BRST approach to Lagrangian construction for bosonic continuous spin field

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Abstract

We formulate the conditions defining the irreducible continuous spin representation of the four-dimensional Poincaré group based on spin-tensor fields with dotted and undotted indices. Such a formulation simplifies analysis of the Bargmann-Wigner equations and reduces the number of equations from four to three. Using this formulation we develop the BRST approach and derive the covariant Lagrangian for the continuous spin fields.

1 Introduction

Description of the irreducible representations of the Poincaré and AdS groups play important role in the formulation of the higher spin field models (see e.g. the reviews [1], [2]). One of such representations is representation with continuous spin.

Continuous (or infinite) spin representation of the Poincaré group [3–5] being massless has unusual properties such as an infinite number of degrees of freedom and appearance of the dimensional parameter μ in the conditions defining the irreducible representation (see for the review [6]). Lagrangian for the bosonic field in $d = 4$ was first proposed in ref. [7] and its structure was analyzed in ref. [8]. Later Lagrangian for bosonic continuous field was generalized for $d > 4$ and written in terms of a tower of double traceless tensor fields

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in the works [9] and in terms of two towers of traceless fields in the works [10] (see also the approaches to Lagrangian constriction in the works [11–16] for bosonic and fermionic fields and relation to the string theory in [17, 18]). Interaction of continuous spin fields with finite spin massive fields was considered in the papers [19, 20]. Model of relativistic particle corresponding to continuous spin field has been constructed in the recent work [21].

In the present paper we develop the BRST approach to derive the Lagrangian for the continuous spin fields in four-dimensional Minkowski space. This approach is a direct generalization of the general BRST construction which was used in our works for deriving the Lagrangians for the free fields of different types in flat and AdS spaces (see e.g. [22–26] and the references therein, see also the review [27])[1]. The crucial element of our approach is the implementation of the two-component spinor description for the continuous spin fields. We suppose that the BRST construction in terms of spin-tensor fields, considered in the given paper, essentially simplifies an clarifies the derivation of the Lagrangian and its analysis.

The paper is organized as follows. In the next section we consider a new, in comparison with ref. [3], possibility of realization for the spin momentum operators in terms of two-component spinors and obtain new equations for the field realizing the continuous spin irreducible representation of the Poincaré group. After this we solve one of these equations and in section 3 we construct Lagrangian on the base of the BRST method. Then we show that after removing one auxiliary field and rescaling the other fields and the gauge parameter the Lagrangian obtained exactly coincides with Lagrangian derived by Metsaev [10].

2 Spin-tensor representation

According to Bargmann and Wigner [3], the continuous spin field are characterized by the following eigenvalues of the Casimir operators

$$P^2\Psi = 0 \qquad\qquad W^2\Psi = \mu^2\Psi. \tag{1}$$

In order to obtain these equations in explicit form we need the explicit expressions for the operators entering into the Casimir operators. Such expressions for the operators depend on how Ψ transforms under action of the Poincaré group.

In the works on continuous spin field it usually was done in the following way. Let us introduce an auxiliary 4-dimensional vector w^μ and define

$$\varphi_s(x,w) \;\;=\;\; \varphi_{\mu_1\ldots\mu_s}(x)\, w^{\mu_1}\ldots w^{\mu_s}, \tag{2}$$

where $\varphi_{\mu_1\ldots\mu_s}(x)$ is a totally symmetric tensor field. Then one realizes the spin momentum operator for $\varphi_s(x,w)$ as follows

$$M_{\mu\nu} = w_\mu i\frac{\partial}{\partial w^\nu} - w_\nu i\frac{\partial}{\partial w^\mu}. \tag{3}$$

[1]The various applications of the BRST-BFV construction in the continuous spin field theory have been studied in refs. [9, 10, 14, 15, 19, 28, 29].

Using this relation for the spin momentum operator one can write the Wigner equations [3]

$$\begin{aligned} &p^2\Psi(p,w) = 0, \\ &(p_\nu w^\nu + \mu)\Psi(p,w) = 0, \\ &p^\nu \frac{\partial}{\partial w^\nu}\Psi(p,w) = 0, \\ &\left(\frac{\partial}{\partial w^\nu}\frac{\partial}{\partial w_\nu} + 1\right)\Psi(p,w) = 0. \end{aligned} \quad (4)$$

If the four equations (4) are satisfied, then the field Ψ will satisfy (1) and thus will describe the irreducible representation of the Poincare group with continuous spin.

We want to turn attention that there is another possibility to realize the spin momentum operator and corresponding representation. Instead of usual tensor fields (2) we consider the spin-tensor field $\varphi_{a_1\ldots a_n\dot{b}_1\ldots\dot{b}_k}(x)$ with n undotted and k dotted indices and define

$$\varphi_{n,k}(x,\xi,\bar{\xi}) \;=\; \varphi_{a_1\ldots a_n\dot{b}_1\ldots\dot{b}_k}(x)\,\xi^{a_1}\ldots\xi^{a_n}\,\bar{\xi}^{\dot{b}_1}\ldots\bar{\xi}^{\dot{b}_k}, \quad (5)$$

where we have introduced two auxiliary bosonic 2-dimensional spinors ξ^a and $\xi^{\dot{b}}$ of the Lorentz group[2]. In this representation the spin momentum operator looks like this[3]

$$M_{\mu\nu} \;=\; \sigma^{ab}_{\mu\nu}M_{ab} - \bar{\sigma}^{\dot{a}\dot{b}}_{\mu\nu}M_{\dot{a}\dot{b}}, \quad (6)$$

where

$$M_{ab} \;=\; \frac{1}{2}\xi^c\varepsilon_{ca}\frac{\partial}{\partial\xi^b} + \frac{1}{2}\xi^c\varepsilon_{cb}\frac{\partial}{\partial\xi^a} = -\frac{i}{2}\Big(\xi_a\pi_b + \xi_b\pi_a\Big), \quad (7)$$

$$\bar{M}_{\dot{a}\dot{b}} \;=\; \frac{1}{2}\bar{\xi}^{\dot{c}}\varepsilon_{\dot{c}\dot{a}}\frac{\partial}{\partial\bar{\xi}^{\dot{b}}} + \frac{1}{2}\bar{\xi}^{\dot{c}}\varepsilon_{\dot{c}\dot{b}}\frac{\partial}{\partial\bar{\xi}^{\dot{a}}} = -\frac{i}{2}\Big(\bar{\xi}_{\dot{a}}\bar{\pi}_{\dot{b}} + \bar{\xi}_{\dot{b}}\bar{\pi}_{\dot{a}}\Big) \quad (8)$$

$$\pi_a = -i\frac{\partial}{\partial\xi^a} \qquad \bar{\pi}_{\dot{a}} = -i\frac{\partial}{\partial\bar{\xi}^{\dot{a}}}\;. \quad (9)$$

One can prove that the operator (6) satisfies the commutation relation for the Lorentz group generators.

To construct the second Casimir operator for the spin-tensor representation one uses the explicit expression for the spin operator (6) acting on the spin-tensor fields. In this case the second Casimir operator becomes

$$W^2 \;=\; -M_{ab}\,\bar{M}_{\dot{a}\dot{b}}\,P^{a\dot{a}}P^{b\dot{b}} + \frac{1}{2}\Big(M_{ab}M^{ab} + \bar{M}_{\dot{a}\dot{b}}\bar{M}^{\dot{a}\dot{b}}\Big)P^2 \quad (10)$$

After some transformations the second Casimir operator takes the form

$$\begin{aligned} W^2 \;=\; &\frac{1}{2}(\xi\sigma^\mu\bar{\xi})(\bar{\pi}\bar{\sigma}^\nu\pi)P_\mu P_\nu + \frac{1}{2}(\bar{\xi}\bar{\sigma}^\mu\pi)(\bar{\pi}\bar{\sigma}^\nu\xi)P_\mu P_\nu \\ &+ \frac{1}{2}\Big(M_{ab}M^{ab} + \bar{M}_{\dot{a}\dot{b}}\bar{M}^{\dot{a}\dot{b}} + i\bar{\xi}^{\dot{a}}\bar{\pi}_{\dot{a}}\Big)P^2 \end{aligned} \quad (11)$$

[2]The two-component spinors have been used for description of the representations with continuous spin in ref. [4]. However they were applied for the other aims.

[3]We use the notation as in book [30]

Using the identity

$$(\bar\xi\bar\sigma^\mu\pi)(\bar\pi\bar\sigma^\nu\xi)P_\mu P_\nu = (\xi\sigma^\mu\bar\xi)(\bar\pi\bar\sigma^\nu\pi)P_\mu P_\nu + \bar\xi^{\dot a}\bar\pi_{\dot a}(i+\pi_a\xi^a)P^2 \tag{12}$$

we can write the operator W^2 in two equivalent forms

$$\begin{aligned} W^2 &= (\xi\sigma^\mu\bar\xi)(\bar\pi\bar\sigma^\nu\pi)P_\mu P_\nu \\ &+ \frac{1}{2}\Big(M_{ab}M^{ab} + \bar M_{\dot a\dot b}\bar M^{\dot a\dot b} + \bar\xi^{\dot a}\bar\pi_{\dot a}\xi^a\pi_a\Big)P^2 \end{aligned} \tag{13}$$

or

$$\begin{aligned} W^2 &= (\bar\xi\bar\sigma^\mu\pi)(\bar\pi\bar\sigma^\nu\xi)P_\mu P_\nu \\ &+ \frac{1}{2}\Big(M_{ab}M^{ab} + \bar M_{\dot a\dot b}\bar M^{\dot a\dot b} - \bar\xi^{\dot a}\bar\pi_{\dot a}\pi_a\xi^a\Big)P^2. \end{aligned} \tag{14}$$

We will consider the irreducible representation with continuous spin on the fields $\Psi(p,\xi,\bar\xi)$, depending on the momentum p_μ and spin-tensor variables ξ^a and $\xi^{\dot a}$. Let the field $\Psi(p,\xi,\bar\xi)$ satisfies the constraints

$$p^2\Psi(p,\xi,\bar\xi) = 0, \tag{15}$$

$$((\bar\pi\bar\sigma^\nu\pi)p_\nu + i\mu)\Psi(p,\xi,\bar\xi) = 0, \tag{16}$$

$$((\xi\sigma^\mu\bar\xi)p_\mu - i\mu)\Psi(p,\xi,\bar\xi) = 0. \tag{17}$$

Then on can show that conditions (1) are satisfied and hence the field $\Psi(p,\xi,\bar\xi)$ describes the irreducible representation with continuous spin. Thus we have obtained the equations on the field Ψ in the spin-tensor representation. Equations (15)–(17) are similar to the "modified Wigner's equations" in the paper [16]. In the case under consideration analog of the fourth equation in (4) are resolved automatically due to the properties of the two component spinors.

We will construct the Lagrangian for the continuous spin field. To do that one should somehow decompose $\Psi(p,\xi,\bar\xi)$ in a series of ξ and $\bar\xi$ and get the spin-tensor fields. However, one can see that, because of equation (17), $\Psi(p,\xi,\bar\xi)$ such a direct decomposition is impossible. Therefore we first solve (17) in the form

$$\Psi(p,\xi,\bar\xi) = \delta((\xi\sigma^\mu\bar\xi)p_\mu - i\mu)\;\varphi(p,\xi,\bar\xi). \tag{18}$$

One can prove that if the field $\varphi(p,\xi,\bar\xi)$ obeys equations

$$\partial^2\varphi(x,\xi,\bar\xi) = 0, \tag{19}$$

$$\left(\bar\sigma^{\mu\dot a a}\frac{\partial}{\partial\xi^a}\frac{\partial}{\partial\bar\xi^{\dot a}}\frac{\partial}{\partial x^\mu} + \mu\right)\varphi(x,\xi,\bar\xi) = 0, \tag{20}$$

then the field $\Psi(p,\xi,\bar\xi)$ will satisfy the rest equations (15) and (16). Here we have made Fourier transform from momentum p^μ representation into the coordinates x^μ representation. Equations (19) and (20) have a solution in the form of an expansion in ξ and $\bar\xi$

$$\varphi(x,\xi,\bar\xi) = \sum_{n,k=0}^{\infty}\frac{1}{\sqrt{n!k!}}\,\varphi_{a_1\ldots a_n\dot b_1\ldots\dot b_k}(x)\;\xi^{a_1}\ldots\xi^{a_n}\;\bar\xi^{\dot b_1}\ldots\bar\xi^{\dot b_k}. \tag{21}$$

Since we are going to construct Lagrangian for real bosonic fields we will consider $n=k$ case in (21).

3 Lagrangian construction

Following the general BRST approach in higher spin field theory we begin with realization of the equations (19) and (20) in auxiliary Fock space.

Let us introduce creation and annihilation operators

$$\langle 0|\bar{c}_{\dot{b}} = \langle 0|c^a = 0, \qquad \bar{a}^{\dot{b}}|0\rangle = a_a|0\rangle = 0, \qquad \langle 0|0\rangle = 1$$

with the following nonzero commutation relations

$$[\bar{a}^{\dot{\alpha}}, \bar{c}_{\dot{\beta}}] = \delta^{\dot{\alpha}}_{\dot{\beta}}, \qquad [a_\alpha, c^\beta] = \delta^\beta_\alpha.$$

The states in the auxiliary Fock space are defined as follows

$$|\varphi\rangle = \sum_{k,l=0}^{\infty} |\varphi_{kl}\rangle \qquad |\varphi_{kl}\rangle = \frac{1}{\sqrt{k!l!}}\, \varphi_{a(k)}{}^{\dot{b}(l)}(x)\, c^{a(k)}\, \bar{c}_{\dot{b}(l)}|0\rangle. \tag{22}$$

We determine the Hermitian conjugation in the Fock space by the rule

$$(a_a)^+ = \bar{c}_{\dot{a}} \qquad (\bar{c}_{\dot{a}})^+ = a_a \qquad (\bar{a}_{\dot{a}})^+ = c_a \qquad (c_a)^+ = \bar{a}_{\dot{a}}$$

Then the state which is Hermitin conjugate to state (22) is written as follows

$$\langle\bar{\varphi}| \;=\; \sum_{k,l=0}^{\infty} \frac{1}{\sqrt{k!l!}}\, \langle 0|\, \bar{a}^{\dot{a}(k)}\, a_{b(l)} \bar{\varphi}^{b(l)}{}_{\dot{a}(k)}. \tag{23}$$

Let us introduce the following operators

$$l_0 = \partial^2 \qquad l_1 = a^a \partial_{a\dot{b}} \bar{a}^{\dot{b}} \qquad l_1^+ = -c^b \partial_{b\dot{a}} \bar{c}^{\dot{a}}. \tag{24}$$

Here $\partial_{a\dot{b}} = \sigma^\mu_{a\dot{b}}\partial_\mu$. One can show that these operators satisfy the commutation relation

$$[l_1^+, l_1] = (N + \bar{N} + 2) l_0, \tag{25}$$

where

$$N = c^a a_a \qquad \bar{N} = \bar{c}_{\dot{a}} \bar{a}^{\dot{a}} \tag{26}$$

All other commutators among these operators (24) vanish.

One can show that in order for a state $|\varphi\rangle$ describe the continuous spin representation it is necessary that the following constraints on the vector $|\varphi\rangle$ will satisfied

$$l_0|\varphi\rangle = 0 \qquad (l_1 - \mu)|\varphi\rangle = 0 \tag{27}$$

where $k = l$ in (22) is assumed.

Now we turn to construction of the BRST charge and the Lagrangian. Taking into account that the Lagrangian is real, we should get the Hermitian BRST charge. However, the system of constraints (27) is not invariant under the Hermitian conjugation. This situation is similar with BRST Lagrangian construction for free higher spin fields. Construction of the BRST charge for such a case was studied in works [22], [24] and we will follow these works. First of all we introduce the operator $l_1^+ - \mu$ and then add it to the set of constraints (27). Thus set of operators l_0, $l_1 - \mu$, $l_1^+ - \mu$ will be invariant under Hermitian conjugation. Moreover this set of operators will form an algebra with the only nonzero commutator (25)

$$[l_1^+ - \mu, l_1 - \mu] = (N + \bar{N} + 2)l_0. \tag{28}$$

Now we can apply the procedure described in the works [22], [24] and construct Hermitian BRST charge on the base of operators l_0, $l_1 - \mu$, $l_1^+ - \mu$. As a result we arrive at the Hermitian BRST charge in the form

$$Q = \eta_0 l_0 + \eta_1^+(l_1 - \mu) + \eta_1(l_1^+ - \mu) + \eta_1^+\eta_1(N + \bar{N} + 2)\mathcal{P}_0. \tag{29}$$

Here we have extended the Fock space by introducing η_0, η_1, η_1^+ which are the fermionic ghost "coordinates" and $\mathcal{P}_0$, $\mathcal{P}_1^+$, $\mathcal{P}_1$ which are their canonically conjugated ghost "momenta" respectively. These operators obey the anticommutation relations

$$\{\eta_1, \mathcal{P}_1^+\} = \{\mathcal{P}_1, \eta_1^+\} = \{\eta_0, \mathcal{P}_0\} = 1 \tag{30}$$

and act on the vacuum state as follows

$$\eta_1|0\rangle = \mathcal{P}_1|0\rangle = \mathcal{P}_0|0\rangle = 0. \tag{31}$$

They possess the standard ghost numbers, $gh(\eta^i) = -gh(\mathcal{P}_i) = 1$, providing the property $gh(\tilde{Q}) = 1$.

The operator (29) acts in the extended Fock space of the vectors

$$|\Phi\rangle = |\varphi\rangle + \eta_0\mathcal{P}_1^+|\varphi_1\rangle + \eta_1^+\mathcal{P}_1^+|\varphi_2\rangle \tag{32}$$

and realizes the gauge transformations

$$|\Phi'\rangle = |\Phi\rangle + Q|\Lambda\rangle, \tag{33}$$

for the equation of motion

$$Q|\Phi\rangle = 0, \tag{34}$$

where $|\Lambda\rangle$ is the gauge parameter

$$|\Lambda\rangle = \mathcal{P}_1^+|\lambda\rangle. \tag{35}$$

The fields $|\varphi_1\rangle$, $|\varphi_2\rangle$ and and the gauge parameter $|\lambda\rangle$ in relations (32), (35) have similar decomposition like $|\varphi\rangle$ (22) with $k = l$. In case of $\mu = 0$ the BRST charge (29) becomes BRST charge for the massless higher spin fields [31].

The equations of motion $Q|\Phi\rangle = 0$ and gauge transformations $\delta|\Phi\rangle = Q|\Lambda\rangle$ in terms of states $|\varphi_i\rangle$ and gauge parameter $|\lambda\rangle$ look like

$$l_0|\varphi\rangle - l_1^+|\varphi_1\rangle + \mu|\varphi_1\rangle = 0 \tag{36}$$
$$l_1|\varphi\rangle - l_1^+|\varphi_2\rangle + (N+\bar{N}+2)|\varphi_1\rangle - \mu|\varphi\rangle + \mu|\varphi_2\rangle = 0 \tag{37}$$
$$l_0|\varphi_2\rangle - l_1|\varphi_1\rangle + \mu|\varphi_1\rangle = 0 \tag{38}$$
$$\delta|\varphi\rangle = l_1^+|\lambda\rangle - \mu|\lambda\rangle \qquad \delta|\varphi_1\rangle = l_0|\lambda\rangle \qquad \delta|\varphi_2\rangle = l_1|\lambda\rangle - \mu|\lambda\rangle \tag{39}$$

The Lagrangian for the continuous spin field is constructed in the framework of the BRST approach as follows (see e.g. [24])

$$\begin{aligned}
\mathcal{L} \;&=\; \int d\eta_0\, \langle\Phi|Q|\Phi\rangle \;= \\
&=\; \langle\bar{\varphi}|\Big\{l_0|\varphi\rangle - l_1^+|\varphi_1\rangle\Big\} - \langle\bar{\varphi}_1|\Big\{l_1|\varphi\rangle - l_1^+|\varphi_2\rangle + (N+\bar{N}+2)|\varphi_1\rangle\Big\} \\
&\quad - \langle\bar{\varphi}_2|\Big\{l_0|\varphi_2\rangle - l_1|\varphi_1\rangle\Big\} \\
&\quad + \mu\Big\{\langle\bar{\varphi}|\varphi_1\rangle + \langle\bar{\varphi}_1|\varphi\rangle - \langle\bar{\varphi}_1|\varphi_2\rangle - \langle\bar{\varphi}_2|\varphi_1\rangle\Big\}
\end{aligned} \tag{40}$$

Lagrangian (40) consists of the sum of Lagrangians for massless bosonic fields plus μ-dependent terms responsible for the continuous spin.

Now we rewrite the Lagrangian (40) in terms of the component fields. Using the equation of motion (37) we remove the field $|\varphi_1\rangle$ from the Lagrangian (40). Then calculating the "average values" over Fock space vectors, converting the spin-tensor fields into traceless tensor fields and converting the spin-tensor gauge parameters into traceless tensor field parameters we arrives at the Lagrangian

$$\begin{aligned}
\mathcal{L} \;=\; &\sum_{s=0}^{\infty} 2^s \varphi^{\mu(s)} \Big[\partial^2\varphi_{\mu(s)} - s\,\partial_\mu\partial^\nu\varphi_{\nu\mu(s-1)} - \frac{s-1}{2}\partial_\mu\partial_\mu\varphi_{2\mu(s-2)} \\
&\qquad + \frac{\mu^2}{2(s+1)}(\varphi_{\mu(s)} - \varphi_{2\mu(s)}) + \mu\partial^\nu\varphi_{\nu\mu(s)} \\
&\qquad - \frac{\mu}{2}\partial_\mu\varphi_{\mu(s-1)} + \frac{\mu}{2}\,\frac{2s+1}{s+1}\partial_\mu\varphi_{2\mu(s-1)}\Big] \\
&- \sum_{s=0}^{\infty} 2^s \varphi_2^{\mu(s)} \Big[\frac{2s+3}{s+2}\partial^2\varphi_{2\,\mu(s)} + \frac{s^2}{s+2}\partial_\mu\partial^\nu\varphi_{2\nu\mu(s-1)} + 2(s+1)\partial^\nu\partial^\tau\varphi_{\nu\tau\mu(s)} \\
&\qquad + \frac{\mu^2}{2(s+1)}(\varphi_{\mu(s)} - \varphi_{2\mu(s)}) + \mu\frac{2s+3}{s+2}\partial^\nu\varphi_{\nu\mu(s)} \\
&\qquad - \mu\frac{s+1}{s+2}\partial^\nu\varphi_{2\nu\mu(s)} + \frac{\mu}{2}\,\frac{s}{s+1}\partial_\mu\varphi_{2\mu(s-1)}\Big]
\end{aligned} \tag{41}$$

and gauge transformations

$$\delta\varphi_{\mu(s)} = s\,\partial_{\mu_s}\lambda_{\mu(s-1)} - \frac{s-1}{2}\,\eta_{\mu(2)}\,\partial^\nu\lambda_{\nu\mu(s-2)} - \mu\,\lambda_{\mu(s)}, \tag{42}$$

$$\delta\varphi_{2\,\mu(s)} = -2(s+1)\,\partial^\nu\lambda_{\nu\mu(s)} - \mu\,\lambda_{\mu(s)}. \tag{43}$$

We note that the set of the fields in Lagrangian (41) is the same like in the Metsaev's Lagrangian (2.14) in [10] in the case $d = 4$, $m = 0$. These Lagrangians will be exactly the same if we make the redefinition of the fields $\varphi^{\mu(n)} \to A_n \phi_I^{a(n)}$, $\varphi_2^{\mu(n)} \to -A_n \phi_{II}^{a(n)}$ and gauge parameters $\lambda^{\mu(n)} \to A_{n+1} \xi^{a(n)}$ where $A_n = (2^{n+1} n!)^{-1/2}$ and also $\mu \to \kappa$. As a result we conclude that the BRST Lagrangian construction works perfectly for the continuous spin fields as well as for all other higher spin fields.

4 Summary

We have developed the Lagrangian BRST construction for the continuous spin field theory.

- We have reformulated the Wigner equations, defining the irreducible representation with continuous spin, in terms of two auxiliary bosonic 2-component spinor variables. The spin operator (6) for such an representation has been introduced. The representation is realized on fields satisfying the constraints (15), (16), (17). The number of the constraints turns out to be less than in case of the usually used representation in terms of vectorial auxiliary variable.

- The constraints are reformulated in terms of operators acting on the vectors of the auxiliary Fock space. Extra operator has been introduced to provide the real Lagrangian. The algebra of all operastors has been calculated.

- Hermitian BRST charge is constructed (29) for the continuous spin field theory, the Lagrangian and gauge transformations in terms of the Fock space vectors (40), and in terms of traceless tensor fields (41), (42), (43) have been derived. The Lagrangian coincides with Metsaev Lagrangian [10] after some redefinitions of the fields and gauge parameters.

Thus, the BRST Lagrangian construction, developed in refs. [22–26] is generalized for the continuous spin field.

The results of the paper can be applied for deriving the Lagrangians for fermionic continuous spin field and for supersymmetric continuous spin field theory. It would be also interesting to generalize these results for the continuous spin fields in the AdS space.

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