PACKED VOTERS AND CRACKED VOTERS

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Abstract. The actions of packing and cracking are central to the construction of gerrymandered district plans. The US Supreme Court opinion in *Gill* v. *Whitford* makes clear that vote dilution arguments require showing that individual voters have been packed or cracked. In this article we provide precise definitions of what it means for a voter to be packed or cracked. These definitions, which depend crucially on the existence of at least one comparator plan, are illustrated using a simple hypothetical example. We also explore who might be considered packed or cracked for congressional plans in Maryland and North Carolina, and for the current state assembly plan in Wisconsin.

1. Introduction

A gerrymander is a district plan in which the lines have been illicitly drawn so as to (dis)advantage one or more groups. This rather nebulous definition is indicative of the fact that there is no simple, concrete way to identify what is a gerrymander and what is not. This fact is an unavoidable problem for anyone — state courts, federal courts, redistricting commissions, legislatures, individuals — attempting to scrutinize district plans for fairness. For racial gerrymanders, the Voting Rights Act of 1965 has (at least until recently) clarified the legal landscape in many ways. However, for partisan gerrymandering claims, the constitutional issues are much murkier. While the federal courts have been considering partisan gerrymandering claims for several decades, little progress has been made in clarifying how a successful constitutional argument against a partisan gerrymander could be marshaled.

The case of *Gill* v. *Whitford* was the first major partisan gerrymandering case to be reviewed by the US Supreme Court in the last decade. The federal district court had ruled that the Wisconsin state legislative district plan was an unconstitutional partisan gerrymander [Whi]. However, the Supreme Court determined that the case, as presented, depended on vote dilution claims that require individual voters to show direct injury in order for standing to be satisfied. To do this, a given plaintiff needed to show that he or she had been packed or cracked.

In this article we explore what it means to show that a voter has been packed or cracked. We begin in Section 2 by reviewing the existing literature on legislative redistricting as it pertains to partisan gerrymandering, particularly with respect to packing and cracking. In Section 3 we review the most pertinent statements from the opinions in *Gill* v. *Whitford*. This is followed in Section 4 by precise criteria for when a voter has been packed or cracked. As Justice Kagan makes clear in her concurring opinion, determining whether a voter has been packed or cracked in a given plan requires having a comparator plan in mind. We present a simple concrete example highlighting the dependence of the answer (i.e., on who has been packed or cracked) on the comparator plan chosen. We close in Section 5 with various analyses of packed and cracked voters for Maryland, North Carolina and Wisconsin.

2. Redistricting and partisan gerrymandering

In this section we consider two aspects of the partisan gerrymandering problem. The first is that of how one can identify gerrymanders. The second is how these techniques can be applied in the service of litigation.

2.1. **Identifying partisan gerrymanders.** The term *gerrymander* derives from the salamander shape of a Massachusetts state senatorial district signed into law by Governor Elbridge Gerry in 1812. That the extended shape of this district was critiqued implicitly supports the notion that regular, or "compact," shapes are the most desirable and natural for districts. Indeed, "[c]ongressionally imposed standards providing that districts be compact, contiguous, and essentially equal in population existed throughout most of the 19th and early 20th centuries, until 1929." [Cro12]. For example, the Apportionment Act of 1911 directs that a district be a "contiguous and compact territory, and containing as nearly as practicable an equal number of inhabitants" [App]. While the third criterion has survived via *Wesberry* v. *Sanders* [Wes] and related cases, the first two have survived primarily at the state level [NCS18]. (An important exception being the appearance of compactness in the Gingles conditions arising in *Thornburg* v. *Gingles* [Gin], as applied to Section 2 of the Voting Rights Act of 1965.)

The three properties of compactness, contiguity and equal population are often referred to as "traditional districting principles." See [Alt98] for an in-depth history of the phrase; additional desiderata such as the preservation of county boundaries or of communities of interest are often included. As a group, these principles provide desired and expected characteristics for districts. It is generally accepted that any failure to follow these principles should be justified. One approach to identifying gerrymanders is therefore to look for violations of these traditional principles.

The most common principle considered through this lens is that of compactness. Distorted shapes have frequently been seen by the federal courts as necessary characteristics of partisan gerrymanders[1]. However, this point of view has not been held unanimously. Justice Souter, for example, in his dissent [Viea] in *Vieth* v. *Jubelirer* allows for the possibility that some traditional principles might be adhered to while others are not. Regardless, there is a large literature on ways to measure compactness. Two of the more famous ones include the Polsby-Popper score [PP91] and the Reock score [Reo61], although dozens of other measures have been proposed. Each measure has its pros and cons (see [NGCH90] for a review). Researchers have addressed this state of affairs by either using ensembles of measures (see, e.g., [Aza10, Aza12] or by simply considering multiple measures in conjunction with each other (see, e.g., [HG90, FLWM15]). There are numerous examples from the popular media in which the "worst" gerrymandered districts have been identified by seeing which districts do the least well on various (combinations of) compactness metrics (e.g., [Ing14]).

There are several issues with using lack-of-compactness as the mark of a gerrymander. The first issue is the aforementioned one that there is a zoo of metrics one might use. A second issue is the high rate of positives. According to one study [AP16], approximately 20% of historical districts are less compact than the original 1812 gerrymander. Finally, contorted district boundaries should be thought of as a *symptom* of gerrymandering, rather than the mechanism by which gerrymandering occurs.

Lack of adherence to other traditional districting principles can also be used as evidence to support allegations of gerrymandering. Contiguity is required for legislative districts in a large number of states [NCS18], though it is not required for many congressional plans. Nonetheless, contiguity is useful only in theory — in practice, modern districts are all contiguous. Population equality can, and is, used. However, the fact that the Supreme Court has ruled in *Wesberry* v. *Sanders* [Wes] that there is no *de minimis* allowable deviation appears to have consistently resulted in equipopulous maps

Scores of traditional districting principles as evidence for or against gerrymandering are provided by the plaintiffs' brief in *Whitford* v. *Gill*. In this brief, they compare the enacted plan to ones from prior decades. The enacted plan *does* score worse on two compactness scores "0.39 versus 0.41, and 0.28 versus 0.29," [Gil16, pg. 36], but the differences are modest. On the other hand,

[1] "Without evidence of any distortion of otherwise legitimate district boundaries, there is no gerrymander, at least as the term is traditionally understood." [Gri]

computer simulations are utilized in the same brief to argue that the Act 43 plan is an outlier with respect to the number of split municipalities [Gil16, Figure 11, pg. 40].

In recent decades there has been significant interest in developing metrics specifically tailored to identifying gerrymandering. Many of these metrics are based on finding asymmetries in the *seats-votes curve*. This curve, studied since at least [Tuf73], relates the fraction of the statewide vote each party gets to the fraction of seats it wins. So, for example, the curve might pass through the points (0.3,0.2) and (0.5,0.45) if winning 30% or 50% of the statewide vote would lead the Democrats to winning 20% or 45%, respectively, of the total seats available. Notwithstanding the fact that a number of assumptions must be made in order to compute the seats-votes curve, several partisan gerrymandering metrics have been derived from it. Gelman and King [GK94] suggest partisan bias, which considers the fraction of seats each party wins under the assumption that they win 50% of the statewide vote. The mean-median bias, which has been advocated for by a number of authors in various forms [Nag15, Wan16, BDK$^+$17], returns a related deviation. Other aspects of the seats-votes curve such as responsiveness and overall competitiveness have also been considered as means of identifying gerrymanders. More recent suggestions that are less directly related to the seats-votes curve include the efficiency gap [McG14] (and many variations; see [War18a] for a summary) and the declination [War18c].

2.2. **Litigation.** The aforementioned measures are simple mathematical functions. They consider a limited amount of electoral and/or redistricting data and attempt to provide insight into the extent to which a district plan is unfair. The first real test for such measures arrived in in 1986 in *Davis* v. *Bandemer*. Indiana Democrats argued that the 1981 Indiana legislative district plan violated the Equal Protection Clause (EPC) of the 14th amendment. The court determined that "political gerrymandering... is properly justiciable under the Equal Protection Clause," [Dav]. However, the justices were unable to agree on a standard.

The next significant partisan gerrymandering case was *Vieth* v. *Jubelirer*. In this case, the 2001 Pennsylvania congressional plan was challenged on both EPC grounds as well as on First Amendment grounds. There was little consensus in the court's ruling; a plurality of four justices determined such cases are nonjusticiable, four found them justiciable and proposed various standards, and Justice Kennedy wrote that such cases are justiciable but was not satisfied with any yet proposed standard. In 2006, the court heard *LULAC* v. *Perry* regarding alleged violations of the EPC and VRA in relation to mid-decade congressional redistricting in Texas. In the opinion of the court, written by Kennedy, some support was shown for partisan asymmetry metrics, in particular partisan bias, but ultimately the court determined that no one had yet proposed a suitable, manageable standard.

Central to any manageable standard is the legal argument on which it is based. For racial gerrymandering cases, while the details are intricate, the general legal argument is straightforward. As described in [Tok17]: "The U.S. Supreme Court has issued three major decisions on [racial gerrymandering] since 2015. All these cases... [are] challenged under the Equal Protection Clause on the ground that they packed racial minorities into districts in a way that was not justified by the interest in complying with the Voting Rights Act (VRA)." For partisan gerrymandering, the best approach has been less clear. Not surprisingly, success in the courts for partisan cases has been, correspondingly, much more limited.

In the subsequent decades, a number of proposals have been put forth for what such a manageable standard should look like. Recent examples include [MS15, Wan16, Gro18]. Contemporaneously, several partisan gerrymandering cases have been wending their ways through the federal court system.

The Supreme Court heard two partisan gerrymandering cases in 2017–2018. While the court reviewed *Benisek* v. *Lamone* [Ben], it did not address justiciability of partisan gerrymandering

claims. However, a more substantive response resulted from *Whitford* v. *Gill* [Gil]. In this case, the Wisconsin state legislative plan was challenged on both EPC and First Amendment grounds. In its opinion striking down the plan, the federal district court focuses its discussion of standing on the EPC. The Supreme Court, in its review of the case, adjudged that the plaintiffs did not, in how it presented the case, show the "injury in fact" necessary for Article III standing. Both Chief Justice Roberts, writing for the court, and Justice Kagan, in a concurring opinion, described what was necessary to show the injury required. We turn to a discussion of those remarks in the next section.

3. Packing and cracking in *Gill* v. *Whitford*

The problem of recognizing a partisan gerrymander reduces to that of recognizing the presence of packing and cracking: "For packing and cracking are the ways in which a partisan gerrymander dilutes votes" (*Gill* v. *Whitford,* concurring opinion). This fact necessitates a careful definition of these terms.

A widely cited definition for packing and cracking is the following given by Justice Scalia [Vieb]:

> "'Packing' refers to the practice of filling a district with a supermajority of a given group or party. 'Cracking' involves the splitting of a group or party among several districts to deny that group or party a majority in any of those districts."

Unfortunately, this definition is not consistent in its treatment of the two actions. As defined by Justice Scalia, packing happens whenever a supermajority arises, regardless of the circumstances; cracking only arises in the context of intent. There are two obvious modifications one could apply to make these definitions consistent with each other.

One option is to remove the intent criterion from cracking. However, this leaves us with definitions for packing and cracking that are less directly connected to the creation of partisan gerrymanders. (Some locales are primarily populated by the supporters of a single party. A supermajority could easily arise without any attempt to create a gerrymander.) As a consequence, we prefer to modify the definition of packing to be the following: "Packing" is the practice of filling a district with extra members of a given group or party so as to prevent those extra members from contributing to majorities in other districts.

Under these modified definitions, the recognition that packing or cracking has occurred contains an implicit acknowledgment that different choices could have been made. Consequently, to show that cracking or packing has occurred, one should be able to point to an acceptable, alternative district plan in which a reconfiguration of districts leads to additional majorities by the disadvantaged group or party. This is essentially the procedure Justice Kagan describes in *Whitford* v. *Gill* (concurring opinion):

> "For example, a Democratic plaintiff living in a 75%-Democratic district could prove she was packed by presenting a different map, drawn without a focus on partisan advantage, that would place her in a 60%-Democratic district. Or conversely, a Democratic plaintiff residing in a 35%-Democratic district could prove she was cracked by offering an alternative, neutrally drawn map putting her in a 50-50 district. The precise numbers are of no import. The point is that the plaintiff can show, through drawing alternative district lines, that partisan-based packing or cracking diluted her vote."

We note that the above argument implies that voters suffer harm merely by being placed in less competitive districts.

In our above discussion, we have only considered packing and cracking as actions. From such a viewpoint, one is naturally provided with at least two district plans: a starting district plan $\mathcal{D}_0$ and a district plan $\mathcal{D}$ that is the result of the packing and/or cracking. In this situation, which districts

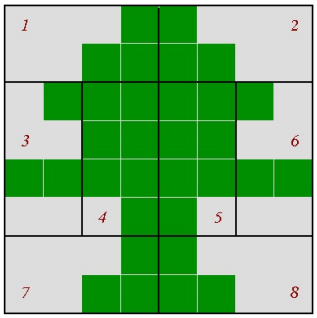

FIGURE 1. Hypothetical district plan $\mathcal{D}$ for a state with 64 voters and eight districts. Each party holds equal statewide support.

have been packed or cracked follows directly from where the packing and cracking have taken place. However, the designation is most cleanly described in terms of districts of the original plan $\mathcal{D}_0$. As we discuss in Section 4, there are some ambiguities regarding how to label the districts in $\mathcal{D}$. As such, the references to packed and cracked districts in the following quotation from Kagan's *Gill* v. *Whitford* concurring opinion must be considered carefully.

> "Consider the perfect form of each variety. When a voter resides in a packed district, her preferred candidate will win no matter what; when a voter lives in a cracked district, her chosen candidate stands no chance of prevailing. But either way, such a citizen's vote carries less weight — has less consequence — than it would under a neutrally drawn map."

Another problem with the phrase "packed district" is that it obscures the dependency on a precursor district plan. So, for example, Chief Justice Roberts' majority opinion in *Gill* v. *Whitford* makes the situation sounds simpler than it actually is:

> "To the extent the plaintiffs' alleged harm is the dilution of their votes, that injury is district specific. An individual voter in Wisconsin is placed in a single district. He votes for a single representative. The boundaries of the district, and the composition of its voters, determine whether and to what extent a particular voter is packed or cracked."

The dilution does not depend *only* on the district and the voters in it. It also depends on the suite of comparator plans being used to determine which districts have been packed or cracked.

In the next section we shall address what it means for a voter to be packed or cracked. (The case of districts will be treated briefly.) As illustrated by the above *Gill* v. *Whitford* excerpts, identifying voters as having been packed or cracked is what is needed for vote dilution claims.

4. Packed and cracked voters

In this section we explore what it means for individual voters to be packed or cracked. We will do so in the context of a hypothetical state with 64 voters living in an 8×8 grid as shown in Figure 1. The Dark-green Party and the Light-gray Party each enjoy equal statewide support. We have illustrated in the figure an undeniably compact district plan $\mathcal{D}$ consisting of eight districts. From the vantage point of seats-votes proportionality, this appears to be an unfair district plan. Even though statewide support for the two parties is equal, the Dark-green Party wins only two of the eight seats. And indeed, partisan asymmetry metrics such as the efficiency gap [McG14] and the declination [War18c] agree with this intuition (see Figure 4.D). But it is not clear *which* districts have been packed or cracked. The analogous questions for individual voters are not any clearer. As discussed in Section 3, such determinations require a comparator plan.

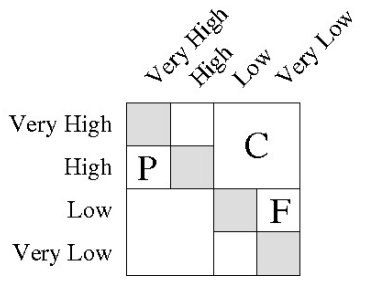

FIGURE 2. Matrix of possible transitions for a voter between a precursor plan $\mathcal{D}_0$ and a resultant district plan $\mathcal{D}$. Row labels indicate level of support for the voter's party in the voter's district in plan $\mathcal{D}_0$; column labels indicate level of support for the voter's party in the voter's district in plan $\mathcal{D}$. High and Very High indicate majority support; Low and Very Low indicate minority support. The P marks the class of packed voters; the C marks the class of cracked voters; and the F marks the class of forsaken voters.

We emphasize the necessity of a comparator plan by considering some of the districts and voters in Figure 1 in detail. Consider, for example, the Dark-green voters in the overwhelmingly Dark-green Districts 4 and 5. The Dark-green Party's overwhelming support in these two districts certainly isn't very efficient, but if these districts cover a community of interest, then it may be appropriate. A verdict that these districts were packed requires the demonstration of, at the least, an acceptable plan in which similar districts have lower proportions of Dark-green voters. Similarly, consider the minority Dark-green District 2. Modest changes to its boundary could make it a majority Dark-green district. But describing one of the Dark-green voters in this or a nearby district in $\mathcal{D}$ as having been cracked presupposes that there is a reasonable district plan — and hence a reasonable district — with those voters in the majority in that single district. Without the explicit demonstration of such a plan, one must allow for the possibility that external constraints or priorities make District 2 defensible as part of a fair district plan. An analogous argument applies when considering whether an individual voter was cracked.

Now assume that we have identified a fair precursor plan, $\mathcal{D}_0$. We can suggest at least tentative definitions for what it means for a voter to be packed or cracked in $\mathcal{D}$ relative to $\mathcal{D}_0$ as follows. To do so, without loss of generality we focus on Dark-green Party voters. First note that four classes of Dark-green voters naturally arise by considering whether a given voter starts in a majority or minority Dark-green district in $\mathcal{D}_0$ and whether they end up in a majority or minority Dark-green district in $\mathcal{D}$. For voters who remain in a majority Dark-green district or remain in a minority Dark-green district, we obtain a finer classification by considering whether the support for the Dark-green Party in the given voter's district goes up or down in the transition from $\mathcal{D}_0$ to $\mathcal{D}$. The resulting transitions are summarized in Figure 2.

With the universe of possible transitions now identified, we are now ready to explore which Dark-green voters should be identified as having been packed or cracked. We first consider those transitions that do not correspond to any form of packing or cracking. Four of the transitions are completely uninteresting: A voter starts and ends in a district with the same level of Dark-green Party support. These are indicated in Figure 2 by the grayed-out squares. Next we note that there are a number of redundancies when a voter transitions from a majority Dark-green district to a minority Dark-green district or vice versa. For example, it does not make sense for our purposes to distinguish a voter transitioning from a majority Dark-green district to a Low Dark-green district from a voter transitioning from a majority Dark-green district to a Very Low Dark-green district. These redundancies are indicated in Figure 2 by the 2×2 squares. For the lower left square, the

voter transitions from a district in which the Dark-green Party candidate loses to one in which she wins; it is hard to see how this voter could be construed as having been packed or cracked.

The final two blank squares in Figure 2 correspond to transitioning from a Very High Dark-green district to a High Dark-green district or from Very Low Dark-green district to Low Dark-green district. It is hard to interpret either as a harm to the voter as an individual. In both cases, he remains in the same type of district in which he started — either the Dark-green Party candidate wins or she doesn't. And in each case, the voter's vote is, if anything, *more* likely to be consequential. In the Very High-to-High case, the Dark-green support in the district is lessened and the given voter's support can be viewed as even more important than before. In the Very Low-to-Low case, the Dark-green candidate still loses, but had things been slightly different, perhaps the given voter's vote would have been the one to push the Dark-green candidate to victory. Irrespective of how efficiently or inefficiently the Dark-green Party's votes are being utilized in this district, the individual voter certainly shouldn't feel that his support is less valuable in $\mathcal{D}$ than in $\mathcal{D}_0$.

We are left with three types of transitions to classify. Transitioning from High to Very High is consistent with the notion of a voter being "packed." This case is denoted by a "P" in Figure 2. Similarly, transitioning from majority Dark-green to minority Dark-green is consistent with the voter being cracked; this is denoted by a "C" in Figure 2. The final case occurs when a voter transitions from a Low Dark-green district to a Very Low Dark-green district. This is in some sense analogous to what happens to the packed voter, except that the voter's candidate of choice suffers a *worse loss* rather than a stronger win. As such, we consider this transition a disadvantage for the voter; we denote this class of voters by an "F" for "forsaken." While Dark-green voters are likely to be packed or cracked when the Light-gray Party is creating the gerrymander, it is Light-gray voters who are more likely to be forsaken in such a scenario: Forsaking Dark-green voters would result in the distribution of additional Dark-green voters into other districts the Light-gray party was trying to win. As suggested by this last observation, the classification in Figure 2 applies, *mutatis mutandis*, equally well to both Light-gray voters and Dark-green voters. Finally, we note that packed voters of one party will, locally, be surrounded by forsaken voters of the other party and vice versa.

Now that we know how to classify individual voters, we are ready to see how voters of Figure 1 get classified under various precursor plans. This is illustrated in Figure 3 for the three precursor plans $\mathcal{D}_0$ depicted in the first row. The second row consists of three copies of $\mathcal{D}$, each indicating the corresponding packed/cracked/forsaken voters according to the given precursor plan as determined by Figure 2. As should be clear from the example, the classification of any individual voter in $\mathcal{D}$ as being packed, cracked, forsaken or neither is highly dependent on the precursor plan chosen.

We now return to the matter of identifying which districts in the plan $\mathcal{D}$ have been packed or cracked. It follows from the definitions of packed and cracked voters that a single district in $\mathcal{D}$ cannot contain *both* packed voters and cracked voters: A Dark-green packed voter must end up in a majority Dark-green district while a cracked voter must end up in a minority Dark-green district. So one possibility is to define a packed (cracked) district in $\mathcal{D}$ relative to $\mathcal{D}_0$ as a district containing at least one packed (cracked) voter. For example, in Figure 3.A, there are six cracked districts (1, 2, 3, 6, 7 and 8) in $\mathcal{D}$ and no packed districts while in Figure 3.C there are two packed districts (4 and 5) and two cracked districts (3 and 6).

There is another approach to identifying packed and cracked districts that is in some ways more natural, but depends on a (partial) matching between districts of $\mathcal{D}_0$ and those of $\mathcal{D}$. It is not necessarily consistent with the above, bottom-up approach. It works by applying the transition matrix of Figure 2 directly to districts rather than descending down to the level of voters. So, for example, in Figure 3.A, if Districts c and 2 are paired, then District 2 would qualify as a cracked district since District c is majority Dark-green while District 2 is not. However, if District 2 is instead paired with District d, then District 2 would not qualify as cracked.

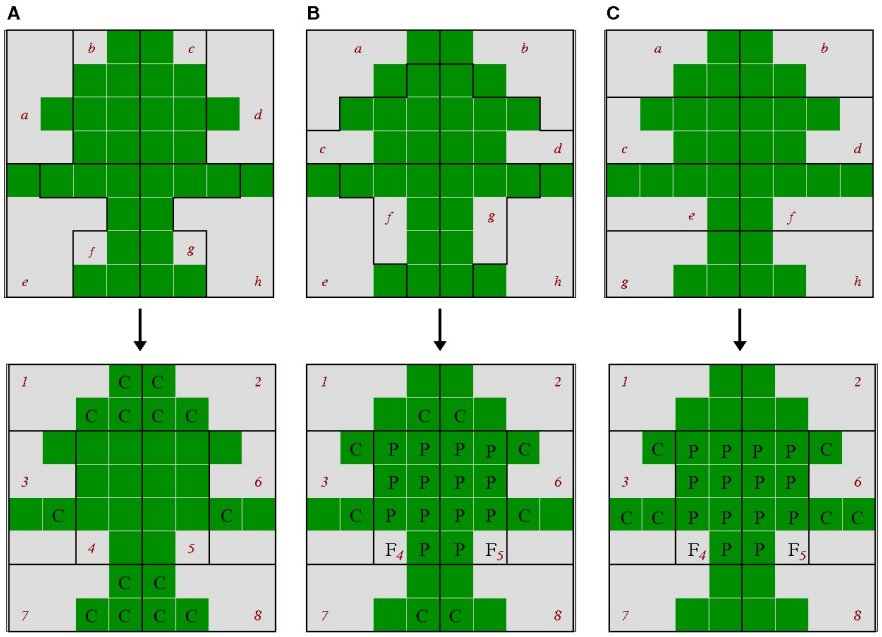

FIGURE 3. Classification of voters in $\mathcal{D}$ as packed (**P**), cracked (**C**), forsaken (**F**), or neither for three possible precursor plans $\mathcal{D}_0$ using the assignments in Figure 2.

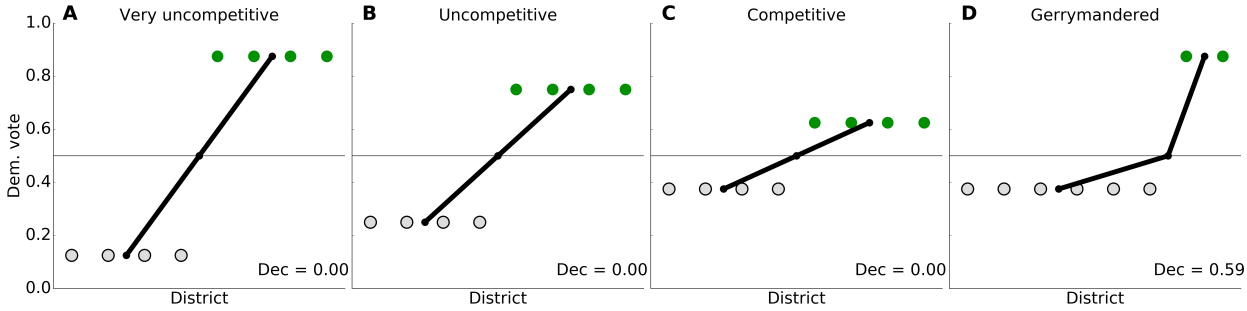

FIGURE 4. Vote distributions for plans from Figure 3. Plotted are the Dark-green Party vote fraction for each district, sorted in increasing order of support. Also shown are the value of the declination and the line segments used to compute its value.

We end this section by illustrating, in Figure 4, the vote distributions for the various district plans depicted in Figure 3 along with the values of one of the partisan asymmetry metrics, the declination (introduced in [War18c]). The symmetry of the first three plans illustrates that the plans we have used for $\mathcal{D}_0$ treat the parties symmetrically. In the fourth plan, one can see that the Dark-green Party loses six close districts while winning two districts overwhelmingly.

5. Packed and cracked voters in Maryland, North Carolina and Wisconsin

In this section we use recent electoral data to explore which voters have been packed or cracked in the states of Maryland, Wisconsin and North Carolina. Before we do so, it is worth elaborating on how forsaken voters are related to packed voters.

Consider an atomic geographic unit such as a ward as it relates to a comparator plan $\mathcal{D}_0$ and a final plan $\mathcal{D}$. Suppose the Dark-green voters in the ward are packed relative to these two plans. This

means that the ward lies in a Dark-green majority district in both plans and that the proportion of Dark-green voters is higher in $\mathcal{D}$. This characterization is equivalent to saying that the ward lies in a Light-gray minority district in both plans and that the proportion of Light-gray voters is lower in $\mathcal{D}$. As such, any Light-gray voters in this ward are automatically forsaken. In light of this connection, we will refrain from referring explicitly to forsaken voters in the following analyses. Their presence for one party can be immediately inferred from the presence of opposing-party packed voters.

5.1. **Maryland.** In *Benisek* v. *Lamone*, Republican voters challenged the Maryland 2011 congressional plan on the grounds that Republican voters of the 6th congressional district had been cracked. We investigate this claim by pairing the district plan with two different precursor plans. The plans we use were generated as part of the *Atlas of Redistricting Project* [Fiv18]. The first comparator plan we use was generated with the goal of making each district as competitive as possible while still respecting county boundaries. In Figure 5.A we illustrate this plan with each district colored according to how strongly it leans Democratic or Republican. The *Atlas* project computed this lean using the Cook Political Report's Partisan Voter Index (PVI) and we use their values here. The competitiveness of the districts is borne out by the light shading of each district. In Figure 5.E we have displayed the current Maryland congressional plan with regions colored according to PVI values taken from [PVI17]. The high saturation levels are indicative of relatively uncompetitive districts. In the middle, as Figure 5.C, we illustrate the regions of the state containing Republican voters who were packed or cracked in the enacted plan relative to the competitive plan. In support of the plaintiffs' claims, much of the Western part of state (i.e., the 6th district) is shown in orange as a result of those voters having been cracked. This is consistent with the fact that the 6th (westernmost) district is slightly Republican leaning in the competitive plan, but is Democratic leaning in the enacted plan. The comparison indicates packing of Republicans in the 1st (easternmost) district as well as some cracking centered on the 2nd district in the vicinity of Baltimore.

In Figure 5.B we illustrate a hypothetical plan from the *Atlas* project that maximized the number of Democratic seats. In this plan, the Democrats are projected to win all seats. So, with this as a precursor plan, there is no possibility of cracked or packed Republicans since the Republicans don't win any seats in the precursor plan. This is illustrated by the lack of either orange or purple regions in in Figure 5.D.

5.2. **North Carolina.** North Carolina has seen two very recent partisan gerrymandering cases. *League of Women Voters* v. *Rucho* and *Common Cause* v. *Rucho* both alleged partisan gerrymandering in the 2016 remedial congressional plan. These cases were consolidated in 2017. After a series of maneuvers and developments, the Supreme Court vacated and remanded a district court decision striking down the map as an unconstitutional gerrymander. In response to the remand, the court ordered parties to respond to a number of issues in light of *Gill* v. *Whitford*. The plaintiffs' response focused on providing additional information to establish standing. They do this by using a simulated plan, Plan 2–297, generated by plaintiffs' expert Jowei Chen. In line with the guidance from the Supreme Court, for each district (except the 3rd), they identify either a plaintiff or a member of the League of Women Voters of North Carolina who consistently votes Democratic living in that district.

In this article we identify packed and packed regions of the *2012* North Carolina congressional plan (i.e., not the plan under discussion in the *Rucho* cases). Instead of identifying individual voters we identify packed and cracked VTDs using 1,000 simulated plans from among the 24,000 plans generated for [HKL$^+$18] and made available by the authors at [Her17a]. For each voting tabulation district (VTD) — comparable for our purposes to a *ward* or *precinct* — and each simulated plan used as a comparator, we considered whether the voters in the VTD had been packed or cracked relative to the two plans. For this analysis and the one for Wisconsin in Section 5.3, we take a

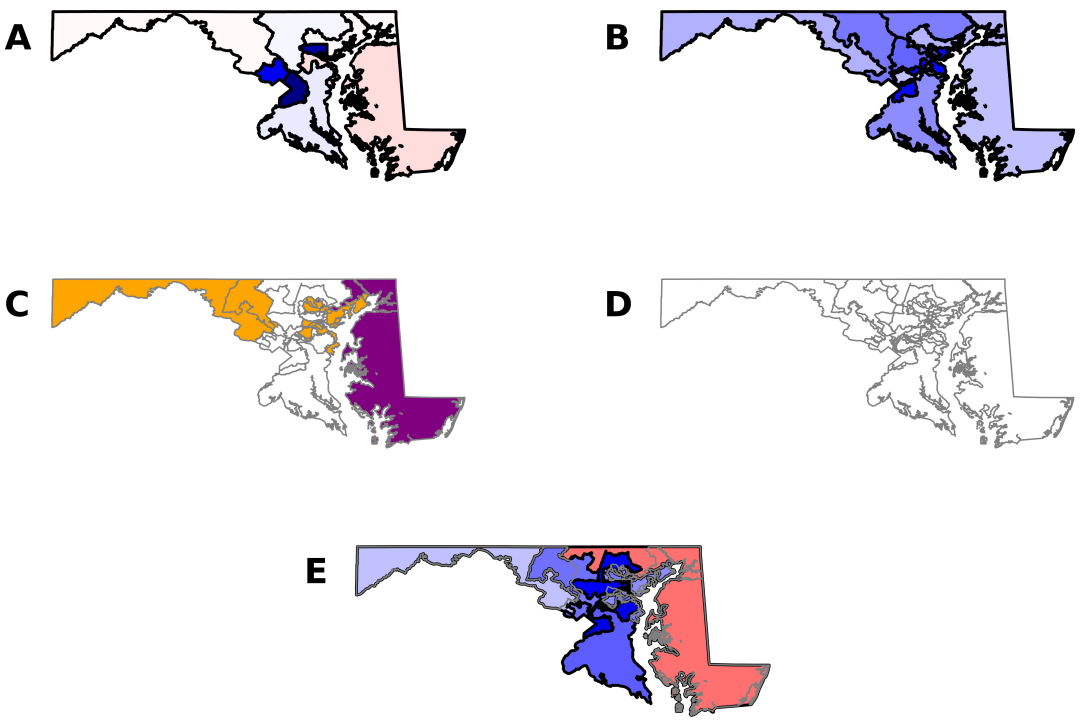

FIGURE 5. Illustration of which Republican voters in Maryland would be considered packed or cracked relative to two different comparator plans.

more conservative approach than that taken in the matrix of Figure 2 and require the support for a given party to increase or decrease by at least 5 points before triggering a classification of packed/cracked. Finally, note that our approach uses the first definition we given in Section 3 for what it means for a region to be packed/cracked (i.e., we do not require any sort of mapping of districts between the comparator plan and the plan of interest).

In Figure 6 we illustrate how frequently Democrats, respectively, in each VTD were characterized as packed or cracked. For example, the darkest orange (stippled) shading shown indicates that the Democratic voters in that region were characterized as cracked in the enacted plan relative to the comparator plans at least 80% of the time.

Note that according to the criteria set out in Section 3, voters of both parties are injured by partisan gerrymandering. In light of this, in Figure 7 we illustrate how frequently Republicans were packed or cracked relative to the comparator plans. According to those criteria, what matters for standing is not the coverage of packed and cracked VTDs within a district, but merely the fact that there exist packed and cracked voters (or VTDs). In fact, for more than 80% of the computer simulations, there exists at least one VTD packed with Democrats in each of the 1st, 4th and 12th congressional districts. The analogous fact is true for cracked Democratic voters in the 2nd, 3rd, 6th, 8th, 9th and 13th districts. If we include the Republican voters in 11th who are packed 80% of the time and the Republican voters in the 1st (and 12th) who are cracked 80% of the time, we see that only the 5th, 7th and 10th districts lack packed/cracked voters this consistently. Note that the plaintiffs' brief does not identify any packed/cracked voters in the year-2016 3rd district.

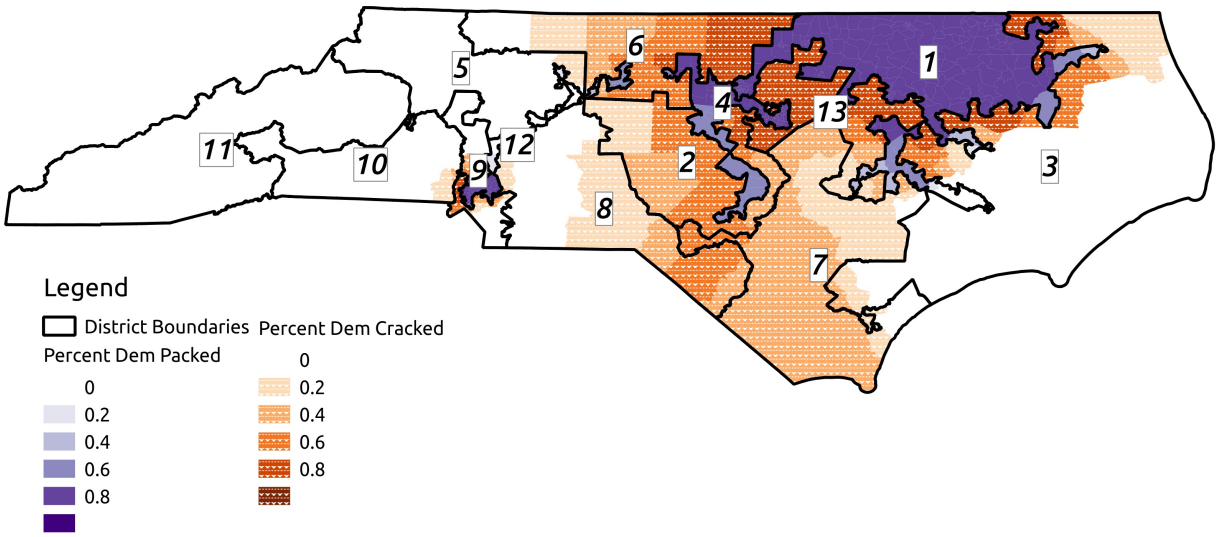

FIGURE 6. Illustration of how frequently Democratic voters in a given VTD as part of the 2012 congressional district plan would be considered to be packed/cracked relative to 1,000 randomly selected comparator plans. Orange (stippled) values indicate cracked regions with darker colors indicating higher frequencies; purple (solid) values indicate packed regions.

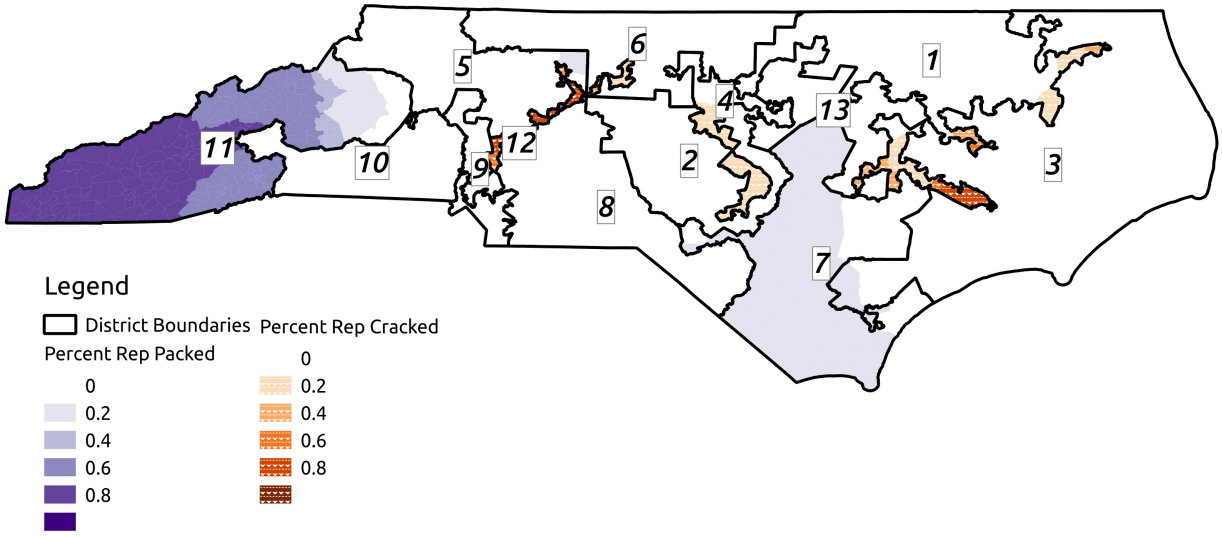

FIGURE 7. Illustration of how frequently Republican voters in a given VTD as part of the 2012 congressional district plan would be considered to be packed/cracked relative to 1,000 randomly selected comparator plans. Orange (stippled) values indicate cracked regions with darker colors indicating higher frequencies; purple (solid) values indicate packed regions.

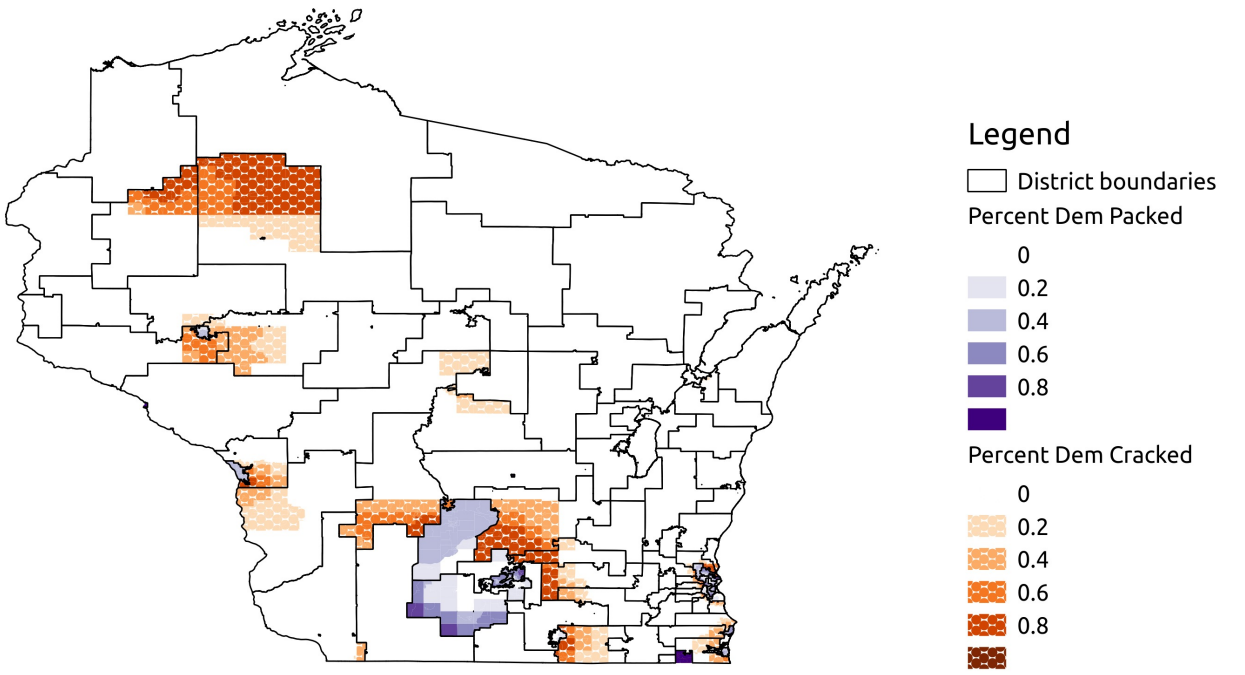

FIGURE 8. Illustration of how frequently Democratic voters in a given ward as part of the Act 43 district plan would be considered to be packed/cracked relative to 1,000 randomly selected comparator plans. Orange (stippled) values indicate cracked regions with darker colors indicating higher frequencies; purple (solid) values indicate packed regions.

From Figure 7, we see that Republican voters in the western half of the 11th district are packed relative to about half of the simulated plans. This would be consistent with a partisan gerrymandering that shores up the 11th district as a relatively safe Republican seat relative to its natural status (as far as the computer simulations are concerned) as a swing district. We also see Republican voters in the 1st and 12th districts who look to have been cracked. Cracking Republican voters does nothing directly to help Republicans win more seats. However, there are several reasons while small populations of Republicans might be moved from Republican districts into Democratic districts. It might be necessary to equal out populations. Or, as is more likely the case in these instances, the narrow strips of Republican-leaning areas are required for contiguity, connecting disparate areas of Democratic support. The 12th district is bounded by Democratic areas of Charlotte on the southwest and the Piedmont Triad of Winston-Salem, Greensboro and Highpoint on the northeast. Similarly, the arm of the 1st district filled with cracked Republicans extends down to the city of New Bern.

5.3. **Wisconsin.** In [Her17b], the authors generate over 19,000 simulated districting plans for the 99 districts of the Wisconsin state assembly. We use a random subset of 1,000 of these plans to use as comparators for the current, Act 43, Wisconsin state assembly plan analogously to our analysis of the North Carolina congressional plan from Section 5.2. The results are displayed in Figure 8 (for packed/cracked Democrats) and in Figure 9 (for packed/cracked Republicans). As for North Carolina, to cut down on noise and spurious results, we only counted a region as having been packed or cracked if the vote values between the comparator plan and the Act 43 plan differed by at least 5%.

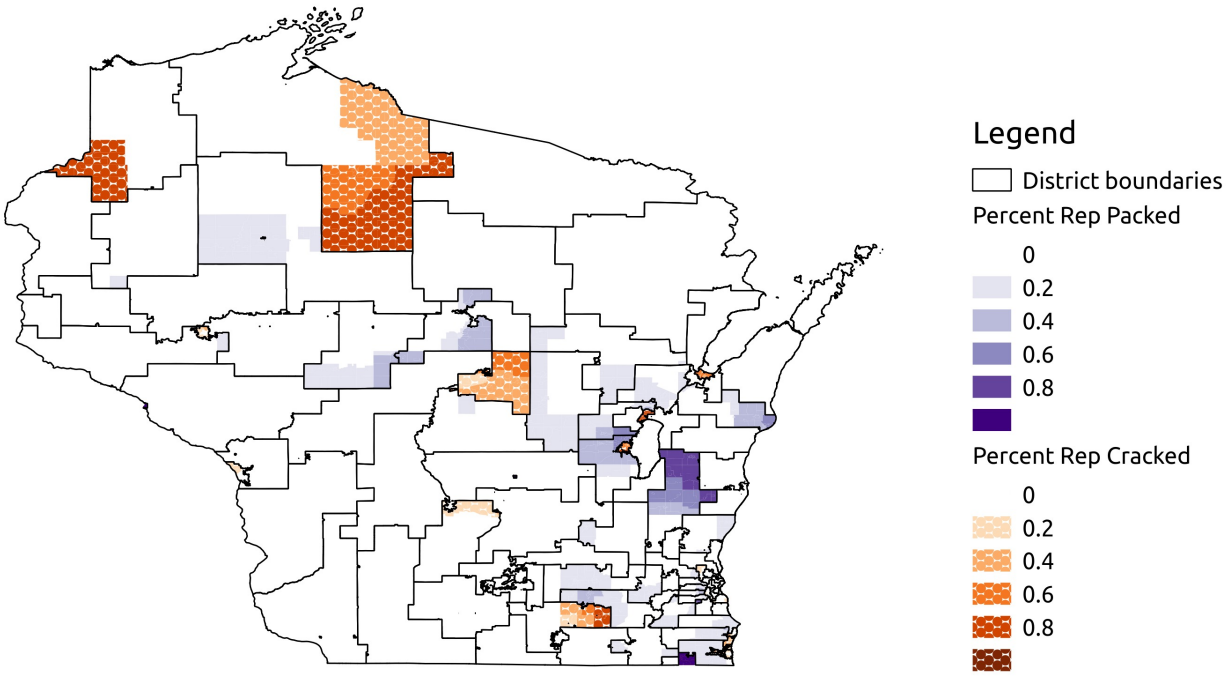

FIGURE 9. Illustration of how frequently Republican voters in a given ward as part of the Act 43 district plan would be considered to be packed/cracked relative to 1,000 randomly selected comparator plans. Orange (stippled) values indicate cracked regions with darker colors indicating higher frequencies; purple (solid) values indicate packed regions.

In Table 1 we record the frequencies with which each Act 43 district was packed or cracked relative to the 1,000 computer-simulated comparator districts.

TABLE 1. Distribution of 99 Wisconsin state legislative districts relative to how frequently they contained a packed/cracked Democratic/Republican ward. The first column indicates the percentage of precursor plans for which a given district at least one ward of the indicated type (containing packed Democrats, cracked Democrats, etc.). One thousand precursor plans were chosen randomly from the data [Her17b] used in [HRM17].

Max frequency	Dem Packed	Dem Cracked	Rep Packed	Rep Cracked
0.0 to 0.19	74	67	68	82
0.2 to 0.39	6	4	19	4
0.4 to 0.59	5	7	7	4
0.6 to 0.79	5	9	3	3
0.8 to 1.00	9	12	2	6

6. DISCUSSION

There are two basic approaches one can take to identifying a gerrymander. The first approach is to show that individual districts have been gerrymandered and, as a result (or by definition), the

vote of certain residents in those districts has been diluted. This approach seems best suited to when the alleged harm is restricted to a relatively small portion of the entire district plan, such as the 6th District and its surroundings in Maryland as alleged in *Benisek* v. *Lamone*. However, as described by Justice Kagan in her concurring opinion in *Whitford* v. *Gill*, this approach could also be applied repeatedly in order to address instances in which the alleged gerrymandering is close to statewide.

However, it is important to note that partisan gerrymanders don't target individual voters. *Every* district plan will have winners and losers as measured by who gets to be in the majority and who does not. The effectiveness of a partisan gerrymander is measured by the net effect, leading to the second approach of considering a gerrymander from a statewide perspective from the outset.

The statewide approach is attractive because the districts in a district plans are typically drawn as parts of a cohesive whole and because the ultimate goal of a partisan gerrymander is to reduce the statewide representation of the opposition party. Partisan asymmetry measures such as the efficiency gap and the declination are well suited to such a statewide analysis. While such analyses have been common in the academic literature and popular press, they have had limited success in the courts. One disadvantage of this approach is that partisan asymmetry measures seem likely to miss "localized" gerrymandering (though perhaps one could apply metrics to only a portion of the statewide map).

While there are arguments in favor of both approaches, which can be successful in the courts is ultimately a legal question and one we do not delve into here. We have had two main goals in this article. The first has been to make as explicit as possible definitions for, and some of the pitfalls inherent in, the local approach outlined by both Chief Justice Roberts and Justice Kagan in their opinions in *Whitford* v. *Gill*. Other definitions for packed/cracked voters and districts are certainly possible. The second has been to illustrate how simulations can be used to provide a more robust picture than can be provided by a single comparator plan, of what has been packed or cracked. Of course, the cost of using simulations is that one must take care to show that they have been appropriately drawn from the universe of possible plans.

7. Data collection and Methods

The electoral data and geometry for Maryland used in Section 5.1 are taken from the repository [Fiv18] created as part of the Atlas of Redistricting project. The district plan shown in Figure 5.E is the one used since 2012.

The North Carolina electoral data were obtained from the Harvard Dataverse [APL15]. Partisan lean for each VTD was computed by averaging the results for the 2012 presidential election and the North Carolina races for governor, attorney general, treasurer and secretary of state. The boundaries for congressional districts are from the Dataverse data and are the boundaries from 2012. These boundaries were updated in 2016 and hence are *not* the boundaries being litigated in the *Rucho* cases. The actual geometries of the VTDs are taken from the Census Bureau [Unib]. County codes were taken from [Unia].

The Wisconsin ward and state Assembly district geometries were obtained from [LTS]. Ward boundaries are for 2017, district boundaries are the Act 43 districts used in the 2012–2016 elections. Electoral data (including fixed ward-district associations) were taken from [Her17b]. Partisan lean for each ward was computed by averaging the results for six elections: the 2016 presidential and US senatorial elections, the 2014 US senatorial election and the 2014 gubernatorial, secretary-of-state, and treasurer elections.

The data were analyzed using python code [War18b] written by the author in a Jupyter notebook environment [KRKP$^+$16]. Python packages utilized were Pandas [McK10], GeoPandas [dev18], Matplotlib [Hun07], NumPy [Oli15] and Shapely [G$^+$]. The depictions of packed and cracked votes for North Carolina and Wisconsin illustrated in Figures 6 to 9 were created using QGis [QGI09].

8. Acknowledgments

The author is grateful to the authors of [HKL+18] and [HRM17] for making their collection of computer-simulated district plans for North Carolina and Wisconsin publicly available in a format usable by other researchers; the author would like to thank Greg Herschlag in particular for his help in accessing these plans.

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References

[Alt98] Micah Altman. *Districting principles and democratic representation*. PhD thesis, California Institute of Technology, 1998.

[AP16] S. Ansolabehere and M. Palmer. A two-hundred year statistical history of the gerrymander. *Ohio State Law Journal*, 77(4):741–762, 2016.

[APL15] Stephen Ansolabehere, Maxwell Palmer, and Amanda Lee. *Precinct-Level Election Data, 2002-2012*, 2015. `https://hdl.handle.net/1902.1/21919`.

[App] Apportionment Act of 1911, Pub. L. No. 5, 37 Stat. 13, 14 (codified at 2 U.S.C. §3 (1935)). Accessed at `https://en.wikipedia.org/wiki/Apportionment_Act_of_1911` on July 24, 2018.

[Aza10] Redrawing the map on redistricting 2010: A national study. 2010. `https://cdn.azavea.com/com.redistrictingthenation/pdfs/Redistricting_The_Nation_White_Paper_2010.pdf`.

[Aza12] Redrawing the map on redistricting 2012: Addendum. 2012. `https://cdn.azavea.com/com.redistrictingthenation/pdfs/Redistricting_The_Nation_Addendum.pdf`.

[BDK+17] Robin E. Best, Shawn J. Donahue, Jonathan Krasno, Daniel B. Magleby, and Michael D. McDonald. Considering the prospects for establishing a packing gerrymandering standard. *Elect. Law J.*, 17(1), 2017.

[Ben] *Benisek* v. *Lamone,* 585 U.S. __ (2018).

[Cro12] Royce Crocker. Congressional redistricting: an overview. November 21, 2012. 7–5700; R42831. `https://fas.org/sgp/crs/misc/R42831.pdf`.

[Dav] *Davis* v. *Bandemer*, 478 U.S. 109 (1986).

[dev18] GeoPandas developers. *GeoPandas v. 0.4.0*, 2018. `http://geopandas.org/index.html`.

[Fiv18] FiveThirtyEight. *Atlas of Redistricting*, 2018. Online; accessed July 26, 2018. `https://github.com/fivethirtyeight/redistricting-atlas-data`.

[FLWM15] Chao Fan, Wenwen Li, Levi J. Wolf, and Soe W. Myint. A spatiotemporal compactness pattern analysis of congressional districts to assess partisan gerrymandering: A case study with california and north carolina. *Annals of the Association of American Geographers*, 105(4):736–753, 2015.

[G+] Sean Gillies et al. Shapely: manipulation and analysis of geometric objects, 2007–. `https://github.com/Toblerity/Shapely`.

[Gil] *Gill* v. *Whitford,* 585 U.S. __ (2018).

[Gil16] Plaintiffs' Trial Brief, Filed May 16, 2016. *Whitford v. Gill*, No. 15-cv-00421-bbc. `https://www.brennancenter.org/sites/default/files/legal-work/Whitford-PlaintiffsTrialBrief051616.pdf`.

[Gin] *Thornburg* v. *Gingles*, 478 U.S. 30 (1986).

[GK94] Andrew Gelman and Gary King. Enhancing democracy through legislative redistricting. *Am. Political Sci. Rev.*, 88:541–559, 1994.

[Gri] *Whitford v. Gill*, No. 15-cv-421, F. Supp. 3d (2016). Griesbach, dissenting, 940.

[Gro18] Bernard Grofman. Crafting a judicially manageable standard for partisan gerrymandering: Five necessary elements. *Election Law Journal: Rules, Politics, and Policy*, 17(2):117–136, 2018.

[Her17a] NC redistricting data, 2017. Online; accessed July 26, 2018. `https://git.math.duke.edu/gitlab/gjh/districtingDataRepository.git`.

[Her17b] Wi redistricting data, 2017. Online; accessed July 26, 2018. `https://git.math.duke.edu/gitlab/gjh/WIRedistrictingData.git`.

[HG90] T. Hofeller and B. Grofman. Comparing the compactness of California congressional districts under three different plans: 1980, 1982, and 1984. In B. Grofman, editor, *Political gerrymandering and the courts*, pages 281–288. Algora, 1990.

[HKL+18] G. Herschlag, H. S. Kang, J. Luo, C. Vaughn Graves, S. Bangia, R. Ravier, and J. C. Mattingly. Quantifying Gerrymandering in North Carolina. *ArXiv e-prints*, January 2018.

[HRM17] G. Herschlag, R. Ravier, and J. C. Mattingly. Evaluating Partisan Gerrymandering in Wisconsin. *ArXiv e-prints*, September 2017.

[Hun07] J. D. Hunter. Matplotlib: A 2d graphics environment. *Computing In Science & Engineering*, 9(3):90–95, 2007.

[Ing14] Christopher Ingraham. Wonkblog: America's most gerrymandered congressional districts. *The Washington Post*, May 15, 2014. Online; accessed February 10, 2017. https://www.washingtonpost.com/news/wonk/wp/2014/05/15/americas-most-gerrymandered-congressional-districts/?utm_term=.e55ff6296765.

[KRKP+16] Thomas Kluyver, Benjamin Ragan-Kelley, Fernando Pérez, Brian Granger, Matthias Bussonnier, Jonathan Frederic, Kyle Kelley, Jessica Hamrick, Jason Grout, Sylvain Corlay, Paul Ivanov, Damián Avila, Safia Abdalla, and Carol Willing. Jupyter notebooks – a publishing format for reproducible computational workflows. In F. Loizides and B. Schmidt, editors, *Positioning and Power in Academic Publishing: Players, Agents and Agendas*, pages 87 – 90. IOS Press, 2016.

[LTS] LTSB. *2012-2020 WI Election Data with 2017 Wards*. https://data-ltsb.opendata.arcgis.com/datasets/2012-2020-wi-election-data-with-2017-wards.

[McG14] Eric McGhee. Measuring partisan bias in single-member district electoral systems. *Legis. Stud. Q.*, 39:55–85, 2014.

[McK10] Wes McKinney. Data structures for statistical computing in python. In Stéfan van der Walt and Jarrod Millman, editors, *Proceedings of the 9th Python in Science Conference*, pages 51 – 56, 2010.

[MS15] Eric McGhee and Nicholas Stephanopoulos. Partisan gerrymandering and the efficiency gap. *82 University of Chicago Law Review*, 831, 2015. 70 pages. U of Chicago, Public Law working Paper No. 493. Available at SSRN: https://ssrn.com/abstract=2457468.

[Nag15] John F. Nagle. Measures of partisan bias for legislating fair elections. *Election Law Journal*, 14(4):346–360, December 2015.

[NCS18] Redistricting criteria, June 14, 2018. http://www.ncsl.org/research/redistricting/redistricting-criteria.aspx. Online; accessed July 24, 2018.

[NGCH90] Richard G. Niemi, Bernard Grofman, Carl Carlucci, and Thomas Hofeller. Measuring compactness and the role of a compactness standard in a test for partisan and racial gerrymandering. *J. of Pol.*, 52:1155–1181, Nov. 1990.

[Oli15] Travis E. Oliphant. *Guide to NumPy*. CreateSpace Independent Publishing Platform, USA, 2nd edition, 2015.

[PP91] Daniel D. Polsby and Robert D. Popper. The third criterion: Compactness as a procedural safeguard against partisan gerrymandering. *Yale Law & Policy Review*, 9(2):301–353, 1991.

[PVI17] Cook political report partisan voter index, 2017. Online; accessed July 26, 2018. https://www.cookpolitical.com/introducing-2017-cook-political-report-partisan-voter-index.

[QGI09] QGIS Development Team. *QGIS Geographic Information System*. Open Source Geospatial Foundation, 2009. http://qgis.osgeo.org.

[Red] Redacted.

[Reo61] Ernest C. Reock. A note: Measuring compactness as a requirement of legislative apportionment. *Midwest Journal of Political Science*, 5(1):70–74, 1961.

[Tok17] Daniel Tokaji. Restricting race-conscious redistricting, July 31, 2017. Online; accessed July 24, 2018. https://www.theregreview.org/2017/07/31/tokaji-restricting-race-conscious-redistricting/.

[Tuf73] Edward R. Tufte. The relationship between seats and votes in two-party systems. *American Political Science Review*, 67(3):540–54, 1973.

[Unia] United States Census Bureau. *2010 FIPS Codes for Counties and County Equivalent Entities*. https://www.census.gov/geo/reference/codes/cou.html.

[Unib] United States Census Bureau. *2012 TIGER/Line Shapefiles: Voting Districts*. https://www.census.gov/cgi-bin/geo/shapefiles/index.php?year=2012&layergroup=Voting+Districts.

[Viea] *Vieth* v. *Jubelirer*, 541 U.S. 267 (2004) III.A.

[Vieb] *Vieth* v. *Jubelirer,* 541 U.S. 267, 287 n.7 (2004) (plurality opinion).

[Wan16] Samuel S.-H. Wang. Three tests for practical evaluation of partisan gerrymandering. *Stanford Law Review*, 68:1263–1321, June 2016.

[War18a] Gregory S. Warrington. A comparison of gerrymandering metrics. 2018. http://arxiv.org/abs/1805.12572.

[War18b] Gregory S. Warrington. Jupyter notebook for analyzing packing and cracking. 2018. http://www.cems.uvm.edu/~gswarrin/gerrymandering/.

[War18c] Gregory S. Warrington. Quantifying gerrymandering using the vote distribution. *Election Law Journal*, 17(1), Mar 2018.

[Wes] *Wesberry* v. *Sanders*, 376 U.S. 1 (2016) 8.

[Whi] *Whitford* v. *Gill,* 218 F. Supp. 3d 837 (2016).

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