the graph of Fig 39(b) and this subgraph is counted only once in M. Consequently, $F_{10} = 12 \sum_{k\neq j,\ j,k\neq i} a^{(2)}_{jk} a_{ij}a_{ik}a_{jk} - \frac{4}{7}\ F_1$.

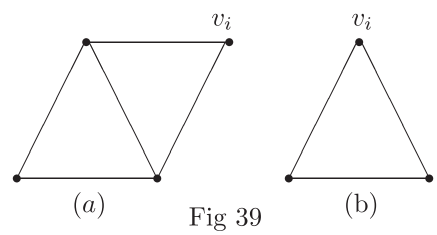


Fig 39

Case 11: For the configuration of Fig 40(a), N= 14, $M = \frac{1}{2}\sum_{j\neq i} a^{(3)}_{jj} a_{ij}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 40(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}F_1)$, where $\frac{1}{42}\ F_1$ is the number of subgraphs of G that have the same configuration as the graph of Fig 40(b) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{11} = 7\sum_{j\neq i} a^{(3)}_{jj} a_{ij} - \frac{2}{3}\ F_1$.

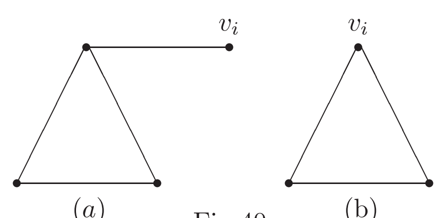
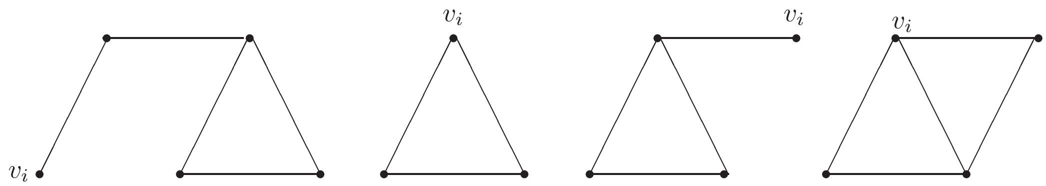


Fig 40

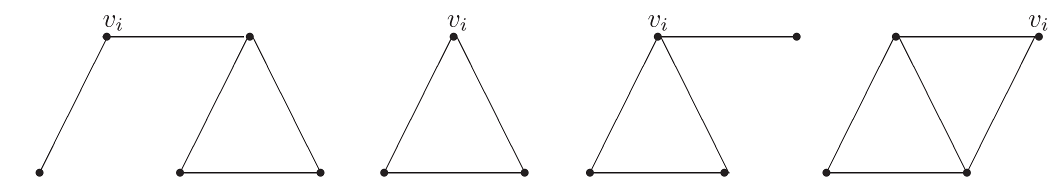
Case 12: For the configuration of Fig 41(a), N= 2, $M = \frac{1}{2}\sum_{j\neq i} a^{(2)}_{ij} a^{(3)}_{jj}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 41(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{42}F_1)$, where $\frac{1}{42}\ F_1$ is the number of subgraphs of G that have the same configuration as the graph of Fig 41(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 41(c) and are counted in M. Thus $P_2 = 2 \times (\frac{1}{14}\ F_{11})$, where $\frac{1}{14}\ F_{11}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 41(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 41(d) and are counted in M. Thus $P_3 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Fig 41(d) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{12} = \sum_{j\neq i} a^{(2)}_{ij} a^{(3)}_{jj} - \frac{2}{21}\ F_1 - \frac{2}{7}\ F_{11} - \frac{1}{8}\ F_7$.



(a) (b) (c) (d)

Fig 41

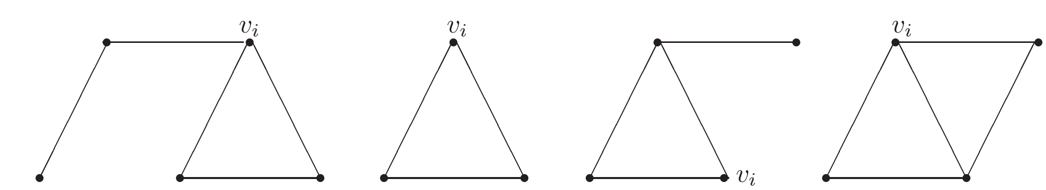
Case 13: For the configuration of Fig 42(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i} a_{jj}^{(3)} a_{ij}(d_i-1)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 42(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}F_1)$, where $\frac{1}{42}$ F_1 is the number of subgraphs of G that have the same configuration as the graph of Fig 42(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 42(c) and are counted in M. Thus $P_2 = 2\times(\frac{1}{34}$ $F_2)$, where $\frac{1}{34}$ F_2 is the number of subgraphs of G that have the same configuration as the graph of Fig 42(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 42(d) and are counted in M. Thus $P_3 = 2\times(\frac{1}{24}$ $F_{10})$, where $\frac{1}{24}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 42(d) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{13}= 2\sum_{j\neq i} a_{jj}^{(3)} a_{ij}(d_i-1) - \frac{4}{21}$ $F_1 - \frac{4}{17}$ $F_2 - \frac{1}{3}$ F_{10}.



(a) (b) (c) (d)

Fig 42

Case 14: For the configuration of Fig 43(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 43(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}F_1)$, where $\frac{1}{42}$ F_1 is the number of subgraphs of G that have the same configuration as the graph of Fig 43(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 43(c) and are counted in M. Thus $P_2 = 1\times(\frac{1}{18}$ $F_3)$, where $\frac{1}{18}$ F_3 is the number of subgraphs of G that have the same configuration as the graph of Fig 43(c) and this subgraph is counted only once in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 43(d) and are counted in M. Thus $P_3 = 2\times(\frac{1}{32}$ $F_7)$, where $\frac{1}{32}$ F_7 is the number of subgraphs of G that have the same configuration as the graph of Fig 43(d) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{14}= 2\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} - \frac{4}{21}$ $F_1 - \frac{2}{9}$ $F_3 - \frac{1}{4}$ F_7.



(a) (b) (c) (d)

Fig 43

Case 15: For the configuration of Fig 44(a), N= 2, M= $\sum_{j\neq k,\ j,k\neq i} a_{ij}^{(2)} a_{jk}^{(2)} a_{ij}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 44(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{42}F_1)$, where $\frac{1}{42}$ F_1 is the number of subgraphs of G that have the same configuration as the graph of Fig 44(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 44(c) and are counted in M. Thus $P_2 = 2 \times (\frac{1}{34}\ F_2)$, where $\frac{1}{34}$ F_2 is the number of subgraphs of G that have the same configuration as the graph of Fig 44(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 44(d) and are counted in M. Thus $P_3 = 2 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 44(d) and 2 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 44(e) and are counted in M. Thus $P_4 = 1 \times (\frac{1}{18}\ F_3)$, where $\frac{1}{18}$ F_3 is the number of subgraphs of G that have the same configuration as the graph of Fig 44(e) and 1 is the number of times that this subgraph is counted in M. Consequently, $F_{15}= 2 \sum_{j \neq k,\ j,k \neq i} a_{ij}^{(2)} a_{jk}^{(2)} a_{ij} - \frac{2}{21}\ F_1 - \frac{2}{17}\ F_2 - \frac{1}{6}\ F_{10} - \frac{1}{9}\ F_3$.

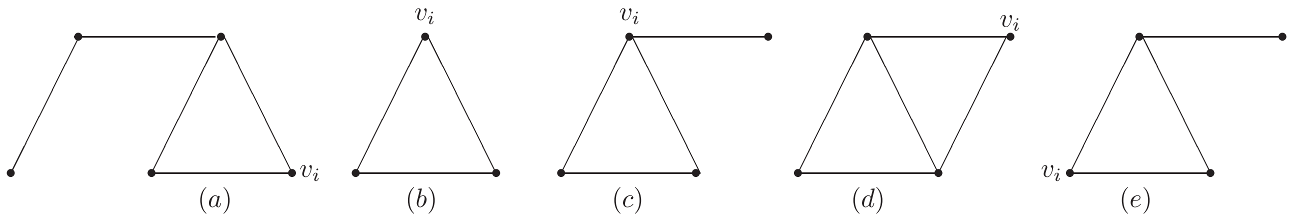


Fig 44

Case 16: For the configuration of Figure 45(a), N= 2, M= $\sum_{j \neq k,\ j,k \neq i} a_{jk}^{(2)} a_{jk} a_{ij} (d_k - 2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 45(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Fig 45(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 45(c) and are counted in M. Thus $P_2 = 1 \times (\frac{1}{18}\ F_3)$, where $\frac{1}{18}$ F_3 is the number of subgraphs of G that have the same configuration as the graph of Fig 45(c) and 1 is the number of times that this subgraph is counted in M.

Consequently, $F_{16}= 2 \sum_{j \neq k,\ j,k \neq i} a_{jk}^{(2)} a_{jk} a_{ij} (d_k - 2) - \frac{1}{6}\ F_{10} - \frac{1}{9}\ F_3$.

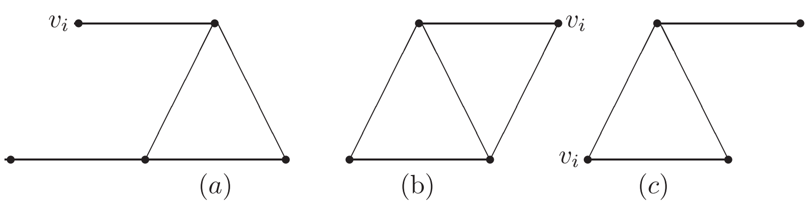


Fig 45

Case 17: For the configuration of Figure 46(a), N= 4, M= $\sum_{j \neq i} a_{ij}^{(2)} a_{ij} (d_j - 2)(d_i - 2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 46(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}$ F_7 is the number of subgraphs of G that have the same configuration as the graph of Figure 46(b) and 2 is the number of times that this subgraph is counted in M. Consequently, $F_{17}= 4 \sum_{j \neq i} a_{ij}^{(2)} a_{ij} (d_j - 2)(d_i - 2) - \frac{1}{4}\ F_7$.

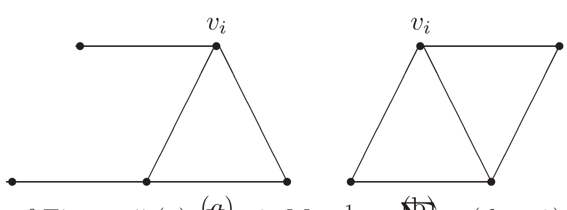
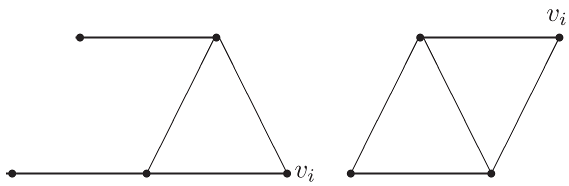


Fig 46

Case 18: For the configuration of Figure 47(a), N= 2, M= $\frac{1}{2} \sum_{k \neq j,\ j,k \neq i} (d_j - 2)(d_k - 2) a_{ij} a_{ik} a_{jk}$. Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Figure 47(b) and are counted in

M. Thus $P_1 = 1 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Figure 47(b) and 1 is the number of times that this subgraph is counted in M.
Consequently, $F_{18} = \sum_{k\neq j,\ j,k\neq i} (d_j - 2)(d_k - 2)a_{ij}a_{ik}a_{jk} - \frac{1}{12}\ F_{10}$.



(a) (b)

Fig 47

Case 19: For the configuration of Figure 48, N= 14, $M=\frac{1}{2}\ [a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i - 2)a_{ii}^{(3)} - 2 \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)} a_{ij}(d_j - 2) - 2 \sum_{j=1, j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$ (See Theorem 1.11) and $F_{19} = 7\ [a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i - 2)a_{ii}^{(3)} - 2 \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)} a_{ij}(d_j - 2) - 2 \sum_{j=1, j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$.

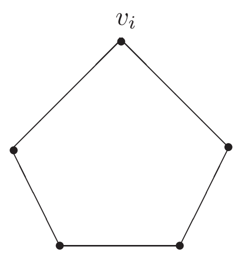
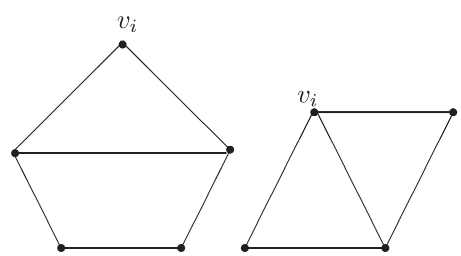


Fig 48

Case 20: For the configuration of Fig 49(a), N= 6, $M= \frac{1}{2} \sum_{k\neq j,\ j,k\neq i} a_{jk}^{(2)}(d_k - a_{jk} - 1)(a_{jk}a_{ij}a_{ik})$ (See Theorem 1.5). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 49(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Figure 49(b) and 2 is the number of times that this subgraph is counted in M.
Consequently, $F_{20} = 3 \sum_{k\neq j,\ j,k\neq i} a_{jk}^{(2)}(d_k - a_{jk} - 1)(a_{jk}a_{ij}a_{ik}) - \frac{3}{8}\ F_7$.



(a) (b)

Fig 49

Case 21: For the configuration of Fig 50(a), N= 12, $M=\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}^{(2)}a_{ij})$ (See Theorem 1.7). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 50(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Figure 50(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 50(c) and are counted in M. Thus $P_2 = 2 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Figure 50(c) and 2 is the number of times that this subgraph is counted in M.
Consequently, $F_{21} = 12\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}^{(2)}a_{ij}) - \frac{3}{4}\ F_7 - F_{10}$.

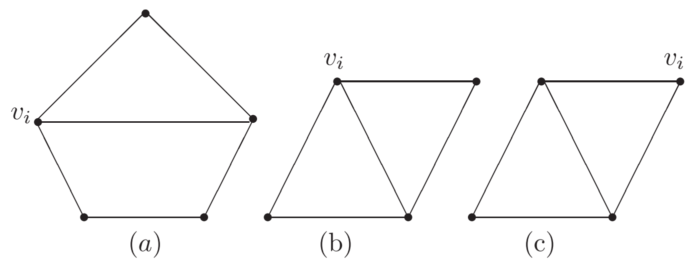


Fig 50

Case 22: For the configuration of Fig 51(a), N= 6, M=$\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{jj}^{(3)}a_{ij})$ (See theorem 1.7). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 51(b) and are counted in M. Thus P_1= $2\times(\frac{1}{24}$ $F_{10})$, where $\frac{1}{24}$ F_{10} is the number of subgraphs of G that have the same configuration as the graph of Figure 51(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 51(c) and are counted in M. Thus P_2= $6\times(\frac{1}{12}$ $F_9)$, where $\frac{1}{12}$ F_9 is the number of subgraphs of G that have the same configuration as the graph of Figure 51(c) and 6 is the number of times that this subgraph is counted in M. Let P_3 denotes the number of all subgraphs of G that have the same configuration as the graph of Figure 51(d) and are counted in M. Thus P_3= $1\times(\frac{1}{12}$ $F_{21})$, where $\frac{1}{12}$ F_{21} is the number of subgraphs of G that have the same configuration as the graph of Figure 51(d) and 1 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 51(e) and are counted in M. Thus P_4= $2\times(\frac{1}{32}$ $F_7)$, where $\frac{1}{32}$ F_7 is the number of subgraphs of G that have the same configuration as the graph of Figure 51(e) and 2 is the number of times that this subgraph is counted in M. Let P_5 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 51(f) and are counted in M. Thus P_5= $1\times(\frac{1}{4}$ $F_{29})$, where $\frac{1}{4}$ F_{29} is the number of subgraphs of G that have the same configuration as the graph of Figure 51(f) and 1 is the number of times that this subgraph is counted in M. Consequently, F_{22}= $3\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{jj}^{(3)}a_{ij}) - \frac{1}{2}$ $F_{10} - 3$ $F_9 - \frac{1}{2}$ $F_{21} - \frac{3}{8}$ $F_7 - \frac{3}{2}$ F_{29}.

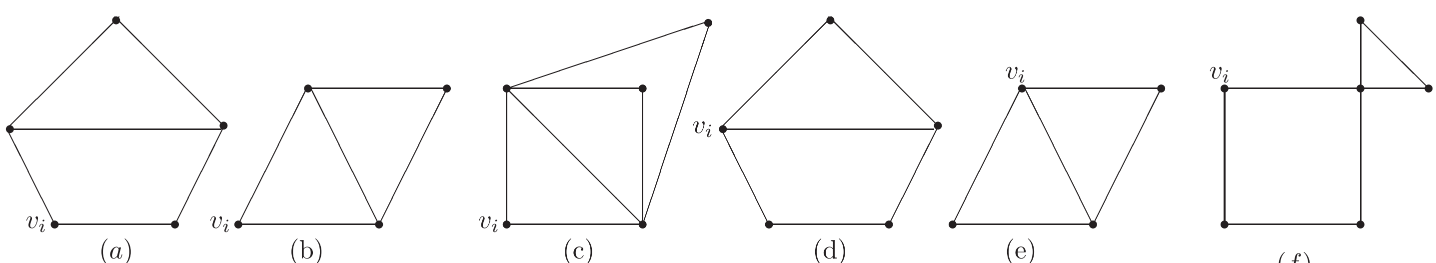


Fig 51

Case 23: For the configuration of Figure 52(a), N= 2, M= $\frac{1}{2}$ $[\sum_{j\neq i}[a_{jj}^{(5)} - 5a_{jj}^{(3)} - 2(d_j - 2)a_{jj}^{(3)} - 2\sum_{k\neq j} a_{jk}^{(2)}a_{jk}(d_k - 2) - 2\sum_{k\neq j} a_{jk}(\frac{1}{2}a_{kk}^{(3)} - a_{jk}a_{jk}^{(2)})]a_{ij}]$ (See Theorem 1.11). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 52(b) and are counted in M. Thus $P_1 = 2\times(\frac{1}{14}$ $F_{19})$, where $\frac{1}{14}$ F_{19} is the number of subgraphs of G that have the same configuration as the graph of Figure 52(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Figure 52(c) and are counted in M. Thus P_2= $1\times(\frac{1}{12}$ $F_{21})$, where $\frac{1}{12}$ F_{21} is the number of subgraphs of G that have the same configuration as the graph of Figure 52(c) and 1 is the number of times that this subgraph is counted in M. Consequently, F_{23}= $\sum_{j\neq i}[a_{jj}^{(5)} - 5a_{jj}^{(3)} - 2(d_j - 2)a_{jj}^{(3)} - 2\sum_{k\neq j} a_{jk}^{(2)}a_{jk}(d_k - 2) - 2\sum_{k\neq j} a_{jk}(\frac{1}{2}a_{kk}^{(3)} - a_{jk}a_{jk}^{(2)})]a_{ij} - \frac{2}{7}$ $F_{19} - \frac{1}{6}$ F_{21}.

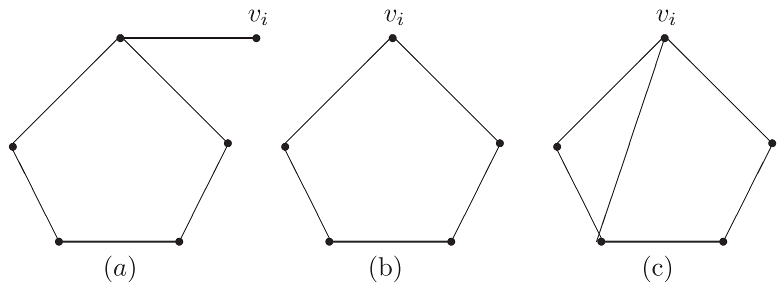


Fig 52

Case 24: For the configuration of Figure 53(a), N= 4, M= $\frac{1}{2}$ $[(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)})(d_i-2) - 2\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j - 2)(d_i-2) - 2\sum_{j\neq i} a_{ij}(d_i-2)(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$ (See Theorem 1.11). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 53(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}$ $F_{21})$, where $\frac{1}{12}$ F_{21} is the number of subgraphs of G that have the same configuration as the graph of Figure 53(b) and 1 is the number of times that this Fig is counted in M. Consequently,
$F_{24}= 2[(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)})(d_i-2) - 2\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - 2\sum_{j\neq i} a_{ij}(d_i-2)(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})] - \frac{1}{3}$ F_{21}.

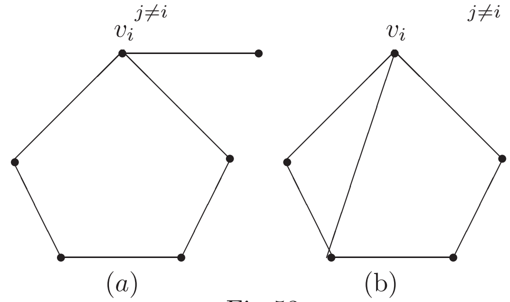


Fig 53

Case 25: For the configuration of Figure 54(a), N= 2, M= $\sum_{j\neq i}[a_{ij}^{(4)} - (d_i + d_j - 3a_{ij})a_{ij}^{(2)} - (a_{ii}^{(3)} + a_{jj}^{(3)} + 2\binom{d_j-1}{2})a_{ij}](d_j-2)a_{ij}$ (See Theorem 1.8). Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 54(b) and are counted in M. Thus $P_1 = 2 \times (\frac{1}{6}$ $F_{20})$, where $\frac{1}{6}$ F_{20} is the number of subgraphs of G that have the same configuration as the graph of Figure 54(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denote the number all subgraphs of G that have the same configuration as the graph of Figure 54(c) and are counted in M. Thus $P_2 = 1 \times (\frac{1}{6}$ $F_{22})$, where $\frac{1}{6}$ F_{22} is the number of subgraphs of G that have the same configuration as the graph of Figure 54(c) and 1 is the number of times that this subgraph is counted in M. Consequently, $F_{25} = 2\sum_{j\neq i}[a_{ij}^{(4)} - (d_i + d_j - 3a_{ij})a_{ij}^{(2)} - (a_{ii}^{(3)} + a_{jj}^{(3)} + 2\binom{d_j-1}{2})a_{ij}](d_j-2)a_{ij} - \frac{2}{3}$ $F_{20} - \frac{1}{3}$ F_{22}.

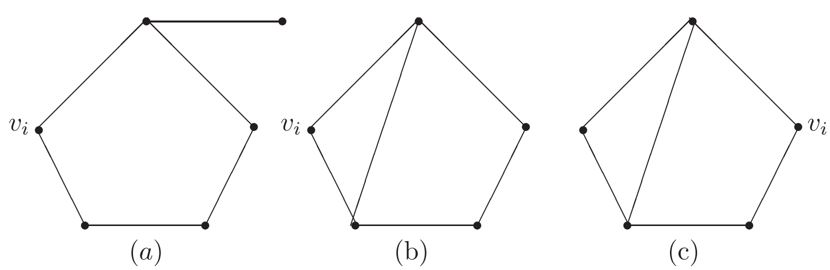
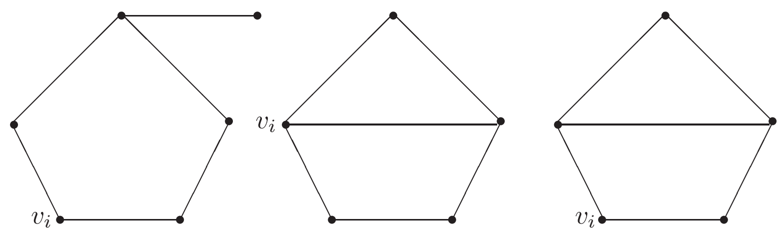


Fig 54

Case 26: For the configuration of Figure 55(a), N= 2, M= $\sum_{j\neq i}[(a_{ij}^{(3)} - (d_i + d_j - 1)a_{ij})a_{ij}^{(2)} - \sum_{k\neq i,j}(a_{ik}^{(2)}a_{ik} -$

$a_{ik}a_{ij}a_{jk})a_{jk} - \sum_{k\neq i,j}(a^{(2)}_{jk}a_{jk} - a_{ik}a_{ij}a_{jk})a_{ik}](d_j - 2)$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 55(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}\ F_{21})$, where $\frac{1}{12}\ F_{21}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 55(b) and 1 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 55(c) and are counted in M. Thus $P_2 = 1 \times (\frac{1}{6}\ F_{22})$, where $\frac{1}{6}\ F_{22}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 55(c) and 1 is the number of times that this subgraph is counted in M. Consequently,

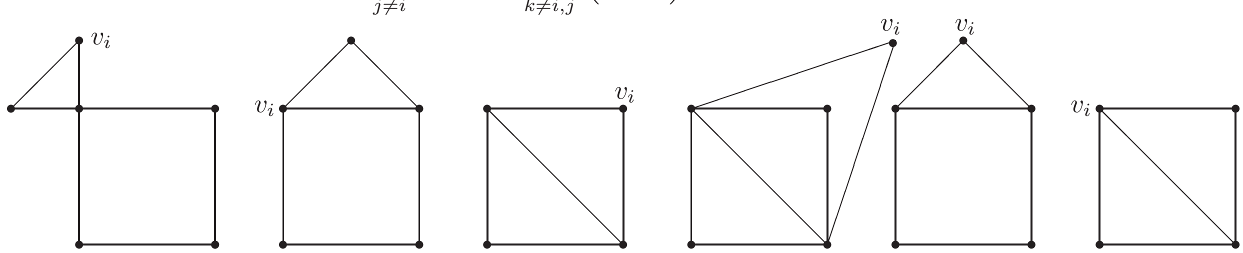
$$F= 2\sum_{j\neq i}[(a^{(3)}_{ij} - (d_i + d_j - 1)a_{ij})a^{(2)}_{ij} - \sum_{k\neq i,j}(a^{(2)}_{ik}a_{ik} - a_{ik}a_{ij}a_{jk})a_{jk} - \sum_{k\neq i,j}(a^{(2)}_{jk}a_{jk} - a_{ik}a_{ij}a_{jk})a_{ik}](d_j - 2) - \frac{1}{6}\ F_{21} - \frac{1}{3}\ F_{22}\ .$$



(a) (b) (c)

Fig 55

Case 27: For the configuration of Figure 56(a), N= 4, M= $\sum_{j\neq i}[a^{(2)}_{ij}a_{ij} \times \sum_{k\neq i,j}\binom{a^{(2)}_{ij}}{2}]$. Let Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 56(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}\ F_{21})$, where $\frac{1}{12}\ F_{21}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 56(b) and 1 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 56(c) and are counted in M. Thus $P_2 = 2 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 56(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 56(d) and are counted in M. Thus $P_3 = 2 \times (\frac{1}{12}\ F_9)$, where $\frac{1}{12}\ F_9$ is the number of subgraphs of G that have the same configuration as the graph of Fig 56(d) and 2 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 56(e) and are counted in M. Thus $P_4 = 2 \times (\frac{1}{6}\ F_{20})$, where $\frac{1}{6}\ F_{20}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 56(e) and 2 is the number of times that this subgraph is counted in M. Let P_5 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 56(f) and are counted in M. Thus $P_5 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Fig 56(f) and 2 is the number of times that this subgraph is counted in M. Consequently, $F= 4\sum_{j\neq i}[a^{(2)}_{ij}a_{ij} \times \sum_{k\neq i,j}\binom{a^{(2)}_{ij}}{2}] - \frac{1}{3}\ F_{21} - \frac{1}{3}\ F_{10} - \frac{2}{3}\ F_9 - \frac{4}{3}\ F_{20} - \frac{1}{4}\ F_7$.

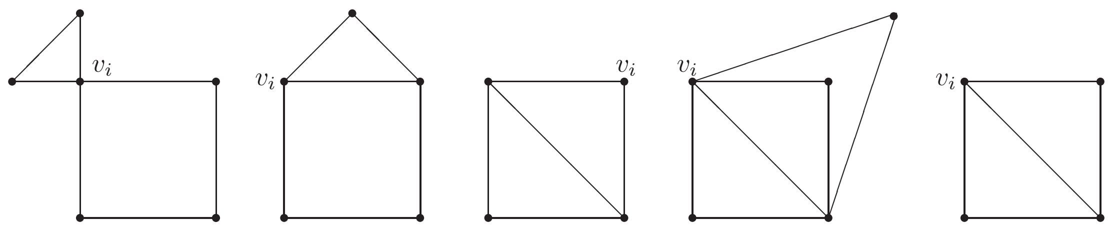


(a) (b) (c) (d) (e) (f)

Fig 56

Case 28: For the configuration of Figure 57(a), N=8, M= $(\frac{1}{2}a^{(3)}_{ii})\sum_{j\neq i}\binom{a^{(2)}_{ij}}{2}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 57(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}\ F_{21})$, where $\frac{1}{12}\ F_{21}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 57(b) and

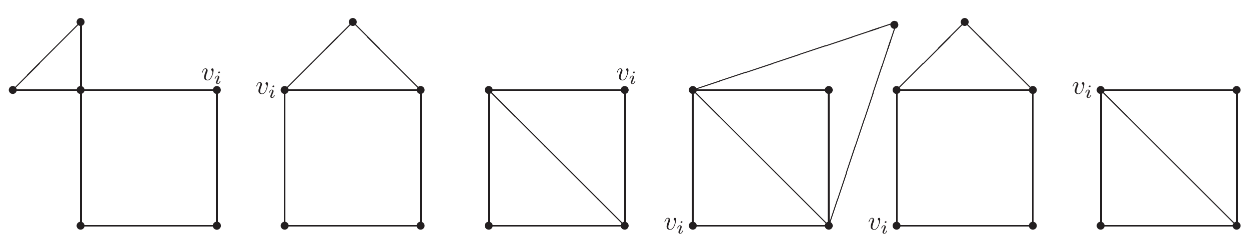
1 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 57(c) and are counted in M. Thus $P_2 = 1 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 57(c) and 1 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 57(d) and are counted in M. Thus $P_3 = 3 \times (\frac{1}{24}\ F_8)$, where $\frac{1}{24}\ F_8$ is the number of subgraphs of G that have the same configuration as the graph of Fig 57(d) and 3 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 57(e) and are counted in M. Thus $P_4 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Fig 57(e) and 2 is the number of times that this subgraph is counted in M. Consequently, F= $(4a_{ii}^{(3)})\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} - \frac{2}{3}\ F_{21} - \frac{1}{3}\ F_{10} - F_8 - \frac{1}{2}\ F_7$.



(a) (b) (c) (d) (e)

Fig 57

Case 29: For the configuration of Figure 58(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i}[(d_j - a_{ij} - 1)a_{jj}^{(3)}a_{ij}^{(2)}a_{ij}]$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 58(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{12}\ F_{21})$, where $\frac{1}{12}\ F_{21}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 58(b) and 1 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 58(c) and are counted in M. Thus $P_2 = 4 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 58(c) and 4 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 58(d) and are counted in M. Thus $P_3 = 4 \times (\frac{1}{12}\ F_9)$, where $\frac{1}{12}\ F_9$ is the number of subgraphs of G that have the same configuration as the graph of Fig 58(d) and 4 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 58(e) and are counted in M. Thus $P_4 = 1 \times (\frac{1}{6}\ F_{22})$, where $\frac{1}{6}\ F_{22}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 58(e) and 1 is the number of times that this subgraph is counted in M. Let P_5 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 58(f) and are counted in M. Thus $P_5 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Fig 58(f) and 2 is the number of times that this subgraph is counted in M. Consequently, F= $2\sum_{j\neq i}[(d_j - a_{ij} - 1)a_{jj}^{(3)}a_{ij}^{(2)}a_{ij}] - \frac{1}{3}\ F_{21} - \frac{2}{3}\ F_{10} - \frac{4}{3}\ F_9 - \frac{2}{3}\ F_{22} - \frac{1}{4}\ F_7$.



(a) (b) (c) (d) (e) (f)

Fig 58

Case 30: For the configuration of Figure 59(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{jj}^{(3)}$. Let P_1 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 59(b) and are counted in M. Thus $P_1 = 1 \times (\frac{1}{6}\ F_{22})$, where $\frac{1}{6}\ F_{22}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 59(b) and

1 is the number of times that this subgraph is counted in M. Let P_2 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 59(c) and are counted in M. Thus $P_2 = 1 \times (\frac{1}{24}\ F_{10})$, where $\frac{1}{24}\ F_{10}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 59(c) and 1 is the number of times that this subgraph is counted in M. Let P_3 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 59(d) and are counted in M. Thus $P_3 = 3 \times (\frac{1}{24}\ F_8)$, where $\frac{1}{24}\ F_8$ is the number of subgraphs of G that have the same configuration as the graph of Fig 59(d) and 3 is the number of times that this subgraph is counted in M. Let P_4 denote the number of all subgraphs of G that have the same configuration as the graph of Fig 59(e) and are counted in M. Thus $P_4 = 2 \times (\frac{1}{32}\ F_7)$, where $\frac{1}{32}\ F_7$ is the number of subgraphs of G that have the same configuration as the graph of Fig 59(e) and 2 is the number of times that this subgraph is counted in M. Consequently, F$= 2\sum_{j \neq i} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{jj}^{(3)} - \frac{2}{3}\ F_{22} - \frac{1}{6}\ F_{10} - \frac{1}{2}\ F_8 - \frac{1}{4}\ F_7$.

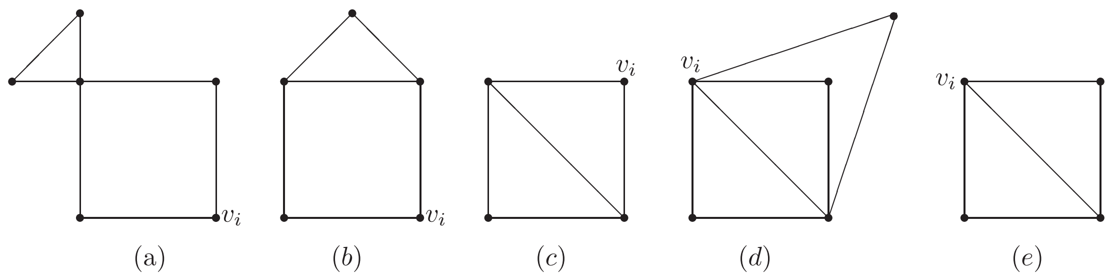


Fig 59

Now we add the values of F_n arising from the above cases and determine x. Substituting the value of x in $\frac{1}{2}(a_{ii}^{(7)} - x)$ and simplifying, we get the number of 7−cycles each of which contains a specific vertex v_i of G. □

Example 3.4 *In the graph of Fig* 29 *we have,* $F_1 = 420$, $F_2 = 1020$, $F_3 = 1080$, $F_4 = 240$, $F_5 = 360$, $F_6 = 240$, $F_7 = 960$, $F_8 = 480$, $F_9 = 360$, $F_{10} = 720$, $F_{11} = 420$, $F_{12} = 120$, $F_{13} = 240$, $F_{14} = 240$, $F_{15} = 240$, $F_{16} = 240$, $F_{17} = 480$, $F_{18} = 120$, $F_{19} = 840$, $F_{20} = 360$, $F_{21} = 1440$, $F_{22} = 720$, $F_{23} = 120$, $F_{24} = 240$, $F_{25} = 240$, $F_{26} = 240$, $F_{27} = 240$, $F_{28} = 240$, $F_{29} = 240$, $F_{30} = 120$,. *So, we have* $x = 13020$. *Consequently, by Theorem* 3.3*, the number of* 7−*cycles each of which contains the vertex* v_1 *in the graph of Fig* 29 *is* 0.

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