$a_1(1420)$ resonance as a tetraquark state and its isospin partner

Hua-Xing Chen and Er-Liang Cui

*School of Physics and Nuclear Energy Engineering and International Research Center
for Nuclei and Particles in the Cosmos, Beihang University, Beijing 100191, China*

Wei Chen[*] and T. G. Steele[†]

*Department of Physics and Engineering Physics,
University of Saskatchewan, Saskatoon, SK, S7N 5E2, Canada*

Xiang Liu[‡]

*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
Research Center for Hadron and CSR Physics, Lanzhou University
and Institute of Modern Physics of CAS, Lanzhou 730000, China*

Shi-Lin Zhu[§]

*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
Center of High Energy Physics, Peking University, Beijing 100871, China*

We systematically construct tetraquark currents of $I^G J^{PC} = 1^- 1^{++}$ and classify them into types **A** (antisymmetric), **S** (symmetric) and **M** (mixed), based on flavor symmetries of diquarks and antidiquarks composing the tetra quark currents. We use tetraquark currents of type **M** to perform QCD sum rule analyses, and find a tetraquark current $\eta_{5\mu}^M$ with quark contents $qs\bar{q}\bar{s}$ ($q = u$ or d) leading to a mass of 1.44 ± 0.08 GeV consistent with the $a_1(1420)$ state recently observed by the COMPASS collaboration. Our results support tetraquark explanations for both $a_1(1420)$ and $f_1(1420)$, assuming that they are isospin partners. We also study their possible decay patterns. As tetraquark candidates, the possible decay modes of $a_1(1420)$ are S-wave $a_1(1420) \to K^*(892)K$ and P-wave $a_1(1420) \to f_0(980)\pi$ while the possible decay patterns of $f_1(1420)$ are S-wave $f_1(1420) \to K^*(892)K$ and P-wave $f_1(1420) \to a_0(980)\pi$. We speculate that $a_1(1420)$ is partly responsible for the large isospin violation in the $\eta(1405) \to f_0(980)\pi_0$ decay mode which is reported by BESIII collaboration in the $J/\psi \to \gamma 3\pi$ process.

I. INTRODUCTION

Recently, the COMPASS collaboration at CERN observed a narrow $J^{PC} = 1^{++}$ signal in the $f_0(980)\pi$ channel, and identified a new a_1 state with mass 1414^{+15}_{-13} MeV and width 153^{+8}_{-23} MeV [1–5]. Including $a_1(1260)$, $a_1(1420)$, $a_1(1640)$, $a_1(1930)$, $a_1(2095)$ and $a_1(2270)$, there are as many as six a_1 states of quantum numbers $I^G J^{PC} = 1^- 1^{++}$ (see Ref. [6] and references therein), which is much richer compared with conventional quark model predictions. Moreover, this new $a_1(1420)$ state was observed in the $f_0(980)\pi$ channel, suggesting that $a_1(1420)$ has a large $\bar{s}s$ component, since $f_0(980)$ is usually interpreted as a $K\bar{K}$ molecule, diquark-antidiquark tetraquark, or other models with an $\bar{s}s$ component [7–15]. In particular, Refs. [13–15] used the diquark-antidiquark model to interpret the light scalar meson spectrum. Accordingly, $a_1(1420)$ can be an exotic multiquark state, which makes it quite an interesting subject.

To date, there are only a few theoretical studies of $a_1(1420)$. In Ref. [16], it was interpreted as an axial-vector two-quark-tetraquark mixed state using QCD sum rule methods, but the analysis is incomplete because it did not include the interference terms in the calculation of the mixed correlator. Ref. [17] interpreted the $a_1(1450)$ as a

dynamical effect due to the singularity in the triangle diagrams formed by the processes $a_1(1260) \to K^\star \bar{K}$, $K^\star \to K\pi$ and $K\bar{K} \to f_0(980)(+c.c)$. It was also briefly discussed using lattice QCD in Ref. [18].

In this paper we shall study the $a_1(1420)$ state in the framework of QCD sum rules, which has proven to be a successful and powerful nonperturbative method over the past few decades [19, 20]. We shall systematically construct local interpolating tetraquark currents of $I^G J^{PC} = 1^- 1^{++}$, and classify them into types **A**, **S** and **M**, respectively based on antisymmetric, symmetric, and mixed flavor symmetries of diquarks and antidiquarks. Tetraquark currents of types **A** and **S** have been investigated in Ref. [21], while in this paper we shall use tetraquark currents of type **M** to perform QCD sum rule analyses. We shall find a tetraquark current with quark contents $qs\bar{q}\bar{s}$ ($q = u$ or d), which leads to a mass result consistent with the $a_1(1420)$ state observed by the COMPASS collaboration [5]. Possible decay patterns based on this current will also be studied.

This paper is organized as follows. In Sec. II, we systematically construct tetraquark currents of $I^G J^{PC} = 1^- 1^{++}$. Then in Sec. III we use these currents of type **M** to perform QCD sum rule analyses. In Sec. IV we summarize our results and discuss possible $a_1(1420)$ decay patterns.

II. INTERPOLATING FIELDS

The flavor structure of light tetraquarks is

$$
\begin{aligned}
\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} &= \left(\bar{\mathbf{3}} \oplus \mathbf{6} \right)_{(qq)} \otimes \left(\mathbf{3} \oplus \bar{\mathbf{6}} \right)_{(\bar{q}\bar{q})} \\
&= \left(\bar{\mathbf{3}} \otimes \mathbf{3} \right)_{(\mathbf{A})} \oplus \left(\bar{\mathbf{3}} \otimes \bar{\mathbf{6}} \right)_{(\mathbf{M_1})} \oplus \left(\mathbf{6} \otimes \mathbf{3} \right)_{(\mathbf{M_2})} \oplus \left(\mathbf{6} \otimes \bar{\mathbf{6}} \right)_{(\mathbf{S})} \\
&= \left(\mathbf{1} \oplus \mathbf{8} \right) \oplus \left(\mathbf{8} \oplus \overline{\mathbf{10}} \right) \oplus \left(\mathbf{8} \oplus \mathbf{10} \right) \oplus \left(\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27} \right),
\end{aligned}
\tag{1}
$$

where the subscripts **A**, **M₁/M₂** and **S** denote that the diquarks and antidiquarks inside have antisymmetric, mixed-symmetric and symmetric flavor structures, respectively.

The tetraquark currents of $J^P = 1^+$ have been constructed in Ref. [21]. Now we need to take the charge-conjugation parity into account in order to construct tetraquark currents of $I^G J^{PC} = 1^- 1^{++}$. The charge-conjugation transformation changes diquarks into antidiquarks, and vice versa, while keeping their flavor symmetries unchanged. Therefore, tetraquark currents themselves can have definite charge-conjugation parities when the diquark and antidiquark fields inside have a symmetric flavor structure $\mathbf{6_f}(qq) \otimes \bar{\mathbf{6}}_\mathbf{f}(\bar{q}\bar{q})$ (**S**) or an antisymmetric flavor structure $\bar{\mathbf{3}}_\mathbf{f}(qq) \otimes \mathbf{3_f}(\bar{q}\bar{q})$ (**A**). These currents have been constructed in Ref. [21], i.e., there are two independent tetraquark currents of quantum numbers $J^{PC} = 1^{++}$ and type **S**:

$$
\begin{aligned}
\psi_{\mathbf{S},1\mu} &= q_A^{aT} \mathbb{C} q_B^b (\bar{q}_C^a \gamma_\mu \gamma_5 \mathbb{C} \bar{q}_D^{bT} + \bar{q}_C^b \gamma_\mu \gamma_5 \mathbb{C} \bar{q}_D^{aT}) + q_A^{aT} \mathbb{C} \gamma_\mu \gamma_5 q_B^b (\bar{q}_C^a \mathbb{C} \bar{q}_D^{bT} + \bar{q}_C^b \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{S},2\mu} &= q_A^{aT} \mathbb{C} \gamma^\nu q_B^b (\bar{q}_C^a \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{q}_D^{bT} - \bar{q}_C^b \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{q}_D^{aT}) + q_A^{aT} \mathbb{C} \sigma_{\mu\nu} \gamma_5 q_B^b (\bar{q}_C^a \gamma^\nu \mathbb{C} \bar{q}_D^{bT} - \bar{q}_C^b \gamma^\nu \mathbb{C} \bar{q}_D^{aT}),
\end{aligned}
\tag{2}
$$

and two independent tetraquark currents of quantum numbers $J^{PC} = 1^{++}$ and type **A**:

$$
\begin{aligned}
\psi_{\mathbf{A},1\mu} &= q_A^{aT} \mathbb{C} q_B^b (\bar{q}_C^a \gamma_\mu \gamma_5 \mathbb{C} \bar{q}_D^{bT} - \bar{q}_C^b \gamma_\mu \gamma_5 \mathbb{C} \bar{q}_D^{aT}) + q_A^{aT} \mathbb{C} \gamma_\mu \gamma_5 q_B^b (\bar{q}_C^a \mathbb{C} \bar{q}_D^{bT} - \bar{q}_C^b \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{A},2\mu} &= q_A^{aT} \mathbb{C} \gamma^\nu q_B^b (\bar{q}_C^a \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{q}_D^{bT} + \bar{q}_C^b \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{q}_D^{aT}) + q_A^{aT} \mathbb{C} \sigma_{\mu\nu} \gamma_5 q_B^b (\bar{q}_C^a \gamma^\nu \mathbb{C} \bar{q}_D^{bT} + \bar{q}_C^b \gamma^\nu \mathbb{C} \bar{q}_D^{aT}).
\end{aligned}
\tag{3}
$$

In these expressions, $q_A^a(x) = [u_a(x), d_a(x), s_a(x)]$ denotes the flavor triplet quark field; $A \cdots D$ are flavor indices; a and b are color indices; $\mathbb{C}$ is the charge-conjugation operator; and the superscript T denotes the transpose of the Dirac indices.

Tetraquark currents constructed using combinations of $\bar{\mathbf{3}}_\mathbf{f} \otimes \bar{\mathbf{6}}_\mathbf{f}$ (**M₁**) and $\mathbf{6_f} \otimes \mathbf{3_f}$ (**M₂**) can also have definite charge-conjugation parities (see Ref. [22] for detailed discussions). Tetraquark currents of $J^{PC} = 1^+$ and types **M₁/M₂** have been constructed in Ref. [21]:

$$
\begin{aligned}
\psi_{\mathbf{M_1},1\mu} &= (q_A^{aT} \mathbb{C} \gamma_5 q_B^b)(\bar{q}_A^a \gamma_\mu \mathbb{C} \bar{q}_D^{bT} - \bar{q}_A^b \gamma_\mu \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_1},2\mu} &= (q_A^{aT} \mathbb{C} \gamma_\mu q_B^b)(\bar{q}_A^a \gamma_5 \mathbb{C} \bar{q}_D^{bT} + \bar{q}_A^b \gamma_5 \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_1},3\mu} &= (q_A^{aT} \mathbb{C} \gamma_\nu \gamma_5 q_B^b)(\bar{q}_A^a \sigma_{\mu\nu} \mathbb{C} \bar{q}_D^{bT} - \bar{q}_A^b \sigma_{\mu\nu} \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_1},4\mu} &= (q_A^{aT} \mathbb{C} \sigma_{\mu\nu} q_B^b)(\bar{q}_A^a \gamma_\nu \gamma_5 \mathbb{C} \bar{q}_D^{bT} + \bar{q}_A^b \gamma_\nu \gamma_5 \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_2},1\mu} &= (q_A^{aT} \mathbb{C} \gamma_\mu q_B^b)(\bar{q}_A^a \gamma_5 \mathbb{C} \bar{q}_D^{bT} - \bar{q}_A^b \gamma_5 \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_2},2\mu} &= (q_A^{aT} \mathbb{C} \gamma_5 q_B^b)(\bar{q}_A^a \gamma_\mu \mathbb{C} \bar{q}_D^{bT} + \bar{q}_A^b \gamma_\mu \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_2},3\mu} &= (q_A^{aT} \mathbb{C} \sigma_{\mu\nu} q_B^b)(\bar{q}_A^a \gamma_\nu \gamma_5 \mathbb{C} \bar{q}_D^{bT} - \bar{q}_A^b \gamma_\nu \gamma_5 \mathbb{C} \bar{q}_D^{aT}), \\
\psi_{\mathbf{M_2},4\mu} &= (q_A^{aT} \mathbb{C} \gamma_\nu \gamma_5 q_B^b)(\bar{q}_A^a \sigma_{\mu\nu} \mathbb{C} \bar{q}_D^{bT} + \bar{q}_A^b \sigma_{\mu\nu} \mathbb{C} \bar{q}_D^{aT}).
\end{aligned}
\tag{4}
$$

We can use these currents to construct positive charge-conjugation parity currents ($J^{PC} = 1^{++}$)

$$\psi_{\mathbf{M},i\mu} = \psi_{\mathbf{M_1},i\mu} + \psi_{\mathbf{M_2},i\mu}, i = 1, \cdots, 4, \tag{5}$$

as well as negative charge-conjugation parity currents ($J^{PC} = 1^{+-}$)

$$\psi_{\mathbf{M'},i\mu} = \psi_{\mathbf{M_1},i\mu} - \psi_{\mathbf{M_2},i\mu}, i = 1, \cdots, 4, \tag{6}$$

where we have denoted them as types $\mathbf{M}$ and $\mathbf{M'}$.

We now consider the isospin degree of freedom (see Fig. 1 of Ref. [22] and related discussions). There are two isospin triplets of type $\mathbf{S}$, whose quark contents are

$$qq\bar{q}\bar{q}(\mathbf{S}), qs\bar{q}\bar{s}(\mathbf{S}) \sim \mathbf{6_f} \otimes \mathbf{\bar{6}_f}\,(\mathbf{S}), \tag{7}$$

one isospin triplet of type $\mathbf{A}$, whose quark contents are

$$qs\bar{q}\bar{s}(\mathbf{A}) \sim \mathbf{\bar{3}_f} \otimes \mathbf{3_f}\,(\mathbf{A}), \tag{8}$$

and two isospin triplets of type $\mathbf{M}$, whose quark contents are

$$qq\bar{q}\bar{q}(\mathbf{M}), qs\bar{q}\bar{s}(\mathbf{M}) \sim (\mathbf{\bar{3}_f} \otimes \mathbf{\bar{6}_f}) \oplus (\mathbf{6_f} \otimes \mathbf{3_f})\,(\mathbf{M}). \tag{9}$$

In these expressions q represents an up or down quark, and s represents a strange quark.

With all these analyses, we can construct tetraquark currents of quantum numbers $I^G J^{PC} = 1^- 1^{++}$ and types $\mathbf{S}/\mathbf{A}/\mathbf{M}$ and collect them as follows.

1. For the two isospin triplets belonging to $\mathbf{6}_f \otimes \mathbf{\bar{6}}_f\,(\mathbf{S})$, there are altogether four independent tetraquark currents. Among them, two contain only light flavors, and the other two contain one $s\bar{s}$ quark pair:

$$\eta_{1\mu}^S \equiv \psi_{\mathbf{S},1\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} d_b(\bar{u}_a \gamma_\mu \gamma_5 \mathbb{C} \bar{d}_b^T + \bar{u}_b \gamma_\mu \gamma_5 \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \gamma_\mu \gamma_5 d_b(\bar{u}_a \mathbb{C} \bar{d}_b^T + \bar{u}_b \mathbb{C} \bar{d}_a^T),$$

$$\eta_{2\mu}^S \equiv \psi_{\mathbf{S},2\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} \gamma^\nu d_b(\bar{u}_a \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \sigma_{\mu\nu} \gamma_5 d_b(\bar{u}_a \gamma^\nu \mathbb{C} \bar{d}_b^T - \bar{u}_b \gamma^\nu \mathbb{C} \bar{d}_a^T), \tag{10}$$

$$\eta_{3\mu}^S \equiv \psi_{\mathbf{S},1\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} s_b(\bar{u}_a \gamma_\mu \gamma_5 \mathbb{C} \bar{s}_b^T + \bar{u}_b \gamma_\mu \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \gamma_\mu \gamma_5 s_b(\bar{u}_a \mathbb{C} \bar{s}_b^T + \bar{u}_b \mathbb{C} \bar{s}_a^T),$$

$$\eta_{4\mu}^S \equiv \psi_{\mathbf{S},2\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \gamma^\nu s_b(\bar{u}_a \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \sigma_{\mu\nu} \gamma_5 s_b(\bar{u}_a \gamma^\nu \mathbb{C} \bar{s}_b^T - \bar{u}_b \gamma^\nu \mathbb{C} \bar{s}_a^T).$$

2. For the isospin triplet belonging to $\mathbf{\bar{3}}_f \otimes \mathbf{3}_f\,(\mathbf{A})$, there are two independent currents. They both contain one $s\bar{s}$ quark pair:

$$\eta_{1\mu}^A \equiv \psi_{\mathbf{A},1\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} s_b(\bar{u}_a \gamma_\mu \gamma_5 \mathbb{C} \bar{s}_b^T - \bar{u}_b \gamma_\mu \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \gamma_\mu \gamma_5 s_b(\bar{u}_a \mathbb{C} \bar{s}_b^T - \bar{u}_b \mathbb{C} \bar{s}_a^T), \tag{11}$$

$$\eta_{2\mu}^A \equiv \psi_{\mathbf{A},2\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \gamma^\nu s_b(\bar{u}_a \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \sigma_{\mu\nu} \gamma_5 s_b(\bar{u}_a \gamma^\nu \mathbb{C} \bar{s}_b^T + \bar{u}_b \gamma^\nu \mathbb{C} \bar{s}_a^T).$$

Note that the two corresponding currents with quark contents $ud\bar{u}\bar{d}$, $\psi_{\mathbf{A},1\mu}(qq\bar{q}\bar{q})$ and $\psi_{\mathbf{A},2\mu}(qq\bar{q}\bar{q})$, both have isospin zero, as shown in Fig. 1 of Ref. [22].

3. For the two isospin triplets belonging to $(\mathbf{\bar{3}}_f \otimes \mathbf{\bar{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)\,(\mathbf{M})$, there are eight independent currents. Among them, four contain only light flavors, and the other four contain one $s\bar{s}$ quark pair:

$$\eta_{1\mu}^M \equiv \psi_{\mathbf{M},1\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} \gamma_5 d_b(\bar{u}_a \gamma_\mu \mathbb{C} \bar{d}_b^T - \bar{u}_b \gamma_\mu \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \gamma_\mu d_b(\bar{u}_a \gamma_5 \mathbb{C} \bar{d}_b^T - \bar{u}_b \gamma_5 \mathbb{C} \bar{d}_a^T),$$

$$\eta_{2\mu}^M \equiv \psi_{\mathbf{M},2\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} \gamma_\mu d_b(\bar{u}_a \gamma_5 \mathbb{C} \bar{d}_b^T + \bar{u}_b \gamma_5 \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \gamma_5 d_b(\bar{u}_a \gamma_\mu \mathbb{C} \bar{d}_b^T + \bar{u}_b \gamma_\mu \mathbb{C} \bar{d}_a^T),$$

$$\eta_{3\mu}^M \equiv \psi_{\mathbf{M},3\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} \gamma_\nu \gamma_5 d_b(\bar{u}_a \sigma_{\mu\nu} \mathbb{C} \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \sigma_{\mu\nu} d_b(\bar{u}_a \gamma_\nu \gamma_5 \mathbb{C} \bar{d}_b^T - \bar{u}_b \gamma_\nu \gamma_5 \mathbb{C} \bar{d}_a^T),$$

$$\eta_{4\mu}^M \equiv \psi_{\mathbf{M},4\mu}(qq\bar{q}\bar{q}) \sim u_a^T \mathbb{C} \sigma_{\mu\nu} d_b(\bar{u}_a \gamma_\nu \gamma_5 \mathbb{C} \bar{d}_b^T + \bar{u}_b \gamma_\nu \gamma_5 \mathbb{C} \bar{d}_a^T) + u_a^T \mathbb{C} \gamma_\nu \gamma_5 d_b(\bar{u}_a \sigma_{\mu\nu} \mathbb{C} \bar{d}_b^T + \bar{u}_b \sigma_{\mu\nu} \mathbb{C} \bar{d}_a^T),$$

$$\eta_{5\mu}^M \equiv \psi_{\mathbf{M},1\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \gamma_5 s_b(\bar{u}_a \gamma_\mu \mathbb{C} \bar{s}_b^T - \bar{u}_b \gamma_\mu \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \gamma_\mu s_b(\bar{u}_a \gamma_5 \mathbb{C} \bar{s}_b^T - \bar{u}_b \gamma_5 \mathbb{C} \bar{s}_a^T), \tag{12}$$

$$\eta_{6\mu}^M \equiv \psi_{\mathbf{M},2\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \gamma_\mu s_b(\bar{u}_a \gamma_5 \mathbb{C} \bar{s}_b^T + \bar{u}_b \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \gamma_5 s_b(\bar{u}_a \gamma_\mu \mathbb{C} \bar{s}_b^T + \bar{u}_b \gamma_\mu \mathbb{C} \bar{s}_a^T),$$

$$\eta_{7\mu}^M \equiv \psi_{\mathbf{M},3\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \gamma_\nu \gamma_5 s_b(\bar{u}_a \sigma_{\mu\nu} \mathbb{C} \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \sigma_{\mu\nu} s_b(\bar{u}_a \gamma_\nu \gamma_5 \mathbb{C} \bar{s}_b^T - \bar{u}_b \gamma_\nu \gamma_5 \mathbb{C} \bar{s}_a^T),$$

$$\eta_{8\mu}^M \equiv \psi_{\mathbf{M},4\mu}(qs\bar{q}\bar{s}) \sim u_a^T \mathbb{C} \sigma_{\mu\nu} s_b(\bar{u}_a \gamma_\nu \gamma_5 \mathbb{C} \bar{s}_b^T + \bar{u}_b \gamma_\nu \gamma_5 \mathbb{C} \bar{s}_a^T) + u_a^T \mathbb{C} \gamma_\nu \gamma_5 s_b(\bar{u}_a \sigma_{\mu\nu} \mathbb{C} \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} \mathbb{C} \bar{s}_a^T).$$

In these expressions the quark content is not exactly correct, so we use "$\sim$" instead of "=". As an example, the current $\eta_{5\mu}^{M}$ contains quark content $us\bar{u}\bar{s}$ and so does not have $I = 1$. To be isovector, it should have quark content $(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$, while its isoscalar partner should have quark content $(us\bar{u}\bar{s} + ds\bar{d}\bar{s})$. However, we do not study effects of isospin breaking in this paper, i.e., we work in the limit of $SU(2)$ isospin symmetry and ignore the difference between up and down quarks, such as their masses and the quark condensates $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$. Similarly, isospin-violating effects from instantons, which are important in the scalar channels [23], are suppressed for the vector channel under consideration. Accordingly, the QCD sum rule results for these two currents with quark contents $(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$ and $(us\bar{u}\bar{s} + ds\bar{d}\bar{s})$ are both the same as the result for $\eta_{5\mu}^{M}$ with quark contents $us\bar{u}\bar{s}$ and similarly for the other currents listed above. This suggests that we would obtain the same sum rule for an isovector tetraquark current of $I^{G}J^{PC} = 1^{-}1^{++}$ and its isoscalar partner of $I^{G}J^{PC} = 0^{+}1^{++}$, which would consequently result in the same mass result for the relevant isovector state and its isoscalar partner.

The tetraquark currents of types **A/S** and quantum numbers $I^{G}J^{PC} = 1^{-}1^{++}$, $\eta_{1\mu}^{S}(qq\bar{q}\bar{q})$, $\eta_{2\mu}^{S}(qq\bar{q}\bar{q})$, $\eta_{3\mu}^{S}(qs\bar{q}\bar{s})$, $\eta_{4\mu}^{S}(qs\bar{q}\bar{s})$, $\eta_{1\mu}^{A}(qs\bar{q}\bar{s})$ and $\eta_{2\mu}^{A}(qs\bar{q}\bar{s})$, have been used to perform QCD sum rule analyses in Ref. [21], and respectively result in similar masses, $1.51 - 1.57$ GeV, $1.52 - 1.57$ GeV, $1.56 - 1.62$ GeV, $1.56 - 1.62$ GeV, $1.57 - 1.63$ GeV and $1.57 - 1.63$ GeV(see Sec. 5.2 and Fig. 4 of Ref. [21] for detailed discussions, where all mass curves have a minimum around 1.5-1.6 GeV against the threshold value s_0). One conclusion of Ref. [21] is that these tetraquark currents couple to the $a_1(1640)$ state. Recently, a new $a_1(1420)$ state was observed, with mass 1414_{-13}^{+15} MeV and width 153_{-23}^{+8} MeV [5]. The masses of these two states are not far from each other, so that if a current couples to both of them and we still use a one-pole parametrization (see Eq. (16) below and related discussion), a prediction between these two masses would be obtained for this single pole model. This may be the reason why the mass prediction 1.5–1.6 GeV is obtained in Ref. [21].

To better understand the properties of $a_1(1420)$, we need to differentiate it from $a_1(1640)$. To do this one can either adopt a two-pole parametrization, or use a current mainly coupling to $a_1(1420)$. The former is impractical because one needs detailed phenomenological models for such closely-spaced resonances, so in this paper we shall try the latter approach. Considering that only tetraquark currents of types **A** and **S** were investigated in Ref. [21], we shall use tetraquark currents of type **M**, $\eta_{i\mu}^{M}$ $(i = 1 \cdots 8)$, to perform QCD sum rule analyses and check whether such a current exists or not. We assume that they couple to the a_1 state of $I^{G}J^{PC} = 1^{-}1^{++}$ through

$$\langle 0|\eta_{i\mu}^{M}|a_1\rangle = f_{M,i}\epsilon_\mu\,, \ i = 1\cdots 8\,, \tag{13}$$

where $f_{M,i}$ is the decay constant.

III.   QCD SUM RULE ANALYSIS

We consider the following two-point correlation function

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iqx}\langle 0|TJ_\mu(x)J_\nu^\dagger(0)|0\rangle \tag{14}$$
$$= \Pi(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi'(q^2)\frac{q_\mu q_\nu}{q^2}\,,$$

in which $J_\mu(x)$ is an interpolating current carrying the same quantum numbers as the hadron state we want to study. Because $J_\mu(x)$ is not a conserved current, there are two different Lorentz structures in $\Pi_{\mu\nu}$, $\Pi(q^2)$ and $\Pi'(q^2)$ related to spin-1 and spin-0 states, respectively.

The two-point function $\Pi_{\mu\nu}(q^2)$ can be calculated in the QCD operator product expansion (OPE) up to certain order in the expansion, which is then matched with a hadronic parametrization to extract information about hadron properties. To do this, we express Eq. (14) at the hadron level as

$$\Pi(q^2) = \frac{1}{\pi}\int_{s_<}^{\infty} \frac{\text{Im}\Pi(s)}{s - q^2 - i\varepsilon}ds\,, \tag{15}$$

where we have used the form of the dispersion relation with a spectral function with $s_<$ denoting the physical threshold. We can write the imaginary part of Eq. (15) as

$$\text{Im}\Pi(s) \equiv \pi\sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle\,. \tag{16}$$

As usual, we adopt a parametrization of one-pole dominance for the ground state and a continuum contribution, but note that the masses of $a_1(1420)$ and $a_1(1640)$ are not far from each other so that it may be more reasonable to adopt

a two-pole parametrization, which is, however, impractical because one needs detailed phenomenological models for such closely-spaced resonances. After performing Borel transform at both the hadron and QCD levels, the two-point correlation function can be expressed as

$$\Pi^{(all)}(M_B^2) \equiv \mathcal{B}_{M_B^2} \Pi(p^2) = \frac{1}{\pi} \int_{s_<}^{\infty} e^{-s/M_B^2} \mathrm{Im}\Pi(s) ds \,. \tag{17}$$

Finally, we assume that the contribution from the continuum states can be approximated well by the OPE spectral density above a threshold value s_0 (duality), and arrive at the sum rule relation which can be used to perform numerical analyses. Here we again use the current $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qs\bar{q}\bar{s})$ as an example, whose quark contents are $qs\bar{q}\bar{s}$. We assume it couples to the a_1 state through Eq. (13), and the obtained sum rule relation is listed in Eq. (A5). The results for other currents are shown in Appendix. A. We note that the *Mathematica* FEYNCALC package [24] is used to calculate these OPEs. In these equations, there are dimension $D = 3$ quark condensates $\langle\bar{q}q\rangle$ and $\langle\bar{s}s\rangle$, $D = 4$ gluon condensate $\langle g^2GG\rangle$, and $D = 5$ mixed condensates $\langle g\bar{q}\sigma Gq\rangle$ and $\langle g\bar{s}\sigma Gs\rangle$. The vacuum saturation for higher dimensional condensates are assumed as usual, such as $\langle 0|\bar{q}q\bar{q}q|0\rangle \sim \langle 0|\bar{q}q|0\rangle\langle 0|\bar{q}q|0\rangle$. We have neglected the chirally suppressed contributions from current up and down quark masses because they are numerically insignificant. Moreover, we consider only leading-order contributions of α_s from the two-gluon condensate ($\langle g_s^2GG\rangle$) because the terms containing quark-related condensates are found to be significantly larger than those containing gluon-related condensates.

To study the convergence of Eq. (A5), we use the following values for various condensates [25–33]:

$$
\begin{aligned}
\langle\bar{q}q\rangle &= -(0.240 \pm 0.010)^3 \text{ GeV}^3 \,, \\
\langle\bar{s}s\rangle &= -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3 \,, \\
\langle g_s^2GG\rangle &= (0.48 \pm 0.14) \text{ GeV}^4 \,, \\
\langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle \,, \\
\langle g_s\bar{s}\sigma Gs\rangle &= -M_0^2 \times \langle\bar{s}s\rangle \,, \\
M_0^2 &= 0.8 \text{ GeV}^2 \,, \\
m_s(1 \text{ GeV}) &= 125 \pm 20 \text{ MeV} \,.
\end{aligned}
\tag{18}
$$

Note that there is a minus sign implicitly included in the definition of the coupling constant g_s in this work. We find that the $D = 6$ and $D = 8$ terms are dominant power corrections, while the $D = 10$ and $D = 12$ terms are much smaller. Actually, the $D = 6$ and $D = 8$ terms in Eq. (A5) are mainly contributed by the condensates $\langle\bar{q}q\rangle\langle\bar{s}s\rangle$ and $\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle/\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle$, respectively. Accordingly, our first criterion is to require that the $D = 10$ and $D = 12$ terms be less than 10%:

$$\text{Convergence (CVG)} \equiv |\frac{\Pi_{M,5}^{\text{high-order}}(\infty, M_B^2)}{\Pi_{M,5}(\infty, M_B^2)}| \leq 10\% \,, \tag{19}$$

where $\Pi_{M,5}^{\text{high-order}}(s_0, M_B^2)$ is the sum of the $D = 10$ and $D = 12$ terms. We show this in the left panel of Fig. 1, which shows that the OPE convergence improves with the increase of M_B. This criterion has a limitation on the Borel mass that $M_B^2 \geq 1.1 \text{ GeV}^2$. We note that this criterion gives almost no limitations if we assume $\Pi_{M,5}^{\text{high-order}}(s_0, M_B^2)$ to only contain the $D = 12$ terms, which implies that the contribution of the $D = 12$ terms is numerically small.

Our second criterion is to require that the pole contribution be larger than 10% (see discussions below for the limitation 20%):

$$\text{Pole contribution (PC)} \equiv \frac{\Pi_{M,5}(s_0, M_B^2)}{\Pi_{M,5}(\infty, M_B^2)} \geq 10\% \,. \tag{20}$$

We note that the pole contribution is usually quite small in the multi-quark sum rule analyses due to the large powers of s in the spectral function. We show the variation of the pole contribution with respect to the Borel mass M_B in the right panel of Fig. 1, when s_0 is chosen to be 2.5 GeV^2. It shows that the PC decreases with the increase of M_B. This criterion has a limitation on the Borel mass that $M_B^2 \leq 1.5 \text{ GeV}^2$. Finally we obtain the working region of Borel mass $1.1 \text{ GeV}^2 < M_B^2 < 1.5 \text{ GeV}^2$ for the current $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qs\bar{q}\bar{s})$ with the continuum threshold $s_0 = 2.5 \text{ GeV}^2$.

Our final expression for the mass of the a_1 state is obtained via:

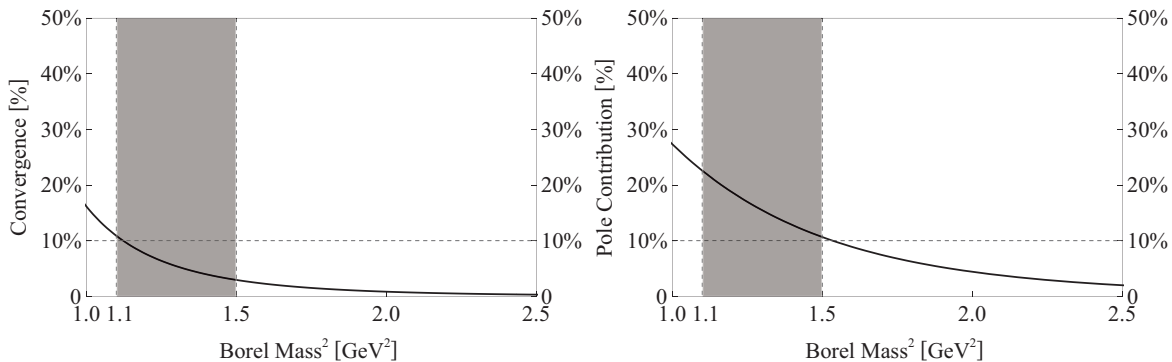$$M_{a_1}^2 = \frac{\frac{\partial}{\partial(-1/M_B^2)}\Pi_{M,5}(s_0, M_B^2)}{\Pi_{M,5}(s_0, M_B^2)} \,, \tag{21}$$

FIG. 1: In the left panel we show CVG, as defined in Eq. (19), as a function of the Borel mass M_B. In the right panel we show the variation of PC, as defined in Eq. (20), as a function of the Borel mass M_B. The current $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qs\bar{q}\bar{s})$ is used here and the threshold value is chosen to be $s_0 = 2.5$ GeV^2.
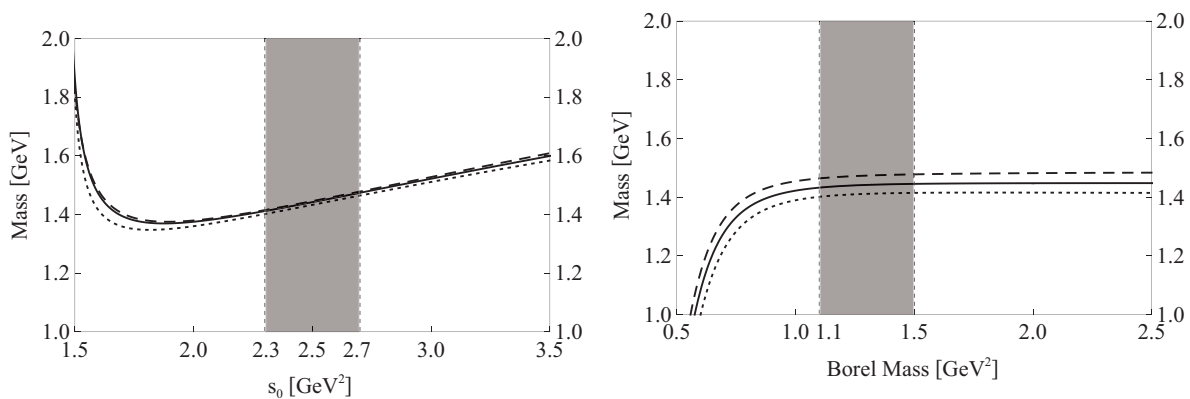

FIG. 2: The mass calculated using the current $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qs\bar{q}\bar{s})$, is shown with respect to the threshold value s_0 (left panel) for $M_B^2 = 1.1$ (dotted), 1.3 (solid) and 1.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right panel) for $s_0 = 2.3$ (dotted), 2.5 (solid), and 2.7 GeV^2 (dashed). The working region is 1.1 GeV$^2 < M_B^2 < 1.5$ GeV^2.

in which s_0 is the continuum threshold. To choose a reasonable value of s_0, we show the variation of M_{a_1} with respect to the threshold value s_0 in the left panel of Fig. 2, in a large region 1.5 GeV$^2 < s_0 < 3.5$ GeV^2. We find that the dependence of the mass curves with respect to the Borel parameter M_B^2 is very weak when the continuum threshold s_0 is chosen to be around 2.5 GeV^2, which is thus a reasonable value of s_0 to give a reliable mass prediction.

The variation of M_{a_1} with respect to the Borel mass M_B is shown in the right panel of Fig. 2, in a large region 0.5 GeV$^2 < M_B^2 < 2.5$ GeV^2. The mass curves increase quickly with M_B^2 from 0.5 GeV^2 to 1.0 GeV^2, but they are quite stable against M_B^2 as it continues increasing from 1 GeV^2. This suggests that the limitation of the second criterion, Eq. (20), can be slightly modified to be 20%, and then the mass obtained is almost the same, but with a much narrower working region. Finally, we choose 2.3 GeV$^2 < s_0 < 2.7$ GeV^2 and use the Borel window 1.1 GeV$^2 < M_B^2 < 1.5$ GeV^2 as our working region resulting in the following numerical results

$$M_{a_1} = 1.44 \pm 0.08 \text{ GeV}, \tag{22}$$
$$f_{M,5} = (1.9 \pm 0.5) \times 10^{-3} \text{ GeV}^5, \tag{23}$$

where the central values correspond to $s_0 = 2.5$ GeV^2 and $M_B^2 = 1.3$ GeV^2. The errors come from the uncertainties of s_0, M_B^2 and the various parameters in Eq. (18). The coupling constant $f_{M,5}$ defined in Eq. (13) gives the strength of the overlap between the interpolating current $\eta_{5\mu}^M$ and the $a_1(1420)$ state.

The sum rule using the current $\eta_{6\mu}^M \equiv \psi_{2\mu}^M(qs\bar{q}\bar{s})$ is similar to the previous sum rule obtained using the current $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qs\bar{q}\bar{s})$. The results are shown in Appendix A and Fig. 3. We again choose 2.3 GeV$^2 < s_0 < 2.7$ GeV^2 and
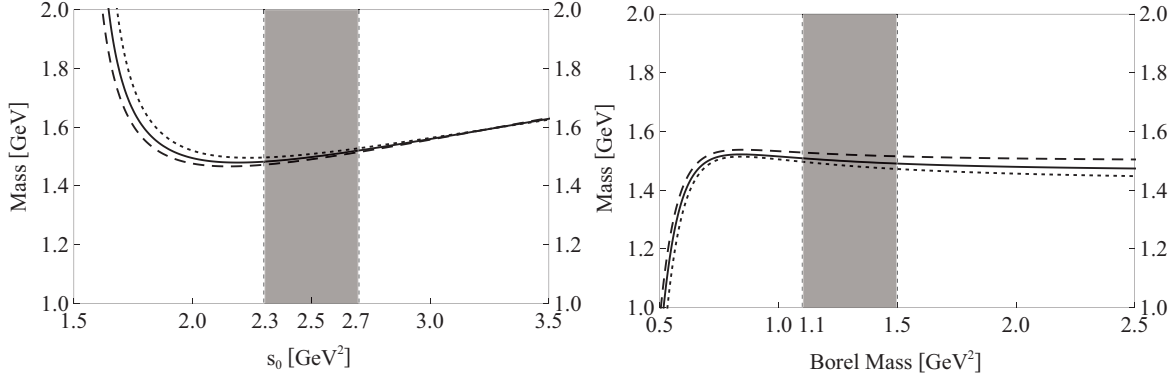
FIG. 3: The mass calculated using the current $\eta_{6\mu}^M \equiv \psi_{2\mu}^M(qs\bar{q}\bar{s})$, is shown with respect to the threshold value s_0 (left panel) for $M_B^2 = 1.1$ (dotted), 1.3 (solid) and 1.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right panel) for $s_0 = 2.3$ (dotted), 2.5 (solid), and 2.7 GeV^2 (dashed). The working region is 1.1 GeV$^2 < M_B^2 < 1.5$ GeV^2.

use the interval 1.1 GeV$^2 < M_B^2 < 1.5$ GeV^2 as our working region, and obtain the following numerical results:

$$M_{a_1} = 1.50 \pm 0.08 \text{ GeV}, \tag{24}$$
$$f_{M,6} = (2.6 \pm 0.7) \times 10^{-3} \text{ GeV}^5, \tag{25}$$

where the central values correspond to $s_0 = 2.5$ GeV^2 and $M_B^2 = 1.3$ GeV^2.

The sum rules using the currents $\eta_{1\mu}^M \equiv \psi_{1\mu}^M(qq\bar{q}\bar{q})$ and $\eta_{2\mu}^M \equiv \psi_{2\mu}^M(qq\bar{q}\bar{q})$ lead to larger masses around 1.6 GeV, as shown in Appendix A and Fig. 4. We choose 2.8 GeV$^2 < s_0 < 3.2$ GeV^2 (we note that s_0 should be larger than $M_{a_1}^2$) and use the interval 1.2 GeV$^2 < M_B^2 < 1.8$ GeV^2 as our working region. We obtain the following numerical results for $\eta_{1\mu}^M$:

$$M_{a_1} = 1.61 \pm 0.06 \text{ GeV}, \tag{26}$$
$$f_{M,1} = (3.0 \pm 0.8) \times 10^{-3} \text{ GeV}^5, \tag{27}$$

and the following numerical results for $\eta_{2\mu}^M$:

$$M_{a_1} = 1.64 \pm 0.08 \text{ GeV}, \tag{28}$$
$$f_{M,2} = (4.2 \pm 1.3) \times 10^{-3} \text{ GeV}^5, \tag{29}$$

where the central values correspond to $s_0 = 3.0$ GeV^2 and $M_B^2 = 1.5$ GeV^2. One notes that the central-value masses obtained in Eqs. (27) and (29) for the non-strange $qq\bar{q}\bar{q}$ tetraquarks are heavier than those in Eqs. (23) and (25) for the strange-flavor $qs\bar{q}\bar{s}$ tetraquarks. This counter-intuitive behavior also appears in the scalar meson sector, where the isovector $a_0(1450)$ is a bit heavier than the strange isospinor $K_0^*(1430)$ [25]. However, it is important to note that the dominant sources of theoretical uncertainty in the QCD input parameters Eq. (18) are uncorrelated in the strange and non-strange cases, and hence we cannot rule out a near degeneracy from our mass predictions.

The sum rules using the currents $\eta_{3\mu}^M \equiv \psi_{3\mu}^M(qq\bar{q}\bar{q})$, $\eta_{4\mu}^M \equiv \psi_{4\mu}^M(qq\bar{q}\bar{q})$, $\eta_{7\mu}^M \equiv \psi_{3\mu}^M(qs\bar{q}\bar{s})$ and $\eta_{8\mu}^M \equiv \psi_{4\mu}^M(qs\bar{q}\bar{s})$ do not have reasonable working regions to give reliable mass results. We show these results in Appendix A. The sum rules using the former two currents lead to mass results roughly around 1.6 GeV, suggesting that they may couple to the $a_1(1640)$ state. The sum rules using the latter two currents lead to mass results around 1.8 GeV.

IV. SUMMARY AND DISCUSSIONS

In summary, we have systematically constructed tetraquark currents of $I^G J^{PC} = 1^- 1^{++}$. These currents can be classified into types **A** (anti-symmetric), **S** (symmetric) and **M** (mixed structure), based on flavor symmetries of diquarks and antidiquarks. Tetraquark currents of types **A** and **S** had been studied in Ref. [21], and in this paper we have used the tetraquark currents of type **M** to perform QCD sum rule analyses and investigated the newly observed $a_1(1420)$ state.
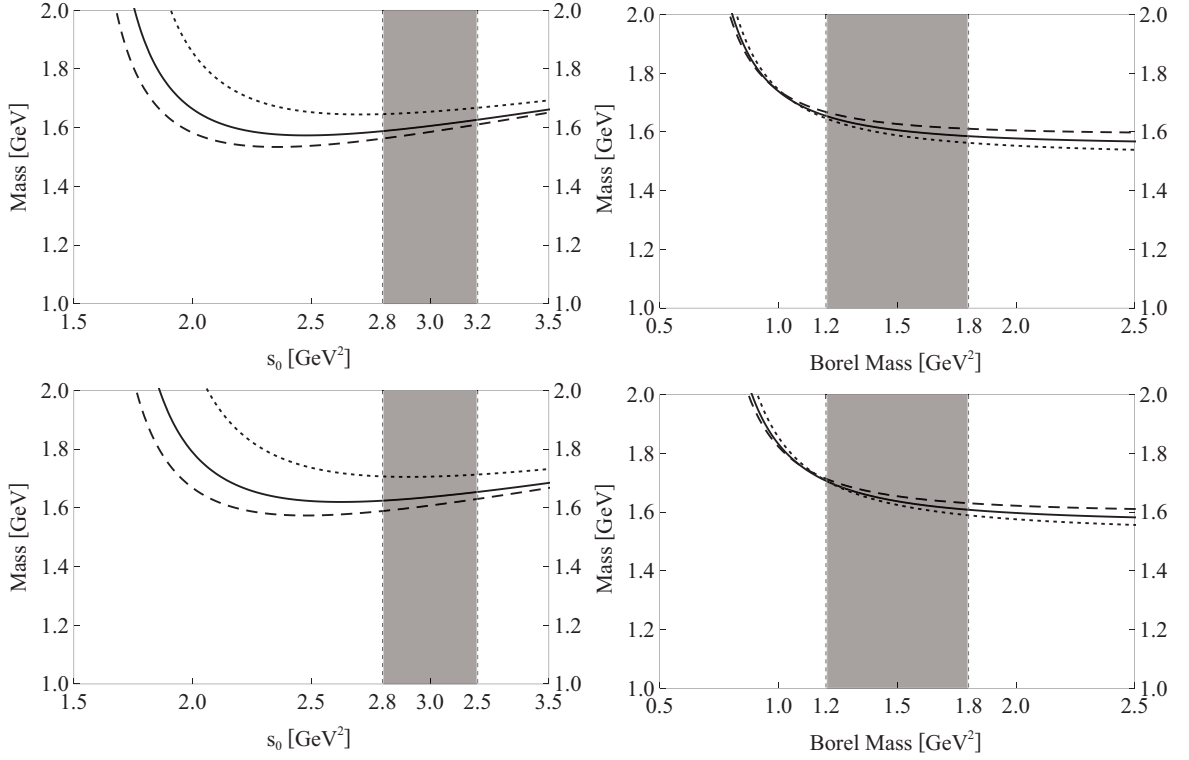
FIG. 4: The mass calculated using the currents $\eta_{1\mu}^M \equiv \psi_{1\mu}^M(qq\bar{q}\bar{q})$ (upper figures) and $\eta_{2\mu}^M \equiv \psi_{2\mu}^M(qq\bar{q}\bar{q})$ (lower figures), is shown with respect to the threshold value s_0 (left figures) for $M_B^2 = 1.2$ (dotted), 1.5 (solid) and 1.8 GeV^2 (dashed), and with respect to the Borel mass M_B (right figures) for $s_0 = 2.8$ (dotted), 3.0 (solid), and 3.2 GeV^2 (dashed). The working region is 1.0 GeV$^2 < M_B^2 < 1.8$ GeV^2. However, The mass curves decrease quickly with M_B^2 from 1.0 GeV^2 to 1.2 GeV^2. Therefore, we choose the new interval 1.2 GeV$^2 < M_B^2 < 1.8$ GeV^2 as our working region.

Combining the results of Ref. [21] and the results obtained in this paper, we found that a mass prediction around 1.5–1.6 GeV is often obtained (with respect to the threshold value s_0). This may be a reasonable result: there are two a_1 states, $a_1(1420)$ and $a_1(1640)$, whose masses are close to each other, so that if a current couples to both of them and one still uses a one-pole parametrization, a mass value between these two masses would be obtained for this single pole. However, in the absence of definitive phenomenological models it is impractical to develop a two-pole parametrization that differentiates $a_1(1420)$ from $a_1(1640)$. In this paper we have used another approach, i.e., finding a current mainly coupling to $a_1(1420)$. We have used tetraquark currents of type **M** to perform QCD sum rule analyses, and found that the current $\eta_{5\mu}^M$ leads to a mass of 1.44 ± 0.08 GeV. The good agreement of this result with the experimental value suggests that this current couples to the $a_1(1420)$ state supporting a tetraquark interpretation.

We note that the quark content $us\bar{u}\bar{s}$ of the current $\eta_{5\mu}^M$ means that it does not have a definite value of isospin(i.e, it is neither isospin one nor isospin zero). The isovector tetraquark current and its isoscalar partner can be constructed by changing the quark contents to be $(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$ and $(us\bar{u}\bar{s} + ds\bar{d}\bar{s})$, respectively. However, the same sum rule and mass prediction would be obtained for all these three currents under $SU(2)$ isospin symmetry. As noted in Ref. [5], there is another isoscalar state, $f_1(1420)$, which has been well established in experiments [25]. It strongly couples to $K\bar{K}^\star$, and is likely to be the isoscalar partner of $a_1(1420)$. If this is the case, our analyses would support tetraquark explanations for both of them.

To conclude this paper, we study the possible decay channels of $a_1(1420)$. To do this, we use the Firez transformation and change the current $\eta_{5\mu}^M$ with quark contents $(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$, i.e., $\psi_{M,1\mu}(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$:

$$\begin{aligned}
\psi_{M,1\mu}(us\bar{u}\bar{s} - ds\bar{d}\bar{s}) &= u_a^T\mathbb{C}\gamma_5 s_b(\bar{u}_a\gamma_\mu\mathbb{C}\bar{s}_b^T - \bar{u}_b\gamma_\mu\mathbb{C}\bar{s}_a^T) + u_a^T\mathbb{C}\gamma_\mu s_b(\bar{u}_a\gamma_5\mathbb{C}\bar{s}_b^T - \bar{u}_b\gamma_5\mathbb{C}\bar{s}_a^T) \\
&\quad - d_a^T\mathbb{C}\gamma_5 s_b(\bar{d}_a\gamma_\mu\mathbb{C}\bar{s}_b^T - \bar{d}_b\gamma_\mu\mathbb{C}\bar{s}_a^T) - d_a^T\mathbb{C}\gamma_\mu s_b(\bar{d}_a\gamma_5\mathbb{C}\bar{s}_b^T - \bar{d}_b\gamma_5\mathbb{C}\bar{s}_a^T) \,,
\end{aligned} \tag{30}$$

into a combination of $(\bar{q}q)(\bar{s}s)$ and $(\bar{q}s)(\bar{s}q)$ currents:

$$
\begin{aligned}
\psi_1^{(\bar{q}q)\bar{s}s} &= (\bar{u}_a u_a)(\bar{s}_b \gamma_\mu \gamma_5 s_b) - (\bar{u}_a \gamma_\mu \gamma_5 u_a)(\bar{s}_b s_b) - (\bar{d}_a d_a)(\bar{s}_b \gamma_\mu \gamma_5 s_b) + (\bar{d}_a \gamma_\mu \gamma_5 d_a)(\bar{s}_b s_b) \,, \\
\psi_2^{(\bar{q}s)\bar{s}q} &= (\bar{u}_a \gamma_\mu s_a)(\bar{s}_b \gamma_5 u_b) - (\bar{u}_a \gamma_5 s_a)(\bar{s}_b \gamma_\mu u_b) - (\bar{d}_a \gamma_\mu s_a)(\bar{s}_b \gamma_5 d_b) + (\bar{d}_a \gamma_5 s_a)(\bar{s}_b \gamma_\mu d_b) \,, \\
\psi_3^{(\bar{q}s)\bar{s}q} &= (\bar{u}_a \gamma^\nu \gamma_5 s_a)(\bar{s}_b \sigma_{\mu\nu} u_b) - (\bar{u}_a \sigma_{\mu\nu} s_a)(\bar{s}_b \gamma^\nu \gamma_5 u_b) - (\bar{d}_a \gamma^\nu \gamma_5 s_a)(\bar{s}_b \sigma_{\mu\nu} d_b) + (\bar{d}_a \sigma_{\mu\nu} s_a)(\bar{s}_b \gamma^\nu \gamma_5 d_b) \,, \\
\psi_4^{(\bar{q}q)\bar{s}s} &= (\bar{u}_a \gamma^\nu u_a)(\bar{s}_b \sigma_{\mu\nu} \gamma_5 s_b) - (\bar{u}_a \sigma_{\mu\nu} \gamma_5 u_a)(\bar{s}_b \gamma^\nu s_b) - (\bar{d}_a \gamma^\nu d_a)(\bar{s}_b \sigma_{\mu\nu} \gamma_5 s_b) + (\bar{d}_a \sigma_{\mu\nu} \gamma_5 d_a)(\bar{s}_b \gamma^\nu s_b) \,,
\end{aligned}
\tag{31}
$$

through

$$
\psi_{M,1\mu}(us\bar{u}\bar{s} - ds\bar{d}\bar{s}) = \frac{1}{2}\psi_1^{(\bar{q}q)\bar{s}s} + \frac{1}{2}\psi_2^{(\bar{q}s)\bar{s}q} - \frac{i}{2}\psi_3^{(\bar{q}s)\bar{s}q} + \frac{i}{2}\psi_4^{(\bar{q}q)\bar{s}s} \,.
\tag{32}
$$

We note that $\psi_1^{(\bar{q}q)\bar{s}s}$ and $\psi_4^{(\bar{q}q)\bar{s}s}$ both contain one $\bar{q}q$ meson and one $\bar{s}s$ meson, while $\psi_2^{(\bar{q}s)\bar{s}q}$ and $\psi_3^{(\bar{q}s)\bar{s}q}$ both contain one $\bar{q}s$ meson and one $\bar{s}q$ meson, where q represents an *up* or *down* quark, and s represents a *strange* quark. This equation suggests that $a_1(1420)$ may naively fall apart to:

$$
\begin{aligned}
\psi_1^{(\bar{q}q)\bar{s}s} &: a_1(1420) \to 0^+ (\sigma(600), a_0(980), f_0(980)\cdots) + 1^+ (b_1(1235), a_1(1260)\cdots) \,, \\
\psi_2^{(\bar{q}s)\bar{s}q} &: a_1(1420) \to 1^- (K^*(892)\cdots) + 0^- (K\cdots) \,, \\
\psi_3^{(\bar{q}s)\bar{s}q} &: a_1(1420) \to 1^+ (K_1(1270)\cdots) + 1^+ (K_1(1270)\cdots) \,, \\
\psi_4^{(\bar{q}q)\bar{s}s} &: a_1(1420) \to 1^- (\rho(770), \omega(782), \phi(1020)\cdots) + 1^- (\rho(770), \omega(782), \phi(1020)\cdots) \,.
\end{aligned}
\tag{33}
$$

These are all S-wave decay channels, while the possible P-wave decay channels can be obtained by naively relating $\bar{q}\gamma_\mu\gamma_5 q$ and $\partial_\mu\pi$:

$$
\begin{aligned}
\psi_1'^{(\bar{q}q)\bar{s}s} &: a_1(1420) \to 0^+ (\sigma(600), a_0(980), f_0(980)\cdots) + 0^- (\pi, \eta, \eta'\cdots) \,, \\
\psi_3'^{(\bar{q}s)\bar{s}q} &: a_1(1420) \to 0^- (K\cdots) + 1^+ (K_1(1270)\cdots) \,,
\end{aligned}
\tag{34}
$$
$$
\tag{35}
$$

One extra constraint is that the final states of $a_1(1420)$ should contain one $\bar{s}s$ pair. Then the kinematically allowed decay channels are S-wave $a_1(1420) \to K^*(892)K$ and P-wave $a_1(1420) \to \sigma(600)\eta$, $a_1(1420) \to a_0(980)\pi$ and $a_1(1420) \to f_0(980)\pi$. However, the P-wave decay channel $a_1(1420) \to \sigma(600)\eta$ is forbidden by the conservation of isospin symmetry, and the P-wave decay channel $a_1(1420) \to a_0(980)\pi$ is forbidden by carefully checking the detailed expression of $\psi_1^{(\bar{q}q)\bar{q}q}$.

Summarizing all the above constrains, the possible decay patterns of $a_1(1420)$ are S-wave $a_1(1420) \to K^*(892)K$ and P-wave $a_1(1420) \to f_0(980)\pi$, the latter of which is observed by the COMPASS experiment [5]. Similarly, the possible decay patterns of $f_1(1420)$ can also be studied, and they are S-wave $f_1(1420) \to K^*(892)K$ and P-wave $f_1(1420) \to a_0(980)\pi$, also consistent with the experiments [25, 35–37] (see Refs. [38–40] for related theoretical studies).

BESIII is a good platform to carry out the search for $a_1(1420)$ by the J/ψ radiative decay $J/\psi \to \gamma f_0(980)\pi$ if we take into consideration the strong interaction between $a_1(1420)$ and $f_0(980)\pi$ indicated by the COMPASS measurement [1–5]. Of course, the production ratio of $J/\psi \to \gamma a_1(1420) \to \gamma f_0(980)\pi$ is related to the inner structure of $a_1(1420)$, which determines the strength of J/ψ decaying into $\gamma a_1(1420)$. We note a former BESIII result in Ref. [34], where the $J/\psi \to \gamma\pi^+\pi^-\pi^0$ and $J/\psi \to \gamma\pi^0\pi^0\pi^0$ decays were studied and a large isospin violating process $\eta(1405) \to f_0(980)\pi^0$ was observed [34]. If there exists a new state $a_1(1420)$, it may be revealed in a BESIII re-analysis of the $\eta(1405) \to f_0(980)\pi^0$ branching ratio. This will provide a definitive test of whether $\eta(1405) \to f_0(980)\pi^0$ still has a large branching ratio when including the $a_1(1420)$ contribution in the $J/\psi \to \gamma\pi^+\pi^-\pi^0$ and $J/\psi \to \gamma\pi^0\pi^0\pi^0$ decays.

Appendix A: Other sum rules

The sum rules using the currents $\eta_{1\mu}^M \equiv \psi_{1\mu}^M(qq\bar{q}\bar{q})$ is

$$
\begin{aligned}
f_{M,1}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,1}(s_0, M_B^2) \\
&= \int_0^{s_0} \left(\frac{1}{36864\pi^6}s^4 + \frac{\langle g_s^2 GG\rangle}{18432\pi^6}s^2 + \frac{5\langle\bar{q}q\rangle^2}{36\pi^2}s + \frac{\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} \right) e^{-s/M_B^2} ds \\
&\quad + \left(\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{96\pi^2} + \frac{\langle g_s^2 GG\rangle\langle\bar{q}q\rangle^2}{864\pi^2} \right) - \frac{1}{M_B^2}\frac{\langle g_s^2 GG\rangle\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{576\pi^2} \,.
\end{aligned}
\tag{A1}
$$

The sum rules using the currents $\eta_{2\mu}^M \equiv \psi_{2\mu}^M(qq\bar{q}\bar{q})$ is

$$
\begin{aligned}
f_{M,2}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,2}(s_0, M_B^2) \\
&= \int_0^{s_0} \left(\frac{1}{18432\pi^6}s^4 - \frac{\langle g_s^2 GG \rangle}{18432\pi^6}s^2 + \frac{5\langle \bar{q}q \rangle^2}{18\pi^2}s + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{4\pi^2} \right) e^{-s/M_B^2} ds \\
&\quad + \left(\frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{48\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{864\pi^2} \right) + \frac{1}{M_B^2} \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{576\pi^2} .
\end{aligned}
\tag{A2}
$$

The sum rules using the currents $\eta_{3\mu}^M \equiv \psi_{3\mu}^M(qq\bar{q}\bar{q})$ is

$$
\begin{aligned}
f_{M,3}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,3}(s_0, M_B^2) \\
&= \int_0^{s_0} \left(\frac{1}{12288\pi^6}s^4 + \frac{\langle g_s^2 GG \rangle}{18432\pi^6}s^2 - \frac{5\langle \bar{q}q \rangle^2}{36\pi^2}s - \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{8\pi^2} \right) e^{-s/M_B^2} ds \\
&\quad + \left(-\frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{96\pi^2} + \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{288\pi^2} \right) - \frac{1}{M_B^2} \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{192\pi^2} ,
\end{aligned}
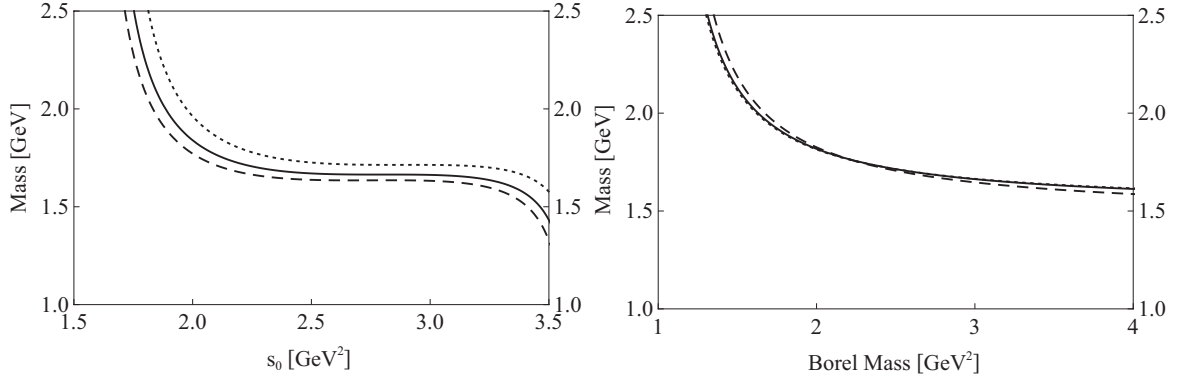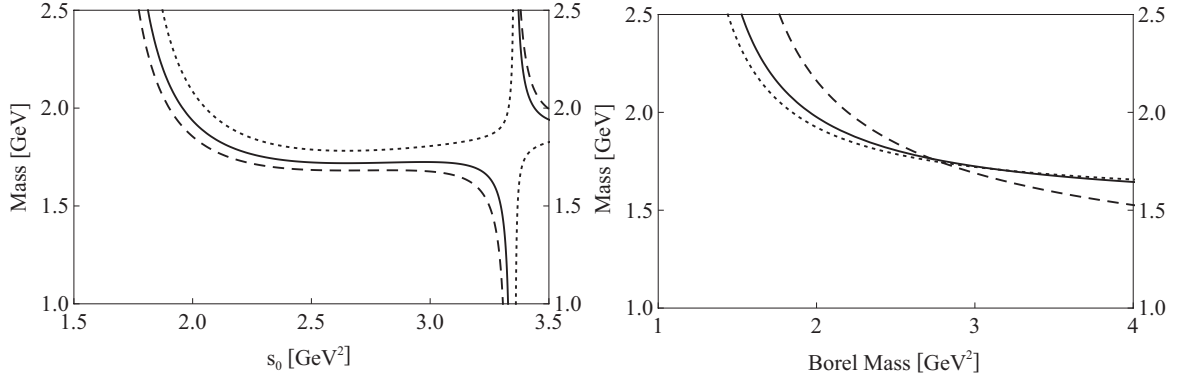\tag{A3}
$$

and the results are shown in Fig. 5.

FIG. 5: The mass calculated using the current $\eta_{3\mu}^M \equiv \psi_{3\mu}^M(qq\bar{q}\bar{q})$ is shown with respect to the threshold value s_0 (left panel) for $M_B^2 = 2.5$ (dotted), 3.0 (solid) and 3.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right panel) for $s_0 = 2.8$ (dotted), 3.0 (solid), and 3.2 GeV^2 (dashed).

The sum rules using the currents $\eta_{4\mu}^M \equiv \psi_{4\mu}^M(qq\bar{q}\bar{q})$ is

$$
\begin{aligned}
f_{M,4}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,4}(s_0, M_B^2) \\
&= \int_0^{s_0} \left(\frac{1}{6144\pi^6}s^4 + \frac{11\langle g_s^2 GG \rangle}{18432\pi^6}s^2 - \frac{5\langle \bar{q}q \rangle^2}{18\pi^2}s - \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{4\pi^2} \right) e^{-s/M_B^2} ds \\
&\quad + \left(-\frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{48\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{288\pi^2} \right) + \frac{1}{M_B^2} \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{192\pi^2} ,
\end{aligned}
\tag{A4}
$$

and the results are shown in Fig. 6.

FIG. 6: The mass calculated using the current $\eta_{4\mu}^M \equiv \psi_{4\mu}^M(qq\bar{q}\bar{q})$ is shown with respect to the threshold value s_0 (left panel) for $M_B^2 = 2.5$ (dotted), 3.0 (solid) and 3.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right panel) for $s_0 = 2.8$ (dotted), 3.0 (solid), and 3.2 GeV^2 (dashed).

The sum rules using the currents $\eta_{5\mu}^M \equiv \psi_{1\mu}^M(qq\bar{q}\bar{q})$ is

$$
\begin{aligned}
f_{M,5}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,5}(s_0, M_B^2) \\
&= \int_{4m_s^2}^{s_0} \left[\frac{1}{36864\pi^6}s^4 - \frac{m_s^2}{960\pi^6}s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} - \frac{7m_s\langle \bar{q}q \rangle}{384\pi^4} + \frac{m_s\langle \bar{s}s \rangle}{128\pi^4} \right)s^2 \right. \\
&\quad + \left(\frac{5\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{36\pi^2} - \frac{5m_s\langle g_s\bar{q}\sigma Gq \rangle}{192\pi^4} - \frac{13m_s^2\langle g_s^2 GG \rangle}{36864\pi^6} \right)s + \frac{\langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{16\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{16\pi^2} \\
&\quad \left. - \frac{m_s\langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{512\pi^4} + \frac{m_s\langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1536\pi^4} + \frac{m_s^2\langle \bar{q}q \rangle^2}{6\pi^2} - \frac{3m_s^2\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{8\pi^2} + \frac{m_s^2\langle \bar{s}s \rangle^2}{48\pi^2} \right] e^{-s/M_B^2} ds \\
&\quad + \left(\frac{\langle g_s\bar{q}\sigma Gq \rangle \langle g_s\bar{s}\sigma Gs \rangle}{96\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{3456\pi^2} + \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{576\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{3456\pi^2} - \frac{4m_s\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} \right. \\
&\quad + \frac{m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{9} - \frac{m_s\langle g_s^2 GG \rangle \langle g_s\bar{q}\sigma Gq \rangle}{3072\pi^4} + \frac{m_s\langle g_s^2 GG \rangle \langle g_s\bar{s}\sigma Gs \rangle}{9216\pi^4} + \frac{m_s^2\langle \bar{q}q \rangle \langle g_s\bar{q}\sigma Gq \rangle}{12\pi^2} \\
&\quad \left. - \frac{m_s^2\langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{16\pi^2} - \frac{m_s^2\langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{24\pi^2} \right) + \frac{1}{M_B^2} \left(\frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s\bar{q}\sigma Gq \rangle}{2304\pi^2} \right. \\
&\quad - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{768\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{768\pi^2} + \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle \langle g_s\bar{s}\sigma Gs \rangle}{2304\pi^2} \\
&\quad + \frac{m_s\langle \bar{q}q \rangle^2 \langle g_s\bar{s}\sigma Gs \rangle}{9} + \frac{2m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{9} - \frac{m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle g_s\bar{s}\sigma Gs \rangle}{12} \\
&\quad \left. - \frac{m_s\langle \bar{s}s \rangle^2 \langle g_s\bar{q}\sigma Gq \rangle}{12} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq \rangle^2}{96\pi^2} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq \rangle \langle g_s\bar{s}\sigma Gs \rangle}{32\pi^2} \right).
\end{aligned}
\tag{A5}
$$

The sum rules using the currents $\eta_{6\mu}^M \equiv \psi_{2\mu}^M(qs\bar{q}\bar{s})$ is

$$
\begin{aligned}
f_{M,6}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,6}(s_0, M_B^2) \\
&= \int_{4m_s^2}^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{m_s^2}{480\pi^6} s^3 + \left(-\frac{\langle g_s^2 GG \rangle}{18432\pi^6} - \frac{7m_s\langle \bar{q}q \rangle}{192\pi^4} + \frac{m_s\langle \bar{s}s \rangle}{64\pi^4} \right) s^2 \right. \\
&\quad + \left(\frac{5\langle \bar{q}q \rangle\langle \bar{s}s \rangle}{18\pi^2} - \frac{5m_s\langle g_s\bar{q}\sigma Gq \rangle}{96\pi^4} - \frac{17m_s^2\langle g_s^2 GG \rangle}{36864\pi^6} \right) s \\
&\quad + \frac{\langle \bar{q}q \rangle\langle g_s\bar{s}\sigma Gs \rangle}{8\pi^2} + \frac{\langle \bar{s}s \rangle\langle g_s\bar{q}\sigma Gq \rangle}{8\pi^2} - \frac{m_s\langle g_s^2 GG \rangle\langle \bar{q}q \rangle}{512\pi^4} + \frac{5m_s\langle g_s^2 GG \rangle\langle \bar{s}s \rangle}{1536\pi^4} + \frac{m_s^2\langle \bar{q}q \rangle^2}{3\pi^2} \\
&\quad \left. - \frac{3m_s^2\langle \bar{q}q \rangle\langle \bar{s}s \rangle}{4\pi^2} + \frac{m_s^2\langle \bar{s}s \rangle^2}{24\pi^2} \right] e^{-s/M_B^2}\, ds + \left(\frac{\langle g_s\bar{q}\sigma Gq \rangle\langle g_s\bar{s}\sigma Gs \rangle}{48\pi^2} \right. \\
&\quad - \frac{5\langle g_s^2 GG \rangle\langle \bar{q}q \rangle^2}{3456\pi^2} + \frac{\langle g_s^2 GG \rangle\langle \bar{q}q \rangle\langle \bar{s}s \rangle}{576\pi^2} - \frac{5\langle g_s^2 GG \rangle\langle \bar{s}s \rangle^2}{3456\pi^2} - \frac{m_s\langle g_s^2 GG \rangle\langle g_s\bar{q}\sigma Gq \rangle}{3072\pi^4} \\
&\quad + \frac{5m_s\langle g_s^2 GG \rangle\langle g_s\bar{s}\sigma Gs \rangle}{9216\pi^4} - \frac{8m_s\langle \bar{q}q \rangle^2\langle \bar{s}s \rangle}{9} + \frac{2m_s\langle \bar{q}q \rangle\langle \bar{s}s \rangle^2}{9} + \frac{m_s^2\langle \bar{q}q \rangle\langle g_s\bar{q}\sigma Gq \rangle}{6\pi^2} \\
&\quad \left. - \frac{m_s^2\langle \bar{q}q \rangle\langle g_s\bar{s}\sigma Gs \rangle}{12\pi^2} - \frac{m_s^2\langle \bar{s}s \rangle\langle g_s\bar{q}\sigma Gq \rangle}{8\pi^2} \right) + \frac{1}{M_B^2}\left(\frac{5\langle g_s^2 GG \rangle\langle \bar{q}q \rangle\langle g_s\bar{q}\sigma Gq \rangle}{2304\pi^2} \right. \\
&\quad - \frac{\langle g_s^2 GG \rangle\langle \bar{q}q \rangle\langle g_s\bar{s}\sigma Gs \rangle}{768\pi^2} - \frac{\langle g_s^2 GG \rangle\langle \bar{s}s \rangle\langle g_s\bar{q}\sigma Gq \rangle}{768\pi^2} + \frac{5\langle g_s^2 GG \rangle\langle \bar{s}s \rangle\langle g_s\bar{s}\sigma Gs \rangle}{2304\pi^2} \\
&\quad + \frac{2m_s\langle \bar{q}q \rangle^2\langle g_s\bar{s}\sigma Gs \rangle}{9} + \frac{4m_s\langle \bar{q}q \rangle\langle \bar{s}s \rangle\langle g_s\bar{q}\sigma Gq \rangle}{9} - \frac{m_s\langle \bar{q}q \rangle\langle \bar{s}s \rangle\langle g_s\bar{s}\sigma Gs \rangle}{6} \\
&\quad \left. - \frac{m_s\langle \bar{s}s \rangle^2\langle g_s\bar{q}\sigma Gq \rangle}{6} - \frac{m_s^2\langle g_s\bar{q}\sigma Gq \rangle^2}{48\pi^2} + \frac{m_s^2\langle g_s\bar{q}\sigma Gq \rangle\langle g_s\bar{s}\sigma Gs \rangle}{16\pi^2} \right).
\end{aligned}
$$

(A6)

The sum rules using the currents $\eta_{7\mu}^M \equiv \psi_{3\mu}^M(qs\bar{q}\bar{s})$ is

$$f_{M,7}^2 e^{-M_{a_1}^2/M_B^2} = \Pi_{M,7}(s_0, M_B^2) \tag{A7}$$

$$= \int_{4m_s^2}^{s_0} \left[\frac{1}{12288\pi^6} s^4 - \frac{11m_s^2}{2560\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} - \frac{7m_s\langle \bar{q}q \rangle}{384\pi^4} + \frac{23m_s\langle \bar{s}s \rangle}{384\pi^4} \right) s^2 \right.$$

$$+ \left(-\frac{5\langle \bar{q}q \rangle^2}{36\pi^2} + \frac{5\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{36\pi^2} - \frac{5\langle \bar{s}s \rangle^2}{36\pi^2} - \frac{5m_s\langle g_s\bar{q}\sigma Gq \rangle}{192\pi^4} + \frac{5m_s\langle g_s\bar{s}\sigma Gs \rangle}{96\pi^4} - \frac{23m_s^2\langle g_s^2 GG \rangle}{36864\pi^6} \right.$$

$$\left. \right) s - \frac{\langle \bar{q}q \rangle \langle g_s\bar{q}\sigma Gq \rangle}{8\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{16\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{16\pi^2}$$

$$- \frac{\langle \bar{s}s \rangle \langle g_s\bar{s}\sigma Gs \rangle}{8\pi^2} - \frac{3m_s\langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{512\pi^4} + \frac{m_s\langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{512\pi^4} + \frac{m_s^2\langle \bar{q}q \rangle^2}{\pi^2} - \frac{3m_s^2\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{8\pi^2}$$

$$\left. + \frac{m_s^2\langle \bar{s}s \rangle^2}{16\pi^2} \right] e^{-s/M_B^2} ds + \left(-\frac{\langle g_s\bar{q}\sigma Gq \rangle^2}{96\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{1152\pi^2} + \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{192\pi^2} \right.$$

$$- \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1152\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq \rangle \langle g_s\bar{s}\sigma Gs \rangle}{96\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs \rangle^2}{96\pi^2} - \frac{m_s\langle g_s^2 GG \rangle \langle g_s\bar{q}\sigma Gq \rangle}{1024\pi^4}$$

$$+ \frac{m_s\langle g_s^2 GG \rangle \langle g_s\bar{s}\sigma Gs \rangle}{3072\pi^4} - \frac{14m_s\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} + \frac{m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{9} + \frac{5m_s^2\langle \bar{q}q \rangle \langle g_s\bar{q}\sigma Gq \rangle}{12\pi^2}$$

$$\left. - \frac{m_s^2\langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{24\pi^2} - \frac{m_s^2\langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{16\pi^2} \right)$$

$$+ \frac{1}{M_B^2} \left(\frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s\bar{q}\sigma Gq \rangle}{768\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle g_s\bar{s}\sigma Gs \rangle}{256\pi^2} - \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle}{256\pi^2} \right.$$

$$+ \frac{\langle g_s^2 GG \rangle \langle \bar{s}s \rangle \langle g_s\bar{s}\sigma Gs \rangle}{768\pi^2} + \frac{m_s\langle \bar{q}q \rangle^2 \langle g_s\bar{s}\sigma Gs \rangle}{3} + m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle g_s\bar{q}\sigma Gq \rangle$$

$$- \frac{m_s\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle g_s\bar{s}\sigma Gs \rangle}{12} - \frac{m_s\langle \bar{s}s \rangle^2 \langle g_s\bar{q}\sigma Gq \rangle}{12} - \frac{m_s^2\langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1152\pi^2} - \frac{3m_s^2\langle g_s\bar{q}\sigma Gq \rangle^2}{32\pi^2}$$

$$\left. + \frac{m_s^2\langle g_s\bar{q}\sigma Gq \rangle \langle g_s\bar{s}\sigma Gs \rangle}{32\pi^2} \right),$$

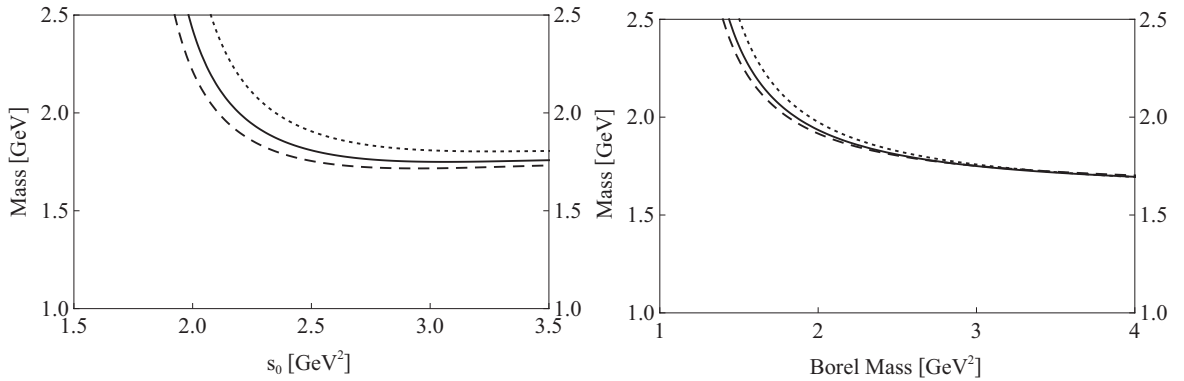and the results are shown in Fig. 7.

FIG. 7: The mass calculated using the current $\eta_{7\mu}^M \equiv \psi_{3\mu}^M(qs\bar{q}\bar{s})$ is shown with respect to the threshold value s_0 (left panel) for $M_B^2 = 2.5$ (dotted), 3.0 (solid) and 3.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right panel) for $s_0 = 2.8$ (dotted), 3.0 (solid), and 3.2 GeV^2 (dashed).

The sum rules using the currents $\eta_{8\mu}^M \equiv \psi_{4\mu}^M(qs\bar{q}\bar{s})$ is

$$
\begin{aligned}
f_{M,8}^2 e^{-M_{a_1}^2/M_B^2} &= \Pi_{M,8}(s_0, M_B^2) \tag{A8} \\
&= \int_{4m_s^2}^{s_0} \left[\frac{1}{6144\pi^6}s^4 - \frac{11m_s^2}{1280\pi^6}s^3 + \left(\frac{11\langle g_s^2 GG\rangle}{18432\pi^6} - \frac{7m_s\langle\bar{q}q\rangle}{192\pi^4} + \frac{23m_s\langle\bar{s}s\rangle}{192\pi^4} \right)s^2 \right. \\
&\quad + \left(-\frac{5\langle\bar{q}q\rangle^2}{18\pi^2} + \frac{5\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{18\pi^2} - \frac{5\langle\bar{s}s\rangle^2}{18\pi^2} - \frac{5m_s\langle g_s\bar{q}\sigma Gq\rangle}{96\pi^4} + \frac{5m_s\langle g_s\bar{s}\sigma Gs\rangle}{48\pi^4} - \frac{163m_s^2\langle g_s^2 GG\rangle}{36864\pi^6} \right. \\
&\quad \left.\vphantom{\frac{\bar{q}q}{4\pi^2}} \right)s - \frac{\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{4\pi^2} + \frac{\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8\pi^2} + \frac{\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} \\
&\quad - \frac{\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{4\pi^2} - \frac{3m_s\langle g_s^2 GG\rangle\langle\bar{q}q\rangle}{512\pi^4} + \frac{5m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{512\pi^4} + \frac{2m_s^2\langle\bar{q}q\rangle^2}{\pi^2} - \frac{3m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{4\pi^2} \\
&\quad \left. + \frac{m_s^2\langle\bar{s}s\rangle^2}{8\pi^2} \right] e^{-s/M_B^2}\,ds + \left(-\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{48\pi^2} - \frac{5\langle g_s^2 GG\rangle\langle\bar{q}q\rangle^2}{1152\pi^2} + \frac{\langle g_s^2 GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{192\pi^2} \right. \\
&\quad - \frac{5\langle g_s^2 GG\rangle\langle\bar{s}s\rangle^2}{1152\pi^2} + \frac{\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{48\pi^2} - \frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{48\pi^2} - \frac{m_s\langle g_s^2 GG\rangle\langle g_s\bar{q}\sigma Gq\rangle}{1024\pi^4} \\
&\quad + \frac{5m_s\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{3072\pi^4} - \frac{28m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{9} + \frac{2m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2}{9} + \frac{5m_s^2\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{6\pi^2} \\
&\quad \left. - \frac{m_s^2\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{12\pi^2} - \frac{m_s^2\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{8\pi^2} \right) \\
&\quad + \frac{1}{M_B^2}\left(\frac{5\langle g_s^2 GG\rangle\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle}{768\pi^2} - \frac{\langle g_s^2 GG\rangle\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{256\pi^2} - \frac{\langle g_s^2 GG\rangle\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle}{256\pi^2} \right. \\
&\quad + \frac{5\langle g_s^2 GG\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{768\pi^2} + \frac{2m_s\langle\bar{q}q\rangle^2\langle g_s\bar{s}\sigma Gs\rangle}{3} + 2m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{q}\sigma Gq\rangle \\
&\quad - \frac{m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{6} - \frac{m_s\langle\bar{s}s\rangle^2\langle g_s\bar{q}\sigma Gq\rangle}{6} - \frac{5m_s^2\langle g_s^2 GG\rangle\langle\bar{s}s\rangle^2}{1152\pi^2} - \frac{3m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{16\pi^2} \\
&\quad \left. + \frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{16\pi^2} \right),
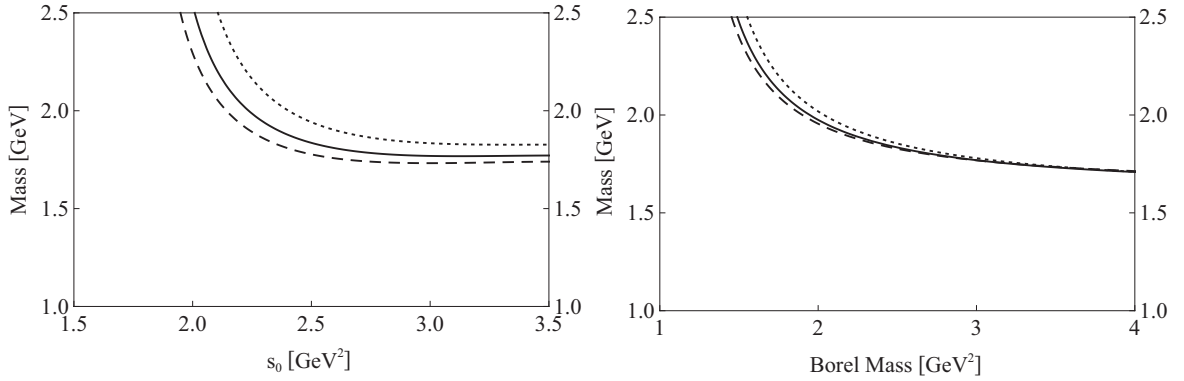\end{aligned}
$$

and the results are shown in Fig. 8.

FIG. 8: The mass calculated using the current $\eta_{8\mu}^M \equiv \psi_{4\mu}^M(qs\bar{q}\bar{s})$, with respect to the threshold value s_0 (left) for $M_B^2 = 2.5$ (dotted), 3.0 (solid) and 3.5 GeV^2 (dashed), and with respect to the Borel mass M_B (right) for for $s_0 = 2.8$ (dotted), 3.0 (solid), and 3.2 GeV^2 (dashed).

ACKNOWLEDGMENTS

[1] S. Paul [COMPASS Collaboration], EPJ Web Conf. **73**, 03002 (2014).
[2] S. Uhl [COMPASS Collaboration], PoS Hadron **2013**, 087 (2013).
[3] F. Krinner, PoS Bormio **2014**, 031 (2014).
[4] F. Nerling [COMPASS Collaboration], arXiv:1403.2667 [hep-ex].
[5] C. Adolph *et al.* [COMPASS Collaboration], arXiv:1501.05732 [hep-ex].
[6] K. Chen, C. Q. Pang, X. Liu and T. Matsuki, arXiv:1501.07766 [hep-ph].
[7] R. Jaffe, Phys. Rev. D **15**, 267 (1977).
[8] M. G. Alford and R. L. Jaffe, Nucl. Phys. B **578**, 367 (2000).
[9] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).
[10] C. Amsler and N. A. Tornqvist, Phys. Rept. **389**, 61 (2004).
[11] D. V. Bugg, Phys. Rept. **397**, 257 (2004).
[12] E. Klempt and A. Zaitsev, Phys. Rept. **454**, 1 (2007).
[13] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004).
[14] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B **662**, 424 (2008).
[15] Hua-Xing Chen, A. Hosaka and Shi-Lin Zhu, Phys. Rev. D **76**, 094025 (2007).
[16] Z. G. Wang, arXiv:1401.1134 [hep-ph].
[17] B. Ketzer, M. Mikhasenko and A. Sarantsev, arXiv:1501.07023 [hep-ph].
[18] C. B. Lang, L. Leskovec, D. Mohler and S. Prelovsek, JHEP **1404**, 162 (2014).
[19] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).
[20] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127**, 1 (1985).
[21] H. X. Chen, Eur. Phys. J. C **73**, 2628 (2013).
[22] H. X. Chen, A. Hosaka and S. L. Zhu, Phys. Rev. D **78**, 054017 (2008).
[23] J. Zhang, and H. Y. Jin, Z. F. Zhang, T. G. Steele and D. H. Lu, Phys. Rev. D **79**, 114033 (2009).
[24] http://www.feyncalc.org/.
[25] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).
[26] K. C. Yang, W. Y. P. Hwang, E. M. Henley and L. S. Kisslinger, Phys. Rev. D **47**, 3001 (1993).
[27] V. Gimenez, V. Lubicz, F. Mescia, V. Porretti and J. Reyes, Eur. Phys. J. C **41**, 535 (2005).
[28] M. Jamin, Phys. Lett. B **538**, 71 (2002).
[29] B. L. Ioffe and K. N. Zyablyuk, Eur. Phys. J. C **27**, 229 (2003).
[30] A. A. Ovchinnikov and A. A. Pivovarov, Sov. J. Nucl. Phys. **48**, 721 (1988) [Yad. Fiz. **48**, 1135 (1988)].
[31] P. Colangelo and A. Khodjamirian, *"At the Frontier of Particle Physics/Handbook of QCD"* (World Scientific, Singapore, 2001), Volume 3, 1495.
[32] W. Y. P. Hwang and K. C. Yang, Phys. Rev. D **49**, 460 (1994).
[33] S. Narison, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **17**, 1 (2002).
[34] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **108**, 182001 (2012).
[35] C. Dionisi *et al.* [CERN-College de France-Madrid-Stockholm Collaboration], Nucl. Phys. B **169**, 1 (1980).
[36] Z. Bai *et al.* [MARK-III Collaboration], Phys. Rev. Lett. **65**, 2507 (1990).
[37] A. Bertin *et al.* [OBELIX Collaboration], Phys. Lett. B **400**, 226 (1997).
[38] R. S. Longacre, Phys. Rev. D **42**, 874 (1990).
[39] S. Ishida, M. Oda, H. Sawazaki and K. Yamada, Prog. Theor. Phys. **82**, 119 (1989).
[40] D. O. Caldwell, Mod. Phys. Lett. A **02**, 771 (1987).