

Relativistic quantum motion of spin-0 particles under the influence of non-inertial effects in the cosmic string space-time

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Abstract

We study solutions for the Klein–Gordon equation with vector and scalar potentials of the Coulomb types under the influence of non-inertial effects in the space-time of topological defects. We also investigate a quantum particle described by the Klein–Gordon oscillator in the background space-time generated by a string. An important result obtained is that the non-inertial effects restrict the physical region of the space-time where the particle can be placed. In addition, we show that these potentials can form bound states for the relativistic wave equation equation in this kind of background.

I. INTRODUCTION

The quantum field theory (QFT) in curved space-time can be considered as a first approximation to quantum gravity. Moreover, to make a consistent quantum field theory in a gravitational background, it is necessary to analyze the single particle states, in this way, efforts have been applied in order to find an adequate formulation of the relativistic equation of motion for particles in a curved space-time. In recent years, there has been a significant increase of interest in the study of gravitational effects on quantum mechanical systems (single particle states) [1–8]. In addition, the physics of a neutrino in a curved metric is considered in [9] by Wheeler and Brill who presented a detailed analysis of the interaction of neutrinos and gravitational fields.

Topological defects are other kind of system that may be studied with this purpose. These defects are intriguing systems, that are supposed to be created during a symmetry breaking phase transition in the early universe [10–12] and may be considered as topological defects in the space-time structure predicted by a large class of theories. These structures are candidates for the generation of observable astrophysical phenomena such as high energy cosmic rays, gamma ray burst and gravitational waves [13]. The recent discovery of gravitational waves by the LIGO collaboration [14] suggests that a promising way to detect cosmic strings is to search for the gravitational-wave radiation they would produce.

A key feature is that the geometry around them is locally flat, but this is not a global feature. In [15], the effects of magnetic fields in the metric have been considered. A relativistic wave equation for spin 1/2 particles in the Melvin space-time, a space-time where the metric is determined by a magnetic field, has been obtained and the effects of very intense magnetic fields in the energy levels, as intense as the ones expected to be produced in ultra-relativistic heavy-ion collisions, has been investigated.

Non-inertial effects on physical systems is another kind of aspect that have been studied in many works in the literature [16–24]. A special case of a non-inertial system is the rotating reference system. In [25, 26], a rotational reference system in the Minkowski space-time, is investigate. Notably, in those papers it was shown that the geometry of the space-time can play the role of a hard-wall potential. Another example of a non-inertial system is the Mashhoon effect, that is the coupling of the spin of the particles with the angular velocity of the rotating reference system and it arises from the influence of these non-inertial frames

when interference effects are considered [27].

In this contribution, we will study scalar bosons in a cosmic string space-time by considering the relativistic wave equation with a vector potential $v(r) = \kappa/r$ and a scalar potential $s(r) = \eta/r$, where r is the radial coordinate with η and κ constants. Afterwards, we will examine a similar problem, the Klein–Gordon oscillator inside a topological defect space-time. Moreover, a rotational reference system in the conical space-time will be considered in both cases, and we will show that non-inertial effects reduce the physical region of the space-time where the quantum particles can be placed, and furthermore the energy levels are shifted by the non-inertial effects on the particle. This feature is an indicator of a nontrivial phenomenon: the coupling between the angular quantum number and the angular velocity of the rotating reference system. Afterwards, we will show that these potentials can form bound states for the spin-0 equation in this space-time. This paper is part of a study where we are interested in making a systematic exploration of the properties of quantum systems inside spaces with different kinds of structures [15, 28].

The paper is organized as follows: In Section II, we will describe the the space-time of topological defect and the transformation from space-time coordinates to rotating coordinates. In Section III, the relativistic wave equation with vector and scalar potentials of the Coulomb types in the space-time of topological defects will be determined and in Section IV the Klein–Gordon oscillator will be solved. Finally, Section V presents our conclusions. In this work, we use natural units where $c = G = \hbar = 1$.

II. THE COSMIC STRING AND THE NON-INERTIAL REFERENCE FRAME

In this section we will describe the relationship between the metric of a topological defect and the effects of the rotation of a reference frame. The cosmic string space-time is a solution of Einstein’s field equations and it describes a space-time determined by an infinitely long straight string. The string space-time is assumed to be static and cylindrically symmetric, and then, the distance element representing this system can be written in the form [2, 29]

$$ds^2 = -dt'^2 + dr'^2 + \alpha^2 r'^2 d\phi'^2 + dz'^2, \quad (1)$$

where $\alpha = 1 - 4G\mu$ and μ is the mass density of the string. In this space-time, the coordinates range is represented in the following way: the azimuthal angle range is $\phi' \in [0, 2\pi)$ while r'

and z' are $r' \in [0, \infty)$ and $z' \in (-\infty, \infty)$ respectively. The parameter α is related to the curvature of space-time. It may assume values in which $\alpha \leq 1$ or $\alpha > 1$, and in this case, it corresponds to a space-time of topological defect with negative curvature. In this work, we are interested in studying the case $0 < \alpha < 1$. The transformation of the metric (1) for a rotational reference system may be made by considering a coordinate transformation [30, 31]

$$t' = t, \quad r' = r, \quad \phi' = \phi + \omega t, \quad z' = z, \quad (2)$$

where ω is angular velocity of the rotational reference system, which we assume to be positive. Inserting this transformation into eq. (1) we obtain the line element

$$ds^2 = - (1 - \alpha^2 r^2 \omega^2) dt^2 + 2\alpha^2 r^2 \omega dt d\phi + dr^2 + \alpha^2 r^2 d\phi^2 + dz^2, \quad (3)$$

that may be associated with the covariant metric tensor

$$g_{\mu\eta} = \begin{pmatrix} -(1 - \alpha^2 r^2 \omega^2) & 0 & 0 & \alpha^2 r^2 \omega \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha^2 r^2 \omega & 0 & 0 & \alpha^2 r^2 \end{pmatrix}. \quad (4)$$

It is possible to see that $g_{\mu\eta}$ is a non-diagonal metric tensor where effects of the topology of the space and the rotation of the reference system are taken into account. An interesting feature of the equation (3) is the condition

$$0 < r < 1/\alpha\omega. \quad (5)$$

This is related to the result that for $r > 1/\alpha\omega$ the velocity of particles is greater than the velocity of the light, for this reason, is convenient to restrict r to the range $(0, 1/\alpha\omega)$. Hence, the radial wave function must vanish at $r = 1/\alpha\omega$ and consequently the system presents two classes of solutions that depend on the value of the product $\alpha\omega$. Thus the first case is obtained by adopting the limit $\alpha\omega \ll 1$ ($1/\alpha\omega \rightarrow \infty$), that provides an analytical solution to the wave equation and as a second case, an arbitrary relation $\alpha\omega$ can be considered.

III. SPIN-0 EQUATION WITH VECTOR AND SCALAR POTENTIALS OF THE COULOMB TYPES IN THE SPACE-TIME OF A TOPOLOGICAL DEFECT

The Dirac equation is a wave equation that represents very well spin-1/2 particles in Minkowski space-time. The spin-0 particles are represented by the usual Klein–Gordon equation which can be generalized to the curved space-time case. In order to determine the generalization of the wave equation one may replace the ordinary derivatives by covariant derivative [28] in the spin-0 equation in Minkowski space-time, the result is

$$-\frac{1}{\sqrt{-g}}D_\mu(g^{\mu\nu}\sqrt{-g}D_\nu\psi) + m^2\psi = 0, \quad (6)$$

that is the Klein–Gordon equation in a curved space-time [32], where m is the particle mass, $D_\mu = \partial_\mu - ieA_\mu$, and e is the electric charge. A scalar potential $V(r)$ may be taken into account by making a modification on the mass term: $m \rightarrow m + V(r)$. Substituting this mass term into (6) we obtain the following differential equation

$$-\frac{1}{\sqrt{-g}}D_\mu(g^{\mu\nu}\sqrt{-g}D_\nu\psi) + (m + V)^2\psi = 0. \quad (7)$$

This differential equation takes into account a scalar potential V and a potential vector A_μ [33, 34]. In the following, we will obtain two classes of solutions of equation (7). First, we will consider a slow rotation regime, and then we consider an arbitrary relation $\alpha\omega$ in the space-time.

By considering the line element (3) and the potential vector A_0 , we obtain the following differential equation

$$\left[-\left(\frac{\partial}{\partial t} - ieA_0\right)^2 + \frac{1}{r}\left(\frac{\partial}{\partial r}\right)r\left(\frac{\partial}{\partial r}\right) \right. \quad (8)$$

$$\left. + \frac{\partial^2}{\partial z^2} + \left(\frac{1 - \alpha^2 r^2 \omega^2}{\alpha^2 r^2}\right) \frac{\partial^2}{\partial \phi^2} \right. \quad (9)$$

$$\left. + 2\omega\left(\frac{\partial}{\partial \phi}\right)\left(\frac{\partial}{\partial t} + ieA_0\right) - (m + V)^2 \right] \phi = 0 \quad (10)$$

that is the spin-0 equation in the space-time of topological defects. One can see that equation (8) is independent of t , z and ϕ , so it is reasonable to write the solution as

$$\psi(t, r, z, \phi) = e^{-i\varepsilon t} e^{i l \phi} e^{i p_z z} R(r), \quad (11)$$

where $l = 0, \pm 1, \pm 2, \pm 3, \dots$, and ε can be interpreted as the energy of the particle, p_z is the momentum. Substituting (11) into Eq. (8), and by considering $A_0 = \kappa/r$, we obtain the radial differential equation

$$\left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + \frac{e^2 \kappa^2 - l^2 / \alpha^2}{r^2} - (m + V)^2 \right. \\ \left. \frac{2\varepsilon e \kappa + 2\omega e \kappa l}{r} + (\varepsilon + \omega l)^2 - p_z^2 \right] R(r) = 0, \quad (12)$$

where the parameter α represents the deficit angle of the space-time and $\alpha = 1$ corresponds to the Minkowski space-time. In this paper, we are interested in studying the case $\alpha < 1$.

In this stage, we consider a scalar potential of the type $V(r) = \eta/r$, where η is a constant, so, substituting this potential into Eq. (12), we get

$$\left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{\beta^2}{r^2} - \frac{2\gamma}{r} - \delta^2 \right] R(r) = 0, \quad (13)$$

where

$$\delta^2 = m^2 + p_z^2 - (\varepsilon + \omega l)^2, \quad \beta = \sqrt{l^2 / \alpha^2 + \eta^2 - e^2 \kappa^2}, \quad \gamma = -\varepsilon e \kappa - \omega e \kappa l + m \eta. \quad (14)$$

We assume the relation $e^2 \kappa^2 < \eta^2$ so that β is a real number. Now, we will consider a transformation of the radial coordinate

$$\rho = 2\delta r, \quad (15)$$

and as a result, Eq.(13) will take the form

$$\left[\frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} - \frac{\beta^2}{\rho^2} - \frac{\gamma}{\delta\rho} - \frac{1}{4} \right] R(\rho) = 0. \quad (16)$$

Normalizable eigenfunctions may be obtained if we propose the solution

$$R(\rho) = \rho^\beta e^{-\frac{\rho}{2}} F(\rho), \quad (17)$$

then substituting $R(\rho)$ (17) into eq. (16), we obtain the differential equation that can be associated with the radial equation

$$\rho \frac{d^2 F}{d\rho^2} + (2\beta + 1 - \rho) \frac{dF}{d\rho} + \left(-\beta - \frac{\gamma}{\delta} - \frac{1}{2} \right) F = 0. \quad (18)$$

This is the confluent hypergeometric equation, which is a second order homogeneous differential equation where two independent solutions can be obtained. The solution of Eq. (18), regular at $\rho = 0$, is given by the confluent hypergeometric function that is denoted by

$$F(\rho) = {}_1F_1 \left(\beta + \frac{\gamma}{\delta} + \frac{1}{2}, 2\beta + 1; \rho \right). \quad (19)$$

If we consider the limit $\alpha\omega \ll 1$, that is a slow rotation regime, the boundary condition implies that the solution

$${}_1F_1\left(\beta + \frac{\gamma}{\delta} + \frac{1}{2}, 2\beta + 1; \frac{2\delta}{\alpha\omega} \rightarrow \infty\right) \quad (20)$$

must be a finite as $\rho_0 = 1/\alpha\omega \rightarrow \infty$. So, due to asymptotic behavior of the hypergeometric function, it is necessary that the ${}_1F_1$ function be a polynomial function of degree N and the parameter $\beta + \frac{\gamma}{\delta} + \frac{1}{2}$ should be a negative integer. These conditions implies that

$$\beta + \frac{\gamma}{\delta} + \frac{1}{2} = -N, \quad N = 0, 1, 2, \dots, \quad (21)$$

and combining this equation and equation (14) we finally obtain the spectrum of energy

$$\varepsilon = \frac{e\kappa m\eta}{\zeta + e^2\kappa^2} \pm \sqrt{\zeta \frac{[(p_z^2 + m^2)(\zeta + e^2\kappa^2) - m^2\eta^2]}{(\zeta + e^2\kappa^2)^2}} - \omega |l|, \quad (22)$$

where $\zeta = \left(N + \frac{1}{2} + \sqrt{l^2/\alpha^2 + \eta^2 - e^2\kappa^2}\right)^2$.

Observing Eq. (22), we can see that the energy spectrum depends on α , the deficit angle of the conical space-time. The first and second terms are associated to the Coulomb-like potentials embedded in a cosmic string background and the third term is associated to the non-inertial effect of rotational frames, that is a Page-Werner et al. term [35–38]. For $l = 0$ or $\omega = 0$ the discrete set of energies are symmetrical about $\varepsilon = 0$, in this way, the presence of

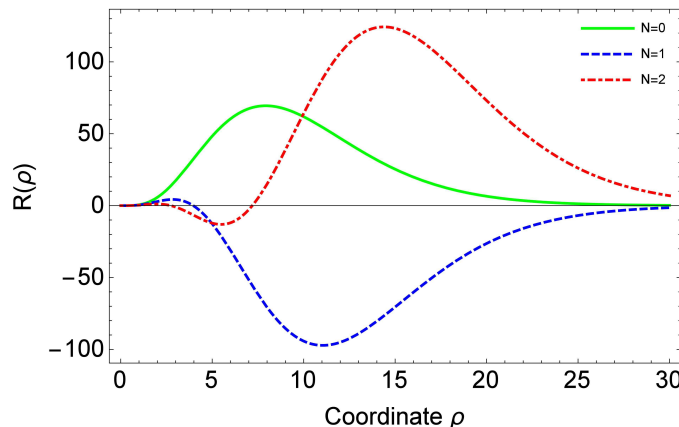


FIG. 1. The plots of the radial coordinate R as the function of the variable ρ displayed for three different values of N with the parameters $\alpha = 0.9$, $\omega = 0.6$, $\eta = 1$ and $l = 1$.

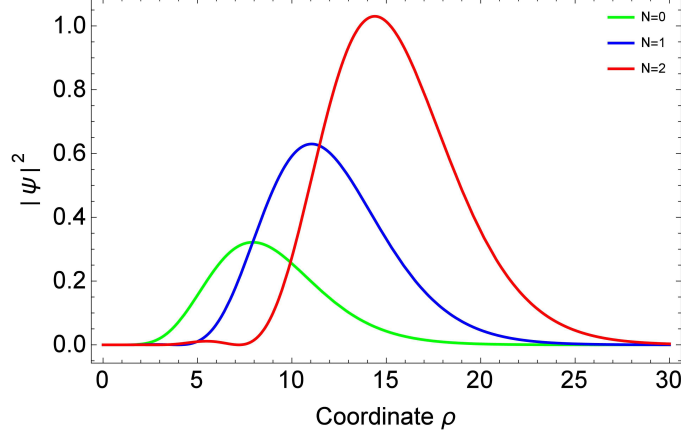


FIG. 2. The plots of $|\psi|^2$ as the function of the variable ρ displayed for three different values of N with the parameters $\alpha = 0.9$, $\omega = 0.6$, $\eta = 1$ and $l = 1$.

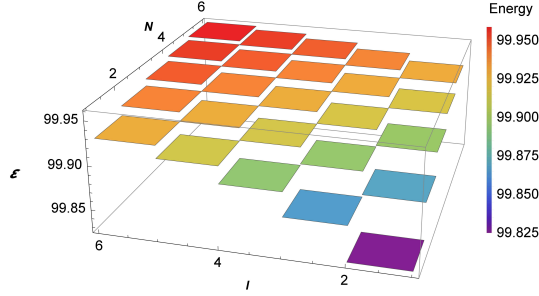


FIG. 3. The plots of particle energy spectrum ϵ as the function of variables N and l .

non-inertial effects of rotational frames in space-time breaks the symmetry of energy levels about $\epsilon = 0$ because ϵ_+ , in general, is greater than ϵ_- .

From equation (22), it is possible to see that the energy depends on the constant α , thus the presence of the topological defect modifies the energy of the particle.

Fig. 1 and 2 show that the radial solution $R(\rho)$ decreases with the coordinate ρ and becomes negligible far away from the topological defect as $\rho \rightarrow \infty$. For clarity, the plots of the energy spectrum ϵ as function of the variables N and l are shown in Figures 5 and 6.

In the next section we will discuss an arbitrary relation $\omega\alpha$ for the Klein–Gordon oscillator where the shape of the potential is adequate for this purpose.

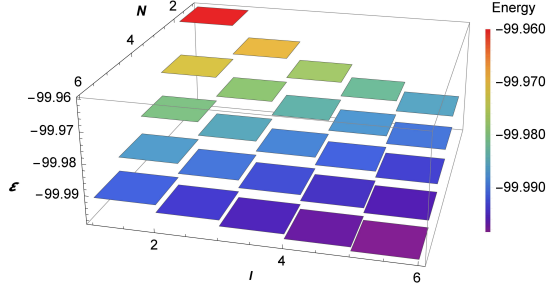


FIG. 4. The plots of negative energy spectrum ε as the function of variables N and l .

IV. KLEIN–GORDON OSCILLATOR IN THE THE SPACE-TIME OF TOPOLOGICAL DEFECTS

Another system of interest that may be considered is the Klein–Gordon oscillator [39] in the background of the cosmic string space-time. In recent years, several studies have addressed the Klein–Gordon oscillator in quantum systems [40–48]. It has a formulation similar to the vector potential in the previous section, so to study its solutions we will use the following change in momentum operator:

$$p_\mu \rightarrow (p_\mu + im\Omega X_\mu), \quad (23)$$

where m is the particle mass at rest, Ω is the frequency of the oscillator and $X_\mu = (0, r, 0, 0)$, with r being the distance from the particle to the string. In this way, the wave equation becomes

$$\left[-\frac{1}{\sqrt{-g}} (\partial_\mu + m\Omega X_\mu) g^{\mu\nu} \sqrt{-g} (\partial_\nu - m\Omega X_\nu) + (m + V)^2 \right] \psi = 0. \quad (24)$$

Taking $V = 0$ in above equation and by considering the line element (3), we obtain the following equation

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{1}{r} \left(\frac{\partial}{\partial r} + m\Omega \right) r \left(\frac{\partial}{\partial r} - m\Omega \right) + \frac{\partial^2}{\partial z^2} + \left(\frac{1 - \alpha^2 r^2 \omega^2}{\alpha^2 r^2} \right) \frac{\partial^2}{\partial \phi^2} + 2\omega \frac{\partial^2}{\partial t \partial \phi} - m^2 \right] \psi = 0. \quad (25)$$

Similar to the case of the Coulomb potential in last section, the eq. (25) is independent of t , z and ϕ , so it is appropriate to choose the ansatz

$$\psi(t, r, z, \phi) = e^{-i\varepsilon t} e^{il\phi} e^{ip_z z} R^l(r), \quad (26)$$

with $l = 0, \pm 1, \pm 2, \pm 3$, and ε being the energy of the particle. Substituting (26) into Eq. (25), we obtain the radial differential equation

$$\left[+\frac{1}{r} \left(\frac{\partial}{\partial r} + m\Omega \right) r \left(\frac{\partial}{\partial r} - m\Omega \right) + \frac{l^2}{\alpha^2 r^2} + (\varepsilon + \omega l)^2 - p_z^2 - m^2 \right] R'(r) = 0 \quad (27)$$

At this stage, we can consider the substitution $R'(r) = R(r)/\sqrt{r}$ in the equation (27) the result is

$$\left[\frac{d^2}{dr^2} - m^2 \Omega^2 r^2 - \frac{(l^2/\alpha^2 - 1/4)}{r^2} + K^2 \right] R(r) = 0, \quad (28)$$

with $K = \sqrt{(\varepsilon + \omega l)^2 - p_z^2 - m^2 - 2m\Omega}$. That is the radial equation that describes the Klein–Gordon oscillator in the space-time of a topological defect. In order to obtain the solution to the above differential equation it is necessary to analyze its asymptotic behavior for $r \rightarrow 0$ and $r \rightarrow r_0$ where $r_0 \equiv 1/\omega\alpha$. In this way, a regular solution at the origin is obtained if the solution of equation (28) has the form

$$R(r) = r^{|\frac{l}{\alpha}| + \frac{1}{2}} e^{-m\Omega r^2/2} F(r). \quad (29)$$

Substituting the above expression in Eq. (28) and by introducing the following new variable $\rho = m\Omega r^2$, we can rewrite the radial Eq. (28) in the form

$$\rho \frac{d^2 F(\rho)}{d\rho^2} + \left(\left| \frac{l}{\alpha} \right| + 1 - \rho \right) \frac{dF(\rho)}{d\rho} - \left(\frac{l}{2\alpha} + \frac{1}{2} - \frac{K^2}{4m\Omega} \right) F(\rho) = 0. \quad (30)$$

The solution of Eq. (30) is given by the confluent hypergeometric function that is denoted by

$$F(\rho) = {}_1F_1(A, B; \rho), \quad (31)$$

where the parameters A , B and ρ are given by

$$A = \frac{1}{2} \left| \frac{l}{\alpha} \right| + \frac{1}{2} - \frac{K^2}{4m\Omega}, \quad (32)$$

$$B = \left| \frac{l}{\alpha} \right| + 1, \quad (33)$$

$$\rho = m\Omega r^2. \quad (34)$$

A. Limit $\alpha\omega \ll 1$ ($1/\alpha\omega \rightarrow \infty$)

Following the discussions of the Sect. 3, we proceed now to find the eigenfunction for this problem.

Considering again the limit $\alpha\omega \ll 1$, we have a change in the boundary condition on the radial coordinate, i.e, when $\alpha\omega \ll 1$ the radial coordinate tends to infinity at $r = 1/\omega\alpha$. Consequently the hypergeometric function must be a polynomial function of degree N , and the parameter $A = \frac{1}{2} \left| \frac{l}{\alpha} \right| + \frac{1}{2} - \frac{K^2}{4m\Omega}$, must be a negative integer. This condition implies that

$$\frac{1}{2} \left| \frac{l}{\alpha} \right| + \frac{1}{2} - \frac{K^2}{4m\Omega} = -N, \quad (35)$$

and by the use of the definition of

$$K = \sqrt{(\varepsilon + \omega l)^2 - p_z^2 - m^2 - 2m\Omega}, \quad (36)$$

we finally obtain the set of energies

$$\varepsilon = \pm \sqrt{2m\Omega \left(2n' + \left| \frac{l}{\alpha} \right| \right) + m^2 + p_z^2 - \omega |l|}, \quad n' \equiv N + 1 = 1, 2, 3, \dots \quad (37)$$

We can see that the energy spectrum associated with the Klein–Gordon Oscillator in the conical space-time depends on α , that is the deficit angle of the conical space-time. It increases the energy of the system if $\alpha < 1$. It is easy to see that for $l = 0$ or $\omega = 0$

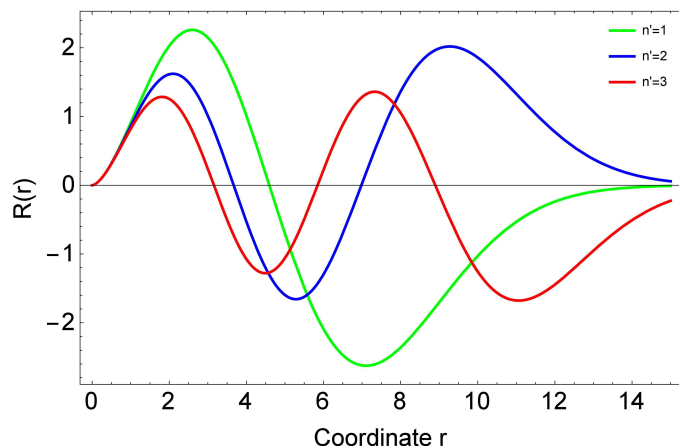


FIG. 5. The plots of radial coordinate R as the function of variable r displayed for three different n' with the parameters $\alpha = 0.9$, $\omega = 0.6$, $\Omega = 0.1$, $m = 1$ and $l = 1$.

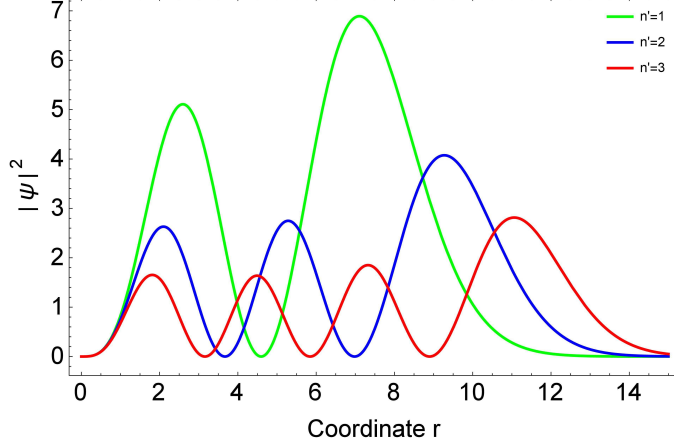


FIG. 6. The plots of $|\psi|^2$ as functions of the variable ρ displayed for three different values of n' with the parameters $\alpha = 0.9$, $\omega = 0.6$, $\Omega = 0.1$, $m = 1$ and $l = 1$.

the energy is symmetrical about $\varepsilon = 0$. In this way, the rotating reference system breaks the symmetry of the energy about $\varepsilon = 0$. The first term in Eq. (37) is associated to the Klein–Gordon Oscillator embedded in a conical space and the second one is associated with the non-inertial effect, which in turn is a coupling between the angular quantum number and the angular velocity of the rotational reference system. As it may be seen in Fig. 5, the radial eigenfunction becomes negligible far away from the string as $\rho \rightarrow \infty$. Fig. 6 presents $|\psi|^2$ as function of the variable ρ for three different values of n' . The energy spectrum as a function of the variables n' and l are shown in the plots of Figures 7 and 8. We note that the result obtained in eq. 37 is similar to the one reported in [2] for scalar bosons described by the Duffin–Kemmer–Petiau (DKP) formalism.

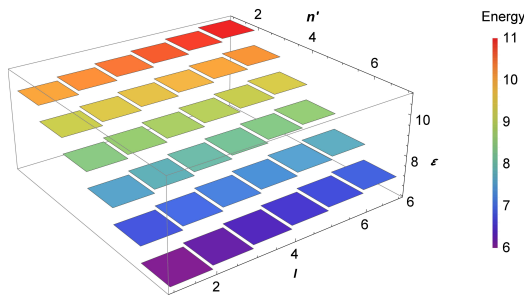


FIG. 7. The plots the of particle energy spectrum ε as function of the variables n' and l .

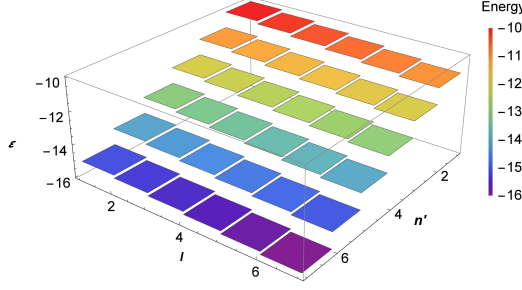


FIG. 8. The plots of the negative energy spectrum ε as function of the variables n' and l .

B. Arbitrary $\omega\alpha$

Now let us study an arbitrary relation between the parameters α and ω . In this case we discuss the behaviour of the Klein–Gordon oscillator without assuming the Limit $\alpha\omega \ll 1$. The physical condition implies that the wave function vanishes at $r_0 = 1/\alpha\omega$, i.e.,

$${}_1F_1(A, B; \rho_0 = m\Omega r_0^2) = 0. \quad (38)$$

If one assumes that $m\Omega \ll 1$, the parameter A of the hypergeometric function can be considered large and the parameters B and r_0 remain fixed. So, we can use these results to expand the hypergeometric function in the form [49, 50]

$$\begin{aligned} {}_1F_1(A, B; \rho_0) &\approx \frac{\Gamma(B)}{\sqrt{\pi}} e^{\frac{\rho_0}{2}} \left(\frac{B\rho_0}{2} - A\rho_0 \right)^{\frac{1-B}{2}} \times \\ &\times \cos \left(\sqrt{2B\rho_0 - 4A\rho_0} - \frac{B\pi}{2} + \frac{\pi}{4} \right), \end{aligned} \quad (39)$$

here $\Gamma(B)$ is the gamma function. By considering the condition (38) and the eq. (39), we finally obtain the set of energies for an arbitrary relation between the parameters α and ω

$$\varepsilon \approx \sqrt{p_z^2 + m^2 + 2m\Omega + \frac{1}{r_0^2} \left(\frac{l\pi}{2\alpha} + \frac{3\pi}{4} + n\pi \right)^2} - \omega l \quad (40)$$

where $n = 0, 1, 2, \dots$ is the radial quantum number of solution. Then equation (40) corresponds to the set of energies for an arbitrary relation between α and ω where the non-inertial effects play a role of an hard-wall confining potential [50]. The first term of eq. (40) is associated to the Klein–Gordon Oscillator embedded in a cosmic string background and the

second term is associated to the non-inertial effect of rotational frames, which in turn is a Sagnac-type effect.

V. CONCLUSIONS

In this work, we have determined the spin-0 equation in the presence of a vector and a scalar potential and examined the wave equation in the presence of a Klein–Gordon oscillator in a cosmic string space-time. Despite the complexity of the studied systems, we obtained compact expressions for the energy spectrum and for the particles wave functions. It has been shown that the potentials studied allow the formation of bound states and the energy spectrum associated with the relativistic wave equation in a cosmic string space depends on the deficit angle α , this fact shows that the topological defect modifies the energy of physical systems.

An important result that we have shown is that the non-inertial effect restrict the region of the space-time where the particle can be observed and beyond that, it shifts the energy levels. This feature reveals the existence of a coupling between the angular quantum number and the angular velocity of the rotational reference system.

We have shown that the wave equation presents two different classes of solutions that depend on the value of the α and ω . In the first case it is assumed the limit $\alpha\omega \ll 1$, that is a slow rotation regime, and as a second case, it is considered an arbitrary relation $\alpha\omega$. For both classes of solutions, we have found the energy spectrum and the eigenfunctions and we have shown that the discrete set of energies in general is composed of two contributions. The first term is associated to the external potential embedded in a cosmic string background and the second one is associated to non-inertial effects. With these results it is possible to have an idea about the general aspects of the quantum dynamics of scalar bosons inside a cosmic string background.

So, in this paper, we have shown some results about quantum systems where general relativistic effects are taken into account, that in addition with the previous results [15, 28] present many interesting effects. This is a fundamental subject in physics, and the connections between these theories are not well understood.

VI. ACKNOWLEDGMENTS

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