

Gravitational Dust Collapse in $f(R)$ Gravity

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Abstract

This paper is devoted to investigate gravitational collapse of dust in metric $f(R)$ gravity. We take FRW metric for the interior region while the Schwarzschild spacetime is considered for the exterior region of a star. The junction conditions have been derived to match interior and exterior spacetimes. The assumption of constant scalar curvature is used to find a solution of field equations. Gravitational mass is found by using the junction conditions. It is concluded that the constant curvature term $f(R_0)$ plays the role of the cosmological constant involved in the field equations of general relativity.

Keywords: Gravitational collapse; $f(R)$ gravity.

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1 Introduction

The most interesting topic in the gravitational physics today is the expansion of our universe. The support to this argument comes from different sources such as Supernovae Ia (SNIa) experiments [1], cosmic microwave background fluctuations [2] and X-ray experiments [3]. All these observations indicate that our universe is expanding with an accelerated rate. The phenomenon of dark matter and dark energy is another topic of discussion [4]. Einstein gave the concept of dark energy in 1917 by introducing a small positive cosmological constant in the field equations. But later on, he rejected it. However, it is now believed that the cosmological constant may be a suitable candidate for dark energy. Higher dimensional theories [5] such as M-theory or string theory may also be helpful to explain this cosmic expansion. Another justification comes from modification of general theory (GR) involving some inverse curvature terms [6]. However, modified gravity with inverse curvature terms seems to be unstable and may not pass solar system tests [7]. This discrepancy can be addressed by including higher derivative terms. Moreover, the viability can be achieved by considering squared curvature terms [8]. It has been suggested that the current expansion may be justified if we add some suitable powers of curvature in the usual Einstein-Hilbert action [9]. Thus it seems interesting to study the universe in the context of alternative or modified theories of gravity.

Among various modification, $f(R)$ gravity is a possible candidate which gives a natural gravitational alternative to dark energy [10]. This theory may provide an easy unification of early time inflation and late time acceleration. The cosmic acceleration can be explained by introducing the term $1/R$ at small curvatures. It was Buchdahl [11] who introduced $f(R)$ gravity using non-linear Lagrangians. The $f(R)$ theory of gravity seems most suitable due to its cosmologically important $f(R)$ models. These models consist of higher order curvature terms as functions of Ricci scalar R . Some viable $f(R)$ gravity models [9] have been suggested which show the unification of late-time acceleration and early-time inflation. It is now expected that dark matter problem can be addressed using viable $f(R)$ models. In recent years, many authors have shown keen interest to investigate this theory in different context [12]-[17]. Some detailed reviews are available to better understand the theory [18]. Multamäki and Vilja [19] investigated static spherically symmetric vacuum solutions in $f(R)$ theory. They established that the field equations in $f(R)$ gravity gave the Schwarzschild de Sitter solution. Exact spherically

symmetric interior solutions in metric $f(R)$ gravity have been studied by Shojai and Shojai [20]. Hollenstein and Lobo [21] analyzed static spherically symmetric solutions in $f(R)$ gravity coupled to non-linear electrodynamics. $f(R)$ gravity at one-loop level in de Sitter universe has been investigated by Cognola et al. [22]. Cylindrical symmetry has also been widely used to investigate $f(R)$ gravity in different contexts [23]. Recently developed $f(T)$ gravity is another alternative theory which is the generalization of teleparallel gravity. This theory also seems interesting as it may explain the cosmic acceleration without involving the dark energy. A considerable amount of work has been done in this theory so far [24].

Gravitational collapse is an interesting and important issue in GR. It is involved in the structure formation of the universe causing the existence of galaxies, stars and planets. The singularity theorem suggests that the occurrence of spacetime singularity is a general feature of any cosmological model under some reasonable conditions. So the solutions with singularities can be produced by the gravitational collapse of non-singular and asymptotically flat initial data [25]. The classification of spacetime singularities is based on two facts whether they can be observed or not. If a spacetime singularity is locally observable then it is termed as naked. A black hole is a spacetime singularity which can not be observed. Penrose [26] proposed a cosmic censorship conjecture that the singularities appearing in the gravitational collapse are always covered by an event horizon. Formation of compact stellar objects like neutron stars and white dwarf is the result of gravitational collapse. Spherical symmetry has been extensively used to study gravitational collapse. Dust gravitational collapse was first explored by Oppenheimer and Snyder [27]. Markovic and Shapiro [28] extended their work by considering positive cosmological constant. Many researchers [29] have investigated gravitational collapse by considering interior and exterior regions.

In the recent years, many authors have shown keen interest to explore gravitational collapse in alternative theories of gravity [30]-[34]. It has been shown that farther one goes from GR, there is a greater chance of having a naked singularity [30]. Ghosh and Maharaj [35] obtained a condition for the occurrence of a naked singularity in the collapse of null dust in higher dimensional $f(R)$ gravity. Openheimer-Snyder collapse in Brans-Dicke theory has been discussed by Scheel [36]. In a recent paper [37], Rudra and Debnath discussed gravitational collapse in Vaidya spacetime for Galileon theory of gravity. Sharif and Abbas [38] studied the dynamics of shearfree dissipative gravitational collapse in $f(G)$ gravity. Spherically symmetric perfect fluid

gravitational collapse has been discussed in metric $f(R)$ gravity by Sharif and Kausar [39]. Cembranos et al. [40] analyzed a general $f(R)$ model with uniformly collapsing cloud of self-gravitating dust particles.

In this paper, we are focussed to discuss the gravitational collapse with dust case in $f(R)$ gravity. We take Friedmann-Robertson-Walker (FRW) spacetime in the interior region and Schwarzschild metric in the exterior region. The paper is organized as follows: Section 2 is used to give general formalism about junction conditions between interior and exterior regions. We introduce field equations in $f(R)$ gravity and solve them using FRW metric for dust case in section 3. In Section 4, we find the apparent horizons and discuss the role of constant curvature term. Finally, we summarize the results in the last section.

2 General Formalism

In this section, we give junction conditions at the surface of a collapsing dust sphere. For this purpose, a 4D spherically symmetric spacetime is divided by a time-like 3D hypersurface Σ into two regions namely interior and exterior regions. Interior and exterior regions are denoted by V^- and V^+ respectively. Interior region represented by FRW spacetime is given by

$$ds_-^2 = dt^2 - a^2(t)dr^2 - a^2(t)b^2(r)[d\theta^2 + \text{Sin}^2\theta d\phi^2], \quad (1)$$

where $a(t)$ is cosmic scale factor and

$$b(r) = \begin{cases} \sin r, & \text{when } k=1, \\ r, & \text{when } k=0, \\ \sinh r, & \text{when } k=-1. \end{cases}$$

For exterior region V^+ , we consider the Schwarzschild spacetime

$$ds_+^2 = \left(1 - \frac{2M}{R}\right)dT^2 - \frac{1}{1 - \frac{2M}{R}}dR^2 - R^2[d\theta^2 + \text{Sin}^2\theta d\phi^2], \quad (2)$$

where M is an arbitrary constant. Using Israel junction conditions, we consider that first and second fundamental forms for interior and exterior spacetimes are same. These conditions are given as:

1. The continuity of first fundamental form over Σ provides

$$(ds_+^2)_\Sigma = (ds_-^2)_\Sigma = (ds^2)_\Sigma, \quad (3)$$

2. The continuity of second fundamental form over Σ yields

$$[K_{ij}] = K_{ij}^+ - K_{ij}^-, \quad (i, j = 0, 2, 3), \quad (4)$$

where the extrinsic curvature tensor K_{ij} is defined as

$$K_{ij}^\pm = -n_\sigma^\pm \left(\frac{\partial^2 x_\pm^\sigma}{\partial \varepsilon^i \partial \varepsilon^j} + \Gamma_{\mu\nu}^\sigma \frac{\partial x_\pm^\mu}{\partial \varepsilon^i} \frac{\partial x_\pm^\nu}{\partial \varepsilon^j} \right), \quad (\sigma, \mu, \nu = 0, 1, 2, 3). \quad (5)$$

Here ε^i and x_\pm^σ correspond to the coordinates on Σ and V^\pm respectively. The christoffel symbols $\Gamma_{\mu\nu}^\sigma$ are calculated using the interior and exterior spacetimes and n^\pm are the components of outward unit normal to Σ in the coordinates x_\pm^σ . Using interior and exterior spacetimes, the equations of hypersurface Σ are written as

$$h_-(r, t) = r - r_\Sigma = 0, \quad (6)$$

$$h_+(R, T) = R - R_\Sigma(T) = 0, \quad (7)$$

where r_Σ is an arbitrary constant. Using these equations, interior and exterior metrics given in Eq.(1) and(2) take the form

$$(ds_-^2)_\Sigma = dt^2 - a^2(t)b^2(r_\Sigma)[d\theta^2 + \text{Sin}^2\theta d\phi^2], \quad (8)$$

$$(ds_+^2)_\Sigma = \left[1 - \frac{2M}{R_\Sigma} - \frac{1}{1 - \frac{2M}{R_\Sigma}} \left(\frac{dR_\Sigma}{dT} \right)^2 \right] dT^2 - R_\Sigma^2 \left[d\theta^2 + \text{Sin}^2\theta d\phi^2 \right]. \quad (9)$$

Here we assume $1 - \frac{2M}{R_\Sigma} - \frac{1}{(1 - \frac{2M}{R_\Sigma})} \left(\frac{dR_\Sigma}{dT} \right)^2 > 0$ so that T remains time-like coordinate. Using junction condition (3), we get

$$R_\Sigma = a(t)b(r_\Sigma), \quad (10)$$

$$\left[1 - \frac{2M}{R_\Sigma} - \frac{1}{1 - \frac{2M}{R_\Sigma}} \left(\frac{dR_\Sigma}{dT} \right)^2 \right]^{\frac{1}{2}} dT = dt. \quad (11)$$

Now using Eqs. (6) and (7), the outward unit normals to the interior and exterior spacetimes are given by

$$n_\mu^- = (0, a(t), 0, 0), \quad (12)$$

$$n_\mu^+ = (-\dot{R}_\Sigma, \dot{T}, 0, 0), \quad (13)$$

while the components of extrinsic curvature K_{ij}^\pm turn out to be

$$K_{00}^- = 0, \quad K_{22}^- = csc^2\theta K_{33}^- = (abb')_\Sigma, \quad (14)$$

$$K_{00}^+ = \left[\dot{R}\ddot{T} - \dot{T}\ddot{R} + \frac{3M\dot{R}^2\dot{T}}{R(R-2M)} - \frac{M(R-2M)\dot{T}^3}{R^3} \right]_\Sigma, \quad (15)$$

$$K_{22}^+ = csc^2\theta K_{33}^+ = (\dot{T}(R-2M))_\Sigma. \quad (16)$$

Here dot and prime denote differentiation with respect to t and r respectively. By applying continuity condition on extrinsic curvatures, we obtain

$$K_{00}^+ = 0, \quad K_{22}^+ = K_{22}^-. \quad (17)$$

Now using Eqs. (14-17) along with (10) and (11), the junction conditions take the form

$$(\dot{b}')_\Sigma = 0, \quad M = \left(\frac{ab + a\dot{a}^2b^3 - abb'^2}{2} \right)_\Sigma \quad (18)$$

It is mentioned here that equations (10), (11) and (18) forms necessary and sufficient conditions to match the interior and exterior regions smoothly.

3 $f(R)$ Gravity and Field Equations

The metric tensor has a key role in GR. One of the main features of GR is the dependence of Levi-Civita connection on the metric tensor. However, the connection does not remain the Levi-Civita connection if we allow the torsion in the theory. Consequently the dependence of connection on metric tensor vanishes. This is the main idea behind different approaches of $f(R)$ theories of gravity.

We get metric version of $f(R)$ gravity if the connection is the Levi-Civita connection. In this approach, the variation of action is done with respect to the metric tensor only. The action for $f(R)$ gravity is

$$S_{f(R)} = \int \sqrt{-g}(f(R) + L_m)d^4x, \quad (19)$$

where $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. It would be worthwhile to mention here that the standard

Einstein-Hilbert action can be achieved when $f(R) = R$. Varying this action with respect to the metric tensor yields the modified field equations

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = \kappa T_{\mu\nu}^m. \quad (20)$$

Here $F(R) \equiv df(R)/dR$, κ is the coupling constant, $T_{\mu\nu}^m$ is the standard energy-momentum tensor and

$$\square \equiv \nabla^\mu \nabla_\mu \quad (21)$$

with ∇_μ is the covariant derivative. We can write the field equations in an alternative form which is familiar with GR field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}^c + \tilde{T}_{\mu\nu}^m, \quad (22)$$

where $\tilde{T}_{\mu\nu}^m = T_{\mu\nu}^m/F(R)$ and the energy-momentum tensor for gravitational fluid is

$$T_{\mu\nu}^c = \frac{1}{F(R)} \left[\frac{1}{2}g_{\mu\nu} \left(f(R) - RF(R) \right) + F(R)^{;\alpha\beta} \left(g_{\alpha\mu}g_{\beta\nu} - g_{\mu\nu}g_{\alpha\beta} \right) \right]. \quad (23)$$

It can be seen from Eq.(23) that energy-momentum tensor for gravitational fluid $T_{\mu\nu}^c$ contributes matter part from geometric origin. This approach is interesting as it may provide all the matter components required to investigate the dark universe. Thus it is hoped that $f(R)$ theory of gravity may give fruitful results to understand the phenomenon of expansion of universe. When we contract Eq.(20), it follows that

$$F(R)R - 2f(R) + 3\square F(R) = \kappa T^m, \quad (24)$$

where T^m is the trace of energy-momentum tensor. Here we are interested in pressureless gravitational collapse. For dust, the energy-momentum tensor is given as

$$T_{\mu\nu}^m = \rho u_\mu u_\nu, \quad (25)$$

where ρ is the matter density and the four velocity vector u_μ satisfies the equation $u_\mu = \delta^0_\mu$. Using this equation along with field equations (20), we get three independent differential equations for the interior spacetime

$$-\frac{3\ddot{a}}{a} = \frac{1}{F} \left[\kappa\rho + \frac{f}{2} - 3\frac{\dot{a}\dot{F}}{a} \right], \quad (26)$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{b''}{a^2b} = \frac{1}{F} \left[-\frac{f}{2} + 2\frac{\dot{a}\dot{F}}{a} + \ddot{F} \right], \quad (27)$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - \frac{b''}{a^2b} - \left(\frac{b'}{ab}\right)^2 + \frac{1}{a^2b^2} = \frac{1}{F} \left[-\frac{f}{2} + 2\frac{\dot{a}\dot{F}}{a} + \ddot{F} \right]. \quad (28)$$

The solution of these highly non-linear differential equations does not seem to be possible straightforwardly. However, we can try to find a solution using the assumption of constant scalar curvature, i.e. $R = R_0$. Using this assumption, left side of Eq.(24) becomes constant which leads to a constant energy density, say $\rho = \rho_0$. Thus the field equations (26-28) take the form

$$-\frac{3\ddot{a}}{a} = \frac{1}{F(R_0)} \left[\kappa\rho_0 + \frac{f(R_0)}{2} \right], \quad (29)$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{b''}{a^2b} = -\frac{f(R_0)}{2F(R_0)}, \quad (30)$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - \frac{b''}{a^2b} - \left(\frac{b'}{ab}\right)^2 + \frac{1}{a^2b^2} = -\frac{f(R_0)}{2F(R_0)}. \quad (31)$$

Manipulating these equations, we obtain

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{1-b'^2}{a^2b^2} = -\frac{1}{2F(R_0)} [\kappa\rho_0 + f(R_0)]. \quad (32)$$

Integrating first equation from (18), it follows that

$$b' = X, \quad (33)$$

where X is an arbitrary integration function of r . Thus Eq.(32) takes the form

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{1-X^2}{a^2b^2} = -\frac{1}{2F(R_0)} [\kappa\rho_0 + f(R_0)]. \quad (34)$$

Integrating this equation with respect to t , we get

$$\dot{a}^2 = \frac{X^2 - 1}{b^2} + \frac{2m}{ab^3} - \frac{a^2}{6F(R_0)} [\kappa\rho_0 + f(R_0)], \quad (35)$$

where $m = m(r)$ is an arbitrary function and is related to the mass of the collapsing system

$$m(r) = \frac{\kappa\rho_0 a^3 b^3}{6F(R_0)}. \quad (36)$$

Using gravitational units, i.e. $\kappa = 8\pi$, the mass of the collapsing system takes the form

$$m(r) = \frac{4\pi\rho_0 a^3 b^3}{3F(R_0)}. \quad (37)$$

It is mentioned here that mass of the system must be positive because negative mass is not acceptable physically. Using Eq.(35) and second junction condition in Eq.(18), we get

$$M = m - \frac{a^3 b^3 [8\pi\rho_0 + f(R_0)]}{12F(R_0)}. \quad (38)$$

Now we calculate the total energy $\tilde{M}(r, t)$ at a time t for the interior hypersurface of radius r using the mass function [41]

$$\tilde{M}(r, t) = \frac{ab}{2} [1 + g^{\mu\nu}(ab)_{,\mu}(ab)_{,\nu}]. \quad (39)$$

Using Eq.(35), the mass function turns out to be

$$\tilde{M}(r, t) = m(r) - \frac{a^3 b^3 [8\pi\rho_0 + f(R_0)]}{12F(R_0)}, \quad (40)$$

where $m(r)$ denotes the energy due to constant matter density in Eq. (37). Now we find the solution with $X(r) = 1$ using Eq.(33). In this case the closed form solution turns out to be

$$ab = \left[\frac{-12mF(R_0)}{8\pi\rho_0 + f(R_0)} \right]^{\frac{1}{3}} \sinh^{\frac{2}{3}} \alpha(r, t), \quad (41)$$

where

$$\alpha(r, t) = \sqrt{-\frac{3[8\pi\rho_0 + f(R_0)]}{8F(R_0)}} [t_s(r) - t]. \quad (42)$$

Here we assume $8\pi\rho_0 + f(R_0) < 0$ to have a realistic solution and $t_s(r)$ is an arbitrary function of r . It is clear that $t = t_s$ is the time formation of singularity for a particular shell at some distance r . In the limiting case when $f(R_0) \rightarrow -8\pi\rho_0$, the above solution takes the form

$$\lim_{f(R_0) \rightarrow -8\pi\rho_0} ab = \left[\frac{9m}{2} (t_s - t)^2 \right]^{\frac{1}{3}}, \quad (43)$$

which correspond to the well known Tolman-Bondi solution [42].

4 Apparent Horizons

We obtain the apparent horizon when the boundary of two trapped spheres is formed. In this section, we find such boundary of two trapped spheres whose outward normals are null. For the interior spacetime (1), this is given as

$$g^{\mu\nu}(ab)_{,\mu}(ab)_{,\nu} = \dot{a}^2 b - b'^2 = 0. \quad (44)$$

Using Eq.(35) in this equation, we get

$$\frac{1}{F(R_0)}[8\pi\rho_0 + f(R_0)]a^3 b^3 + 6ab - 12m = 0. \quad (45)$$

The solutions of this equation for ab yield the apparent horizons. For $f(R_0) = -8\pi\rho_0$, it becomes the Schwarzschild horizon, i.e., $ab = 2m$. When $m = 0$, it yields a de-Sitter horizon, i.e.

$$ab = \sqrt{\frac{-6F(R_0)}{8\pi\rho_0 + f(R_0)}} \quad (46)$$

The case $3m < \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}}$ leads to two horizons,

$$(ab)_c = \sqrt{\frac{-8F(R_0)}{8\pi\rho_0 + f(R_0)}} \cos \frac{\psi}{3} \quad (47)$$

and

$$(ab)_{bh} = -\sqrt{\frac{-8F(R_0)}{8\pi\rho_0 + f(R_0)}} \left[\cos \frac{\psi}{3} - \sqrt{3} \sin \frac{\psi}{3} \right], \quad (48)$$

where the subscripts c and bh represent cosmological and black hole horizons respectively and

$$\cos \psi = -3m \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}}. \quad (49)$$

If we take $m = 0$, the equations (47) and (48) reduce to

$$(ab)_c = \sqrt{\frac{-6F(R_0)}{8\pi\rho_0 + f(R_0)}}, \quad (ab)_{bh} = 0. \quad (50)$$

It is mentioned here that the results can be generalized when $m \neq 0$ and $8\pi\rho_0 + f(R_0) \neq 0$ [43].

For the case when $3m = \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}}$, both horizons coincide, i.e.,

$$(ab)_c = (ab)_{bh} = \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}}. \quad (51)$$

Thus the range of the cosmological horizon and the black hole horizon becomes

$$0 \leq (ab)_{bh} \leq \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}} \leq (ab)_c \leq \sqrt{\frac{-6F(R_0)}{8\pi\rho_0 + f(R_0)}}. \quad (52)$$

There does not exist any apparent horizon in the case $3m > \sqrt{\frac{-2F(R_0)}{8\pi\rho_0 + f(R_0)}}$.

The formation time of the apparent horizon can be calculated using Eqs. (41) and (45) and is given by

$$t_n = t_s - \sqrt{\frac{-8F(R_0)}{3(8\pi\rho_0 + f(R_0))}} \sinh^{-1} \left[\frac{(ab)_n}{2m} - 1 \right]^{\frac{1}{2}}, \quad n = 1, 2. \quad (53)$$

In the limiting case when $f(R_0) \rightarrow -8\pi\rho_0$, the result corresponds to Tolman-Bondi solution

$$t_{ah} = t_s - \frac{4}{3}m. \quad (54)$$

From Eq.(53), it follows that

$$\frac{(ab)_n}{2m} = \cosh^2 \alpha_n, \quad (55)$$

where

$$\alpha_n(r, t) = \sqrt{\frac{-3(8\pi\rho_0 + f(R_0))}{8F(R_0)}} [t_s(r) - tn]. \quad (56)$$

It is clear from Eq.(53) that both the black hole horizon and the cosmological horizon form earlier than the singularity $t = t_s$. Now since Eq. (53) yields $t_1 \leq t_2$ and Eq. (52) implies that $(ab)_{bh} \leq (ab)_c$. This is an indication that cosmological horizon forms earlier than the black hole horizon.

5 Concluding Remarks

This paper is devoted to discuss gravitational collapse in $f(R)$ gravity. For this purpose, we consider the metric approach of this theory to study the field equations. It is observed that the field equations (26)-(28) are complicated and highly non-linear. Thus it seems difficult to solve them analytically without any assumption. The assumption of constant curvature has been used to find a solution.

We investigate the gravitational collapse of dust by considering FRW spacetime as the interior region while for the exterior region we take Schwarzschild metric. The junction conditions have been derived between interior and exterior spacetimes. We get two physical apparent horizons namely cosmological horizon and black hole horizon. It is found that formation time for black hole horizon is more as compared to cosmological horizon. Moreover, both horizons are formed earlier than singularity. This indicates that the singularity is covered, i.e., black hole, which shows that $f(R)$ gravity supports cosmic censorship conjecture. From dynamical equation (35), the rate of gravitational collapse is

$$\ddot{a}b = -\frac{m}{a^2b^2} - \frac{ab}{6F(R_0)}[8\pi\rho_0 + f(R_0)]. \quad (57)$$

The acceleration should be negative for the collapsing process which is possible when

$$ab < \left[-\frac{6mF(R_0)}{8\pi\rho_0 + f(R_0)} \right]^{\frac{1}{3}}. \quad (58)$$

Thus Eq. (57) indicates that $f(R_0)$ slows down the collapsing process when $f(R_0) + 8\pi\rho_0 < 0$. Further, the presence of $f(R_0)$ causes two physical horizons. One is the black hole horizon and the other is cosmological horizon. It also influences the time difference between the formation of the apparent horizon and singularity. It is concluded that $f(R_0)$ affects the process of collapse and hence it limits the size of the black hole. It would be worthwhile to mention here that the term $f(R_0)$ plays the same role as that of the cosmological constant in GR field equations and our results agree with [44].

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