Massive Anti-de Sitter Gravity from String Theory

Constantin Bachas and Ioannis Lavdas

July 10, 2018

Laboratoire de Physique Theorique de l'Ecole Normale Superiéure,
PSL University, CNRS & Sorbonne Universités
24 rue Lhomond, 75231 Paris Cedex, France

Abstract

We study top-down embeddings of massive Anti-de Sitter (AdS) gravity in type-IIB string theory. The supergravity solutions have a AdS_4 fiber warped over a manifold M_6 whose shape resembles that of scottish bagpipes: The 'bag' is a conventional AdS_4-compactification manifold, while the 'pipes' are highly-curved semi-infinite Janus throats. Besides streamlining previous discussions of the problem, our main new result is a formula for the graviton mass which only depends on the effective gravitational coupling of the bag, and on the D3-brane charges and dilaton jumps of the Janus throats. We compare these embeddings to the Karch-Randall model and to other bottom-up proposals for massive-AdS-gravity, and we comment on their holographic interpretation. This is a companion paper to [1], where some closely-related bimetric models with pure $\mathrm{AdS}_5\times\mathrm{S}^5$ throats were analyzed.

1. Introduction

Efforts to endow the graviton with a tiny mass have a long history, going back to the work of Fierz and Pauli [2] and continuing unabated today – for reviews and references see [3][4][5]. The problem is of obvious theoretical interest, and could have far-reaching implications for cosmology. In one recent development it has been argued that a key obstruction to the graviton mass – the appearance of a Boulware-Deser ghost [6], can be removed in certain classical non-linear extensions of the Fierz-Pauli action [7][8](see also [9]). Questions, however, remain both in what concerns the consistency of such classical theories, and with regards to their range of validity if viewed as effective field theories around a given classical background. [1]

One may hope to answer such questions by embedding massive gravity in an ultraviolet-complete theory like string theory. In this paper we will consider four-dimensional Anti-de Sitter (AdS) solutions of IIB string theory in which the lowest spin-2 mode has a tiny mass $m_{\rm g}$. In discussions of massive gravity, AdS is known to be an 'easier case' since it does not suffer from some of the difficulties encountered in the Mikowski and de Sitter backgrounds. There is, in particular, no van Dam-Veltman-Zakharov discontinuity [17][18], and hence no need for the strong non-linearities known as Vainshtein screening [19]. It remains therefore to be seen whether our embeddings of massive AdS gravity carry any lessons for these other backgrounds.

The general idea behind the embeddings, due to Karch and Randall [20][21], is to 'locally localize' the graviton on an AdS_4 brane living in AdS_5 bulk. Using a thin-brane approximation these authors showed that if the ratio of AdS radii is small, $L_5/L_4 \ll 1$, the lightest graviton mode acquires a tiny mass. The proper string-theory realization of the idea had to wait for the derivation of exact solutions describing intersecting D3 , D5 and NS5-branes [22] - [27] . One key departure from the Karch-Randall model is the failure of the thin-brane approximation for the localizing brane, which is a D3-D5-NS5 bound state. As shown in [24] the AdS radius of this composite brane cannot be made parametrically larger than its thickness. As a result the Kaluza-Klein scale (beyond which any $4d$ description must break down) is L_4, and not L_5 as in ref. [21]. This is related to the familiar scale separation problem of AdS flux vacua, for a discussion see [28][29] [30]. [2]

The purpose of the present note is to derive an (almost universal) formula for the graviton mass in these string-theory embeddings. This is a follow-up paper to ref. [1] which analyzed closely related embeddings of bigravity models. Apart from the change in emphasis compared to [1], we will here also extend the results of this reference by allowing the dilaton to vary in the AdS_5 bulk which is deformed to the supersymmetric Janus background [22]. This modifies the graviton mass by a multiplicative factor that we will compute. Our formula for the graviton mass is derived on the gravity side. It is an interesting open problem to match it with a computation of the anomalous dimension of the energy-momentum tensor on the CFT side.

There have been two other proposals in the literature for realizing massive AdS gravity in string theory. They relied either on transparent boundary conditions in AdS [31][32], or on multi-trace deformations in CFT [33][34][35]. In these proposals the graviton mass is a quantum, one-loop effect. Although our embeddings could be possibly rephrased in these other frameworks by integrating out messenger degrees of freedom, they have the advantage of relying on proper classical solutions of $10d$ supergravity. They do not therefore suffer from some difficulties of the above proposals, namely non-locality of the worldsheet theory, or hard-to-control renormalization group flows [36][37][38]. The 'price to pay' is that the graviton mass is quantized and cannot be tuned continuously to zero. We

[1] For a very partial list of references raising or trying to address such questions see [10] - [16] and the above reviews.

[2] As explained in ref. [28] scale non-separation is unavoidable for solutions with a continuous R-symmetry, i.e. for the vast majority of known solutions of $10d$ supergravity including the ones discussed here. The authors of this reference outline possible ways out of the problem.

will return to this point towards the end.

This paper is organized as follows: In section 2 we recall why defect or interface CFT [40] - [43] is the appropriate holographic setup for Higgsing the AdS graviton. Holographic duality is not crucial to our later analysis, but it provides useful insights on the underlying mechanism. Section 3 explains the group theory of the Higgsing, i.e. the recombination of representations of the $\mathcal{N}=4$ superconformal algebra $\mathfrak{osp}(4|4)$ which is the symmetry of the relevant background solutions. This section can be skipped without affecting the flow of the paper.

Section 4 describes the qualitative characteristics of the supergravity solutions that lead to a small graviton mass. These solutions consist of AdS_4 fibers warped over six-dimensional manifolds with the shape of scottish bagpipes. The 'bag' describes a standard AdS_4 compactification, while the non-compact 'pipes' are highly-curved Janus throats. In section 5 we calculate, following [1], the graviton mass to leading order in the throat-to-bag size, and show that it only depends on few parameters of the solutions: the radius and dilaton variation in each throat, and the effective gravitational coupling of the bag solution. This is the main technical result of the paper. To extract its physical significance we reexpress it in three different ways. In section 6 we comment on the relation to bimetric and multi-trace models, while section 7 contains some concluding remarks. Explicit expressions for the metric and dilaton of the 'bagpipes' solutions are collected in the appendix.

2. Mass as holographic leakage

We begin our discussion of massive AdS gravity from the dual CFT side. This sheds instructive light on the Higgsing mechanism and motivates the construction of the dual supergravity solutions. Recall that holographic duality associates to any AdS_4 vacuum of string theory a three-dimensional conformal field theory (CFT_3). The AdS_4 graviton is mapped to the energy-momentum tensor T_{ab} of the CFT_3, and the mass ($m_{\rm g}$) of the former to the scaling dimension ($\Delta_{\rm g}$) of the latter via [39]

$$m_{\rm g}^2\, L_4^2 = \Delta_{\rm g}(\Delta_{\rm g}-3)\ , \tag{1}$$

where L_4 is the AdS_4 radius. [3] The operator T_{ab} and its tower of derivatives arrange themselves in a spin-2 highest-weight representation of the conformal algebra $\mathfrak{so}(2,3)$. Usually the energy-momentum tensor is conserved, $\partial^b T_{ab}=0$, so this representation must be short since it has three null descendant states. A simple algebraic computation then shows that T_{ab} must have canonical scaling dimension $\Delta_{\rm g}=3$, and the dual AdS_4 graviton is hence massless.

To obtain a massive graviton we must therefore allow $3d$ energy-momentum to 'leak out'. [4] There are two possibilities that are consistent with $\mathfrak{so}(2,3)$ symmetry:

(i) Couple the original theory to another $3d$ theory so that conformal symmetry is preserved. The coupling could be a double- trace deformation [33][34], or it could be mediated by messenger degrees of freedom [1]. If it is weak the dual low-energy string theory is a bimetric theory, with one graviton massless and the other obtaining a small mass;

(ii) Consider the original theory as a defect or boundary of some higher-dimensional theory, in the simplest case a CFT_4. The $3d$ energy-momentum can now leak out in the extra dimension $\partial^b T_{ab} = T_{a4}|_{\rm defect} \neq 0$. There is therefore now no shortening condition, and T_{ab} acquires an anomalous dimension [44], $\epsilon = \Delta_{\rm g}-3>0$ (unitarity requires that it be non-negative).

These two options are related – we will here focus on option (ii) which can be obtained as a limit of option (i). Since the graviton mass is proportional to the $3d$ energy-momentum leakage, we want the

[3] In warped compactifications both $m_{\rm g}$ and L_4 vary in the transverse space, but their product is constant – see below.
[4] Leakage of T_{ab} should not be confused with the transparent boundary conditions of [31] on the gravity side.

latter to be weak.[5] In principle this could be achieved by fine tuning a (nearly or exactly) marginal bulk-boundary coupling, but this is not the mechanism at work here. Weak leakage will be instead ensured by the scarcity of the bulk CFT_4 degrees of freedom, as compared to those of the boundary CFT_3. A consequence of this is that the Higgsing will not be a continuous process in these models, even though the graviton mass can be arbitrarily small.

Let us be now specific about the defect CFT. The natural candidate for the bulk CFT_4 is $\mathcal{N}=4$ super Yang-Mills with gauge group $SU(n)$ and coupling g_{YM}. Its half -BPS superconformal boundaries and interfaces have been analyzed by Gaiotto and Witten [45]. Half-maximal supersymmetry guarantees the stability of the solutions, and gives extra technical control, but it is not otherwise essential. The graviton mass, in particular, is not a protected quantity as we will see in a minute. Weak leakage of $3d$ energy-momentum could be achieved in the decoupling limit $g_{\mathrm{YM}} \to 0$, but this limit is singular. A better alternative is to insist that there are much fewer degrees of freedom in the bulk than on the boundary. We will indeed show in section 5 that the anomalous dimension of T_{ab} scales like $\epsilon \sim n^2/\tilde{F}_3$, where $\tilde{F}_3$ is the free energy on the 3-sphere which measures the boundary degrees of freedom [46].

3. Recombination of representations

Before moving to geometry, let us discuss the Higgsing from the point of view of representation theory. Let $D(\Delta, s)$ denote a unitary highest-weight representation of $\mathfrak{so}(2,3)$ with conformal primary of spin s and scaling dimension Δ. Massive gravitons belong to long representations of the algebra. The decomposition of a long spin-s representation at the unitarity threshold reads [31]

$$D(s+1+\epsilon; s) \quad \xrightarrow{\epsilon\to 0} \quad D(s+1; s) \oplus D(s+2; s-1)\ . \tag{2}$$

Thus the AdS_4 graviton ($s=2$) acquires a mass by eating a massive Goldstone vector. In the $10d$ supergravity this vector must be the combination of off-diagonal components of the metric and tensor fields that is dual to the CFT operator T_{a4}.

Since we will here deal with $\mathcal{N}=4$ backgrounds, fields and dual operators fit in representations of the larger superconformal algebra $\mathfrak{osp}(4|4)$. These have been all classified under mild assumptions [47][48]. In the notation of [48] (slightly retouched in [49]) the supersymmetric extension of the above decomposition reads

$$L[0]^{(0;0)}_{1+\epsilon} \quad \xrightarrow{\epsilon\to 0} \quad A_2[0]^{(0;0)}_1 \oplus B_1[0]^{(1;1)}_2\ , \tag{3}$$

where L denotes a long representation, A_i (B_i) a short representation that is marginally (absolutely) protected, and $[s]^{(j;j')}_{\Delta}$ denotes a superconformal primary with spin s, scaling dimension Δ and $\mathfrak{so}(4)$ R-symmetry quantum numbers $(j;j')$. The above decomposition (or recombination) describes the Higgsing of the $\mathcal{N}=4$ graviton multiplet in AdS_4. That this is at all possible is not automatic. For instance $\mathcal{N}=4$ supersymmetry forbids the Higgsing of ordinary gauge symmetries because conserved vector currents transform in absolutely protected representations of $\mathfrak{osp}(4|4)$ [50][51].

The bosonic field content of the above $\mathcal{N}=4$ multiplets is as follows:

$$A_2[0]^{(0;0)}_1 = [0]^{(0;0)}_1 \oplus [0]^{(0;0)}_2 \oplus [1]^{(1;0)\oplus(0;1)}_2 \oplus [2]^{(0;0)}_3 \oplus \text{fermions}\ , \tag{4}$$

$$\begin{aligned} B_1[0]^{(1;1)}_2 \ = \ & [0]^{(1;1)}_2 \oplus [1]^{(1;1)\oplus(1;0)\oplus(0;1)}_3 \oplus [0]^{(2;0)\oplus(0;2)\oplus(1;0)\oplus(0;1)\oplus(1;1)\oplus(0;0)}_3 \\ & \oplus [1]^{(0;0)\oplus(1;0)\oplus(0;1)}_4 \oplus [0]^{(1;1)\oplus(0;0)}_4 \oplus [0]^{(0;0)}_5 \oplus \text{fermions}\ . \end{aligned} \tag{5}$$

[5] In generic defect CFTs the defect-to-bulk leakage is very strong and $\Delta_{\mathrm{g}} - 3 \sim O(1)$. In such cases there is no hope of any effective $4d$ description on the gravity side.

The supergraviton multiplet A_2 has in addition to the spin-2 boson, six vectors and two scalar fields, making a total of 16 physical states. [6] The eaten Goldstone multiplet B_1 contains 112 physical bosonic states and as many fermions. These latter include massive spin-3/2 states which are not part of the spectrum of gauged $4d$ supergravity [49]. Higgsing with that much supersymmetry is thus necessarily a higher dimensional process.

4. Scottish bagpipes

We turn now to the gravity side of the Higgsing. The local form of all solutions of type-IIB supergravity with $\mathfrak{osp}(4|4)$ symmetry has been derived by D'Hoker *et al* [22][23] (see also [52][53] for earlier work). Global solutions and the detailed holographic dictionary have been worked out in [26][27][49]. All solutions are warped products of AdS$_4$ over a base manifold M$_6$,

$$ds^2_{10} = L_4^2(y)\, ds^2_{\rm AdS_4} + \sum_{i,j=1}^{6} g_{ij}(y)\, dy^i dy^j \ , \qquad (6)$$

where $ds^2_{\rm AdS_4}$ is the metric of the unit-radius Anti-de Sitter spacetime, $\{y^i\}$ are the coordinates of M$_6$ with metric g_{ij}, and $L_4(y)$ is the local radius of the AdS$_4$ fiber at a point y. The base manifold M$_6$ is itself the warped product of two 2-spheres over a Riemann surface. The complete AdS$_4\times$S$^2\times\hat{\rm S}^2$ fiber realizes the $\mathfrak{so}(2,3)\times\mathfrak{so}(4)\subset\mathfrak{osp}(4|4)$ bosonic symmetry of the backgrounds.

For the lightest $4d$ graviton to acquire mass, M$_6$ should not be a compact manifold. The manifolds that lead to a small graviton mass actually resemble six-dimensional Scottish bagpipes: they have one or more semi-infinite throats (the 'pipes') attached to a large central core (the 'bag') as illustrated in figure 1. The full $10d$ geometry of the pipes is AdS$_5\times$S^5, or its Janus generalization [23] in which the dilaton is also allowed to vary. A crucial technical remark [25][26] is that under certain mild conditions (existence of both NS5-brane and D5-brane charges in the bag) pipes can be shrank smoothly away [7]

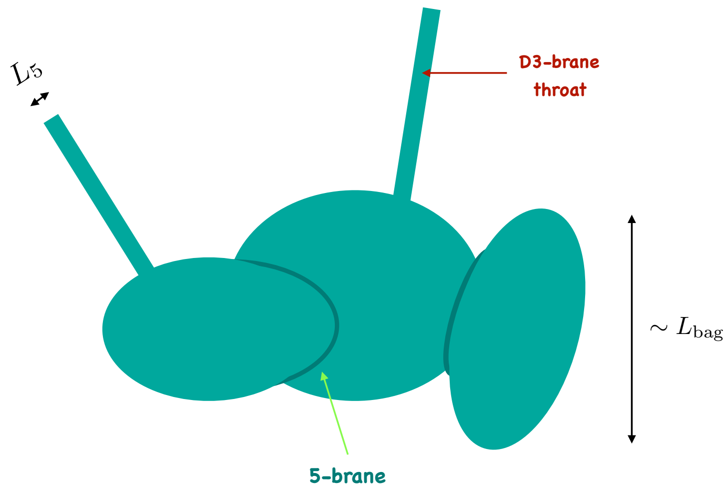

Figure 1: The 'bagpipes' manifold M$_6$ consists of semiinfinite pipes with cross-sectional radius L_5, attached to a compact bag of typical size $\sim L_{\rm bag}\gg L_5$. The dark curves on the bag depict 5-brane singularities. The AdS$_4$ scale factor diverges at infinity in the pipes so that the full $10d$ geometry asymptotes to (AdS$_5/\mathbb{Z}_2$)$\times$S^5.

[6]The CFT operators include T_{ab}, six conserved R-symmetry currents and two scalar operators. Taking into account the conservation laws this gives also a total of $25-9=16$ independent operators.

[7]Strictly-speaking this is possible in the supergravity approximation. In string theory the throat radii are quantized.

leaving behind simple coordinate singularities. Doing this reduces the bag to a compact manifold $\overline{\mathrm{M}}_6$, and $\mathrm{AdS}_4 \times_w \overline{\mathrm{M}}_6$ becomes a standard AdS_4 vacuum with a massless $4d$ graviton. Since we want the graviton to acquire a small mass, we keep the throat radii finite but much smaller than the characteristic bag size.

The exact metric and dilaton backgrounds of the solutions are summarized in the appendix. [8] The bag depends on a set of integer D5-, NS5- and D3-brane charges which can be arranged in two Young tableaux [26]. Most of these will play however no role here. The only relevant bag parameters are (i) an overall measure of its size $\sim L_{\rm bag}$ to be defined below, and (ii) the values of the dilaton at the entries of the throats. Note that the bag is a sort of 'composite Karch-Randall brane'. Tuning the available parameters to make it flat, as in ref. [20], makes it so thick that it ends up occupying (figuratively, not literally) most of space [24]. As stated above this is a facet of the scale non-separation problem of AdS vacua: L_4 is parametrically tied to the characteristic size of $\overline{\mathrm{M}}_6$.

The spectral problem for spin-2 excitations around any AdS_4 supergravity solution was set up in ref. [24] (generalizing mildly [54]). Interestingly this problem only depends on the Einstein-frame metric, and not on the scalar and flux background fields. Mass eigenstates factorize as $\psi(y)\,\chi_{\mu\nu}$, where $\chi_{\mu\nu}$ is an eigenfunction of the wave operator in AdS_4, i.e. $\mathcal{L}^{\rm AdS}_{(2)}\chi_{\mu\nu} = \lambda\chi_{\mu\nu}$ where $\mathcal{L}^{\rm AdS}_{(2)}$ is the (Lichnerowicz-Laplace) operator that acts on spin-2 (transverse-traceless) excitations, and the eigenvalue λ is related to the mass via

$$\lambda + 2 = m^2(y) L_4^2(y)\ . \tag{7}$$

Note that both the mass and the AdS_4 radius may vary as functions of the coordinates y^i, but their product is constant. It is this invariant squared mass that replaces the left-hand side of eq. (1) in warped (as opposed to direct-product) solutions.

The Kaluza-Klein mass spectrum is determined by the elliptic operator acting on the wavefunctions $\psi(y)$ on M_6 [24]

$$\mathcal{M}^2\psi := -\frac{L_4^{-2}}{\sqrt{g}}\,\partial_i\left(L_4^4\sqrt{g}g^{ij}\,\partial_j\,\psi\right) = (\lambda+2)\,\psi\ . \tag{8}$$

For direct-product solutions with constant L_4, $\mathcal{M}^2$ is simply the Laplace-Beltrami operator on M_6. To define the spectral problem we need also to provide a norm. With canonically-normalized fields in ten dimensions [9] the Kaluza-Klein reduction of the inner product reads [24]

$$\langle\psi_1|\psi_2\rangle = \int_{\mathrm{M}_6} d^6y\,\sqrt{g}\,L_4^2\psi_1^*\psi_2\ . \tag{9}$$

The mass-squared operator (8) is thus hermitean and non-negative, as expected.

To summarize this section, we are interested in the smallest eigenvalue of the above mass operator, $\mathcal{M}^2$, for manifolds consisting of a large compact bag ($\overline{\mathrm{M}}_6$) attached to one or more thin semi-infinite Janus throats. It actually turns out that the solutions studied here, whose CFT duals are $\mathcal{N}=4$ linear-quiver gauge theories, admit at most two semi-infinite throats. But more general backgrounds based on star-like quivers could have more throats. As will be clear, each throat makes a separate contribution to the squared mass of the graviton in the $L_5/L_{\rm bag} \ll 1$ limit.

[8] The supergravity solutions are actually singular at certain 2-cycles of M_6 that are wrapped by D5- and NS5-branes. In their vicinity higher-order stringy and loop corrections cannot be neglected, but fortunately they do not affect the computation of the graviton mass at leading order in $L_5/L_{\rm bag}$.

[9] In our convention ψ has dimensions of the $10d$ gravitational coupling $\kappa_{10} \sim [\text{mass}]^4$. An overall multiplicative constant in the norm is irrelevant as it drops out from the expression for the mass.

5. Mass from Janus throats

The general spectral problem defined in (8) and (9) is a difficult one. But we are only interested in the smallest eigenvalue given by the equivalent minimization problem

$$\lambda_0 + 2 = \ \mathbf{min}_\psi \left[\int_{\mathrm{M}_6} d^6y \sqrt{g}\, L_4^4\, (g^{ij}\partial_i\psi^\star\partial_j\psi) \right] \quad \text{with} \quad \int_{\mathrm{M}_6} d^6y \sqrt{g}\, L_4^2\, |\psi|^2 = 1 \ . \tag{10}$$

Here $\lambda_0 + 2 = \Delta_{\rm g}(\Delta_{\rm g} - 3)$ is the lowest eigenvalue of $\mathcal{M}^2$, and the expression in square brackets is $\langle\psi|\mathcal{M}^2|\psi\rangle$ after an integration by parts. If M_6 were replaced by the compact bag $\overline{\mathrm{M}}_6$ (obtained by truncating the pipes) the minimum would have been the constant wavefunction

$$\psi_0(y) = \left(\int_{\overline{\mathrm{M}}_6} d^6y \sqrt{g}\, L_4^2 \right)^{-1/2} := \psi_{\rm bag} \tag{11}$$

corresponding to a massless graviton. But the infinite pipes make the constant ψ non-normalizable. Indeed, $\sqrt{g}\, L_4^2$ reaches a minimum value L_5^8 inside the pipes, then blows up at infinity where the $10d$ geometry asymptotes to that of (half) the boundary of AdS_5. This is explained in the appendix and illustrated in figure 2 . Normalizable wavefunctions should therefore go to zero inside the pipes. Furthermore, it is clear from eq. (10) that in order to minimize the mass ψ should go to zero in the region of minimal $\sqrt{g}L_4^4$ where gradients have lower cost, as shown in fig. 2 .

To make the argument precise, separate the manifold M_6 in three parts: (I) the bag, (II) the infinite throats, and (III) the matching regions where throats are attached to the bag. Minimizing $\mathcal{M}^2$ in region (I) sets ψ to a constant, so the bag does not contribute to the graviton mass. On the other

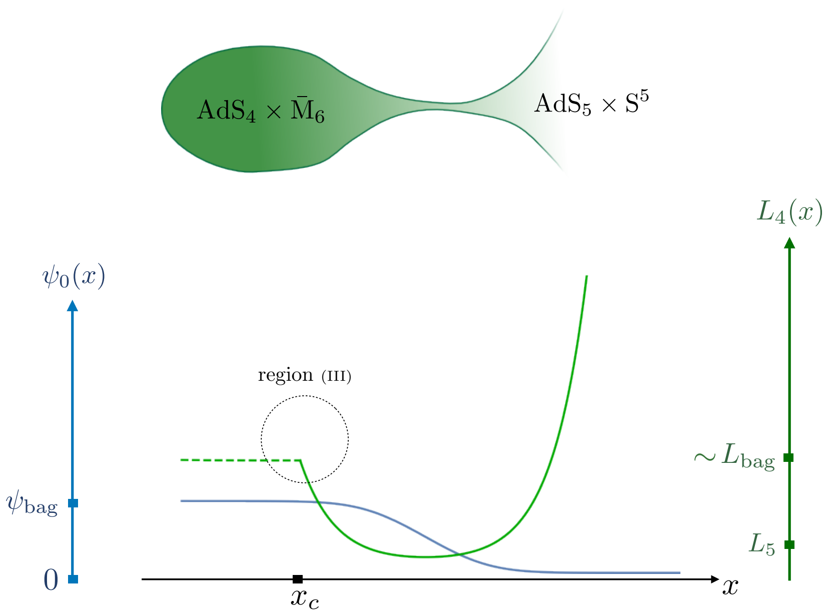

Figure 2: Schematic drawing of the AdS_4 radius L_4 (green curve) and of the graviton wavefunction ψ_0 (blue curve) as functions of the coordinate x of the Janus throat. The radius reaches a minimum value of L_5 inside the throat, and grows as $L_5 \cosh x$ on either side where the geometry asymptotes to $\mathrm{AdS}_5/\mathbb{Z}_2{\times}\mathrm{S}^5$. The left half of the throat is cut-off by the 'bag' at a characteristic radius $\sim L_{\rm bag} \gg L_5$. The graviton wavefunction approaches a constant in this region, and vanishes exponentially at infinity. Since the matching region (III) contributes neither to the norm nor to the mass, it can be shrunk for our purposes to a point.

hand, at leading order in $L_5/L_{\rm bag}$ only the bag contributes to the norm of ψ. The reason is that $\sqrt{g}L_4^2$ decreases exponentially fast in the matching region, and ψ vanishes exponentially fast in the throat as will be shown in a minute. This fixes the constant value $\psi_0 \simeq \psi_{\rm bag}$ in region (I).

The region of minimal $\sqrt{g}L_4^4$, on the other hand, where ψ_0 can vary with minimal mass cost, lies deep inside the throat regions. Only the throats will therefore contribute to the graviton mass at this leading order, an assumption whose validity will be again verified a posteriori.

In summary, the leading-order contribution to the norm comes from the bag, while the leading-order contribution to the mass comes from the bottom of the throat where $\sqrt{g}L_4^4$ is minimal. The matching region (III) contributes to neither and can be neglected. We may thus reformulate the problem as a variational problem in the Janus geometry:

$$\lambda_0 + 2 \simeq \ \mathbf{min}_\psi \left[\int_{\rm throats} d^6y\,\sqrt{g}\,L_4^4 g^{ij}\partial_i\psi^\star\partial_j\psi \right] \qquad \text{with} \quad \psi \to \begin{cases} \psi_{\rm bag} & \text{in matching region,} \\ 0 & \text{at infinity .} \end{cases} \tag{12}$$

The only residual dependence on M_6 (viz. on the composite NS5-D5-D3 brane) is via the boundary value $\psi_{\rm bag}$ whose physical meaning will soon be made clear.

The $\mathcal{N}=4$ supersymmetric Janus solution [23] depends on two parameters, the radius L_5 and the dilaton variation $\delta\phi$. Like all other solutions in this class it has the fibered form

$$ds^2_{10} = L_4^2 ds^2_{\mathrm{AdS}_4} + ds^2_{\mathrm{M}_6} \qquad \text{with} \qquad ds^2_{\mathrm{M}_6} = f^2 ds^2_{\mathrm{S}^2} + \hat{f}^2 ds^2_{\hat{\mathrm{S}}^2} + 4\rho^2 dz d\bar{z}\ . \tag{13}$$

The scale factors $L_4, f, \hat{f}, \rho$ depend on the complex coordinate z that parametrizes the infinite strip. We write $z = x + i\tau$ with $\tau \in [0, \pi/2]$. The metric factors and dilaton are given in the appendix. Here we only need the combination that enters in the square brackets in (12). Things simplify actually further because the spin-2 eigenfunctions in the Janus geometry factorize into spherical harmonics on the 2-spheres, and separate functions of x and τ, and the lightest mode is only function of x [24]. Integrating over the 2-spheres and τ gives [10]

$$\lambda_0 + 2 \ = \ \mathbf{min}_\psi \left[\frac{\pi^3}{4} L_5^8 \int_{x_c}^\infty dx\, \mathcal{G}(x) \left(\frac{d\psi}{dx}\right)^2 \right] \qquad \text{with} \quad \psi(x) \to \begin{cases} \psi_{\rm bag} & \text{at } x = x_c, \\ 0 & \text{at } x = \infty\ , \end{cases} \tag{14}$$

where the function $\mathcal{G}(x)$, computed in the appendix, reads

$$\mathcal{G}(x) := \left(\frac{\cosh 2x + \cosh\delta\phi}{\cosh\delta\phi} \right)^2 . \tag{15}$$

We have cutoff the integral at some large negative value x_c, at the boundary of the matching region. The value of x_c will drop out and could be replaced by $-\infty$, its only role is to remind us that ψ would have been a non-normalizable mode in the complete Janus geometry.

The variational problem (14) can be easily solved,

$$\frac{d}{dx}(\mathcal{G}\frac{d\psi_0}{dx}) = 0 \quad \Longrightarrow \quad \psi_0(x) = c_1 + c_2 \int_0^x \frac{dx'}{\mathcal{G}(x')} \tag{16}$$

where c_1, c_2 are integration constants. We can perform the integral analytically with the result

$$I(x,a) := \int_0^x \frac{a^2\, dx'}{(\cosh 2x' + a)^2} =$$

$$= \frac{a^3}{2(a^2-1)^{3/2}} \log\left[\frac{\sqrt{a+1} + \sqrt{a-1}\,\tanh x}{\sqrt{a+1} - \sqrt{a-1}\,\tanh x} \right] - \frac{a^2}{(a^2-1)} \frac{\tanh x}{[(a+1) - (a-1)\tanh^2 x]}\ . \tag{17}$$

[10] When $\delta\phi = 0$ the 2-spheres and the coordinate τ make up the 5-sphere in $\mathrm{AdS}_5\times\mathrm{S}^5$, and $\mathcal{G}(x) = 4\cosh^4 x$.

We here set $\cosh\delta\phi = a$ and chose the lower integration limit so that I is an odd function of x. Fixing c_1,c_2 so as to satisfy the boundary conditions (14) gives finally the graviton wavefunction in the throat

$$\psi_0(x,a) \;\simeq\; \frac{1}{2}\psi_{\rm bag}\left[1-\frac{I(x,a)}{I(\infty,a)}\right] \,. \tag{18}$$

Note that I approaches its limiting values exponentially, so $\psi_0(x_c)\simeq\psi_{\rm bag}$ up to exponentially small corrections. Furthermore at $x\to+\infty$, $\psi_0 = O(e^{-2x})$ as required for the norm to be finite. The reader can now check that this contribution to the norm is parametrically smaller than that of the bag, and can be neglected as claimed earlier.

Plugging the above wavefunction in the expression (14) leads to the graviton mass. Note that ψ_0 obeys eq. (16), so the integrand is a total derivative and one finds

$$\lambda_0+2 \;=\; \frac{\pi^3}{4}L_5^8\int_{x_c}^{\infty}dx\,\mathcal{G}\left(\frac{d\psi_0}{dx}\right)^2 \;=\; \frac{\pi^3}{4}L_5^8\left[\mathcal{G}\,\frac{d\psi_0}{dx}\psi_0\right]_{x_c}^{\infty} \,. \tag{19}$$

Since $\mathcal{G}(x)d\psi_0/dx = -\psi_{\rm bag}/2I(\infty,a)$ and $[\psi_0]_{x_c}^{\infty} = -\psi_{\rm bag}$, we finally get

$$\lambda_0+2 \;=\; \frac{\pi^3}{8}L_5^8\,\psi_{\rm bag}^2\,/I(\infty,a) \;=\; \frac{3\pi^3}{4}L_5^8\,\psi_{\rm bag}^2\,J(a)\ , \tag{20}$$

where we have introduced the Janus correction factor $J(a)$,

$$J(a)^{-1} := 6\,I(\infty,a) = \frac{3a^3}{(a^2-1)^{3/2}}\,\log\left[a+\sqrt{a^2-1}\right]-\frac{3a^2}{(a^2-1)}\ . \tag{21}$$

As $a=\cosh\delta\phi$ ranges from 1 to ∞, $J(a)$ decreases monotonically from 1 to 0, see figure 3. The function is normalized so as to drop out for $AdS_5\times S^5$ throats, while more generally it has the effect of reducing the graviton mass.

This can be understood intuitively as follows: $\delta\phi$ is the difference between the value of the dilaton at the entry of the throat and its value at infinity. The former is fixed by the bag (see the appendix). The latter is a free parameter that determines the coupling constant $g_{\rm YM}$ of the dual $4d$, $\mathcal{N}=4$ super Yang-Mills theory. Taking $g_{\rm YM}$ to zero (and hence $|\delta\phi|\to\infty$) decouples the bulk CFT from the defect, restores conservation of T_{ab} and sends the graviton mass to zero. The same is true, by S-duality, if $g_{\rm YM}$ is taken to infinity – it is the bulk magnetic theory now that decouples manifestly. These limits are however singular. Not only does supergravity break eventually down, but also the spectrum in the Janus throat becomes quasi-continuous [24] invalidating any effective $4d$ description.

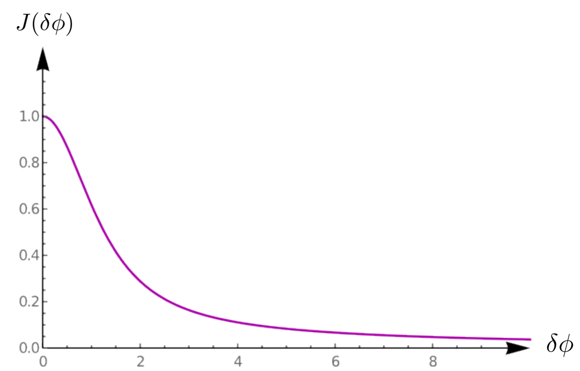

Figure 3: The Janus correction function, J, defined in equation (21).

Eqns. (20), (21) are the main result of this paper. To extract their physical meaning we will rewrite them in three different ways. We first express $\psi_{\rm bag}$, defined in eq. (11), in terms of geometric data,

$$\psi_{\rm bag}^{-2} \;=\; \int_{\bar{\rm M}_6} \sqrt{g}\, L_4^2 \;=\; V_6\, \langle L_4^2\rangle_{\rm bag} \tag{22}$$

where V_6 is the volume of the bag, and $\langle L_4^2\rangle_{\rm bag}$ the average AdS$_4$ squared radius (which is finite after truncating away the throats). The contribution (20) to the graviton mass thus reads [11]

$$\underline{\text{geometric}}: \qquad m_{\rm g}^2\, L_4^2 \;=\; \frac{3\pi^3 L_5^8}{4V_6\langle L_4^2\rangle_{\rm bag}} \times J(\cosh\delta\phi)\ . \tag{23}$$

In the conventional AdS$_4 \times \overline{M}_6$ vacua the AdS radius is of the same order as the Kaluza-Klein scale (this is again the scale non-separation problem), so $(V_6)^{1/3} \sim L_{\rm bag}^2 \sim \langle L_4^2\rangle_{\rm bag}$. Thus $m_{\rm g}^2$ is suppressed by the eighth power of L_5/L_4, as compared to the second power which is the result in the thin-brane Karch-Randall model [55].

For a different rewriting, we use the relation between the compactification volume and the $4d$ effective gravitational coupling, $\kappa_{10}^2 = V_6\,\kappa_4^2$ where κ_{10} is the coupling in ten dimensions. Using also the relations $2\kappa_{10}^2 = (2\pi)^7\alpha'^{\,4}\lambda_s^2$ and $L_5^4 = 4\pi\alpha'^{\,2}\lambda_s\, n$, where α' is the Regge slope, λ_s the string coupling constant (which can be absorbed in the definition of the Einstein-frame metric) and n the quantized D3-brane charge of the throat [56] we get

$$\underline{\text{gravitational}}: \qquad m_{\rm g}^2\, L_4^2 \;=\; \frac{3\,n^2}{16\pi^2}\frac{\kappa_4^2}{\langle L_4^2\rangle_{\rm bag}} \times J(\cosh\delta\phi)\ . \tag{24}$$

This rewriting in terms of four-dimensional parameters brings forth two important points. First, the graviton mass is quantized so the Higgsing cannot be continuous [1]. Of course, to trust the supergravity approximation we need $n \gg 1$. The mass does vanish with two continuous parameters, κ_4 and $(\cosh\delta\phi)^{-1}$, which can be taken to zero, but these limits are singular.

The second remark is that (24) has the same parametric form as the result found in refs. [31][32]. In these references the mass is a quantum effect arising from integrating out matter fields, while in our case it arose from a standard small-fluctuation analysis around a classical supergravity solution. We will comment on this again in the next section.

One final rewriting sheds further light on the dual CFT side. We argued earlier that what ensures a weak $3d$ energy-momentum leakage is the scarcity of the bulk-CFT$_4$ degrees freedom as compared to those on the $3d$ defect. A measure of CFT degrees of freedom for even dimensions is the familiar Weyl-anomaly coefficient a, which for $\mathcal{N}=4$ super Yang Mills with gauge group $U(n)$ is equal to n^2 [in the normalization in which the contribution of a scalar to a is 1/90]. The analog of a in three dimensions is (minus) the free energy on S^3, which is related to the vacuum entanglement entropy across a spatial circle [57] - [60]. These measures have been unified by Giombi and Klebanov in a generalized free energy [46] defined (via dimensional regularization) for arbitrary d, $\tilde{F}_d = \sin(\pi d/2)\log Z({\rm S}^d)$. For even dimensions $\tilde{F}_d = (-)^{d/2} a\pi/2$, while in three dimensions a holographic calculation gives $\tilde{F}_3 = 4\pi^2\langle L_4^2\rangle/\kappa_4^2$ (see for example section 4.2 of [61]). [12] Combining everything we find

$$\underline{\text{defect}}\ \underline{\text{CFT}}: \qquad 3(\Delta_{\rm g}-3) \simeq m_{\rm g}^2\, L_4^2 \;=\; \frac{6\pi^3\tilde{F}_4}{\tilde{F}_3} \times J(\cosh\delta\phi) \tag{25}$$

where $\Delta_{\rm g}$ is the anomalous dimension of the almost conserved $3d$ energy-momentum tensor. This way of expressing the result makes it clear that the expansion parameter is the ratio of generalized free energies between the bulk and the defect CFT.

[11] If there are more than one throats, one adds up their contributions on the right-hand-side.

[12] The CFT calculation has been performed using localization in refs. [62][63].

6. Bimetric and double-trace models

In ref. [1] we analyzed solutions with a highly-curved $AdS_5\times S^5$ throat [13] capped-off at both of its ends by bags of much larger size. The low-energy theory is in this case a two-graviton theory. This is a concrete realization of the idea of 'Weakly Coupled Worlds' [64] in which two or more Universes endowed with separate metrics are coupled through the mixing of their gravitational fields. It is well known that massive gravity can be obtained from bigravity in a decoupling limit, and this is also true for our solutions. Before exhibiting this decoupling limit, we will first generalize the analysis of [1] from $AdS_5\times S^5$ to Janus throats.

The manifold M_6 now consists of a Janus throat capped-off on both sides by two bags, $\overline{M}_6$ and $\overline{M}_6{}'$. For economy of notation we introduce the parameters

$$v := \int_{\overline{M}_6} \sqrt{g}\, L_4^2 \qquad \text{and} \qquad v' := \int_{\overline{M}_6'} \sqrt{g}\, L_4^2 \ .$$

Note that v is just a short-hand for the parameter $V_6\langle L_4^2\rangle_{\rm bag} = \psi_{\rm bag}^{-1/2}$ of the previous section. Using the inner product $\langle\psi_1|\psi_2\rangle = \int_{M_6}\sqrt{g}\, L_4^2\, \psi_1^*\psi_2$ one finds easily two orthogonal, low-lying spin-2 states. A massless state with constant wavefunction throughout M_6 (which is normalizable because M_6 is now compact), and a massive state whose wavefunction is approximately constant in the bags,

$$\psi_0(x) \simeq (v+v')^{-1/2} \times \begin{cases} \sqrt{v'/v} & \text{in } \overline{M}_6, \\ -\sqrt{v/v'} & \text{in } \overline{M}_6' \ . \end{cases} \tag{26}$$

Since the throat makes a subleading contribution to the inner product, the above wavefunction is clearly orthogonal to the constant one, i.e. to the wavefunction of the massless graviton. This second mode is necessarily massive because ψ_0 is forced to vary inside the Janus throat in order to extrapolate between the above values at the exits.

One can now repeat almost verbatim the calculation of the previous section. The wavefunction in the Janus throat with the new boundary conditions reads

$$\psi_0 \ \simeq \ \frac{1}{2\sqrt{vv'(v-v')}} \left[(v'-v) - (v'+v)\frac{I(x,a)}{I(\infty,a)} \right] \ , \tag{27}$$

where $I(x,a)$ has been defined in eq. (17). Inserting the above wavefunction in (19), and reexpressing v and v' in terms of radii and effective couplings gives

$$m_{\rm g}^2\, L_4^2 \ = \ \frac{3\, n^2}{16\pi^2} \left[\frac{\kappa_4^2}{\langle L_4^2\rangle_{\rm bag}} + \frac{\kappa_4^{2\,\prime}}{\langle L_4^2\rangle_{\rm bag'}} \right] \times J(\cosh\delta\phi) \ . \tag{28}$$

This agrees with the result derived in [1] for pure $AdS_5\times S^5$ throats for which $J=1$. [14] It also reduces to our formula of the previous section in the decoupling limit $v'\to\infty$, i.e. $\kappa_4'\to 0$ or equivalently $\langle L_4^2\rangle_{\rm bag'}\to\infty$. In this limit the massless graviton has vanishing wavefunction and decouples, whereas ψ_0 is concentrated entirely in the (unprimed) bag $\overline{M}_6$ and in the throat.

From the perspective of the dual field theory, these bigravity solutions are not $4d$ defect CFTs, but rather $3d$ CFTs of a special kind. They are superconformal gauge theories based on linear quivers with a low-rank 'weak' node [1]. Removing this node breaks the quiver into two disjoint quivers. One could in principle integrate out the scarce messenger fields, thereby generating multitrace couplings

[13] The restriction to $AdS_5\times S^5$ throats is valid for identical bags, or more generally for bags with the same value of the dilaton at the two throat entries. This was implicitly assumed in ref. [1].

[14] In this reference we worked in units of $L_4 = 1$ and absorbed $\langle L_4^2\rangle$ in the definition of κ_4^2. We thank Thibault Damour for suggesting that we reestablish explicitly all units.

between disjoint theories in the spirit of [33][34]. In contrast with these references, the couplings are however non-local (they are generated by massless messengers) and exactly scale invariant (the AdS_4 symmetry is manifest). Conversely, integrating back in the messenger fields restores the interpretation of the multitrace couplings in terms of a classical supergravity background, and resolves the conflicts with string-theory locality discussed in refs. [36][37].

Similar comments apply to the relation of our models with the transparent boundary conditions of [31][32]. These could conceivably mimic the effects of the semi-infinite throats, but they are obscuring the issues of locality and scale invariance. It is nevertheless interesting that they lead to the same parametric dependence of $m_{\rm g}$ on the effective gravitational coupling κ_4.

7. Final remarks

As these top-down embeddings demonstrate, massive AdS_4 gravity is part of the string-theory landscape. String theory is believed to be a consistent theory, so we expect the effective $4d$ low-energy gravity to be free of any pathologies. We have seen that the effective theory must break down at the AdS radius L_4, which is comparable to the Kaluza-Klein scale, a feature that is related to the scale non-separation problem and could be generic. This still leaves a range of energies, $m_{\rm g} < E < L_4^{-1}$, in which to try to compare with effective actions such as those of refs. [7][9]. A technical complication is that string theory is rarely minimal – the low-energy theory would have extra fields in addition to the massive graviton.

Massive Minkowski gravity is harder to embedd and could possibly lie in swampland. One way to see the difficulty is as follows: a key feature of the Karch-Randall model is the existence of a local minimum of the AdS scale factor L_4. The existence of a minimum seems however to be in tension with the holographic c-theorem when the AdS_4 fiber is replaced by Minkowski [65][66][20]. It is an interesting question whether this obstruction can be somehow relaxed.

In a different direction one can look for massive-gravity and bimetric models in other dimensions and/or with different amounts of supersymmetry. Many exact AdS_D solutions with $D > 4$ and half-maximal supersymmetry are known by now, for instance [67] for AdS_7, [68][69][70] for AdS_6, and [71] for AdS_5 (for a review and more references see also [72]). Some cases can be a priori excluded. A prime example is $6d$ $\mathcal{N} = 1$, where the stress tensor belongs to a protected B-series multiplet [48] and cannot acquire an anomalous dimension. Thus massive AdS_7 supergravity is a priori excluded. [15] The stress tensor multiplet is also absolutely protected for $\mathcal{N} = 1$ in $5d$, and for more-than-half-maximal supersymmetry ($\mathcal{N} > 2$ in $4d$ and $\mathcal{N} > 4$ in $3d$). This is consistent with the fact that there exist no candidate defect CFTs with so many unbroken supersymmetries. [16] A situation with no protection is $\mathcal{N} = 2$ AdS_5. It would be interesting to search for embeddings of massive AdS supergravity or bigravity in this case. It would be even more interesting to search for non-supersymmetric embeddings that allow a Kaluza-Klein cutoff $\gg L_4$ along the lines of [28].

Aknowledgement: We have benefited from discussions or email exchanges with Laura Bernard, Ali Chamseddine, Cedric Deffayet, Eric D'Hoker, Gregory Gabadadze, Slava Mukhanov, Alessandro Tomassielo and Christoph Uhlemann. C.B. aknowledges the hospitality of the Mani L. Bhaumik Institute of UCLA during the last stage of this work.

[15] The solutions of [67] resemble those studied here – they are dual to gauge theories based on linear quivers. Anomaly cancellation forces however these latter to be *balanced* quivers. This implies that the gauge-group rank is a concave function along the quiver, which forbids the low-rank 'bridge' nodes of [1].

[16] The relation of anomalous dimensions with charge leakage also implies that whenever the stress-tensor multiplet is protected, so are all conserved-current multiplets. Indeed in a local QFT no charge can leak out without carrying also some energy. Inspection of all superconformal representations in [48] confirms the validity of this assertion.

Appendix

Here we collect some formulae on the exact type-IIB supergravity solutions that we use. For more extensive descriptions of this class of solutions see refs. [22][23] and refs. [26][27].

All solutions of type-IIB supergravity preserving $\mathcal{N}=4$ AdS_4 supersymmetries have the form of a fibration over a Riemann-surface Σ,

$$ds^2_{10} \;=\; L_4^2 ds^2_{\mathrm{AdS}_4} + f^2 ds^2_{\mathrm{S}^2} + \hat{f}^2 ds^2_{\hat{\mathrm{S}}^2} + 4\rho^2 dz d\bar{z} \tag{29}$$

where the scale factors of the three (pseudo)spheres depend only on the coordinate $z = x + i\tau$ of the Riemann surface. For us this latter is the infinite strip, $0 \leq \tau \leq \pi/2$. Solutions based on the annulus do not allow the attachment of semi-infinite Janus throats, and there are no known solutions based on higher-genus Riemann surfaces. The six-dimensional manifold $\mathrm{S}^2 \times \hat{\mathrm{S}}^2 \times \Sigma$ is called M_6 in the main text (or $\overline{\mathrm{M}}_6$ when it is compact).

The expressions for the metric factors and the dilaton, ϕ, depend on a pair of harmonic functions

$$L_4^8 = 16\,\frac{\mathcal{U}\hat{\mathcal{U}}}{W^2}\ , \quad f^8 = 16\,h^8\,\frac{\hat{\mathcal{U}}W^2}{\mathcal{U}^3}\ , \quad \hat{f}^8 = 16\,\hat{h}^8\,\frac{\mathcal{U}W^2}{\hat{\mathcal{U}}^3}\ , \quad \rho^8 = \frac{\mathcal{U}\hat{\mathcal{U}}W^2}{h^4\hat{h}^4}\ , \quad e^{4\phi} = \frac{\hat{\mathcal{U}}}{\mathcal{U}}\ , \tag{30}$$

where h and $\hat{h}$ are the two harmonic functions, from which one computes [17]

$$\mathcal{U} = 2h\hat{h}|\partial_z h|^2 - h^2 W\ , \quad \hat{\mathcal{U}} = 2h\hat{h}|\partial_z \hat{h}|^2 - \hat{h}^2 W\ , \quad \text{and} \quad W = \partial_z\partial_{\bar{z}}(h\hat{h})\ . \tag{31}$$

The solutions also have 5-form and 3-form fluxes whose explicit form we won't need.

After imposing regularity conditions, the most general solutions with the strip as base manifold are given by the following choice for the harmonic functions:

$$\begin{aligned} h &= -i\alpha\sinh(z-\beta) - \sum_{a=1}^{N}\gamma_a \log\tanh\left(\frac{i\pi}{4} - \frac{z}{2} + \frac{\delta_a}{2}\right) \;+\; c.c.\ , \\ \hat{h} &= \hat{\alpha}\cosh(z-\hat{\beta}) - \sum_{b=1}^{\hat{N}}\hat{\gamma}_b \log\tanh\left(\frac{z}{2} - \frac{\hat{\delta}_b}{2}\right) \;+\; c.c.\ . \end{aligned} \tag{32}$$

All parameters in these expressions are real. Furthermore the parameters $\{\alpha, \gamma_a\}$ and $\{\hat{\alpha}, \hat{\gamma}_b\}$ must all have the same sign which can be chosen positive – the harmonic functions are then positive everywhere inside the strip. The logarithmic singularities on the upper (lower) boundary of the strip correspond to D5-brane (NS5-brane) sources wrapping 2-sphere cycles. The regions $z \to \pm\infty$, on the other hand, correspond to semi-infinite D3-brane throats with the geometry of the supersymmetric Janus solution.

The pure Janus solution is found by setting $\gamma_a = \hat{\gamma}_b = 0$. Its radius is $L_5 = 2(\alpha\hat{\alpha}\cosh\delta\phi)^{1/4}$, where $\delta\phi = \beta - \hat{\beta}$ is the change of the dilaton as x goes from $-\infty$ to ∞. By shifting the origin of the x axis one may choose $\beta = -\hat{\beta} = \delta\phi/2$. The combination of scale factors that enters in the expression (12) for the squared mass is

$$\sqrt{g}\,L_4^4\,\rho^{-2} = L_4^4\,f^2\hat{f}^2 = 16\,h^2\hat{h}^2\ . \tag{33}$$

Inserting the Janus harmonic functions gives $16\,h^2\hat{h}^2 = (L_5^8/16)\sin^2(2\tau)\,\mathcal{G}(x)$ where the function $\mathcal{G}$ is the one defined in eq. (15). Using finally the τ-independence of ψ_0 [24], and integrating over the two 2-spheres and over τ, leads to the expression (14) for the mass.

[17] In the previous literature $h, \hat{h}$ are denoted h_1, h_2 (and similarly for the associated functions $\mathcal{U}$,$\hat{\mathcal{U}}$, the 2-spheres S^2, $\hat{\mathrm{S}}^2$, and the parameters $\alpha, \hat{\alpha}$ and $\beta, \hat{\beta}$). We think that the benefit of making S-duality (or mirror symmetry) manifest outweighs the risk of confusion from this change of notation .

A typical 'scottish bagpipes' manifolds M_6 has $\alpha, \hat{\alpha} \ll \gamma_a, \hat{\gamma}_b$ and all other parameters $\sim O(1)$. The bag $\overline{\mathrm{M}}_6$ is obtained in the limit $\alpha, \hat{\alpha} = 0$ which truncates away the asymptotic $\mathrm{AdS}_5/\mathbb{Z}_2 \times \mathrm{S}^5$ regions of the Janus throats. It can be checked that $x = \pm\infty$ become simple coordinate singularities in this limit, and that $\overline{\mathrm{M}}_6$ is compact [26]. The product $\alpha\hat{\alpha}$ that controls the graviton Higgsing is a continuous parameter in supergravity, but it is quantized in string theory where it is related to the D3-brane charges of the throats (see below).

The detailed shape of the bag depends on the parameters $\{\gamma_a, \hat{\gamma}_b, \delta_a, \hat{\delta}_b\}$ which are in one-to-one corerspondence with the NS5-, D5- and D3-brane charges [26] of the 'fat Karch-Randall brane'. In general the bag has many different length scales, but a typical size is $L_{\mathrm{bag}} \sim (\sum \gamma_a \sum \hat{\gamma}_b)^{1/4}$ where the sum runs over 5-brane singularities in some $\delta z \sim O(1)$ region of the strip. In order to stabilize the bag one needs both NS5-brane and D5-brane charges, or else the dilaton runs away and the solution is singular.

Most of the details of the bag play no role in the calculation of the graviton mass at leading order. The only relevant parameters are ψ_{bag} defined in eq.(11), and the four combinations

$$\gamma_\pm := \sum_{a=1}^{N} \gamma_a e^{\pm\delta_a} \qquad \text{and} \qquad \hat{\gamma}_\pm := \sum_{b=1}^{\hat{N}} \hat{\gamma}_b \, e^{\pm\hat{\delta}_b} \ . \tag{34}$$

To see why let us divide the strip in several regions. For $x \sim O(1)$ we may neglect $\alpha, \hat{\alpha}$ altogether. This is the bag region (I) of section 5. For x sufficiently large, on the other hand, we can expand the log tanh functions, and approximate h and $\hat{h}$ as follows

$$2h \simeq -i(\alpha e^{z-\beta} - 4\gamma_+ e^{-z}) + c.c. \ , \qquad 2\hat{h} \simeq (\hat{\alpha} e^{z-\hat{\beta}} + 4\hat{\gamma}_+ e^{-z}) + c.c. \ . \tag{35}$$

This is the throat region (II) of section 5, where the background is the supersymmetric Janus solution. Expressing (35) in terms of hyperbolic sine and cosine leads to the Janus-parameter identification

$$L_5^{(+)} = 2\sqrt{2}\left[\alpha\hat{\alpha}\gamma_+\hat{\gamma}_+ e^{-(\beta+\hat{\beta})}\cosh^2\delta\phi^{(+)}\right]^{1/8} \quad \text{and} \quad e^{2\delta\phi^{(+)}} = \frac{\hat{\alpha}\gamma_+}{\alpha\hat{\gamma}_+}\, e^{\beta-\hat{\beta}} \ . \tag{36}$$

Similar formulae hold for the parameters $L_5^{(-)}, \delta\phi^{(-)}$ of the other throat, in the region $x \to -\infty$. The only thing to retain from this discussion is that for a given bag (i.e. given $\gamma_\pm, \hat{\gamma}_\pm$) we have enough free parameters to choose the radii and dilaton jumps of the two asymptotic Janus throats at will.

References

[1] C. Bachas and I. Lavdas, "Quantum Gates to other Universes," Fortsch. Phys. **66** (2018) no.2, 1700096 [arXiv:1711.11372 [hep-th]].

[2] M. Fierz and W. Pauli, "On relativistic wave equations for particles of arbitrary spin in an electromagnetic field," Proc. Roy. Soc. Lond. A **173** (1939) 211.

[3] K. Hinterbichler, "Theoretical Aspects of Massive Gravity," Rev. Mod. Phys. **84** (2012) 671 [arXiv:1105.3735 [hep-th]].

[4] C. de Rham, "Massive Gravity," Living Rev. Rel. **17** (2014) 7 [arXiv:1401.4173 [hep-th]].

[5] A. Schmidt-May and M. von Strauss, "Recent developments in bimetric theory," J. Phys. A **49** (2016) no.18, 183001 [arXiv:1512.00021 [hep-th]].

[6] D. G. Boulware and S. Deser, "Can gravitation have a finite range?," Phys. Rev. D **6** (1972) 3368.

[7] C. de Rham and G. Gabadadze, "Generalization of the Fierz-Pauli Action," Phys. Rev. D **82** (2010) 044020 [arXiv:1007.0443 [hep-th]] ; C. de Rham, G. Gabadadze and A. J. Tolley, "Resummation of Massive Gravity," Phys. Rev. Lett. **106** (2011) 231101 [arXiv:1011.1232 [hep-th]].

[8] S. F. Hassan and R. A. Rosen, "Resolving the Ghost Problem in non-Linear Massive Gravity," Phys. Rev. Lett. **108** (2012) 041101 [arXiv:1106.3344 [hep-th]].

[9] A. H. Chamseddine and V. Mukhanov, "Ghost Free Mimetic Massive Gravity," JHEP **1806** (2018) 060 [arXiv:1805.06283 [hep-th]]. "Mimetic Massive Gravity: Beyond Linear Approximation," arXiv:1805.06598 [hep-th].

[10] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, "Effective field theory for massive gravitons and gravity in theory space," Annals Phys. **305** (2003) 96 [hep-th/0210184].

[11] A. H. Chamseddine and V. Mukhanov, "Hidden Ghost in Massive gravity," JHEP **1303** (2013) 092 [arXiv:1302.4367 [hep-th]].

[12] S. Deser, K. Izumi, Y. C. Ong and A. Waldron, "Superluminal Propagation and Acausality of Nonlinear Massive Gravity," arXiv:1312.1115 [hep-th].

[13] S. Deser, K. Izumi, Y. C. Ong and A. Waldron, "Problems of massive gravities," Mod. Phys. Lett. A **30** (2015) 1540006 [arXiv:1410.2289 [hep-th]].

[14] S. F. Hassan and M. Kocic, "On the local structure of spacetime in ghost-free bimetric theory and massive gravity," arXiv:1706.07806 [hep-th].

[15] B. Bellazzini, F. Riva, J. Serra and F. Sgarlata, "Beyond Positivity Bounds and the Fate of Massive Gravity," Phys. Rev. Lett. **120** (2018) no.16, 161101 [arXiv:1710.02539 [hep-th]].

[16] C. de Rham, S. Melville, A. J. Tolley and S. Y. Zhou, "Positivity Bounds for Massive Spin-1 and Spin-2 Fields," arXiv:1804.10624 [hep-th].

[17] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, "The m→0 limit for massive graviton in dS(4) and AdS(4): How to circumvent the van Dam-Veltman-Zakharov discontinuity," Phys. Lett. B **503** (2001) 173 [hep-th/0011138].

[18] M. Porrati, "No van Dam-Veltman-Zakharov discontinuity in AdS space," Phys. Lett. B **498** (2001) 92 [hep-th/0011152].

[19] A. I. Vainshtein, "To the problem of nonvanishing gravitation mass," Phys. Lett. **39B** (1972) 393.

[20] A. Karch and L. Randall, "Locally localized gravity," JHEP **0105** (2001) 008 [hep-th/0011156].

[21] A. Karch and L. Randall, "Localized gravity in string theory," Phys. Rev. Lett. **87** (2001) 061601 [hep-th/0105108].

[22] E. D'Hoker, J. Estes and M. Gutperle, "Exact half-BPS Type IIB interface solutions I: Local solution and supersymmetric Janus," JHEP **0706** (2007) 021 [arXiv:0705.0022 [hep-th]].

[23] E. D'Hoker, J. Estes and M. Gutperle, "Exact half-BPS type IIB interface solutions. II: Flux solutions and multi-janus," JHEP **0706** (2007) 022 [arXiv:0705.0024 [hep-th]].

[24] C. Bachas and J. Estes, "Spin-2 spectrum of defect theories," JHEP **1106** (2011) 005 [arXiv:1103.2800 [hep-th]].

[25] O. Aharony, L. Berdichevsky, M. Berkooz and I. Shamir, "Near-horizon solutions for D3-branes ending on 5-branes," Phys. Rev. D **84** (2011) 126003 [arXiv:1106.1870 [hep-th]].

[26] B. Assel, C. Bachas, J. Estes and J. Gomis, "Holographic Duals of D=3 N=4 Superconformal Field Theories," JHEP **1108** (2011) 087 [arXiv:1106.4253 [hep-th]].

[27] B. Assel, C. Bachas, J. Estes and J. Gomis, "IIB Duals of D=3 N=4 Circular Quivers," JHEP **1212**, 044 (2012) [arXiv:1210.2590 [hep-th]].

[28] J. Polchinski and E. Silverstein, "Dual Purpose Landscaping Tools: Small Extra Dimensions in AdS/CFT," arXiv:0908.0756 [hep-th].

[29] D. Tsimpis, "Supersymmetric AdS vacua and separation of scales," JHEP **1208** (2012) 142 [arXiv:1206.5900 [hep-th]].

[30] F. F. Gautason, M. Schillo, T. Van Riet and M. Williams, "Remarks on scale separation in flux vacua," JHEP **1603** (2016) 061 [arXiv:1512.00457 [hep-th]].

[31] M. Porrati, "Higgs phenomenon for 4-D gravity in anti-de Sitter space," JHEP **0204** (2002) 058 [hep-th/0112166] ; "Higgs phenomenon for the graviton in AdS space," Mod. Phys. Lett. A **18** (2003) 1793 [hep-th/0306253].

[32] M. J. Duff, J. T. Liu and H. Sati, "Complementarity of the Maldacena and Karch-Randall pictures," Phys. Rev. D **69** (2004) 085012 [hep-th/0207003].

[33] E. Kiritsis, "Product CFTs, gravitational cloning, massive gravitons and the space of gravitational duals," JHEP **0611** (2006) 049 [hep-th/0608088].

[34] O. Aharony, A. B. Clark and A. Karch, "The CFT/AdS correspondence, massive gravitons and a connectivity index conjecture," Phys. Rev. D **74**, 086006 (2006) [hep-th/0608089].

[35] E. Kiritsis and V. Niarchos, "Interacting String Multi-verses and Holographic Instabilities of Massive Gravity," Nucl. Phys. B **812** (2009) 488 [arXiv:0808.3410 [hep-th]].

[36] O. Aharony, M. Berkooz and E. Silverstein, "Multiple trace operators and nonlocal string theories," JHEP **0108** (2001) 006 [hep-th/0105309].

[37] O. Aharony, M. Berkooz and B. Katz, "Non-local effects of multi-trace deformations in the AdS/CFT correspondence," JHEP **0510** (2005) 097 [hep-th/0504177].

[38] O. Aharony, G. Gur-Ari and N. Klinghoffer, "The Holographic Dictionary for Beta Functions of Multi-trace Coupling Constants," JHEP **1505** (2015) 031 [arXiv:1501.06664 [hep-th]].

[39] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323** (2000) 183 [hep-th/9905111].

[40] A. Karch and L. Randall, "Open and closed string interpretation of SUSY CFT's on branes with boundaries," JHEP **0106** (2001) 063 [hep-th/0105132].

[41] O. DeWolfe, D. Z. Freedman and H. Ooguri, "Holography and defect conformal field theories," Phys. Rev. D **66** (2002) 025009 [hep-th/0111135].

[42] C. Bachas, J. de Boer, R. Dijkgraaf and H. Ooguri, "Permeable conformal walls and holography," JHEP **0206** (2002) 027 [hep-th/0111210].

[43] J. Erdmenger, Z. Guralnik and I. Kirsch, "Four-dimensional superconformal theories with interacting boundaries or defects," Phys. Rev. D **66** (2002) 025020 [hep-th/0203020].

[44] O. Aharony, O. DeWolfe, D. Z. Freedman and A. Karch, "Defect conformal field theory and locally localized gravity," JHEP **0307** (2003) 030 [hep-th/0303249].

[45] D. Gaiotto, E. Witten, "Janus Configurations, Chern-Simons Couplings, And The theta-Angle in N=4 Super Yang-Mills Theory," JHEP **1006**, 097 (2010). [arXiv:0804.2907 [hep-th]]; 'S-Duality of Boundary Conditions In N=4 Super Yang-Mills Theory," [arXiv:0807.3720 [hep-th]].

[46] S. Giombi and I. R. Klebanov, "Interpolating between a and F," JHEP **1503** (2015) 117 [arXiv:1409.1937 [hep-th]].

[47] F. A. Dolan, "On Superconformal Characters and Partition Functions in Three Dimensions," J. Math. Phys. **51** (2010) 022301 [arXiv:0811.2740 [hep-th]].

[48] C. Cordova, T. T. Dumitrescu and K. Intriligator, "Multiplets of Superconformal Symmetry in Diverse Dimensions," arXiv:1612.00809 [hep-th].

[49] C. Bachas, M. Bianchi and A. Hanany, "N=2 Moduli of AdS4 vacua: A fine-print study," arXiv:1711.06722 [hep-th].

[50] J. Louis and H. Triendl, "Maximally supersymmetric AdS_4 vacua in N = 4 supergravity," JHEP **1410** (2014) 007 [arXiv:1406.3363 [hep-th]].

[51] C. Cordova, T. T. Dumitrescu and K. Intriligator, "Deformations of Superconformal Theories," JHEP **1611** (2016) 135 [arXiv:1602.01217 [hep-th]].

[52] O. Lunin, "On gravitational description of Wilson lines," JHEP **0606** (2006) 026 [hep-th/0604133].

[53] J. Gomis and C. Romelsberger, "Bubbling Defect CFT's," JHEP **0608** (2006) 050 [hep-th/0604155].

[54] C. Csaki, J. Erlich, T. J. Hollowood and Y. Shirman, "Universal aspects of gravity localized on thick branes," Nucl. Phys. B **581** (2000) 309 [hep-th/0001033].

[55] A. Miemiec, "A Power law for the lowest eigenvalue in localized massive gravity," Fortsch. Phys. **49** (2001) 747 [hep-th/0011160].

[56] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond,"

[57] R. C. Myers and A. Sinha, "Seeing a c-theorem with holography," Phys. Rev. D **82** (2010) 046006 [arXiv:1006.1263 [hep-th]]. "Holographic c-theorems in arbitrary dimensions," JHEP **1101** (2011) 125 doi:10.1007/JHEP01(2011)125 [arXiv:1011.5819 [hep-th]].

[58] H. Casini, M. Huerta and R. C. Myers, "Towards a derivation of holographic entanglement entropy," JHEP **1105** (2011) 036 [arXiv:1102.0440 [hep-th]]. "On the RG running of the entanglement entropy of a circle," Phys. Rev. D **85** (2012) 125016 [arXiv:1202.5650 [hep-th]].

[59] D. L. Jafferis, I. R. Klebanov, S. S. Pufu and B. R. Safdi, "Towards the F-Theorem: N=2 Field Theories on the Three-Sphere," JHEP **1106** (2011) 102 [arXiv:1103.1181 [hep-th]].

[60] H. Liu and M. Mezei, "A Refinement of entanglement entropy and the number of degrees of freedom," JHEP **1304** (2013) 162 [arXiv:1202.2070 [hep-th]].

[61] B. Assel, J. Estes and M. Yamazaki, "Large N Free Energy of 3d N=4 SCFTs and AdS_4/CFT_3," JHEP **1209** (2012) 074 [arXiv:1206.2920 [hep-th]].

[62] S. Benvenuti and S. Pasquetti, "3D-partition functions on the sphere: exact evaluation and mirror symmetry," JHEP **1205** (2012) 099 [arXiv:1105.2551 [hep-th]].

[63] T. Nishioka, Y. Tachikawa and M. Yamazaki, "3d Partition Function as Overlap of Wavefunctions," JHEP **1108** (2011) 003 [arXiv:1105.4390 [hep-th]].

[64] T. Damour and I. I. Kogan, "Effective Lagrangians and universality classes of nonlinear bigravity," Phys. Rev. D **66**, 104024 (2002) [hep-th/0206042].

[65] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, "Renormalization group flows from holography supersymmetry and a c theorem," Adv. Theor. Math. Phys. **3** (1999) 363 [hep-th/9904017].

[66] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, "Novel local CFT and exact results on perturbations of N=4 superYang Mills from AdS dynamics," JHEP **9812** (1998) 022 [hep-th/9810126].

[67] F. Apruzzi, M. Fazzi, A. Passias, A. Rota and A. Tomasiello, "Six-Dimensional Superconformal Theories and their Compactifications from Type IIA Supergravity," Phys. Rev. Lett. **115** (2015) no.6, 061601 [arXiv:1502.06616 [hep-th]].

[68] E. D'Hoker, M. Gutperle, A. Karch and C. F. Uhlemann, "Warped $AdS_6 \times S^2$ in Type IIB supergravity I: Local solutions," JHEP **1608** (2016) 046 [arXiv:1606.01254 [hep-th]].

[69] E. D'Hoker, M. Gutperle and C. F. Uhlemann, "Warped $AdS_6 \times S^2$ in Type IIB supergravity II: Global solutions and five-brane webs," JHEP **1705** (2017) 131 [arXiv:1703.08186 [hep-th]].

[70] F. Apruzzi, M. Fazzi, A. Passias, D. Rosa and A. Tomasiello, "AdS_6 solutions of type II supergravity," JHEP **1411** (2014) 099 Erratum: [JHEP **1505** (2015) 012] [arXiv:1406.0852 [hep-th]].

[71] D. Gaiotto and J. Maldacena, "The Gravity duals of N=2 superconformal field theories," JHEP **1210** (2012) 189 [arXiv:0904.4466 [hep-th]].

[72] M. Fazzi, "Higher-dimensional field theories from type II supergravity," arXiv:1712.04447 [hep-th].