

Thermalization Process after Inflation and Effective Potential of Scalar Field

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Abstract

We investigate the thermalization process of the Universe after inflation to determine the evolution of the effective temperature. The time scale of thermalization is found to be so long that it delays the evolution of the effective temperature, and the resulting maximal temperature of the Universe can be significantly lower than the one obtained in the literature. Our results clarify the finite density corrections to the effective potential of a scalar field and also processes of heavy particle production. In particular, we find that the maximum temperature of the Universe may be at most electroweak scale if the reheating temperature is as low as $\mathcal{O}(1)$ MeV, which implies that the electroweak symmetry may be marginally restored. In addition, it is noticeable that the dark matter may not be produced from thermal plasma in such a low reheating scenario, since the maximum temperature can be smaller than the conventional estimation by five orders of magnitude. We also give implications to the Peccei-Quinn mechanism and the Affleck-Dine baryogenesis.

Contents

1	Introduction and Summary	2
1.1	Introduction	2
1.2	Summary of our results	3
2	Preliminaries	6
2.1	Hard primaries	6
2.2	Setup	7
2.3	Thermal potentials	8
3	Thermalization History	10
3.1	$\tilde{t}_{\text{ini}} < \tilde{t} < \tilde{t}_{\text{soft}}$	14
3.2	$\tilde{t}_{\text{soft}} < \tilde{t} < \tilde{t}_{\text{max}}$	20
3.3	$\tilde{t}_{\text{max}} < \tilde{t} < \tilde{t}_{\text{RH}}$	23
3.4	$\tilde{t}_{\text{RH}} < \tilde{t}$	25
3.5	Summary	25
4	Applications	28
4.1	PQ symmetry	28
4.2	Electroweak phase transition and DM production	29
4.3	Affleck-Dine field	31
5	Conclusion	32
A	LPM-suppressed Splitting Rate	32
B	Effective Temperature for χ_ϕ-particles	34
B.1	$\tilde{t}_{\text{ini}} < \tilde{t} < \tilde{t}_{\text{soft}}$	34
B.2	$\tilde{t}_{\text{soft}} < \tilde{t} < \tilde{t}_{\text{max}}$	35
B.3	$\tilde{t}_{\text{max}} < \tilde{t} < \tilde{t}_{\text{RH}}$	37

1 Introduction and Summary

1.1 Introduction

Slow roll inflationary scenarios are successful in light of the solution to the horizon problem, flatness problem, and the origin of the large scale structure. Inflation is usually driven by a finite energy density of a slowly rolling scalar field, called inflaton. After the slow roll conditions fail, inflaton starts to oscillate around its potential minimum and the energy density of the Universe is then dominated by that of the oscillating inflaton. In order to proceed the big bang nucleosynthesis, the inflaton has to decay into radiation, which is referred to as reheating [1, 2].

While the inflaton elegantly solves the above problems and provides the primordial density perturbations, there still remains unsolved cosmological issues; dark matter and baryon asymmetry of the Universe. Scalar fields other than the inflaton may play essential roles in solving these remaining problems. For instance, dark matter can be explained by a pseudo-NG boson, called axion [3], which is associated with the spontaneously symmetry breaking (SSB) of PQ symmetry triggered by a PQ-charged scalar field [4, 5]. In supersymmetric (SUSY) theories, the Affleck-Dine mechanism can explain the origin of baryon asymmetry by using a baryonic scalar field [6, 7]. Generally, scalar fields should have interaction terms with radiation in order to successfully lose their energy, otherwise they tend to dominate the energy density of the Universe. This very interaction makes the dynamics of scalar fields non-trivial.

One of prominent effects caused by the interaction between the scalar fields and radiation is modification of their effective potential.^{♣¹} As the thermal plasma grows after inflation, it drastically affects the dynamics of scalar fields. At a sufficiently high temperature, for instance, thermal effects may induce a positive thermal mass for a scalar field and make it stay at the point of the potential where particles coupled to the scalar field remain massless. When the scalar field is responsible to the SSB of some symmetry, like the Higgs boson or a PQ breaking scalar field, the symmetry is restored at that high temperature [14, 15, 16, 17, 18]. Then, a phase transition occurs when the thermal mass decreases down to the zero-temperature mass of the boson field. At the phase transition, topological defects, such as cosmic strings and domain walls, may form and affect the evolution of the Universe. In QCD axion models, there may be the domain wall problem when the PQ symmetry is broken after inflation [19, 20], while there is the isocurvature problem when it is broken before inflation [21, 22, 23]. The abundance of axion DM also depends on whether the PQ symmetry is broken before or after inflation. In SUSY theories, the Affleck-Dine mechanism can generate baryon asymmetry by using a scalar field carrying a nonzero baryon charge [6, 7]. The amount of baryon asymmetry is sometimes affected by the finite temperature effects [7, 24, 25, 26, 27]. These examples show that the thermal effects on boson fields drastically affect the evolutions of their dynamics and change their predictions. Therefore, we should clarify the evolution of the temperature after inflation.

One might assume that the reheating occurs instantaneously and regard the reheating temperature as the maximum temperature of the Universe. More carefully, one may solve the Boltzmann equation for the inflaton and radiation to include the production of radiation before the complete decay of inflaton [28, 29]. Even in this method, “*instantaneous ther-*

^{♣¹} For other relevant effects of background thermal plasma on scalar condensates in the Universe, see Refs. [8, 9, 10, 11, 12, 13] for instance.

malization” of radiation is implicitly assumed. In Ref. [30], however, we have pointed out that the time scale of thermalization of radiation is finite and affects the reheating process of the Universe after inflation.^{♣²}

In this paper, we further investigate the thermalization process of inflaton decay products and provides the evolution of an effective temperature which describes the strength of finite temperature effects. We find that the maximal temperature of the Universe after inflation is much smaller than that obtained by the “*instantaneous thermalization*” assumption, by extending the analysis given in Ref. [30] to the era before the soft sector is thermalized. The discrepancy becomes larger for smaller decay rate of inflaton and can be as large as about five order of magnitude.

Our results clarify the finite density corrections to the effective potential of the scalar field such as so called the thermal mass and the thermal log potential; which are essential so as to determine the beginning time of oscillation of the scalar field and also the condition of the symmetry restoration by finite density corrections. For example, we find that the maximum temperature of the Universe may be at most $\sim 100\text{GeV}$ if the reheating temperature is as low as $\mathcal{O}(1)\text{ MeV}$. In this case, the electroweak phase transition is marginally restored after inflation. We also find that the reheating temperature may have to be two order of magnitude lower than the PQ breaking scale in order to avoid the restoration of PQ symmetry. In addition, we investigate the effect on the AD field, which generates baryon asymmetry by the Affleck-Dine mechanism. We find that the result is consistent with the case that one assumes “*instantaneous thermalization*” when the VEV of the AD field is larger than the mass of inflaton.

In order to study the finite density corrections to the effective potential, we have to discuss “heavy” particle production processes with the mass $\sim |\phi|$. Hence, our discussion can be also applied to heavy DM production processes. Although in the literature it was expected that DM can be produced from the thermal plasma before reheating is completed [28, 29], our result implies that they may not be produced in a low reheating temperature scenario. Instead, DM may be efficiently produced via scatterings between the soft thermalized plasma and hard primaries as studied in Ref. [33], or directly produced via inflaton decay.

In the next section, we briefly explain the finite density effects on boson fields, and derive the relation between the effective temperature which characterizes the finite density corrections to the effective potential and the distribution function of radiation. Then we investigate the evolution of the thermalization process and calculate the effective temperature of radiation in Sec. 3. In Sec. 4, we apply our results to some important scenarios, such as the restoration of PQ symmetry and electroweak symmetry, and the Affleck-Dine baryogenesis. Section 5 is devoted to the conclusion.

1.2 Summary of our results

Before we explain the detail of our calculations, here we summarize our main results. If a system is not in thermal equilibrium, we cannot characterize the system simply by a temperature in general. In this paper, we define an effective temperature T_* such that it describes the finite density effect on the potential of scalar fields because our main purpose is to clarify the effects of radiation on scalar fields. Note that these scalar fields of our

^{♣²} See also Refs. [31, 32].

interests are different from inflaton, and that inflaton is assumed to reheat the Universe via Planck-suppressed decays throughout this paper.^{♣³} See Sec. 2.2 for more details of our setup.

Let us first summarize important equations. If χ_ϕ -particles, which are particles interacting with a scalar field ϕ , are not directly produced from inflaton decay, the thermal potential of ϕ is roughly given by

$$V_{\text{eff}}(\phi) \sim \begin{cases} \alpha_\phi T_*^2 \phi^2 & \text{for } |g_\phi \phi| \ll T_* \\ \alpha^2 T_*^4 \log\left(\frac{\phi^2}{T_*^2}\right) & \text{for } |g_\phi \phi| \gg T_* \end{cases}, \quad (1.1)$$

where $\alpha_\phi (\equiv g_\phi^2/4\pi)$ denotes a typical coupling between the scalar field and χ_ϕ -fields [See Eq. (2.6) and Fig. 1], and α is the relevant coupling constant for thermalization process.

In Sec. 3, we show that the evolution of the effective temperature for the scalar field ϕ is dramatically different from that of conventional studies. The evolution of the effective temperature T_* after inflation is found to be

$$\frac{T_*}{m_I} \sim \begin{cases} \alpha^{1/2} \tilde{\Gamma}_I^{-1/2} \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}}\right)^{-1/4} & \text{for } \tilde{t}_{\text{ini}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{soft}} \\ \alpha \tilde{\Gamma}_I^{-1/2} \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}}\right) & \text{for } \tilde{t}_{\text{soft}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{max}} \\ \alpha^{4/5} \tilde{\Gamma}_I^{-2/5} \left(\frac{\tilde{t}}{\tilde{t}_{\text{max}}}\right)^{-1/4} & \text{for } \tilde{t}_{\text{max}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{RH}} \end{cases}, \quad (1.2)$$

where m_I is inflaton mass, α is the relevant coupling constant for thermalization process, $\tilde{\Gamma}_I$ is the decay rate of inflaton normalized by that of dimension-five operator, and \tilde{t} is a cosmic time normalized by the inflaton mass. The time scales are given by

$$\tilde{t}_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}, \quad (1.3)$$

$$\tilde{t}_{\text{soft}} \equiv \alpha^{-3} \tilde{\Gamma}_I^{-1/2}, \quad (1.4)$$

$$\tilde{t}_{\text{max}} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}, \quad (1.5)$$

$$\tilde{t}_{\text{RH}} \equiv \tilde{\Gamma}_I^{-1} \frac{M_{\text{pl}}^2}{m_I^2}, \quad (1.6)$$

where $M_{\text{pl}} (\simeq 2.4 \times 10^{18} \text{ GeV})$ is the reduced Planck scale. We consider the case of $\tilde{\Gamma}_I \lesssim 1$ so that we can neglect non-perturbative effects of inflaton decay.

To see a rough sketch, we briefly explain the thermalization process after inflation. Figs. 7, 8 and 9 may be helpful to understand the essence of the following discussion. First, inflaton decays into hard primaries, whose energy is of order the mass of inflaton m_I . At the very first stage, a showering of hard primaries produces the soft population. Soon after that, the hard primaries then inelastically scatter with each other and release their energy into soft particles via in-medium cascading processes. Due to the loss of causality, the soft thermal-like distribution with k^{-1} emerges after the time defined by $\tilde{t}_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}$.

^{♣³} If inflaton reheats the Universe via not so small couplings, the preheating and thermal dissipations after that may become important. See Refs. [34, 35, 10] for instance.

The inelastic scattering rate is suppressed by the LPM effect, which is stronger in lower dense environment. Since the density of the hard primaries decreases with time due to the expansion of the Universe, the inelastic scattering rate decreases with time. Thus, the energy density of soft particles also decreases, so that their effective temperature T_* decreases with time. Then, at $\tilde{t} \sim \tilde{t}_{\text{soft}}$, soft particles are completely thermalized. At the same time, the number density of the Universe is dominated by the soft particles, so that the LPM suppression becomes weaker and weaker. Therefore the effective temperature of soft particles increases with time after $\tilde{t} \sim \tilde{t}_{\text{soft}}$. Then, at $\tilde{t} \sim \tilde{t}_{\text{max}}$, the primary particles can lose their energy completely and are thermalized within the Hubble time scale. After that time, the temperature of the Universe can be defined definitely and is determined by the energy injected by the inflaton decay.^{♣4} Since the energy density of inflaton decreases with time, the temperature of the Universe decreases, too. Finally, at $\tilde{t} \sim \tilde{t}_{\text{RH}}$, inflaton completely decays into radiation and reheating is completed.

There are two things to be noted.

1. We cannot rely on the conventional estimation of the temperature, $T \sim \rho_r^{1/4}$, for $\tilde{t} \lesssim \tilde{t}_{\text{max}}$. This is because the energy density is still dominated by the hard primaries for $\tilde{t} \lesssim \tilde{t}_{\text{max}}$, whose distribution is far from thermal equilibrium.
2. The maximum temperature of the Universe may not be achieved before \tilde{t}_{max} , and thus it may be significantly lower than the standard scenario in which the “*instantaneous thermalization*” is assumed. The effective temperature of the soft sector has two local maxima as $T_*/m_I \sim \alpha^{1/2} \tilde{\Gamma}_I^{1/2}$ and $\alpha^{4/5} \tilde{\Gamma}_I^{2/5}$ at the time of $\tilde{t} = \tilde{t}_{\text{ini}}$ and \tilde{t}_{max} , respectively. In particular, for the case of $\tilde{\Gamma}_I \lesssim \alpha^3$, the effective temperature reaches its maximal value at $\tilde{t} \sim \tilde{t}_{\text{max}}$ (see Fig. 11):

$$\alpha^{4/5} \tilde{\Gamma}_I^{2/5} m_I \quad \text{for } \tilde{\Gamma}_I \lesssim \alpha^3. \quad (1.7)$$

If the primary particles was thermalized instantaneously, the temperature of the plasma would be calculated from $T^4 \sim \rho_R \sim H \Gamma_\phi M_{\text{pl}}^2$ [28, 29]. (This is shown as red dotted lines in Figs. 10 and 11, which clarifies that the thermal effect is overestimated in the “*instantaneous thermalization*” approximation.) The most important quantity is the maximal temperature of the Universe because it determines whether a symmetry is restored after inflation or not. It is overestimated in the “*instantaneous thermalization*” approximation by the following factor:

$$\frac{T_{\text{max}}^{(\text{inst})}}{T_*|_{\text{max}}^{(\text{this work})}} \sim \alpha^{-4/5} \tilde{\Gamma}_I^{-3/20} \left(\frac{H_I}{m_I} \right)^{1/4}, \quad (1.8)$$

where we assume $\tilde{\Gamma}_I \lesssim \alpha^3$. This implies that the instantaneous thermalization approximation results in an overestimation by about five order of magnitude for the case of $\alpha = 1/10$, $\tilde{\Gamma}_I = 10^{-27}$, and $H_I = m_I$, for example. Here note that the decay rate of inflaton is bounded from below not to spoil the success of BBN:

$$\tilde{\Gamma}_I \sim \frac{T_{\text{RH}}^2 M_{\text{pl}}}{m_I^3} \sim 10^{-27} \left(\frac{T_{\text{RH}}}{1 \text{ MeV}} \right)^2 \left(\frac{m_I}{10^{13} \text{ GeV}} \right)^{-3}. \quad (1.9)$$

^{♣4} Still, note that there remains a tail of cascading hard particles as can be seen from Fig. 9. Though this tail does not contribute to neither the energy density nor the number density, it can be a source of heavy particle production with masses of $m \gg T$ [33].

2 Preliminaries

2.1 Hard primaries

We focus on the era between the end of inflation and the completion of reheating, during which the energy density of the Universe is dominated by that of the inflaton oscillation. Under the quadratic potential of inflaton, its oscillation amplitude and energy density evolve as

$$I(t) \simeq \left(\frac{a(t_I)}{a(t)} \right)^{3/2} e^{-\Gamma_I t/2} I_0 \cos(m_I t) \quad (2.1)$$

$$\rho_I \simeq \left(\frac{a(t_I)}{a(t)} \right)^3 e^{-\Gamma_I t} 3H_I^2 M_{\text{pl}}^2, \quad (2.2)$$

respectively. Here, $a(t)$ is the scale factor, t_I is the time at which inflation ends, I_0 is the initial amplitude of the inflaton oscillation, Γ_I is inflaton decay rate, and H_I is the Hubble parameter at the end of inflation. We define the following dimensionless parameters, $\tilde{\Gamma}_I$, \tilde{t} , as:

$$\Gamma_I \equiv \tilde{\Gamma}_I \frac{m_I^3}{M_{\text{pl}}^2}, \quad (2.3)$$

$$\tilde{t} \equiv m_I t. \quad (2.4)$$

Note that we have $\tilde{t} \gtrsim 1$ because the inflaton has to oscillate at least once before it decays. We assume that the inflaton decays into radiation perturbatively. This assumption is fulfilled at least for $\tilde{\Gamma}_I \lesssim 1$. To see this, let us consider the case that the inflaton decays into a fermion χ_I through the interaction of $\lambda I \bar{\chi}_I \chi_I$. This interaction term gives an effective mass to the χ_I field, so that its frequency is given by $\omega_{\chi_I}^2 = k^2 + \lambda^2 I^2(t)$. The non-perturbative decay occurs when the adiabatic condition $|\dot{\omega}_{\chi_I}/\omega_{\chi_I}^2| \ll 1$ is violated. Since $\dot{I}|_{t \simeq 0} \lesssim m_I I_0$ and $k \simeq m_I/2$, the adiabatic condition can be rewritten as $\lambda I_0 \ll m_I$. Using the perturbative decay rate given by $\Gamma_I \sim \lambda^2 m_I$, we obtain the condition to the perturbative decay as $\tilde{\Gamma}_I \ll M_{\text{pl}}^2/I_0^2$. Since $I_0 \lesssim M_{\text{pl}}$, the condition of perturbative decay is always satisfied at least for the case of $\tilde{\Gamma}_I \lesssim 1$. Otherwise, the inflaton might decay through non-perturbatively [34, 35], which is beyond the scope of this paper.

Since the energy of inflaton decay products (hereafter we call them as hard primaries) is of the order of inflaton mass m_I , their phase space distribution is given as [36]

$$f_h(t, p) \sim (p/m_I)^{-3/2} \tilde{\Gamma}_I \tilde{t}^{-1} \quad \text{for } \tilde{t}^{-2/3} \lesssim p/m_I \lesssim 1, \quad (2.5)$$

where the factor in the parentheses represents the effect of redshift. It implies smaller number density but harder particles than those of thermal distribution for relevant time scales in consideration; $f_h(t, m_I) \sim \tilde{\Gamma}_I \tilde{t}^{-1} \ll 1$. This is the reason why we call them hard primaries. Obviously, number violating processes are essential in thermalization, and in fact the hard primaries scatter with each other inelastically and emit soft particles. As we will see in the following, these soft particles take crucial roles both in thermalization of hard primaries and in dynamics of a scalar field ϕ which is the main character in this paper. Note that the species of soft particles can be different from that of hard primaries. For example, even if inflaton decays only into Standard-Model gauge bosons through Planck-suppressed operators, soft particles can be quarks and leptons. Thus, almost all of the fields interacting

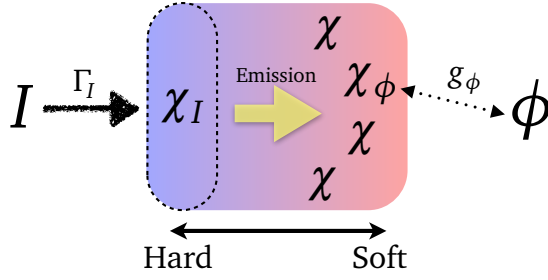


Figure 1: **Schematic figure of our setup:** I is the inflaton which creates hard primaries denoted by χ_I , which has typical momentum of m_I . Hard primaries emit soft populations of SM particles χ (including χ_I) and also χ_ϕ (through SM interactions). The field ϕ denotes the scalar field that we are interested in. It interacts with SM particles via χ_ϕ with a typical coupling g_ϕ .

with a Standard Model particle are affected by the soft sector. In this sense, our calculation can be applied to wide range of models. How the soft particles are produced and affect the effective potential of ϕ -condensate is discussed in the subsequent sections.

2.2 Setup

Before going into details, let us clarify our setup. Throughout this paper, we denote the scalar field in consideration (which is different from the inflaton) as ϕ , and fields which couple to the scalar field and interact with standard model (SM) particles (e.g., charged under SM gauge group) as χ_ϕ . In addition, inflaton, light fields directly produced via inflaton decay, and light fields other than χ_ϕ are represented by I , χ_I , and χ , respectively. For simplicity, we assume that all the fields except for ϕ are charged under SM gauge group with the predominant coupling being denoted as g ,^{♣5} and that masses of all the particles other than χ_ϕ can be neglected. That is, hard χ_I -particles can emit themselves in general and hence χ includes χ_I , but χ_ϕ is treated separately since it can acquire a large effective mass of $|g_\phi \phi|$. Here, symbolically, we represent the coupling between the scalar field ϕ and χ_ϕ as g_ϕ ; concrete examples are^{♣6}

$$-\mathcal{L}_{\text{Yukawa}} = g_\phi \phi \overline{\chi_{\phi R}} \chi_{\phi L} + \text{h.c.}, \quad -\mathcal{L}_{\text{quartic}} = g_\phi^2 \phi^2 |\tilde{\chi}_\phi|^2, \quad (2.6)$$

for instance. Here, $\chi_{\phi i}$ and $\tilde{\chi}_\phi$ are fermion and boson, respectively, which are collectively denoted as χ_ϕ hereafter. A schematic figure of our setup is shown in Fig. 1.

At first, the scalar field condensates almost homogeneously with an expectation value ϕ . Then, when the potential force overcomes the expansion of the Universe, $H \sim \sqrt{|\partial V_{\text{eff}}/\partial \phi^2|}$, it starts to roll down its effective potential V_{eff} against the expansion of the Universe. Eventually, it relaxes to its “equilibrium value” at that time by dissipating its energy into the background plasma. Hence, it is of quite importance to understand the behavior of effective potential so as to know the cosmological fate of scalar condensates. In particular, the effective potential is drastically changed via interactions with a dense and high temperature background plasma.

^{♣5} The field ϕ itself can be either charged or singlet under SM gauge group, but note that if ϕ is charged under SM gauge group, then one should care that SM gauge group is not entirely broken down by the expectation value of ϕ . Otherwise, the t -channel enhancement of massless gauge boson exchange is suppressed, and the following discussion should be altered.

^{♣6} For a complex scalar field theory, ϕ is regarded as its radial component.

2.3 Thermal potentials

There are two contributions to the effective potential depending on the expectation value of ϕ -condensation. The first one comes from the abundant population of χ_ϕ -particles, which typically has the following form:

$$V_M \sim g_\phi^2 \phi^2 \int_{\mathbf{p}} \frac{f_{\chi_\phi}(\mathbf{p})}{E_p}, \quad (2.7)$$

$$\int_{\mathbf{p}} \equiv \int \frac{d^3p}{(2\pi)^3}, \quad (2.8)$$

where f_{χ_ϕ} and $E_p (= \sqrt{|\mathbf{p}|^2 + |g_\phi \phi|^2})$ stand for the distribution function and the energy of χ_ϕ -particles, respectively. For instance, in the case of quartic interaction, $g_\phi^2 \phi^2 \tilde{\chi}_\phi^2$, the effective potential for the homogeneous condensate of ϕ encodes the following term (see Fig. 2):

$$g_\phi^2 \phi^2 \int_P G_H^{\chi_\phi}(P) \simeq g_\phi^2 \phi^2 \int_P [1 + 2f_{\chi_\phi}(P)] G_J^{\chi_\phi}(P) \simeq g_\phi^2 \phi^2 \int_{\mathbf{p}} \frac{1}{E_p} [1 + 2f_{\chi_\phi}(\mathbf{p})], \quad (2.9)$$

with $f_{\chi_\phi}(\mathbf{p}) \equiv f_{\chi_\phi}(E_p, \mathbf{p})$. Here $G_{H/J}^{\chi_\phi}$ is the Hadamard/Jordan propagator defined by the commutator/anti-commutator: $G_{H/J}^{\chi_\phi}(x, y) \equiv \text{Tr} \langle [\hat{\chi}_\phi(x), \hat{\chi}_\phi^\dagger(y)]_{\pm} \rangle$. In the first similarity, we have assumed the Kadanoff-Baym ansatz between the Hadamard and Jordan propagator, and then assumed that the spectrum is dominated by particle-like excitations in the second similarity. One can see that the finite density correction is reproduced aside from the quadratic divergence that should be canceled by the counter term. A similar computation yields the finite density correction from the Yukawa interaction, $g_\phi \phi \overline{\chi_{\phi R}} \chi_{\phi L}$, which is essentially the same as Eq. (2.7). The expression of Eq. (2.7) coincides with the well-known thermal mass, if f_{χ_ϕ} is given by the thermal distribution and if the effective mass of χ_ϕ is smaller than the temperature, $m_{\chi_\phi} \sim |g_\phi \phi| \ll T$. Motivated by this observation, we define the effective temperature for χ_ϕ -particles as follows:

$$T_{*,\chi_\phi}^2 \equiv \int_{\mathbf{p}} \frac{f_{\chi_\phi}(\mathbf{p})}{E_p}. \quad (2.10)$$

Although the distribution f_{χ_ϕ} might be far from thermal equilibrium, we can estimate the effective potential for the scalar field ϕ , which is caused by abundant χ_ϕ -particles. Imitating the expression at thermal equilibrium, we write down this effective mass contribution as

$$V_M \sim \alpha_\phi T_{*,\chi_\phi}^2 \phi^2, \quad (2.11)$$

where $\alpha_\phi = g_\phi^2/4\pi$.

When the field value of ϕ -condensate is larger than the typical production scale of χ_ϕ -particles, they are less likely to be produced from other light particles. Hence, the contribution given in Eq. (2.7) may be suppressed. Even in this case, the effective potential for the ϕ -condensate receives finite density corrections from other light particles. This second contribution can be understood from the fact: the runnings of coupling constants are affected by the expectation value of the scalar field. Suppose that ϕ has a large expectation

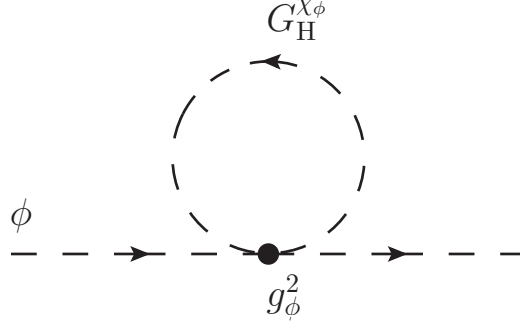


Figure 2: Diagram responsible for the finite density correction to the mass term of ϕ in the case of quartic interaction.

value. Since χ_ϕ has a large effective mass of $|g_\phi \phi|$ and is less likely to be produced, one can integrate out χ_ϕ -fields. As a result, the running coupling constant of gauge group under which χ_ϕ is charged encodes the ϕ -dependence as a threshold correction of χ_ϕ -fields, or in other words, the ϕ -condensate interacts with other light particles via the operator $\sim g^{-2}(\phi) F_{\mu\nu}^a F^{a\mu\nu}$.^{◆7} Here we have assumed that ϕ is singlet under this gauge group. This implies the following contribution to the effective potential for the ϕ -condensate (see Fig. 3):^{◆8}

$$g^2(\phi) \left(\int_p \frac{f_\chi(\mathbf{p})}{p} \right)^2 \equiv g^2(\phi) T_*^4. \quad (2.12)$$

Here we have defined the effective temperature T_* for light particles χ , which does not always coincides with that for χ_ϕ -particles since they have masses of $|g_\phi \phi|$. Extracting ϕ -dependent part, one may obtain

$$V_L \sim \alpha^2 T_*^4 \log \left(\frac{\phi^2}{T_*^2} \right), \quad (2.13)$$

with $\alpha = g^2/4\pi$. This is nothing but the so-called thermal-log potential for the thermal distribution of χ -particles [27].

Thus, in order to discuss the finite density corrections to the effective potential for ϕ , we have to know distribution functions for light χ -particles as well as that for χ_ϕ -particles with the mass of $|g_\phi \phi|$. In the following, we discuss how χ and χ_ϕ are produced from hard primaries during the course of thermalization and calculate their effective temperatures T_* and T_{*,χ_ϕ} .

Before closing this section, we would like to comment on basic assumptions behind our following discussions. First of all, the effective potential listed above is computed perturbatively, which implicitly assumes that the occupation number $f(k_{\max})$ [that dominates the integral $\int_k f/k \sim f(k_{\max})k_{\max}^2$] should be smaller than α^{-1} , since otherwise the perturbative expansion breaks down. Below, we will see that this condition is automatically

^{◆7} Other interactions like Yukawa may also encode the threshold correction of χ_ϕ .

^{◆8} Here we have assumed that χ_ϕ is also charged under the dominant gauge group of coupling g . If this is not the case, a factor should be multiplied, which is defined as the ratio of the coupling under which χ_ϕ is charged to the dominant coupling g . We restrict ourselves to order of magnitude estimation and do not care about this factor in the following discussion to avoid complications.

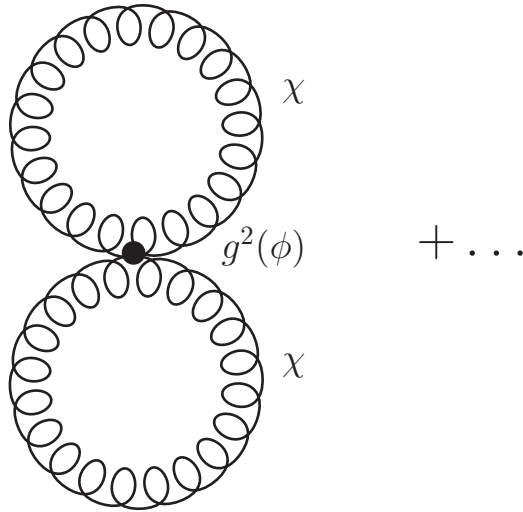


Figure 3: Diagram responsible for the logarithmic correction to the effective potential of ϕ .

satisfied. In addition, although we only focus on the effective potential to discuss the symmetry restoration, it is not trivial that whether or not the effective potential is maintained after the onset of oscillation and also the scalar condensate soon dissipates its energy into background plasma. In the following, we simply assume that ϕ can dissipate its energy into background plasma soon after the onset of its oscillation, since its relaxation process can be strongly model dependent [37].

3 Thermalization History

In this section, we investigate the thermalization process of hard primaries and calculate the effective temperatures for the soft sector T_* and T_{*,χ_ϕ} . Basic parameters which are required to study thermalization after inflation are the mass of inflaton m_I , the decay rate of inflaton $\tilde{\Gamma}_I$ and the dominant couplings of decay produces α ; *e.g.*, for the SM plasma, the predominant interaction is the strong coupling, $\alpha = \alpha_s$. This is because the first two parameters m_I , $\tilde{\Gamma}_I$ determine the typical distribution of decay products, and the last parameter α is essential to estimate their interaction time scale. We are also interested in the production of χ_ϕ -particles and hence the coupling between χ_ϕ and the scalar condensate ϕ , which is given by g_ϕ , is also an important parameter. Thus, we have four parameters essentially to capture this system.

We assume that the inflaton decays into particles χ_I which are charged under the abelian and/or non-abelian gauge groups (SM gauge group). We mainly consider the era after $\tilde{t} = \tilde{t}_{\text{ini}}$, where \tilde{t}_{ini} is defined below [see Eq. (3.22)]. First, the hard primaries emit soft particles but the soft sector is not thermalized soon. As shown in Sec. 3.2, the soft sector is completely thermalized after the time of $\tilde{t} = \tilde{t}_{\text{soft}}$ [Eq. (3.43)]. After that, around the time of $\tilde{t} = \tilde{t}_{\text{max}}$ [Eq. (3.57)], the hard sector as well as the soft sector are completely thermalized and the effective temperature reaches a (local) maximum. Then the reheating is completed at $\tilde{t} = \tilde{t}_{\text{RH}}$ [Eq. (3.63)]. We investigate thermalization processes during these eras in turn, and then we briefly summarize the result of this section in Sec. 3.5. Our procedure is based on the one used in Ref. [38, 39] (see also Refs. [30, 33]).

Figs. 7, 8 and 9 summarize the essence of this section. Hurry readers can skip all the

details and move to these figures.

Kinetic equations

Here we briefly summarize minimal knowledge required for our estimation. Eventually, we will see that the collinear splitting process in medium plays crucial roles in the following discussion. We also summarize basic formulae of this process in $SU(N)$ with N_F -flavor fermions in Sec. A for the sake of completeness.

Our method is based on kinetic equation (Boltzmann-like equation), which can be derived from first principles, *i.e.*, from Schwinger Dyson equations in the closed time path formalism (see for instance [40, 41], reviews [42, 43] and references therein). It was shown that particle-like excitations of $m_s \ll p$ with m_s being the screening mass obey the kinetic equations if these quasi-particles fulfill the following conditions: (i) Typical “size” of quasi-particles are smaller than the mean free path, (ii) Typical duration time of interactions are shorter than the mean free time, (iii) Typical value of distribution function should be smaller than $1/\alpha$. The condition (ii) is sometimes violated for collinear emissions in the medium, and in this case appropriate resummations are required, which is so-called the Landau-Pomeranchuk-Migdal (LPM) effect [44, 45, 46, 47, 48, 49, 50]. After the resummations, we eventually obtain the following kinetic equations for quasi-particles in the plasma [51, 38, 39]:

$$\left[\partial_t - H\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{k}} \right] f_{\bullet}(t, k) = -\mathcal{C}[f_{\bullet}], \quad (3.1)$$

where f_{\bullet} is the distribution function of quasi particles with \bullet being species and \mathcal{C} is the collision term. As we will explain later, at least for the perturbative Planck-suppressed decay of inflaton, these conditions are safely satisfied^{♣⁹} except for the condition (ii). Hence, we can rely on the kinetic equations given in [51, 38, 39]. In our case, we are interested in the distributions of χ and χ_{ϕ} to obtain information of effective temperatures.

The collision term for χ -particles encodes two-to-two scatterings responsible for diffusions and effective “one-to-two” (inelastic) scatterings due to splittings:

$$\mathcal{C}[f_{\chi}] \supset (\mathcal{C}_{2 \text{ to } 2} + \mathcal{C}_{\text{split}})[f_{\chi}], \quad (3.2)$$

where

$$\begin{aligned} \mathcal{C}_{2 \text{ to } 2}[f_a] = & \frac{1}{2\mathcal{N}_a} \sum_{b,c,d} \int_{p,k',p'} \frac{|\mathcal{M}_{cd}^{ab}(K, P; K', P')|^2}{2E_k 2E_p 2E_{k'} 2E_{p'}} (2\pi)^4 \delta^{(4)}(K + P - K' - P') \\ & \times [f_a(k)f_b(p) [1 \pm f_c(k')] [1 \pm f_d(p')] - (\text{inverse process})], \end{aligned} \quad (3.3)$$

^{♣⁹} It is contrary to the preheating case where the fluctuations can be strongly correlated. See Refs. [52, 53] for instance.

$$\begin{aligned}
\mathcal{C}_{\text{split}}[f_a] = & \frac{(2\pi)^3}{2k^2\nu_a} \sum_{b,c} \int dp' dk' \delta(k - p' - k') \gamma_{bc}^a(k; p', k') \\
& \times [f_a(k) [1 \pm f_b(p')] [1 \pm f_c(k')] - (\text{inverse process})] \\
& + \frac{(2\pi)^3}{k^2\nu_a} \sum_{b,c} \int dp' dp \delta(k + p - p') \gamma_{ab}^c(p'; k, p) \\
& \times [f_a(k) f_b(p) [1 \pm f_c(p')] - (\text{inverse process})] \tag{3.4}
\end{aligned}$$

$$\begin{aligned}
\sim & \int d \ln p' \Gamma_{\text{split}}(p') [f_\chi(k) [1 \pm f_\bullet(k - p')] [1 \pm f_\bullet(p')] - (\text{inverse process})] \\
& + \int d \ln p' \left(\frac{p'}{k}\right)^3 \Gamma_{\text{split}}(k) [f_\chi(k) f_\bullet(p' - k) [1 \pm f_\bullet(p')] - (\text{inverse process})], \tag{3.5}
\end{aligned}$$

with ν_a being the number of degree of freedom for a species a (normalized by one real d.o.f.). Here, the signs are taken as $+$ ($-$) for boson fields (fermion fields). Γ_{split} stands for the splitting rate. One can show that Eq. (3.5) can be derived by using the definition of the splitting function, γ_{bc}^a , for $a \leftrightarrow bc$, given in App. A. Instead, below, we give an intuitive derivation of the splitting rate, Γ_{split} , in order to understand its physics. See App. A for more rigorous explanation.

The first term of the right-hand side of Eq. (3.2) imprints elastic scatterings that diffuse distribution functions in momentum space, where the process is dominated by small momentum exchange due to the strong t -channel enhancement of gauge bosons as shown in Fig. 4. The momentum transfer squared obeys the following diffusion equation:

$$(\Delta p)^2 \sim \hat{q}_{\text{el}} t, \tag{3.6}$$

with

$$\hat{q}_{\text{el}} \sim \int d^2 q_\perp \frac{\partial \Gamma_{\text{el}}}{\partial q_\perp^2} q_\perp^2 \sim \alpha^2 \int_{p'} f_\bullet(p') [1 \pm f_\bullet(p')]. \tag{3.7}$$

Here we have used the rate of elastic scatterings

$$\frac{\partial \Gamma_{\text{el}}}{\partial q_\perp^2} \sim \frac{\alpha^2}{q_\perp^2 (q_\perp^2 + m_s^2)} \int_{p'} f_\bullet(p') [1 \pm f_\bullet(p')]. \tag{3.8}$$

Note that the screening mass for χ -particles is given by $m_s^2 \sim \alpha T_*^2$.

Then, let us move to the second term, $\mathcal{C}_{\text{split}}$. As notified in the beginning, in estimating the inelastic scattering rate of the hard primaries, it is necessary to take into account an interference effect between a hard primary and its daughter particle, known as the LPM effect. A Feynman diagram describing the interference effect is shown in Fig. 5. The interference effect forbids subsequent scattering processes during the interval while their phase spaces overlap with each other. Taking this effect into account, we obtain the following rate of inelastic scatterings:

$$\Gamma_{\text{split}}(k) \sim \alpha \text{Min} \left[\Gamma_{\text{el}}, \frac{1}{t_{\text{form}}} \right], \tag{3.9}$$

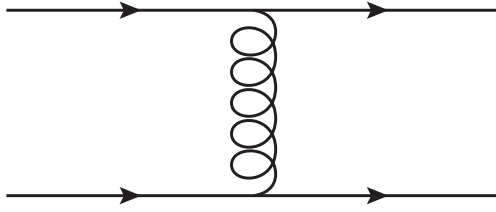


Figure 4: Diagram for elastic 2 to 2 scattering process.

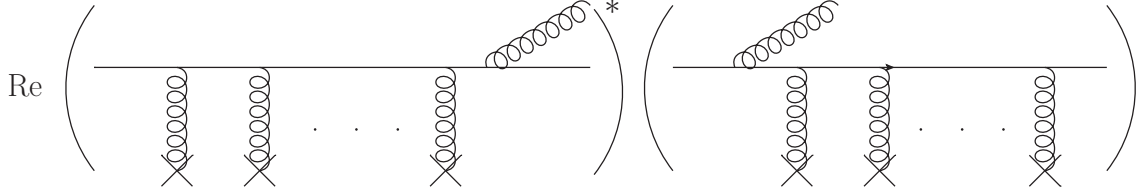


Figure 5: Diagram representing an interference effect, known as the LPM effect.

with

$$t_{\text{form}} \equiv \sqrt{\frac{k}{\hat{q}_{\text{el}}}}. \quad (3.10)$$

Here we omit a factor that comes from model dependence and logarithmic enhancement. See App. A. The time scale of t_{form} denotes the formation time which it takes to resolve the overlap between the daughter and parent particles. k is the momentum for a daughter particle. One can see that the rate is LPM suppressed above the threshold momentum k_{LPM} , which is given by

$$k_{\text{LPM}} \equiv \frac{\hat{q}_{\text{el}}}{\Gamma_{\text{el}}^2}. \quad (3.11)$$

Note that Eq. (3.10) comes from the condition so that the overlap between the parent and daughter should be resolved; $t \gtrsim k/k_{\perp}^2$ with $k_{\perp}^2 \sim \hat{q}_{\text{el}} t$. Therefore, for a given time t , there is an upper bound on the daughter momentum:

$$k_{\text{form}} = \hat{q}_{\text{el}} t^2. \quad (3.12)$$

Above k_{form} , even the diffusion cannot resolve the destructive interference. In this case, the vacuum cascades may become important, though the emitting angle between the parent and the daughter is bounded below $\theta \gtrsim (kt)^{-1/2}$ for a given k and t . See also Eq. (3.28) and discussion about it.

So as to obtain the effective temperature of χ_{ϕ} , we also have to know how χ_{ϕ} -particles are produced. The collision term for χ -particles responsible for χ_{ϕ} -production may be given by

$$\mathcal{C}[f_{\chi_{\phi}}] \supset (\mathcal{C}_{2 \text{ to } 2} + \mathcal{C}_{\text{split}} + \mathcal{C}_{\text{dec}})[f_{\chi_{\phi}}], \quad (3.13)$$

where the last term with the subscript “dec” denotes the decay/inverse decay processes. They are expected in general since the ϕ -condensation can give a sizable mass to χ_{ϕ} -

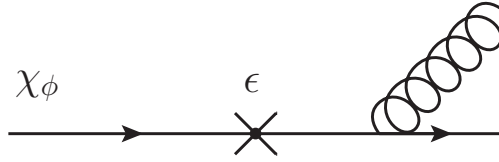


Figure 6: Diagram for the decay of χ_ϕ -particles.

particles; that is, larger than the screening mass, $|g_\phi \phi| > m_s$, and are calculated by

$$\mathcal{C}_{\text{dec}}[f_{\chi_\phi}] \sim \int_{k', p'} \frac{|\mathcal{M}_{\text{dec}}(K; K', P')|^2}{2E_k 2E_{k'} 2E_{p'}} (2\pi)^4 \delta^{(4)}(K - K' - P') \left[f_{\chi_\phi}(k) [1 \pm f_\bullet(k')] [1 \pm f_\bullet(p')] - (\text{inverse process}) \right]. \quad (3.14)$$

These collision terms depend on models how χ_ϕ -particles couple with other light particles; in particular, we have to specify interaction terms which can induce decay of χ_ϕ into light particles for a sizable mass of χ_ϕ , $|g_\phi \phi| > m_s$. For simplicity, we assume that the typical magnitude of this term is proportional to $\epsilon^2 \alpha$ with ϵ being a small parameter and α being the fine structure constant of the gauge group which dominates the thermalization. We assume $\epsilon^2 \lesssim \alpha$ to avoid unnecessary complications. See also discussion given below Eq. (3.34). For instance, if χ_ϕ -particles are charged under the gauge group and if they mix with light particles, then their decay rate may be proportional to $\epsilon^2 \alpha |g_\phi \phi|$, where ϵ is identified as the mixing angle (see Fig. 6). Under this assumption, the dominant contribution to first two terms is essentially the same as that for χ -particles, though we should note that when a daughter particle has a mass larger than the screening mass m_s , we have to replace m_s in the scattering rate of Eq. (3.8) to the mass of the daughter particle. And also note that these terms can be sources of χ_ϕ -particle production.

3.1 $\tilde{t}_{\text{ini}} < \tilde{t} < \tilde{t}_{\text{soft}}$

Apparently, two to three processes with the momentum exchange of the order of m_l seem to be efficient in order to decrease/increase energy/number of hard primaries. However, this is not true since the reaction energy/occupation number is too large/small and the rate is found to be much smaller than the Hubble parameter. This is a typical situation of reheating via Planck suppressed decay, $\tilde{\Gamma}_l \lesssim 1$. In this case, the hard primaries lose their energy mainly through inelastic collinear scatterings with the rate of Eq. (3.9), which are enhanced by the t -channel contribution [31]. Below, we will see how χ -particles are produced via inelastic scatterings, and form the soft population. Then, we discuss production of χ_ϕ -particles, which is essentially different from that of χ -particles due to the mass of $|g_\phi \phi|$.

Production of χ -particles

At the first stage of thermalization process, the hard primaries scatter among themselves and emit soft particles through the inelastic scatterings imprinted in $\mathcal{C}_{\text{split}}$. The phase space distribution of these soft particles might be estimated as

$$f_s(t, k) \sim \Gamma_{\text{split}}(k) n_h k^{-3} t, \quad (3.15)$$

where $n_h \sim \int_p f_h$ is the number density of hard primaries [see Eq. (2.5)]. However, low-momentum modes are so over-occupied that inverse processes and subsequent elastic scattering processes have to be taken into account. In fact, soft particles feel subsequent scatterings with hard primaries^{◆10} and low-momentum modes fall into a thermal-like distribution such as^{◆11}

$$f_s(t, k) \sim \frac{T_s}{k} \text{ for } k < k_{\max}, \quad (3.16)$$

where k_{\max} is determined by the diffusion constant of Eq. (3.7) with $f = f_h$ and is written as

$$k_{\max} \sim \sqrt{\hat{q}_{\text{el}} t} \sim \alpha m_I \sqrt{f_h(t, m_I) \tilde{t}} \sim \alpha m_I \tilde{\Gamma}_I^{1/2}. \quad (3.17)$$

One can show that it is the same as the threshold momentum of LPM suppression, i.e., $k_{\max} \sim k_{\text{LPM}}$. The effective temperature for bosonic soft modes T_s can be estimated from the energy conservation. The energy of soft modes below the scale k_{\max} is given as^{◆12}

$$k_{\max} \Gamma_{\text{split}}(k_{\max}) t n_h \sim \rho_s \sim k_{\max} n_s \sim T_s k_{\max}^3, \quad (3.18)$$

where Γ_{split} is given by Eq. (3.9). Thus, the effective temperature for the soft modes T_s is given as

$$T_s \sim \alpha^{-1/2} \tilde{\Gamma}_I^{1/4} \tilde{t}^{-1/2} m_I, \quad (3.19)$$

from Eq. (3.18) together with Eqs. (3.9) and (3.17). Here, we have used $\Gamma_{\text{split}} \sim \alpha \sqrt{\hat{q}_{\text{el}}/k}$ because the relevant energy scale k_{\max} is so large that such a soft mode are produced by LPM suppressed interactions.

We illustrate the distribution of χ -particles in Fig. 7, where we include the hard sector as well as the soft one. The distribution of smaller wavenumber modes than k_{\max} is a thermal-like one given by Eq. (3.16), while that of larger wavenumber modes in the soft sector is determined by the LPM effect as Eq. (3.15). The distribution of hard sector is given by Eq. (2.5). The distributions of soft and hard sectors coincide with each other at a certain wavenumber, $\tilde{\Gamma}_I^{-1/4} k_{\max} \tilde{t}^{1/4}$, shown in Fig. 7. For clarity, we call the distribution of χ -particles above/below this momentum as the hard/soft sector.

At this regime, the effective temperature T_* is dominated by the contribution from the soft sector

$$T_*^2 \sim \int_k \frac{f_s}{k} \sim T_s k_{\max} \sim \alpha^{1/2} \tilde{\Gamma}_I^{3/4} \tilde{t}^{-1/2} m_I^2, \quad (3.20)$$

$$> \int_k \frac{f_h}{k} \sim \tilde{\Gamma}_I \tilde{t}^{-1} m_I^2, \quad (3.21)$$

◆10 Although there are many processes for the thermalization of soft modes (see Ref. [38, 39]), we explain one of them as an illustration.

◆11 In the case that soft particles are fermions, the distribution f_s cannot exceed unity due to the Pauli blocking effect. In this case, we should truncate f_s when it reaches unity. However, we expect that soft particles contain bosons that dominate the thermal effect on ϕ , so that we focus on the distribution of bosonic particles. Its extension to fermionic case is straightforward.

◆12 One might wonder if the energy conservation within the soft modes [that is, Eq. (3.18)] is violated due to the scatterings between hard primaries and soft modes. However, this does not change our order estimations. The energy for soft modes is dominated by that for the modes around k_{\max} , and the scatterings between hard primaries and such soft modes are inefficient to change the energy-conservation relation of Eq. (3.18).

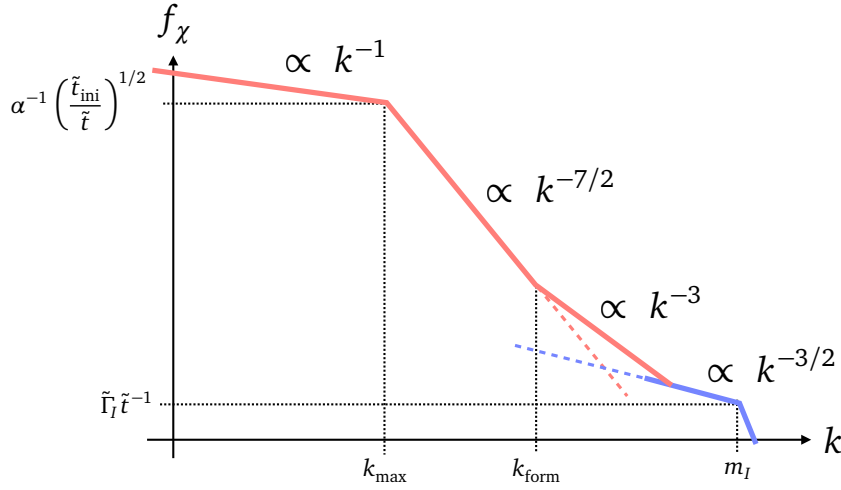


Figure 7: Distribution of χ -particles for $\tilde{t}_{\text{ini}} < \tilde{t} < \tilde{t}_{\text{soft}}$. Recall that $k_{\text{max}} \sim \alpha \tilde{\Gamma}_I^{-1/2} m_\phi$, $k_{\text{form}} \sim k_{\text{max}}(\tilde{t}/\tilde{t}_{\text{ini}})$ and $t_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}$. **Soft (pink)**: The thermal-like distribution dominates below k_{max} , the LPM-suppressed spectrum can be seen between k_{max} and k_{form} , and the vacuum cascade may be relevant above k_{form} . **Hard (blue)**: The hard spectrum is sourced by the direct decay of inflaton and by its red-shifted spectrum of previously produced one. See Eq. (2.5).

where the inequality holds for $\tilde{t} > \tilde{t}_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}$ (see below). This implies that the screening mass $m_s (= \alpha T_*^2)$ is already dominated by the soft sector. Note that $T_s \neq T_*$ because the soft sector is not completely thermalized and T_s and T_* are nothing but effective temperatures with different definitions.

Here, we should ensure that the relevant wavelength scale (e.g., k_{max}^{-1}) is smaller than the Hubble horizon $H^{-1} \sim t$ for a consistent treatment because longer wavelength modes than the Hubble horizon cannot be emerged due to the loss of causality. This implies that we can determine the time scale when the softest mode emerges by comparing the maximum momentum of the soft sector with the Hubble parameter. The inequality $k_{\text{max}} > H$ indicates

$$\tilde{t} > \tilde{t}_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}. \quad (3.22)$$

This inequality immediately means that the occupation number of soft sector at k_{max} , which dominates the energy and number density of the soft sector, is smaller than α^{-1} :

$$f_s(t, k_{\text{max}}) \sim \frac{T_s}{k_{\text{max}}} \sim \alpha^{-1} \left(\frac{\tilde{t}_{\text{ini}}}{t} \right)^{1/2} < \alpha^{-1}. \quad (3.23)$$

Hence, we may treat the soft sector perturbatively. Then, let us compare the screening mass with the maximum momentum k_{max} . Up to here, our arguments rely on the effective kinetic theory of non-Abelian plasma, and hence produced modes should not be too soft, $k > m_s$, for a consistent treatment. In fact, Eq. (3.22) also ensures

$$\frac{k_{\text{max}}}{m_s} \sim \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}} \right)^{1/4} > 1. \quad (3.24)$$

We should also note that

$$\frac{m_s}{H} \sim \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}} \right)^{3/4} > 1, \quad (3.25)$$

which means that the screening length is smaller than the horizon length. Finally, we check that a typical time scale of interactions (e.g., Γ_{el}^{-1}) is smaller than the cosmological time scale H^{-1} :

$$\frac{H^{-1}}{\Gamma_{\text{el}}^{-1}} \sim \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}} \right)^{1/2} > 1, \quad (3.26)$$

where we use Eq. (3.8). Therefore, the condition $\tilde{t} > \tilde{t}_{\text{ini}}$ ensures that $k_{\text{max}} > m_s > H$, $f_s < \alpha^{-1}$, and $\Gamma_{\text{el}} > H$. These inequalities justify our calculations performed in this paper. Otherwise we may have to take into account the effect of causality, the finite cosmological time scale, and non-perturbative effects on production processes.

Finally, we comment on the era before \tilde{t}_{ini} . Since elastic scatterings among decay products does not take place within the Hubble time scale, medium induced cascades can be neglected. In addition, the formation momentum, k_{form} , is smaller than the Hubble parameter before t_{ini} :

$$\frac{k_{\text{form}}}{H} \sim \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}} \right)^2 < 1. \quad (3.27)$$

Still, the vacuum cascades might broaden the spectrum towards the infrared, and hence we roughly estimate their effect on the effective temperature. The quantum formation time scale in vacuum implies that one can find a quanta of k with a probability of $\mathcal{O}(\alpha)$ per $\ln k \ln \theta$ for $\theta > (kt)^{-1/2}$. Omitting the log factor, one may obtain the distribution of the soft sector and the effective temperature as

$$f_s(t, k) \sim \alpha \tilde{\Gamma}_I \tilde{t}^{-1} \left(\frac{m_I}{k} \right)^3 \quad \text{for } k > H \rightarrow T_*^2 \sim \alpha \tilde{\Gamma}_I m_I^2, \quad (3.28)$$

which coincides with Eq. (3.20) for $\tilde{t} \rightarrow \tilde{t}_{\text{ini}}$. Hence, we expect that the effective temperature is at most that of \tilde{t}_{ini} . After \tilde{t}_{ini} , these particles with k^{-3} are soon red-shifted away and LPM suppressed spectrum starts to dominate the soft sector. Thus, we will not consider the case of $\tilde{t} < \tilde{t}_{\text{ini}}$ further in the following.

Production of χ_ϕ -particles

In contrast to χ -particles, χ_ϕ -particles can have sizable masses of $|g_\phi \phi|$ due to the large expectation value of the ϕ -condensate. The distribution in momentum space can be different from that of χ -particles and hence separate discussions are required. Although the calculations are complicated, we find that the effect on the effective potential from χ_ϕ -particles is always subdominant for the case of $|g_\phi \phi| \gtrsim \alpha^{-1} k_{\text{max}}$ and $\tilde{\Gamma}_I \lesssim \alpha^6$. Even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^6$, its effect is roughly the same order with that of thermal log potential from χ -particles, so that we can neglect it for rough estimations. We briefly explain χ_ϕ -particle production processes in this subsection, while detailed calculations are shown in Appendix. ^{◆13}

^{◆13} We conservatively omit the decay of massive χ_ϕ -particles to show that the effect on the effective potential from χ_ϕ -particles is usually subdominant. In other words, we show that the effect from χ_ϕ -particles can be neglected even if they are stable and maximally abundant. Note that the calculation can be applied to the production of massive dark matter, once we forget the field ϕ and identify χ_ϕ as just a dark matter with mass of $|g_\phi \phi|$. Its abundance can be calculated from the resulting distribution function of χ_ϕ -particles given in Appendix.

If the effective mass, $|g_\phi \phi|$, is smaller than the screening mass, m_s , then the results are exactly the same as those for χ -particles. Hence, we concentrate on $|g_\phi \phi| > m_s$ in the following. First, let us consider contributions from splittings which are imprinted in $\mathcal{C}_{\text{split}}$. Note that since we consider the case of $|g_\phi \phi| > m_s$, we should replace m_s in Eq. (3.8) to $|g_\phi \phi|$. The phase space distribution and the resultant effective temperature may be estimated as

$$f_{\chi_\phi}(t, k) \Big|_{\text{hard}} \sim \Gamma_{\text{split}}(k) n_h k^{-3} t \rightarrow T_{*,\chi_\phi}^2 \Big|_{\text{hard}} \sim \Gamma_{\text{split}}(M) n_h M^{-1} t, \quad (3.29)$$

where

$$M \equiv \text{Max} \left[|g_\phi \phi|, k_{\text{max},\chi_\phi} \right]; \quad k_{\text{max},\chi_\phi} \equiv k_{\text{max}} \left(\frac{m_s}{|g_\phi \phi|} \right). \quad (3.30)$$

The ‘‘hard’’ subscript means that splittings occur dominantly through interactions among hard primaries, since the number density is still dominated by the hard sector at this stage, $\tilde{t} < \tilde{t}_{\text{soft}}$. Here Γ_{split} is given by Eq. (3.9) but m_s is replaced by $|g_\phi \phi|$. In this case, the threshold momentum k_{LPM,χ_ϕ} is given by

$$k_{\text{LPM},\chi_\phi} = k_{\text{LPM}} \left(\frac{|g_\phi \phi|}{m_s} \right)^4. \quad (3.31)$$

If the effective mass for χ_ϕ is not so large, that is, $(g_\phi \phi)^2 < m_s k_{\text{max},\chi_\phi}$, then the distribution function for χ_ϕ coincides with that for χ below k_{max,χ_ϕ} .

In addition, χ_ϕ -particles are produced via $\mathcal{C}_{2 \text{ to } 2}$ and \mathcal{C}_{dec} with interactions among the soft sector and also between the soft and hard sector. First, let us concentrate on $\mathcal{C}_{2 \text{ to } 2}$, since it gives model independent contribution.^{◆14} Although the number density is still dominated by the hard sector, the soft sector may produce ‘‘soft’’ χ_ϕ -particles with a larger rate (e.g., via s -channel scattering processes). The effective temperature may be given by

$$T_{*,\chi_\phi}^2 \Big|_{\text{soft}} \sim \frac{\alpha^2}{|g_\phi \phi|^3} t \int d \log k' n_s(k') n_s \left(\frac{|g_\phi \phi|^2}{k'} \right), \quad (3.32)$$

for $|g_\phi \phi| > k_{\text{max}}$, where $n_s \sim \int_p f_s$. Note that, for $|g_\phi \phi| < k_{\text{max}}$, it is given by $T_{*,\chi_\phi}^2 \Big|_{\text{soft}} \sim k_{\text{max}}^2$. They are also produced from interactions between the hard and soft sectors:

$$T_{*,\chi_\phi}^2 \Big|_{\text{int}} \sim \frac{\alpha^2}{|g_\phi \phi|^3} t \int d \log k' n_s(k') n_h \left(\frac{|g_\phi \phi|^2}{k'} \right), \quad (3.33)$$

where the subscript ‘‘int’’ indicates contributions from the interaction between the hard and soft sectors. Note that both n_s and n_h can be calculated from the distribution function illustrated in Fig. 7. Then, we give comments on contributions from \mathcal{C}_{dec} , which strongly depends on details of a model. As an illustration, we have assumed that the magnitude of this process is proportional to $\epsilon^2 \alpha$. The inverse decay can produce χ_ϕ -particles and it may yield

$$T_{*,\chi_\phi}^2 \Big|_{\text{soft/int}} \sim \frac{\epsilon^2 \alpha}{|g_\phi \phi|^3} t \int d \log k' n_s(k') n_{s/h} \left(\frac{|g_\phi \phi|^2}{k'} \right). \quad (3.34)$$

^{◆14} We can drop terms proportional to $\epsilon^2 \alpha^2$ at least for $\epsilon^2 \lesssim \alpha$.

Hence, if the typical mixing parameter ϵ is not so large, say $\epsilon^2 \lesssim \alpha$, then this contribution does not exceed those from Eqs. (3.32) and (3.33). Moreover, this collision term, \mathcal{C}_{dec} , also yields decay of χ_ϕ -particles:

$$\langle \Gamma_{\text{decay}} \rangle t \sim \epsilon^2 \alpha |g_\phi \phi| t \lesssim \left(\frac{|g_\phi \phi|}{k_{\text{max}}} \right) \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}} \right), \quad (3.35)$$

where we use $\tilde{t}_{\text{soft}} \equiv \alpha^{-3} \tilde{\Gamma}_I^{-1/2}$ defined in the next subsection. One can see that the decay is insignificant for $|g_\phi \phi| < k_{\text{max}}$ at this stage, $\tilde{t} \lesssim \tilde{t}_{\text{soft}}$. As explained in the footnote [♠13](#), we can omit the decay term in order to show that the effect on the effective potential from abundant χ_ϕ -particles is subdominant for $|g_\phi \phi| \gtrsim \alpha^{-1} k_{\text{max}}$.

For later convenience, we denote all these contributions where χ_ϕ -particles are produced indirectly (not directly produced by inflaton decay) as

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \equiv \max \left[T_{*,\chi_\phi}^2 \Big|_{\text{hard}}, T_{*,\chi_\phi}^2 \Big|_{\text{soft}}, T_{*,\chi_\phi}^2 \Big|_{\text{int}} \right]. \quad (3.36)$$

The explicit forms of the RHS are shown in Appendix.

In contrast to the above discussion, if χ_ϕ -particles are produced directly from inflaton decay, then the situation turns out to be slightly different. This is because the hard sector still has a sizable contribution to the effective temperature, which tend to dominate for a large field value. In this case, its distribution function has a contribution of Eq. (2.5). As shown below, the effect on the effective potential from χ_ϕ -particles can be efficient compared with the thermal log potential. Therefore, in this case, we should take into account the decay of χ_ϕ -particles to obtain more realistic predictions because χ_ϕ -particles obtain effective masses of $|g_\phi \phi|$ and can decay into light particles in general (see footnote [♠13](#)). Its decay effect can be taken into account by multiplying $\text{Min}[1, (\Gamma_{\text{decay}} t)^{-1}]$ to its distribution of Eq. (2.5), where Γ_{decay} is the decay rate of χ_ϕ -particles. The effective temperature may be estimated as

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} = \tilde{\Gamma}_I \tilde{t}^{-1} m_I^2 \text{Min} \left[1, \frac{1}{\Gamma_{\text{decay}} t} \right], \quad (3.37)$$

where $\Gamma_{\text{decay}} \sim \epsilon^2 \alpha |g_\phi \phi|$.

Thermal potential

Here we summarize the thermal potentials given in Eqs. (2.11) and (2.13), and discuss which contribution dominates for each case. See also Appendix B. Note that for an interval between $k_{\text{max}} < |g_\phi \phi| < \alpha^{-1} k_{\text{max}}$, the effective temperature may depend on ϵ^2 . We do not care about this small interval for our rough estimation.

- Indirect χ_ϕ -production (produced from hard primaries/soft daughters):
 - The simplest case is a large field value regime, $|g_\phi \phi| > \alpha^{-1} k_{\text{max}}$. In the case of $\tilde{\Gamma}_I \lesssim \alpha^6$, the effective mass from abundant χ_ϕ -particles always gives subdominant corrections to the effective potential, meanwhile the log contribution dominates:

$$\alpha^2 \frac{T_*^4}{\phi^2} \sim \alpha_\phi \alpha T_*^2 \left(\frac{\tilde{t}_{\text{ini}}}{\tilde{t}} \right)^{1/2} \left(\frac{k_{\text{max}}}{|g_\phi \phi|} \right)^2. \quad (3.38)$$

Even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^6$, the effect of χ_ϕ -particles is very limited, so that we can neglect its contribution for a rough estimation.

- For a small field value regime, $|g_\phi \phi| < k_{\max}$, the effective mass from abundant χ_ϕ -particles dominates the effective potential:

$$\alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \sim \delta \alpha_\phi T_*^2; \quad \delta \in [\alpha, 1]. \quad (3.39)$$

Here we do not write down complicated parameter dependence of $T_{*,\chi_\phi} \Big|_{\text{indir}}$ for brevity. See Appendix B for details.

- Direct χ_ϕ -production (produced from inflaton decay):

- For the large field value regime, $|g_\phi \phi| > \alpha^{-1} k_{\max}$, the effective potential is given by the competition of two contributions; the log dependence of the running coupling constant and hard χ_ϕ -primaries from the direct decay of inflaton. The contribution from the direct decay of inflaton, which is given by

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} = \alpha \tilde{\Gamma}_I^{1/2} T_*^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^{1/2} \text{Min} \left[1, \frac{1}{\Gamma_{\text{decay}} t} \right], \quad (3.40)$$

dominates the effective temperature for

$$m_I > |g_\phi \phi| > \alpha m_I \text{Max} \left[\alpha^{1/2} \tilde{\Gamma}_I^{1/4}, \alpha \left(\frac{\epsilon^2}{\alpha} \right) \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}} \right) \right]. \quad (3.41)$$

- For the small field value regime, $|g_\phi \phi| < k_{\max}$, the contribution from the abundant χ_ϕ -particles dominates over that from the running coupling constant, as explained in the previous case. Again, we do not write down the explicit parameter dependence of $T_{*,\chi_\phi} \Big|_{\text{indir}}$ since it is complicated. See Appendix B for details.

The trivial case is for $|g_\phi \phi| < m_s$; there the contribution from abundant χ_ϕ -particles always dominate over that from hard χ_ϕ :

$$\alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{indir}} > \alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{dir}}. \quad (3.42)$$

This is simply because the effective temperature for massless modes is dominated by the soft sector as shown in Eq. (3.21).

3.2 $\tilde{t}_{\text{soft}} < \tilde{t} < \tilde{t}_{\max}$

After the time when the effective temperature T_* becomes comparable to k_{\max} , the soft sector is completely thermalized by their own interactions and can be described by a single parameter $T_s \sim T_* \sim k_{\max}$. This occurs at the time of

$$\tilde{t} \sim \tilde{t}_{\text{soft}} \equiv \alpha^{-3} \tilde{\Gamma}_I^{-1/2}. \quad (3.43)$$

At the same time, the number density of soft sector is as large as that of hard primaries, which means that we should substitute f in Eq. (3.7) with f_s .

Production of χ -particles

The energy injection to the soft sector is given by

$$\rho_s \sim k_{\text{split}} n_h. \quad (3.44)$$

The momentum scale k_{split} is given by a criteria $\Gamma(k_{\text{split}}) \sim H(t)$, which means that modes below k_{split} are soon thermalized and fall into soft modes. Equating ρ_s with T_s^4 , we obtain

$$T_s \sim \alpha^4 \tilde{\Gamma}_I \tilde{t} m_I, \quad (3.45)$$

$$k_{\text{split}} \sim \alpha^{16} \tilde{\Gamma}_I^3 \tilde{t}^5 m_I, \quad (3.46)$$

where we use $\hat{q}_{\text{el}} \sim \alpha^2 T_s^3$. The thermal potential is now determined by the temperature of the soft sector:

$$k_{\text{max}} \sim T_* \sim T_s \sim \alpha^4 \tilde{\Gamma}_I \tilde{t} m_I. \quad (3.47)$$

Note that the distribution function for soft χ -particles after $\tilde{t} > \tilde{t}_{\text{soft}}$ is given by

$$f_s(t, k) \sim \begin{cases} \frac{T_s}{k} & \text{for } k \lesssim T_s, \\ \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}}\right)^2 \left(\frac{T_s}{k}\right)^{7/2} & \text{for } T_s \lesssim k \lesssim k_{\text{form}}, \\ \alpha \tilde{\Gamma}_I \tilde{t}^{-1} \left(\frac{m_I}{k}\right)^3 & \text{for } k_{\text{form}} \lesssim k. \end{cases} \quad (3.48)$$

Also note that the ‘‘hard’’ interactions among soft particles with the momentum exchange of T_s is faster than the cosmic expansion:

$$\frac{\alpha^2 n_s / T_s^2}{H} \sim \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}}\right)^2 > 1. \quad (3.49)$$

Hence, the soft sector is thermalized separately.

We illustrate the distribution of χ -particles in Fig. 8, where we include the hard sector as well as the soft one. The distribution of the soft sector is given by Eq. (3.48), while that of the hard sector is given by Eq. (2.5). Eventually, the distributions of soft and hard sectors coincide with each other at the wavenumber of $k \sim m_I (\tilde{t} / \tilde{t}_{\text{max}})^{5/4}$, where $\tilde{t}_{\text{max}} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}$ is defined in the next subsection [See Eq. (3.57)].

Production of χ_ϕ -particles

If the effective mass term, $|g_\phi \phi|$, is smaller than the temperature of the soft sector, $|g_\phi \phi| < T_s$, then the dominant contribution to the distribution function of χ_ϕ -particles is given by

$$f_{\chi_\phi}(t, k) \sim \frac{T_s}{k} \text{ for } k < T_s, \quad (3.50)$$

which yields

$$T_{*, \chi_\phi}^2 \Big|_{\text{indir}} \sim T_s^2, \quad (3.51)$$

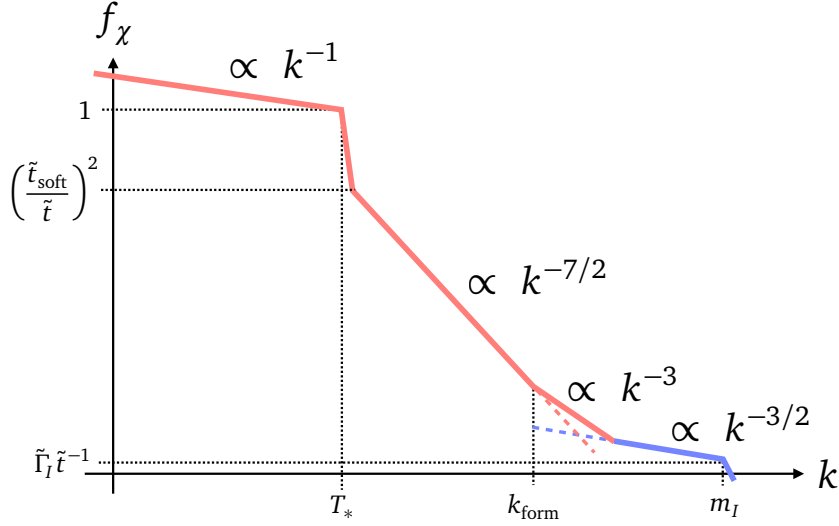


Figure 8: Distribution of χ -particles for $\tilde{t}_{\text{soft}} < \tilde{t} < \tilde{t}_{\text{max}}$. Recall that $T_* \sim \alpha \tilde{\Gamma}_I^{1/2} m_I (\tilde{t}/\tilde{t}_{\text{soft}})$, $k_{\text{form}} \sim \alpha^{-2} T_* (\tilde{t}/\tilde{t}_{\text{soft}})^4$, $t_{\text{soft}} \equiv \alpha^{-3} \tilde{\Gamma}_I^{-1/2}$, and $t_{\text{max}} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}$ [See Eq. (3.57)]. **Soft (pink)**: The thermal-like distribution dominates below $T_*(=T_s)$, the LPM-suppressed spectrum can be seen between k_{max} and k_{form} , and the vacuum cascade may be relevant above k_{form} . **Hard (blue)**: The hard spectrum is sourced by the direct decay of inflaton and by its red-shifted spectrum of previously produced one. Eventually, the distributions of soft and hard sectors become to coincide with each other at the wavenumber of $k \sim m_I (\tilde{t}/\tilde{t}_{\text{max}})^{5/4}$.

for $|g_\phi \phi| < T_s$. Basically, this is because “hard” interactions among the soft sector with the momentum exchange of T_s is much faster than the cosmic expansion at this stage, as shown in Eq. (3.49). Thus, χ_ϕ -particles with a mass lighter than T_s are efficiently produced and participate in the thermal plasma.

The case of $|g_\phi \phi| > T_s$ is calculated in Appendix. The χ_ϕ -particle production processes are the same with the ones in the previous subsection, though the number densities of n_s and n_h are different.

Thermal potential

Here we summarize the thermal potentials and discuss which contribution dominates for each case. See also Appendix B. Again, note that for an interval between $T_* < |g_\phi \phi| < \alpha^{-1} T_*$, the effective temperature may depend on ϵ^2 . We do not care about this small interval for our rough estimation.

- Indirect χ_ϕ -production (produced from hard primaries/soft daughters):
 - First, we consider a large field value regime, $|g_\phi \phi| > \alpha^{-1} T_*$. In the case of $\tilde{\Gamma}_I \lesssim \alpha^6$, the effective mass from abundant χ_ϕ -particles always gives subdominant corrections to the effective potential, meanwhile the log contribution dominates:

$$\alpha^2 \frac{T_*^4}{\phi^2} \sim \alpha_\phi \alpha^2 T_*^2 \left(\frac{T_*}{|g_\phi \phi|} \right)^2. \quad (3.52)$$

Even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^6$, the effect of χ_ϕ -particles is very limited, so that we can neglect its contribution for a rough estimation. For $\tilde{t} \gtrsim \text{Min}[\alpha^{-1} \tilde{t}_{\text{soft}}, t_{\text{max}}]$,

that is, soon after the soft sector is thermalized the running coupling constant contribution dominates for $|g_\phi \phi| > \alpha^{-1} T_*$ independently of $\tilde{\Gamma}_I$. This is because the number density of soft sector increases with time, while the number density of hard primaries decreases due to the red-shift of inflaton number density.

- For a small field value regime, $|g_\phi \phi| < T_*$, the effective mass from abundant χ_ϕ -particles dominates the effective potential:

$$\alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{indir}} = \alpha_\phi T_*^2. \quad (3.53)$$

- Direct χ_ϕ -production (produced from inflaton decay):

- For the large field value regime, $|g_\phi \phi| > \alpha^{-1} T_*$, the effective potential is given by the competition of two contributions; the log dependence of the running coupling constant and hard χ_ϕ -primaries from the direct decay of inflaton. The contribution from the direct decay of inflaton, which is given by

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \sim \alpha \tilde{\Gamma}_I^{1/2} T_*^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^3 \text{Min} \left[1, \left(\frac{\alpha}{\epsilon^2} \right) \left(\frac{T_*}{g_\phi \phi} \right) \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^2 \right], \quad (3.54)$$

dominates the effective temperature for

$$m_I > |g_\phi \phi| > \alpha m_I \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}} \right)^{5/2} \text{Min} \left[\alpha^{1/2} \tilde{\Gamma}_I^{1/4}, \alpha \left(\frac{\epsilon^2}{\alpha} \right) \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}} \right)^{7/2} \right]. \quad (3.55)$$

- For the small field value regime, $|g_\phi \phi| < T_*$, the contribution from abundant χ_ϕ -particles is always larger than other ones; from the running coupling constant and also from the direct decay of inflaton:

$$\alpha_\phi T_{*,\chi_\phi}^2 \simeq \alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{indir}} = \alpha_\phi T_{*,\chi}^2. \quad (3.56)$$

This is simply because the effective temperature for massless modes is dominated by the soft sector as shown in Eq. (3.21).

3.3 $\tilde{t}_{\text{max}} < \tilde{t} < \tilde{t}_{\text{RH}}$

When $\Gamma_{\text{split}}(k \simeq m_\phi) \sim H$ is satisfied, that is, when k_{split} becomes comparable to m_I , primary particles lose their energy completely and are thermalized soon. This occurs at the time of

$$\tilde{t} \sim \tilde{t}_{\text{max}} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}. \quad (3.57)$$

This is the actual thermalization time scale of hard particles.

Production of χ -particles

The energy conservation implies that

$$\rho_r \sim \rho_\phi \Gamma_\phi t \sim H(t) \Gamma_\phi M_{\text{pl}}^2, \quad (3.58)$$

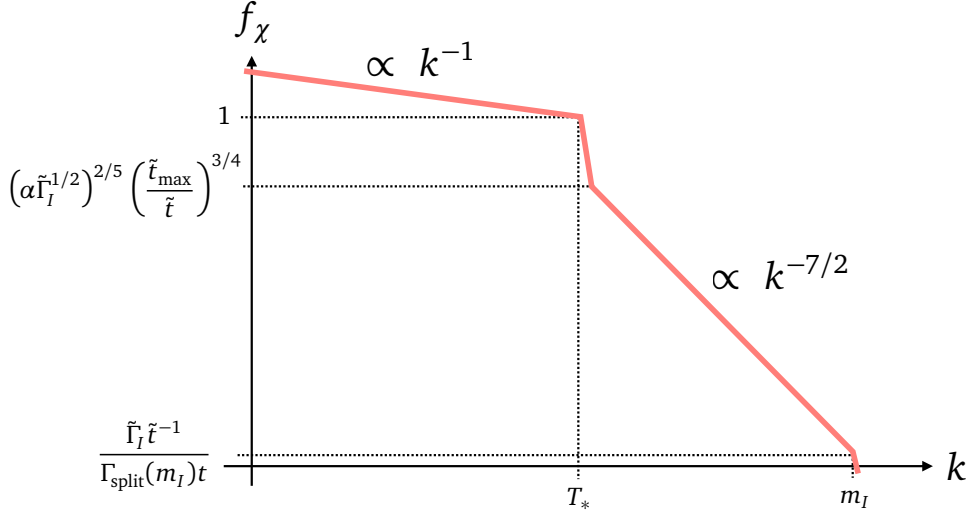


Figure 9: Distribution of χ -particles for $\tilde{t}_{\max} < \tilde{t} < \tilde{t}_{\text{RH}}$. Recall that $T_* \sim \tilde{\Gamma}_I^{1/4} \tilde{t}^{-1/4} m_I$, $t_{\max} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}$, and $t_{\text{RH}} = \tilde{\Gamma}_I^{-1} M_{\text{pl}}^2 / m_I^2$ [See Eq. (3.63)]. Note that there remains the tail of LPM-suppressed spectrum $\propto k^{-7/2}$ because the inflaton continuously produces primaries with the momentum of $p \sim m_I$.

where ρ_r ($\sim T_*^4$) is the energy density of radiation. This gives us the time evolution of the temperature such as

$$T_* \sim \tilde{\Gamma}_I^{1/4} \tilde{t}^{-1/4} m_I. \quad (3.59)$$

Since the hard sector as well as the soft sector are thermalized, they can be described by a single parameter T_* .

We illustrate the distribution of χ -particles in Fig. 9. The distribution of smaller wavenumber modes than T_* is a thermal one, while that of larger wavenumber modes is determined by the LPM effect as Eq. (3.15). Note that the hard sector is completely thermalized, so that it is absent in Fig. 9.

Production of χ_ϕ -particles and thermal potential

The production process of χ_ϕ -particles is the same with the one considered in the previous subsection. However, since the number density of hard primaries decreases efficiently by splittings for $t > t_{\max}$, the contribution from the direct decay of inflaton is now given by

$$f_{\chi_\phi}(t, m_I) \Big|_{\text{dir}} \sim \tilde{\Gamma}_I \tilde{t}^{-1} \text{Min} \left[\frac{1}{\Gamma_{\text{split}}(m_I) t}, \frac{1}{\Gamma_{\text{decay}} t} \right]. \quad (3.60)$$

Next, we discuss which contribution dominates the thermal potential. See also Appendix B. Again, note that for an interval between $T_s < |g_\phi \phi| < \alpha^{-1} T_s$, the effective temperature may depend on ϵ^2 . We do not care about this small interval for our rough estimation.

- Indirect χ_ϕ -production (produced from hard primaries/soft daughters):
 - For the large field value regime, $|g_\phi \phi| > \alpha^{-1} T_s$, we find that the effect on the effective potential from χ_ϕ -particles is always subdominant compared with the thermal log potential of Eq. (2.13).

- For the small field value regime, $|g_\phi \phi| < T_s$, the distribution of χ_ϕ -particles and its effective temperature are given by Eqs. (3.50) and (3.51). In this case, the effective potential of ϕ is given by the usual thermal mass term of Eq. (2.11).
- Direct χ_ϕ -production (produced from inflaton decay):
 - For the large field value regime, $|g_\phi \phi| > \alpha^{-1} T_s$, the effective potential is determined by the competition between the contribution from the running coupling constant and the direct decay of inflaton. The latter yields

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \sim (\alpha \tilde{\Gamma}_I^{1/2})^{8/5} \left(\frac{\tilde{t}_{\text{max}}}{\tilde{t}} \right)^{9/8} T_*^2 \text{Min} \left[1, (\alpha \tilde{\Gamma}_I^{1/2})^{2/5} \left(\frac{\alpha}{\epsilon^2} \right) \left(\frac{T_*}{g_\phi \phi} \right) \left(\frac{\tilde{t}_{\text{max}}}{\tilde{t}} \right)^{1/8} \right]. \quad (3.61)$$

This contribution dominates the effective potential for

$$m_I > |g_\phi \phi| > \alpha m_I \left(\frac{\tilde{t}}{\tilde{t}_{\text{max}}} \right)^{5/16} \text{Max} \left[1, (\alpha \tilde{\Gamma}_I^{1/2})^{-6/5} \epsilon^2 \left(\frac{\tilde{t}}{\tilde{t}_{\text{max}}} \right)^{11/16} \right]. \quad (3.62)$$

- For the small field value regime, $|g_\phi \phi| < T_s$, the effective potential of ϕ is governed by the usual thermal mass term of Eq. (2.11).

3.4 $\tilde{t}_{\text{RH}} < \tilde{t}$

When $\Gamma_I \sim H(t)$ is satisfied, the energy density of radiation becomes to dominate that of the Universe. This is when the reheating is completed:

$$\tilde{t} \sim \tilde{t}_{\text{RH}} \equiv \tilde{\Gamma}_I^{-1} \frac{M_{\text{pl}}^2}{m_I^2}. \quad (3.63)$$

The reheating temperature is given by

$$T_{\text{RH}} \sim \sqrt{\Gamma_I M_{\text{pl}}}. \quad (3.64)$$

After the reheating is completed, the temperature of the Universe decreases with time as

$$T_* \sim T_{\text{RH}} \left(\frac{\tilde{t}_{\text{RH}}}{\tilde{t}} \right)^{1/2}, \quad (3.65)$$

because $H^2 \sim T_*^4/M_{\text{pl}}^2$ in the radiation dominated era.

3.5 Summary

To sum up, we obtain the evolution of the effective temperature of χ -particles as follows:

$$\frac{T_*}{m_I} \sim \begin{cases} \alpha^{1/2} \tilde{\Gamma}_I^{1/2} \left(\frac{\tilde{t}}{\tilde{t}_{\text{ini}}} \right)^{-1/4} & \text{for } \tilde{t}_{\text{ini}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{soft}} \\ \alpha \tilde{\Gamma}_I^{1/2} \left(\frac{\tilde{t}}{\tilde{t}_{\text{soft}}} \right) & \text{for } \tilde{t}_{\text{soft}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{max}} \\ \alpha^{4/5} \tilde{\Gamma}_I^{2/5} \left(\frac{\tilde{t}}{\tilde{t}_{\text{max}}} \right)^{-1/4} & \text{for } \tilde{t}_{\text{max}} \lesssim \tilde{t} \lesssim \tilde{t}_{\text{RH}} \end{cases} \quad (3.66)$$

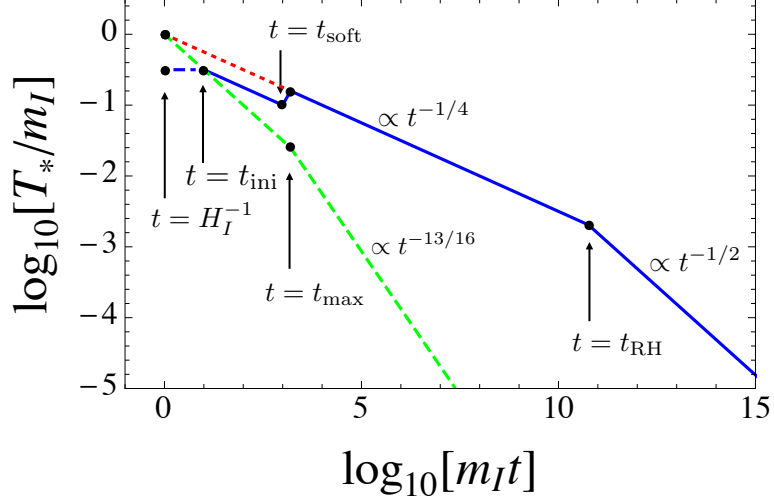


Figure 10: Evolution plots for the effective temperature of the soft sector T_* . We assume $\alpha = 0.1$, $H_I = m_I$, and $m_I = 10^{13}$ GeV. We take $\tilde{\Gamma}_I = 1$, which corresponds to the case of $\tilde{\Gamma}_I > \alpha^3$. The blue lines are results derived in this paper, while the red dotted lines describe the temperature derived in the literature by assuming the “instantaneous thermalization”. We also show the effective temperature of the hard sector as green dashed lines, for the case that they are produced from inflaton decay, where we neglect the decay of χ_ϕ -particles. The blue dot-dashed line is the effective temperature for $\tilde{t} < \tilde{t}_{\text{ini}}$, which is estimated from the discussion of the vacuum cascades [see Eq. (3.28)].

where the time scales are given by

$$\tilde{t}_{\text{ini}} \equiv \alpha^{-1} \tilde{\Gamma}_I^{-1/2}, \quad (3.67)$$

$$\tilde{t}_{\text{soft}} \equiv \alpha^{-3} \tilde{\Gamma}_I^{-1/2}, \quad (3.68)$$

$$\tilde{t}_{\text{max}} \equiv \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}, \quad (3.69)$$

$$\tilde{t}_{\text{RH}} \equiv \tilde{\Gamma}_I^{-1} \frac{M_{\text{pl}}^2}{m_I^2}. \quad (3.70)$$

Here note that we have implicitly assumed $\tilde{t}_{\text{RH}} > \tilde{t}_{\text{max}}$ so far, which implies the thermalization takes place faster than the complete decay of inflaton. As shown in Ref. [30], this inequality is satisfied in most cases: $\alpha \gtrsim 4 \times 10^{-4} (m_I/10^{13} \text{ GeV})^{5/8} \tilde{\Gamma}_I^{1/8}$. These results are illustrated in Figs. 10 and 11 as blue lines. The effective temperature of the soft sector has two local maxima as $T_*/m_I \sim \alpha^{1/2} \tilde{\Gamma}_I^{1/2}$ and $\alpha^{4/5} \tilde{\Gamma}_I^{2/5}$ at the time of $\tilde{t} = \tilde{t}_{\text{ini}}$ and \tilde{t}_{max} , respectively. The global maximum of the effective temperature is given by

$$T_*|_{\text{max}} \sim \begin{cases} \alpha^{1/2} \tilde{\Gamma}_I^{1/2} m_I & \text{for } 1 \gtrsim \tilde{\Gamma}_I \gtrsim \alpha^3 \\ \alpha^{4/5} \tilde{\Gamma}_I^{2/5} m_I & \text{for } \tilde{\Gamma}_I \lesssim \alpha^3 \end{cases} \quad (3.71)$$

There is a local minimum given as $T_* \sim \alpha \tilde{\Gamma}_I^{1/2}$ at $\tilde{t} \sim \tilde{t}_{\text{soft}}$. Note that the effective temperature for $\tilde{t} < \tilde{t}_{\text{ini}}$ is at most that for $\tilde{t} \sim \tilde{t}_{\text{ini}}$ from the discussion of the vacuum cascades [see Eq. (3.28)], so that the maximal temperature is in fact given by Eq. (3.71). The thermalization process described in this section is summarized in the second paragraph in Sec. 1.2.

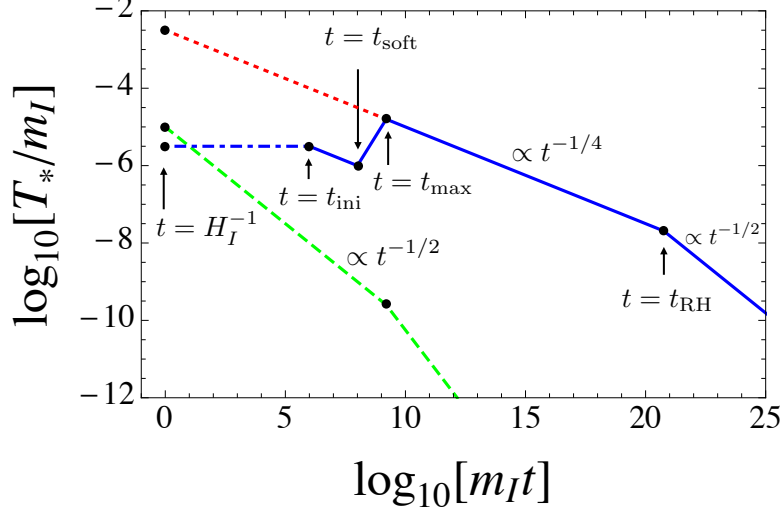


Figure 11: Same as Fig. 10, but we take $\tilde{\Gamma}_I = 10^{-10}$, which corresponds to the case of $\tilde{\Gamma}_I < \alpha^3$.

Now we obtain the evolution of the thermal effects on the potential of ϕ . If χ_ϕ -particles are not directly produced from inflaton decay, the thermal potential of ϕ is roughly given by

$$V_{\text{eff}}(\phi) \sim \begin{cases} \alpha_\phi T_*^2 \phi^2 & \text{for } |g_\phi \phi| \ll T_* \\ \alpha^2 T_*^4 \log\left(\frac{\phi^2}{T_*^2}\right) & \text{for } |g_\phi \phi| \gg T_*, \end{cases} \quad (3.72)$$

for the case of $\tilde{\Gamma}_I \lesssim \alpha^6$. Here, we neglect the complicated parameter dependence of the effective temperature T_{*,χ_ϕ} for $\alpha T_* \lesssim |g_\phi \phi| \lesssim \alpha^{-1} T_*$, and roughly evaluate it as $T_{*,\chi_\phi} \sim T_*$ for $|g_\phi \phi| \ll T_*$. Even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^6$, the deviation of the above formula is very limited, so that one can still use it for a rough estimation. On the other hand, if χ_ϕ -particles are directly produced from inflaton decay, it may affect the thermal potential of ϕ . In particular, for the large field value regime, $|g_\phi \phi| \gtrsim \alpha^{-1} T_*$, the effective potential is given by the competition of two contributions; the thermal log potential from χ -particles and thermal mass from χ_ϕ -particles. The latter contribution is given in Eqs. (3.40), (3.54) and (3.61). If we can neglect the decay of χ_ϕ -particles owing to the smallness of ϵ , T_{*,χ_ϕ} evolves as green dashed lines in Figs. 10 and 11.

Finally, we comment on the uncertainty of our results. We omit the log-enhancement factor of splitting rate in Eq. (3.9) for simplicity, which may be as large as a factor of ten, and also a model dependent factor. See App. A for this issue. There is also numerical factor that is derived from numerical calculations of Boltzmann equations in Ref. [39]. Here we collectively denote these factor as c , i.e., we write $\Gamma_{\text{split}}(k) = c\alpha\sqrt{\hat{q}_{\text{el}}}/k$ and discuss the uncertainty of the maximal temperature of the Universe. Since we are mostly interested in the case of $\tilde{\Gamma}_I \ll \alpha^3 \ll 1$, we focus on the effective temperature at the time of $\tilde{t} \sim \tilde{t}_{\text{max}}$. The effective temperature T_* becomes the maximal value at the time when $\Gamma_{\text{split}}(k \simeq m_I) \simeq H$ is satisfied. Note here that there are another two model dependent factors: (i) total degrees of freedoms of plasma, g_* , and (ii) a factor of the diffusion coefficient, $\hat{q}_{\text{el}} = c_q \alpha^2 T^3$ [See Eq. (A.5)]. As a result, the maximal temperature is given by the second line of Eq. (3.71) times the uncertainty factor of $[(36/\pi^2 g_*) c c_q^{1/2}]^{2/5}$. This dependence is rather

small because the time-dependence of the effective temperature has a small power of $-1/4$. We may conclude that the uncertainty of the maximal temperature is at most a factor of ten.

4 Applications

In this section, we discuss implications of obtained results to the dynamics of scalar condensates in the early Universe. In particular, we focus on their effects on SSB and onset of oscillation, which are important to determine the fate of the Universe.

Suppose that ϕ has a tachyonic mass at zero-temperature and is a field responsible for the SSB of some symmetry. When it acquires a thermal mass larger than its zero-temperature tachyonic mass, it is stabilized at the origin of the potential. As the temperature decreases, the thermal mass becomes comparable to its zero-temperature mass. Then ϕ starts to oscillate around the true vacuum and obtains a nonzero VEV, so that the SSB occurs at that time. The SSB may result in formation of topological defects, such as cosmic strings and domain walls. These topological defects may predict some detectable signals. In particular, the energy density of domain walls decreases as a^{-2} , so that they eventually dominate the Universe and spoil the success of the standard cosmology [19]. However, when the SSB occurs before inflation, these topological defects are washed out by inflation. Thus, it is important to determine the condition that the SSB occurs after inflation.

Also, a scalar field can have a large expectation value during inflation due to the negative Hubble induced mass term [7]. Such a scalar condensate starts to oscillate around its effective potential minimum when the potential force becomes comparable to the Hubble friction. The subsequent evolution of the Universe can crucially depend on the time when the scalar field starts to oscillate. For instance, if its oscillation starts earlier due to the finite density effects, then the scalar field tends to dissipate its energy earlier into the background plasma. More importantly, the baryon asymmetry of the Universe is fixed at the time when the AD field starts to oscillate, as we will see later.

In the next subsection, we calculate the condition that the SSB occurs after inflation. We apply the result to a QCD axion model, which is well motivated in light of solution to the strong CP problem. Then we also apply the result to the electroweak symmetry breaking and comment on the DM production from the thermal plasma. Finally, we consider the dynamics of a flat direction in SUSY theories, especially in the context of the Affleck-Dine baryogenesis.

4.1 PQ symmetry

In this section, we consider the following potential for the field ϕ :

$$V(\phi) = m_T^2 |\phi|^2 + \frac{\lambda^2}{2} \left(|\phi|^2 - v^2 \right)^2, \quad (4.1)$$

where we include the thermal mass term ($m_T^2 \sim \alpha_\phi T_{*,\chi_\phi}^2$). The SSB occurs at the effective temperature of T_{SSB} that is determined by $m_T(T_{\text{SSB}}) \simeq \lambda v$.

When we require $m_T(T) < \lambda v$ throughout the history of the Universe, we obtain an

upper bound on the reheating temperature as

$$T_{\text{RH}} \lesssim 2 \times 10^{10} \text{ GeV} \left(\frac{\alpha}{0.1} \right)^{-1} \left(\frac{\lambda \nu}{10^{12} \text{ GeV}} \right) \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^{1/2}, \quad (4.2)$$

where we assume that $\tilde{\Gamma}_I > \alpha^3$, $\alpha \sim \alpha_\phi$, and that particles which couple to ϕ (PQ-quarks) are not produced directly from inflaton decay. This implies that the reheating temperature has to be much lower than the dynamical scale ν to avoid the symmetry restoration of that symmetry. ^{◆15}

Here, let us consider a QCD axion model with right-handed neutrinos [58, 59, 60, 61] and identify ϕ as the field responsible to the SSB of PQ symmetry. When the SSB occurs after inflation, cosmic strings form at the phase transition [62]. After the QCD phase transition, the non-perturbative effect associated with instantons breaks $U(1)_{\text{PQ}}$ symmetry down to Z_n , where n is an integer depending on models. This implies that domain walls form at the QCD phase transition. While these domain walls are short lived in the case of $n = 1$ [63, 64, 65, 66], they are stable and disastrous in the case of $n \geq 2$ [19, 20]. One of the simplest solution of this domain wall problem is that the PQ symmetry is never restored after inflation. ^{◆16} This scenario requires a reheating temperature lower than the one derived in Eq. (4.2). QCD axion models predict a pseudo-NG boson called axion, which is a good candidate of DM. The abundance of axion is related to the PQ breaking scale ν , so that the observed DM abundance determines its value [67, 68, 69]. The result is given as

$$\nu \simeq 8 \times 10^{11} \text{ GeV} \times n \left| \theta_0 \right|^2, \quad (4.3)$$

where θ_0 is the initial phase of axion field and n is the domain wall number. This implies that the reheating temperature should be lower than 10^{10} GeV [see Eq. (4.2)]. Hence, leptogenesis may be marginally realized to explain the baryon asymmetry of the Universe [70] (see also Ref. [71]).

4.2 Electroweak phase transition and DM production

In order not to spoil the success of the BBN, reheating has to be completed before the BBN epoch, which means $T_{\text{RH}} \gtrsim 1$ MeV. This requires that $\tilde{\Gamma}_I$ has to be larger than the following value:

$$\tilde{\Gamma}_I \sim \frac{T_{\text{RH}}^2 M_{\text{pl}}}{m_I^3} \sim 10^{-27} \left(\frac{T_{\text{RH}}}{1 \text{ MeV}} \right)^2 \left(\frac{m_I}{10^{13} \text{ GeV}} \right)^{-3}. \quad (4.4)$$

In this case, the effective temperature of the soft sector can be as large as

$$T_* \Big|_{\text{max}} \sim \alpha^{4/5} \tilde{\Gamma}_I^{2/5} m_I \quad (4.5)$$

$$\sim 200 \text{ GeV} \times \alpha^{4/5} \left(\frac{\tilde{\Gamma}_I}{10^{-27}} \right)^{2/5} \left(\frac{m_I}{10^{13} \text{ GeV}} \right). \quad (4.6)$$

^{◆15} This is merely a necessary condition to avoid the symmetry restoration. If the field value during inflation is much larger than ν due to the negative Hubble induced mass, the non-thermal phase transition can occur, which may result in formation of topological defects [54, 55, 56, 57]. To study its fate, one may also have to take into account of background plasma, which is involved and beyond the scope of this paper. See [37] for instance.

^{◆16} In this case, the axion DM may predict sizable isocurvature fluctuations [21, 22, 23], which is tightly constrained by the observations of CMB fluctuations. This severely restricts the energy scale of inflation to suppress isocurvature fluctuations.

Assuming $\alpha = 10^{-1}$, we obtain $T_*|_{\max} \sim 30$ GeV for the case of $m_I = 10^{13}$ GeV. Even if χ_ϕ -particles are generated directly from the inflaton decay, the effective temperature of the hard sector is at most

$$T_{*,\chi_\phi}|_{\text{indir}} \sim \tilde{\Gamma}_I^{1/2} m_I \quad (4.7)$$

$$\sim 0.3 \text{ GeV} \times \left(\frac{\tilde{\Gamma}_I}{10^{-27}} \right)^{1/2} \left(\frac{m_I}{10^{13} \text{ GeV}} \right), \quad (4.8)$$

just after inflation, where we assume $\tilde{t} \sim 1$.

Here, let us consider thermal production of DM in a low reheating temperature. Even if the reheating temperature is lower than the mass of DM, it can be produced from the thermal plasma before the reheating is completed [28, 29]. The present energy density of DM from the thermal production divided by the entropy density of the Universe is calculated as

$$\frac{\rho_{\text{DM}}^{\text{th}}}{s} \Big|_{\text{now}} \simeq \frac{\rho_{\text{DM}}^{\text{th}}}{s} \Big|_{\text{F}} \left(\frac{T_{\text{RH}}}{T_{\text{F}}} \right)^5, \quad (4.9)$$

where the subscript ‘‘F’’ represents the corresponding value at the time of DM freeze-out (see, *e.g.*, Ref. [33]). To derive this, it is assumed that the maximal temperature of the Universe after inflation is larger than the mass of DM. However, Eqs. (4.6) and (4.8) show that the maximal temperature of the Universe is at most the electroweak scale for $T_{\text{RH}} = \mathcal{O}(1)$ MeV. This means that in such a low reheating temperature the thermal production of DM is not efficient and Eq. (4.9) cannot be applicable to calculate the amount of DM. Even in this case, DM is produced from interactions between the hard and soft sector and this contribution is actually much more efficient to generate DM [33].

Next, let us identify the field ϕ as the Higgs boson [14, 15, 16, 17, 18]. Then χ_ϕ -particles are identified as the quarks, leptons, and $SU(2)_L \times U(1)_Y$ gauge bosons. In general, they are generated directly from the inflaton decay, so that the case of ‘‘direct χ_ϕ -production’’ should be applied to this case. First, the Higgs boson obtains a thermal mass from hard χ_ϕ -particles just after inflation ($\tilde{t} \sim 1$) as $\alpha_\phi^{1/2} T_*$ with Eq. (4.8), but this may be too small to restore the electroweak symmetry. Then, at the time around $\tilde{t} = \tilde{t}_{\max}$, the effective temperature of the soft sector reaches a maximal value and the Higgs boson obtains a thermal mass from χ -particles as $\alpha_\phi^{1/2} T_{*,\chi_\phi}$ with Eq. (4.6). This can be of order 100 GeV for the case of $m_I = 10^{13}$ GeV if the reheating temperature is slightly larger than 1 MeV, say $T_{\text{RH}} \simeq 5$ MeV, because $\alpha_\phi \approx y_t^2/4\pi \sim 0.1$ and $\alpha \approx \alpha_s \sim 0.1$, where y_t and α_s are the top Yukawa coupling and the strong coupling, respectively.^{◆17} Thus, the electroweak symmetry may be restored at the time around $\tilde{t} = \tilde{t}_{\max}$ even if the reheating temperature is as low as $\mathcal{O}(1)$ MeV and the inflaton mass is 10^{13} GeV. When the Higgs field stays at the symmetric phase, the sphaleron effect, which is a $B + L$ violating process in a finite temperature plasma, is turned on [72]. Our result implies that there may be an era in which the sphaleron effect proceeds efficiently even if the reheating temperature is $\mathcal{O}(1)$ MeV. This is an important result when one determines the baryon asymmetry of the Universe in such a low reheating temperature scenario.

^{◆17} Although the top quark is fermion, its effective temperature at a time around $\tilde{t} \sim \tilde{t}_{\max}$ coincides with the one for bosons, which we calculate in this paper.

4.3 Affleck-Dine field

In SUSY theories, there are many scalar fields whose potentials are absent in the limit of exact SUSY and within the renormalizable level. Focusing on a baryonic scalar field with such a flat potential, called the AD field, baryon asymmetry can be generated by the Affleck-Dine mechanism [6, 7]. In this subsection, we identify ϕ as the AD field.

The AD field ϕ has a large VEV due to a negative Hubble-induced mass during inflation and the inflaton oscillation dominated era. When the Hubble parameter becomes comparable to the curvature of the potential, it starts to oscillate around the origin of the potential. At the same time, it is kicked in the phase direction and generate baryon asymmetry. The resulting baryon-to-entropy ratio Y_b can be calculated from

$$Y_b \sim \frac{m_{3/2} T_{\text{RH}}}{H_{\text{osc}}^2} \left(\frac{|\phi_{\text{osc}}|}{M_{\text{pl}}} \right)^2, \quad (4.10)$$

where we neglect $O(1)$ numerical factors (see, e.g., Ref. [73]). Here, $m_{3/2}$ is gravitino mass, ϕ_{osc} is the VEV of ϕ at the time of beginning of its oscillation. Note that the amount of baryon asymmetry depends on the Hubble parameter at the beginning of oscillation H_{osc} . Thus, we should include thermal effects on the potential of the AD field to determine H_{osc} as

$$H_{\text{osc}}^2 \simeq \text{Max} \left[m_\phi^2, \alpha_s^2 \frac{T^4}{|\phi_{\text{osc}}|^2} \right], \quad (4.11)$$

where we assume $|g_\phi \phi| \gg T$ [26, 27]. In the literature, they use the relation of $T = (T_{\text{RH}}^2 M_{\text{pl}} H)^{1/4}$ to calculate H_{osc} [see Eq. (3.59)]. However, as shown in this paper, the finite time scale of thermalization implies that this relation holds only after the time of $\tilde{t} = \tilde{t}_{\text{max}}$, so that we should check that $H_{\text{osc}}^{-1} m_\phi \gtrsim \tilde{t}_{\text{max}}$ is satisfied.

Assuming $T = (T_{\text{RH}}^2 M_{\text{pl}} H)^{1/4}$, we can rewrite Eq. (4.11) as

$$H_{\text{osc}} \simeq \text{Max} \left[m_\phi, \alpha_s^2 \frac{T_{\text{RH}}^2 M_{\text{pl}}}{|\phi_{\text{osc}}|^2} \right]. \quad (4.12)$$

We require the following condition to use the relation of $T = (T_{\text{RH}}^2 M_{\text{pl}} H)^{1/4}$:

$$H_{\text{osc}} \lesssim \alpha^{16/5} \tilde{\Gamma}_I^{3/5} m_I. \quad (4.13)$$

Using Eq. (4.10) to eliminate $|\phi|_{\text{osc}}$ and assuming $H_{\text{osc}} \simeq \alpha_s^2 T_{\text{RH}}^2 M_{\text{pl}} / |\phi_{\text{osc}}|^2$, we can rewrite the condition as

$$T_{\text{RH}} \gtrsim 4 \text{ GeV} \times \left(\frac{\alpha}{0.1} \right)^{-38/3} \left(\frac{Y_b}{10^{-10}} \right)^{-5/3} \left(\frac{m_I}{10^{13} \text{ GeV}} \right)^4 \left(\frac{m_{3/2}}{1 \text{ TeV}} \right)^{5/3}. \quad (4.14)$$

This is easily satisfied, so that we can consistently use the relation of $T = (T_{\text{RH}}^2 M_{\text{pl}} H)^{1/4}$ to calculate the baryon asymmetry.

Finally, we comment on the case of “direct χ_ϕ -production”. When the VEV of the AD field is smaller than the inflaton mass, χ_ϕ -particles may be produced directly from the decay of inflaton. In this case, the AD field may obtain a large effective mass as Eq. (4.8) just after inflation. Since this thermal mass easily exceeds the mass of the AD field m_ϕ , calculations of the baryon asymmetry may be completely changed in this case. We leave this issue for a future work.

5 Conclusion

We have investigated the reheating process after inflation, and derived the evolutions of distribution function of light particles (χ -particles), and also effective temperatures that describe thermal effects on scalar fields. Our results are summarized in Sec. 3.5 and are illustrated as the blue lines in Figs. 10 and 11. Comparing with the result calculated by assuming the “*instantaneous thermalization*”, which is widely assumed in the literature, we find that the maximal temperature of the Universe is overestimated in the literature, in particular, for the case of $\alpha^3 \gtrsim \tilde{\Gamma}_I = \Gamma_I/(m_I^3/M_{\text{pl}}^2)$. This is mainly because emissions of high momentum particles are necessary for primary particles to be thermalized but such processes are suppressed by the LPM effect in reality.

We have applied the result to some important phenomena: the SSB of PQ symmetry, the electroweak symmetry breaking, and the dynamics of an AD field. We have found that the maximum temperature of the Universe can be much lower than the conventional estimation where the “*instantaneous thermalization*” is assumed. In particular, we have shown that if the reheating temperature is as low as $\mathcal{O}(1)$ MeV, then the maximum temperature of the Universe may be at most electroweak scale. This implies that the electroweak symmetry may be marginally restored after inflation even for the case of such a low reheating temperature. The PQ symmetry might not be restored after inflation in the axion cold dark matter scenario, even if the reheating temperature is as large as the one required by the realization of thermal leptogenesis. Our results are applied to the calculation of the baryon asymmetry in the Affleck-Dine baryogenesis. We have justified the conventional calculations performed in the literature when the VEV of the AD field is larger than the mass of inflaton. Also, our result implies that DM may not be able to be produced from the thermal plasma in a low reheating temperature scenario, contrary to the conventional studies under the “*instantaneous thermalization*” assumption.

We have studied in detail the evolution of distribution function of radiation, and how particles with masses of $|g_\phi\phi|$ are produced from radiation. Thus, our results are also applicable to heavy particle production in the detailed process of thermalization, as partly done in Ref. [33].

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A LPM-suppressed Splitting Rate

Here we summarize basic formulae of LPM-suppressed splitting rate in $SU(N)$ gauge theory with N_F -flavor fundamental fermions to understand the model dependence of splitting rate. We follow Ref. [51] in the following.

Let us first start with a precise form of the collision term for the splitting process. When

the primaries have larger energy than the typical screening mass scale of plasma, like in our case, we have to take into account nearly collinear emissions. The collision term in this case can be expressed as

$$\begin{aligned}
\mathcal{C}_{\text{split}}[f_a] = & \frac{(2\pi)^3}{2k^2\nu_a} \sum_{b,c} \int dp' dk' \delta(k - p' - k') \gamma_{bc}^a(k; p', k') \\
& \times (f_a(k) [1 \pm f_b(p')] [1 \pm f_c(k')] - [1 \pm f_a(k)] f_b(p') f_c(k')) \\
& + \frac{(2\pi)^3}{k^2\nu_a} \sum_{b,c} \int dp' dp \delta(k + p - p') \gamma_{ab}^c(p'; k, p) \\
& \times (f_a(k) f_b(p) [1 \pm f_c(p')] - [1 \pm f_a(k)] [1 \pm f_b(p)] f_c(p')),
\end{aligned} \tag{A.1}$$

where ν_a represents the number of degree of freedom for species a (normalized by one real d.o.f.). The splitting/joining process for $a \leftrightarrow bc$ is characterized by a splitting function γ_{bc}^a . As explained intuitively in Sec. 3, we have to include destructive interference effect, LPM effect, in order to correctly cope with the collinear splitting process in medium. The required splitting function which deals with the LPM effect at the leading logarithmic order is given by [74, 51]

$$\begin{aligned}
\gamma_{qg}^q(P; (1-x)P, xP) &= \gamma_{qg}^{\bar{q}}(P; (1-x)P, xP) \\
&= \frac{d_F C_F \alpha}{4\pi^3 \sqrt{2}} [\hat{q}_{\text{el}} P_{\text{min}}]^{\frac{1}{2}} \frac{1 + (1-x)^2}{x^2(1-x)} \times LL(x; F, A, F),
\end{aligned} \tag{A.2}$$

$$\gamma_{q\bar{q}}^g(P; xP, (1-x)P) = \frac{d_F C_F \alpha}{4\pi^3 \sqrt{2}} [\hat{q}_{\text{el}} P_{\text{min}}]^{\frac{1}{2}} \frac{x^2 + (1-x)^2}{x(1-x)} \times LL(x; A, F, F), \tag{A.3}$$

$$\gamma_{gg}^g(P; xP, (1-x)P) = \frac{d_A C_A \alpha}{4\pi^3 \sqrt{2}} [\hat{q}_{\text{el}} P_{\text{min}}]^{\frac{1}{2}} \frac{1 + x^4 + (1-x)^4}{x^2(1-x)^2} \times LL(x; A, A, A), \tag{A.4}$$

where $P_{\text{min}} \equiv \text{Min}[1, (1-x)]P$, d_a is the dimension of representation for a species a , and C_a is the quadratic Casimirs for a species a . q and g represent a fermion with the fundamental representation and a gauge boson of $SU(N)$ respectively. The diffusion coefficient is defined as

$$\hat{q}_{\text{el}} \equiv \alpha^2 \sum_a \bar{\nu}_a t_a \int_{\mathbf{p}} f_a(\mathbf{p}) [1 \pm f_a(\mathbf{p})], \tag{A.5}$$

with $\bar{\nu}_a \equiv \nu_a/d_a$ being the number of d.o.f. of a species a excluding the ‘‘color’’ factor and t_a being the normalization of representation for a species a . The Leading Logarithmic enhancement factor is given by

$$\begin{aligned}
LL(x; s_1, s_2, s_3) = & \left[\frac{2}{\pi} \text{Max}[x, 1-x] \right]^{\frac{1}{2}} \\
& \times \left[\frac{1}{2} (C_{s_2} + C_{s_3} - C_{s_1}) + \frac{1}{2} (C_{s_3} + C_{s_1} - C_{s_2}) x^2 + \frac{1}{2} (C_{s_1} + C_{s_2} - C_{s_3}) (1-x)^2 \right]^{\frac{1}{2}} \\
& \times \left[\ln \left(\frac{P}{T_{\text{eff}}} \right) \right]^{\frac{1}{2}},
\end{aligned} \tag{A.6}$$

where the effective temperature is defined as

$$T_{\text{eff}} \equiv \frac{\sum_a \bar{v}_a t_a \int_p f_a(\mathbf{p}) [1 \pm f_a(\mathbf{p})]}{2 \sum_a \bar{v}_a t_a \int_p \frac{f_a(\mathbf{p})}{p}}. \quad (\text{A.7})$$

For instance, for a thermal-like distribution $\sim T_s/p$, like Eq. (3.16), the effective temperature, T_{eff} , coincides with the soft temperature, T_s .

It is instructive to reproduce the important equation [Eq. (3.15)], which is responsible for production of the soft population from hard primaries, and also determines \tilde{t}_{max} and hence T_{max} . To be concrete, let us assume that inflaton decays directly into SM gauge bosons. The Boltzmann equation responsible for production of soft particles is given by

$$\frac{1}{(2\pi)^3} \dot{f}_{g,s}(t, k) = \int dp' dp \delta(k + p - p') \frac{1}{k^2 v_g} \gamma_{gg}^g(p'; k, p) f_{g,h}(p') + \dots \quad (\text{A.8})$$

$$\simeq \frac{\sqrt{2} C_A}{\bar{v}_g} LL(0; A, A, A) \times \alpha \left[\frac{\hat{q}_{\text{el}}}{k} \right]^{\frac{1}{2}} \times n_h k^{-3}. \quad (\text{A.9})$$

Thus, the splitting rate can be parametrized as

$$\Gamma_{\text{split}}(k) = c' \alpha \left[\frac{\hat{q}_{\text{el}}}{k} \right]^{\frac{1}{2}} \quad \text{with} \quad c' \equiv \frac{\sqrt{2} C_A}{\bar{v}_g} LL(0; A, A, A). \quad (\text{A.10})$$

The coefficient c' is given by a model dependent factor times the leading log enhancement. One can derive a similar formula when the inflaton decays into fermions and then it emits gauge bosons collinearly. Also, Eq. (3.5) can be derived from Eq. (A.1).

In the main part of this paper, we have omitted this factor to avoid model dependent discussions, but one can see that this factor results in at most $\mathcal{O}(10)$ uncertainty. As discussed in the end of Sec. 3, the maximum temperature has a mild dependence on this uncertainty factor, $\propto c'^{2/5}$. Together with the numerical factor derived from numerical calculations of Boltzmann equation, we conservatively conclude that the uncertainty of maximum temperature is at most a factor of ten.

B Effective Temperature for χ_ϕ -particles

In contrast to χ -particles, χ_ϕ -particles can have sizable masses of $|g_\phi \phi|$ due to the large expectation value of the ϕ -condensate. The distribution in momentum space can be different from that of χ -particles and hence separate discussions are required. In this Appendix, we show the results of T_{*,χ_ϕ} .

B.1 $\tilde{t}_{\text{ini}} < \tilde{t} < \tilde{t}_{\text{soft}}$

If the effective mass of χ_ϕ -particles are smaller than the screening mass, then the effective temperature for χ_ϕ -particles coincide with that for χ -particles:

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \sim T_*^2, \quad (\text{B.1})$$

for $|g_\phi \phi| < m_s$. Here the subscript ‘‘indir’’ indicates χ_ϕ -particles which are not produced directly from inflaton decay. For the case of $m_s < |g_\phi \phi|$, the effective temperature squared T_{*,χ_ϕ}^2 is evaluated as^{◆18}

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \equiv \text{Max} \left[T_{*,\chi_\phi}^2 \Big|_{\text{hard}}, T_{*,\chi_\phi}^2 \Big|_{\text{soft}}, T_{*,\chi_\phi}^2 \Big|_{\text{int}} \right], \quad (\text{B.2})$$

where

$$T_{*,\chi_\phi}^2 \Big|_{\text{hard}} \sim T_*^2 \left(\frac{m_s}{|g_\phi \phi|} \right) \text{Min} \left[1, \left(\frac{k_{\text{max}} m_s}{(g_\phi \phi)^2} \right) \right], \quad (\text{B.3})$$

$$T_{*,\chi_\phi}^2 \Big|_{\text{soft}} \sim \alpha^2 \tilde{\Gamma}_I m_I^2 \text{Min} \left[1, \left(\frac{k_{\text{max}}}{g_\phi \phi} \right)^4 \text{Max} \left[1, \left(\frac{|g_\phi \phi|}{(k_{\text{max}} k_{\text{form}})^{1/2}} \right) \right] \right], \quad (\text{B.4})$$

$$T_{*,\chi_\phi}^2 \Big|_{\text{int}} \sim \alpha^2 \tilde{\Gamma}_I^{3/2} \tilde{t}^{-1/2} m_I^2 \text{Min} \left[1, \left(\frac{k_{\text{max}} m_I}{|g_\phi \phi|^2} \right)^2 \text{Max} \left[1, \left(\frac{|g_\phi \phi|}{(k_{\text{form}} m_I)^{1/2}} \right) \right] \right]. \quad (\text{B.5})$$

By using these equations, one can show that, for $\alpha^{-1} k_{\text{max}} \lesssim |g_\phi \phi|$ with $\tilde{\Gamma}_I \lesssim \alpha^6$, the contribution of abundant χ_ϕ -particles is always subdominant compared with that of the running coupling constant. Even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^6$, its effect is roughly the same order with that of the running coupling constant for $|g_\phi \phi| \gtrsim \alpha^{-1} k_{\text{max}}$. Hence, one can omit the decay of massive χ_ϕ -particles in order to show that the effect on the effective potential of ϕ from χ_ϕ -particles is subdominant for the large field value regime.

However, if χ_ϕ -particles are produced directly from inflaton decay, it has a sizable contribution to the effective temperature, which tend to dominate for a large field value [see Eq. (3.40)]. Therefore, we should take into account the decay of χ_ϕ -particles to obtain more realistic predictions at least in that case:

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} = \alpha \tilde{\Gamma}_I^{1/2} T_*^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^{1/2} \text{Min} \left[1, \frac{1}{\Gamma_{\text{decay}} \tilde{t}} \right], \quad (\text{B.6})$$

where the subscript ‘‘dir’’ indicates contribution from χ_ϕ -particles which are directly produced from inflaton decay. Hence, one should compare it with the log potential from the running coupling constant. The effective mass squared for the large field value regime, $|g_\phi \phi| \gtrsim \alpha^{-1} k_{\text{max}}$, should be given by

$$\text{Max} \left[\alpha^2 \frac{T_*^4}{\phi^2}, \alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \right]. \quad (\text{B.7})$$

B.2 $\tilde{t}_{\text{soft}} < \tilde{t} < \tilde{t}_{\text{max}}$

If the effective mass of χ_ϕ -particles are smaller than the screening mass, then the effective temperature for χ_ϕ -particles coincide with that for χ -particles:

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \sim T_s^2, \quad (\text{B.8})$$

^{◆18} Here we neglect t -channel contributions in the soft sector, because they are always subdominant to calculate the thermal potential.

for $|g_\phi \phi| < m_s$. Here the subscript ‘‘indir’’ indicates χ_ϕ -particles which are not produced directly from inflaton decay. For the case of $m_s < |g_\phi \phi|$, we obtain the effective temperature squared T_{*,χ_ϕ}^2 as

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \equiv \text{Max} \left[T_{*,\chi_\phi}^2 \Big|_{\text{hard}}, T_{*,\chi_\phi}^2 \Big|_{\text{soft}}, T_{*,\chi_\phi}^2 \Big|_{\text{int}} \right], \quad (\text{B.9})$$

where

$$T_{*,\chi_\phi}^2 \Big|_{\text{hard}} \sim T_s^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^3 \left(\frac{m_s}{|g_\phi \phi|} \right) \text{Min} \left[1, \frac{T_s m_s}{(g_\phi \phi)^2} \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^3 \right], \quad (\text{B.10})$$

$$T_{*,\chi_\phi}^2 \Big|_{\text{soft}} \sim T_s^2 \text{Min} \left[1, \left(\frac{T_s}{g_\phi \phi} \right)^4 \text{Max} \left[1, \left(\frac{|g_\phi \phi|}{(T_s k_{\text{form}})^{1/2}} \right) \right] \right], \quad (\text{B.11})$$

$$T_{*,\chi_\phi}^2 \Big|_{\text{int}} \sim \begin{cases} \text{Max} \left[\alpha^{3/2} \tilde{\Gamma}_I^{3/4} T_s^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^{1/2}, \alpha T_s^2 \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^2 \left(\frac{T_s}{|g_\phi \phi|} \right)^3 \right] & \text{for } (k_{\text{th}} T_s)^{1/2} \lesssim |g_\phi \phi| \lesssim (T_s m_I)^{1/2}, \\ \alpha^{3/2} \tilde{\Gamma}_I^{3/4} \left(\frac{\tilde{t}_{\text{soft}}}{\tilde{t}} \right)^{5/2} T_s^2 \left(\frac{T_s m_I}{|g_\phi \phi|^2} \right)^2 \text{Max} \left[1, \left(\frac{|g_\phi \phi|}{(m_I k_{\text{form}})^{1/2}} \right) \right] & \text{for } (T_s m_I)^{1/2} \lesssim |g_\phi \phi|, \end{cases} \quad (\text{B.12})$$

where k_{th} represents the interval boundary the hard and soft sectors. Here we include the contribution coming from t -channel scatterings between the hard and soft sector. Again, one can show that the effective potential from χ_ϕ -particles are always subdominant for the case of $|g_\phi \phi| \gtrsim \alpha^{-1} T_*$ with $\tilde{\Gamma}_I \lesssim \alpha^{-6}$. And also even in the case of $\tilde{\Gamma}_I \gtrsim \alpha^{-6}$, it is roughly the same order of that from the running coupling constant. Thus, we can omit the decay of χ_ϕ -particles in order to show that it is always subdominant compared with that from the running coupling constant for the large field regime.

However, if the inflaton directly decay into χ_ϕ -particles, then the effective temperature from direct inflaton decay tends to dominate for the large field value, $|g_\phi \phi| \gtrsim \alpha^{-1} T_*$.^{◆19} Hence, one should compare it with that from the running coupling constant:

$$\text{Max} \left[\alpha^2 \frac{T_*^4}{\phi^2}, \alpha_\phi T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \right], \quad (\text{B.13})$$

where $T_{*,\chi_\phi}^2 \Big|_{\text{dir}}$ is given by Eq. (B.6) [See also Eq. (3.54)].

^{◆19} For the small field value regime of $|g_\phi \phi| \lesssim T_*$, one can show that the soft χ_ϕ -particles always dominate over that of hard ones produced via direct inflaton decay.

B.3 $\tilde{t}_{\max} < \tilde{t} < \tilde{t}_{\text{RH}}$

Calculating the distribution of χ_ϕ -particles, we obtain the effective temperature squared T_{*,χ_ϕ}^2 as

$$T_{*,\chi_\phi}^2 \Big|_{\text{indir}} \sim \begin{cases} T_*^2 & \text{for } |g_\phi \phi| \lesssim T_* \\ T_*^2 \left(\frac{T_*}{|g_\phi \phi|} \right)^4 & \text{for } T_* \lesssim |g_\phi \phi| \lesssim (T_* m_I)^{1/2} \\ T_*^2 \alpha^2 \tilde{\Gamma}_I \left(\frac{\tilde{t}_{\max}}{\tilde{t}} \right)^{5/4} \left(\frac{(T_* m_I)^{1/2}}{|g_\phi \phi|} \right)^4 & \text{for } (T_* m_I)^{1/2} \lesssim |g_\phi \phi|, \end{cases} \quad (\text{B.14})$$

where the subscript ‘‘indir’’ indicates χ_ϕ -particles which are not produced directly from inflaton decay. At that time, one can show that the contribution from abundant χ_ϕ -particles is always subdominant compared with that from the running coupling constant for the large field value regime $|g_\phi \phi| \gtrsim \alpha^{-1} T_*$. Thus, we can omit the decay of χ_ϕ -particles in order to show that it is always subdominant compared with that from the running coupling constant for the large field regime.

However, if the inflaton directly decay into χ_ϕ -particles, then its contribution tends to dominate for the large field value regime, $|g_\phi \phi| \gtrsim \alpha^{-1} T_*$. Hence, one should compare it with that from the running coupling constant:

$$\text{Max} \left[\alpha \frac{T_*^4}{\phi^2}, T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \right], \quad (\text{B.15})$$

where $T_{*,\chi_\phi}^2 \Big|_{\text{dir}}$ is given by [See also Eq. (3.61)]

$$T_{*,\chi_\phi}^2 \Big|_{\text{dir}} \sim \tilde{\Gamma}_I \tilde{t}^{-1} m_I^2 \text{Min} \left[\frac{1}{\Gamma_{\text{split}}(m_I) t}, \frac{1}{\Gamma_{\text{decay}} t} \right]. \quad (\text{B.16})$$

Here note that the hard particles with the momentum of m_I soon breaks up within the time scale of Γ_{split} , and hence $1/[\Gamma_{\text{split}}(m_I) t]$ should be multiplied.

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