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# $SU(2)_L$ triplet scalar as the origin of the 95 GeV excess?

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We explore the possibility that an $SU(2)_L$ triplet scalar with hypercharge $Y = 0$ is the origin of the 95 GeV diphoton excess. For a small mixing angle with the Standard Model Higgs, its neutral component has naturally a sizable branching ratio to $\gamma\gamma$ such that its Drell-Yan production via $pp \to W^* \to HH^\pm$ is sufficient to obtain the desired signal strength, where $H^\pm$ is the charged Higgs component of the triplet. The predictions of this setup are: 1) The $\gamma\gamma$ signal has a $p_T$ spectrum different from gluon fusion but similar to associated production. 2) Photons are produced in association with tau leptons and jets, but generally do not fall into the vector-boson fusion category. 3) The existence of a charged Higgs with $m_{H^\pm} \approx (95 \pm 5)$ GeV leading to $\sigma(pp \to \tau\tau\nu\nu) \approx 0.4$ pb, which is of the same level as the current limit and can be discovered with Run 3 data. 4) A positive definite shift in the $W$ mass as suggested by the current global electroweak fit.

## I. INTRODUCTION

The Standard Model (SM) is the currently accepted theoretical description of the known constituents and interaction of matter. It has been successfully tested in precision experiments [1–3] and the Brout-Englert-Higgs boson [4–7], the last missing piece, was finally discovered in 2012 at CERN [8–10]. In fact, this 125 GeV particle has properties consistent with the ones predicted by the SM [11–15]. However, this does not exclude the existence of additional scalar bosons, as long as their role in electroweak symmetry breaking is subleading and their production cross sections are smaller than the ones of the SM-like Higgs [16, 17].

The minimality of the SM Higgs sector, *i.e.* the existence of a single $SU(2)_L$ doublet scalar that simultaneously gives mass to the electroweak (EW) gauge bosons and all fermions, is not guaranteed by any theoretical principle or symmetry. A plethora of such extensions have been proposed in the literature, including the addition of $SU(2)_L$ singlets [18–20], doublets [21–25] and triplets [26–31].

While Large Hadron Collider (LHC) searches for new particles did not lead to any discovery (yet), there are interesting hints for new scalar bosons [32]. In particular, CMS [33–35] searches hint toward a neutral scalar $H$ decaying into two photons at 95 GeV. This is compatible with the latest ATLAS result [36] and supported by $Z$-strahlung with $H \to b\bar{b}$ at LEP [37], as well as by $\tau\tau$ [35] and $WW$ [38–40] searches. In fact, combining these channels results in a global significance of $3.8\sigma$ [41].

So far, explanations of the 95 GeV excesses in terms of $SU(2)_L$ singlets and/or $SU(2)_L$ doublets were proposed in the literature [42–67], which all respect custodial symmetry at tree-level. For higher dimensional $SU(2)_L$ representations, the measurement of the $\rho$-parameter restricts the vacuum expectation value (VEV) of the new scalar to be $\lesssim \mathcal{O}(1)$ GeV [3] and except for the $SU(2)_L$ triplet with hypercharge $Y = 0$ multiply charged scalars at the same mass scale are unavoidable which is problematic with respect to LHC searches [68–71].[1] It is well known that this field provides a positive definite shift in the $W$ mass (with respect to the SM prediction) [80–91], as motivated by the current global electroweak fit [92–94] (driven by the CDF II result [95]). However, its collider phenomenology has been barely studied. In this article, we study the viability of $Y = 0$ triplet as an alternative in addressing the hints for a ≈95 GeV scalar.

## II. PHENOMENOLOGY

The SM extended with an $SU(2)_L$ triplet scalar with hypercharge 0, is commonly referred to as the ΔSM [96–103]. It contains an additional charged scalar $H^\pm$ and a

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[1] For small mass-splitting among the $SU(2)_L$ components, LHC searches for multiply charged scalars would exclude scenarios with a neutral Higgs with a mass around ∼ 95 GeV [72, 73]. However, nondegenerate scenarios, with the heavier multiply charged Higgses decaying into (off-shell) neutral Higgses and $W$-bosons, could still be consistent with the LHC searches [74–77]. The phenomenology of such mass spectra has been studied in Refs. [78, 79].

neutral one $H$ which acquires a vacuum expectation value $v_\Delta$ in the process of spontaneous symmetry breaking. Importantly, without mixing $H$ couples only to $W$ bosons at tree-level, while the $CP$-even mixing angle $\alpha$ induces couplings to SM fermions. Furthermore, charged Higgs loops modify both $h \to \gamma\gamma$ and $H \to \gamma\gamma$. A detailed description of the model is provided in the Appendix.

### A. Perturbative unitarity and vacuum stability

The ΔSM parameter space can be constrained by vacuum stability and perturbative unitarity. The region between the red lines in Fig. 1 is allowed by both criteria and the explicit calculation of the constraints is given in the Appendix.

### B. $W$ mass

The latest ATLAS update of $m_W = 80.360(16)$ [104] (superseding the 2017 result [105]) as well as the LHCb result $m_W = 80.354(32)$ [106] are significantly smaller compared to $m_W = 80.4335(94)\,\mathrm{GeV}$ obtained by CDF II. When combined with D0 [95] and LEP [107], this lead to a naive global average of $m_W = 80.406(7)\,\mathrm{GeV}$. Because there is considerable tension between these measurements ($\chi^2/\mathrm{dof} = 4.3$), we inflate the error on $m_W$ to $0.015\,\mathrm{GeV}$ to get a conservative average of [90][2]

$$m_W^{\mathrm{comb}} = (80.406 \pm 0.015)\,\mathrm{GeV}. \quad (1)$$

Comparing this with the SM prediction of $m_W^{\mathrm{SM}} = 80.3499(56)\,\mathrm{GeV}$ [3, 92, 108–114], with $m_t = 172.5(0.7)\,\mathrm{GeV}$ [3], the discrepancy of $56\,\mathrm{MeV}$ amounts to $3.7\sigma$. If we disregarded the CDF II result, we find an average of

$$m_W^{\mathrm{comb\ (w/o\ CDF\ II)}} = (80.372 \pm 0.010)\,\mathrm{GeV}, \quad (2)$$

which corresponds to a discrepancy of $22\,\mathrm{MeV}$ ($2.2\sigma$).

In the ΔSM, we have

$$m_W^2 = \frac{g^2}{4}(v^2 + 4v_\Delta^2), \qquad m_Z^2 = \frac{g^2}{4\cos\theta_W^2}v^2. \quad (3)$$

Therefore, $v_\Delta$ of a few GeV can easily alter the $m_W$ prediction in the desired direction. As such, $m_W^{\mathrm{comb}}$ requires $v_\Delta = 4.60^{+0.58}_{-0.66}\,\mathrm{GeV}$, while $m_W^{\mathrm{comb\ (w/o\ CDF\ II)}}$ requires $v_\Delta = 2.89^{+0.59}_{-0.75}\,\mathrm{GeV}$.

### C. SM Higgs signal strength

Through the quartic interactions $H^\pm$ contributes to the diphoton decay rate of the SM Higgs $h$ (see Fig. 2 left). The corresponding signal strength, with respect to the SM one, is given by

$$\mu_{h,\gamma\gamma} = \Gamma_{h\to\gamma\gamma}/\Gamma^{\mathrm{SM}}_{h\to\gamma\gamma} = |\kappa_\gamma^2|\,, \quad (4)$$

with

$$\kappa_\gamma \approx \cos\alpha + \frac{A_{hH^\pm H^\mp}v}{2m_{H^\pm}^2}\frac{\beta_H^0\left(\frac{4m_{H^\pm}^2}{m_h^2}\right)}{\frac{4}{3}\beta_H^{1/2}\left(\frac{4m_t^2}{m_h^2}\right) + \beta_H^1\left(\frac{4m_W^2}{m_h^2}\right)}, \quad (5)$$

and the loop functions [115] are given in Appendix.

Combining the most recent measurements of CMS [116] and ATLAS [117], $\mu^{\mathrm{CMS}}_{h,\gamma\gamma} = 1.12^{+0.09}_{-0.09}$ and $\mu^{\mathrm{ATLAS}}_{h,\gamma\gamma} = 1.04^{+0.10}_{-0.09}$, respectively, we get the weighted average

$$\mu^{\mathrm{exp}}_{h,\gamma\gamma} = 1.08^{+0.07}_{-0.06}. \quad (6)$$

The resulting preferred regions at the $1\sigma$ and $2\sigma$ level are shown in blue in Fig. 1.

While the $h \to \gamma\gamma$ signal strength is the most precise measured one, it is affected by $h$-$H$ mixing and the $H^\pm$-loop contribution so that cancellations occur. Therefore, the second-best measured SM Higgs signal $h \to ZZ^*$ [118, 119] provides a complementary constraint of [3]

$$\mu^{\mathrm{exp}}_{h,ZZ^*} = 1.02 \pm 0.08\,, \quad (7)$$

which, to a very good approximation, is only sensitive to the mixing angle $\alpha$. The region on the right of the solid vertical line in Fig. 1 is compatible with $\mu^{\mathrm{exp}}_{h,ZZ^*}$ at the $1\sigma$ level.

### D. diphoton excess

While nearly all relevant decay modes of $H$ can be obtained from a rescaling of the widths of a SM-like Higgs with a mass of $95\,\mathrm{GeV}$ by multiplying with $\sin^2\alpha$, the decay $H \to WW^*$ is already generated at tree-level via $v_\Delta$ and $H \to \gamma\gamma$ receives loop contributions from the charged Higgs as well as from $W$ loops:[3]

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[2] This *naive* average agrees well with the one obtained in a sophisticated fit performed by `HEPfit` [92] prior to the ATLAS update.

[3] Only $Z\gamma$ also receives an additional direct contribution from the $W$ loop, which is already present for $\sin\alpha = 0$, but the corresponding branching ratio is negligibly small.

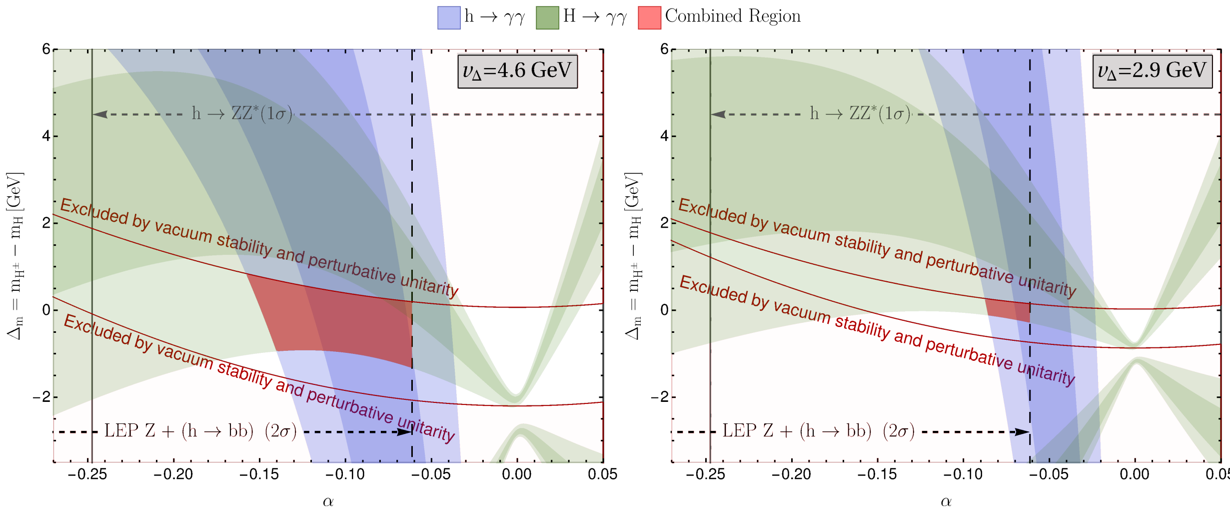


FIG. 1. Preferred regions ($1\sigma$ and $2\sigma$) by the $h \to \gamma\gamma$ signal strength (blue) and the 95 GeV $H \to \gamma\gamma$ excess (green) in the $\alpha - \Delta_m$ plane for the two values of $v_\Delta$ corresponding to the two $m_W$ benchmark points. The region between the two red lines is allowed by vacuum stability and perturbative unitarity. The dashed vertical line indicates the region preferred by the LEP measurement of $Z + (H \to bb)$, and the region to the right of the solid vertical line is preferred by the $h \to ZZ^*$ signal strength at $1\sigma$ level.

$$\Gamma(H \to \gamma\gamma) \approx \frac{\alpha_{\rm em}^2 g_2^2 m_H^3}{1024\pi^3 m_W^2} \left| -\frac{4}{3}\sin\alpha\, \beta_H^{1/2}\left(\frac{4m_t^2}{m_H^2}\right) + \left(-\sin\alpha + \frac{4v_\Delta}{v}\cos\alpha\right)\beta_H^1\left(\frac{4m_W^2}{m_H^2}\right) + \frac{A_{HH^\pm H^\mp} v}{2m_{H^\pm}^2}\beta_H^0\left(\frac{4m_{H^\pm}^2}{m_H^2}\right)\right|^2 . \quad (8)$$

Here, $\alpha_{\rm em}$ at $q^2 = 0$ numerically approximates well the NLO QED corrections.

For a small mixing angle $\alpha$, $H$ is mainly produced via the Drell-Yan (DY) process $pp \to W^* \to H^\pm H$ (see Fig. 2 right) with a leading order (LO) cross section of 1.77 pb for $m_{H^\pm} \approx m_H = 95$ GeV. While the QCD corrections have not been estimated so far for the $\Delta$SM, it is obvious that they pertain dominantly to the hadronic ends of the processes and are thus expected to be the same as for sleptons or $SU(2)_L$ triplet leptons. The latter has been calculated in Ref. [120], resulting in a correction factor of 1.15, by which we naively rescale the LO cross section (computed with `MadGraph5aMC@NLO` [121]) to obtain $\approx 2$ pb. In addition, $H$ is also produced via gluon-gluon fusion (ggF) and vector boson fusion (VBF) processes through the mixing with $h$. The corresponding cross section is calculated by multiplying the production cross section of a SM-like 95 GeV Higgs by $\alpha^2$. Neglecting the subdominant contribution from VBF, and using $\sigma[pp \to h(95)] \approx 68$ pb [122–132], we thus have

$$\sigma[pp \to H \to \gamma\gamma] \approx {\rm Br}[H \to \gamma\gamma] \times \left(2 + 68\alpha^2\right) \text{pb}. \quad (9)$$

Normalizing the signal strength to the one of a hypothetical SM-like Higgs with the same mass [122], we find numerically

$$\mu_{H,\gamma\gamma} \approx (21.5 + 719\alpha^2) \times {\rm Br}[H \to \gamma\gamma]. \quad (10)$$

This has to be compared to the combination of the CMS and ATLAS analyses of a low mass $\gamma\gamma$ searches of [65][4]

$$\mu_{H,\gamma\gamma}^{\rm exp} = 0.27^{+0.10}_{-0.09} . \quad (11)$$

The resulting preferred regions are shown in green in Fig. 1.[5].

[4] Note that the signal strength of $H$ is normalized with respect to an SM-like Higgs with the same mass. While the latter is mainly produced via ggF and VBF processes, the former is dominantly produced via the DY process $pp \to W^* \to H^\pm H$ while the other production modes are too a good approximation only induced via the mixing with $h$. Note that, while in the limit of zero mixing between the SM Higgs and the triplet Higgs, $H$ is fermiophobic, this region in parameter space is, contrary to the setup of Ref. [133], not excluded due to the charged Higgs contribution to $H \to \gamma\gamma$. Furthermore, for $\alpha \neq 0$, couplings to fermions are induced.

[5] Note that our model has similarities with one of the "square" benchmark scenarios of Ref. [44], where the 95 GeV excess was studied in the context of the type-I two-Higgs-doublet model. There, in the fermiophobic limit, $pp \to W^{\pm *} \to H^\pm H$ is the dominant production mode. However, the model in Ref. [44] predicts an additional pseudoscalar with $\approx 80$ GeV while the Higgs potential allows for more freedom than our setup.

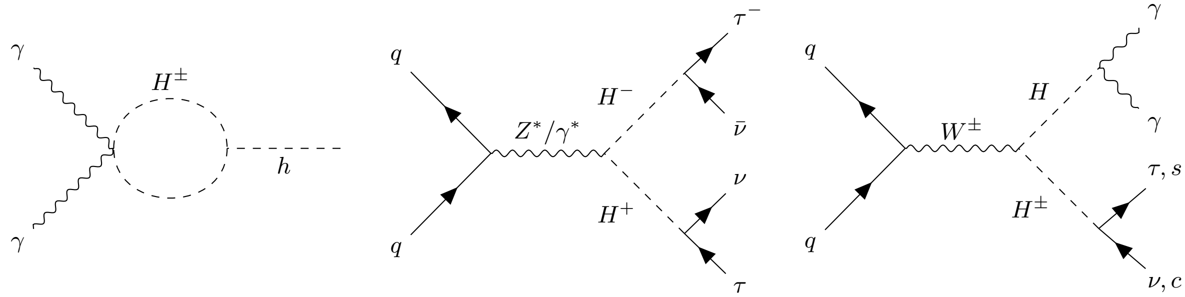


FIG. 2. Feynman diagrams showing the modification of $h\to\gamma\gamma$ (left), the DY processes $pp\to Z^*,\gamma^*\to H^+H^-\to\tau^+\tau^-\nu\bar\nu$ (middle) and $pp\to W^*\to H^\pm H^0$ (right).

### E. $Zbb$, $WW$ and $\tau\tau$

While Br$[H\to WW]$ is large for a very small mixing angle $\alpha$, the resulting effect in $\gamma\gamma$ would be too high if one aims at the central value of the cross section of Ref. [40]. Therefore, $\alpha$ cannot be too small, and it is possible to explain the $Zbb$ excess of LEP which requires

$$\mu_{bb}^{\rm exp}=\frac{\sigma^{\rm exp}\left(e^+e^-\to ZH\right)}{\sigma^{\rm SM}\left(e^+e^-\to ZH\right)}{\rm Br}\left(H\to b\bar b\right)=0.117\pm0.057. \tag{12}$$

For tau decays, the central values of the signal strength $\mu_{\tau\tau}^{\rm exp}=1.2\pm0.5$ cannot be fully explained, which is a general feature of most SM extensions addressing the 95 GeV excess [58], the error is too large to draw a conclusion here.

### F. $pp\to H^+H^-\to\tau^+\tau^-\nu\bar\nu$

The charged Higgs in general dominantly decays to $\tau\nu$. Therefore, its pair production and subsequent decays, *i.e.* $pp\to Z^*,\gamma^*\to H^+H^-\to\tau^+\tau^-\nu\bar\nu$ (see Fig. 2 middle), leads to a collider signature searched for in the context of supersymmetric tau partners [134–137]. While CMS [136] provides an upper bound on the cross section and observes a weaker limit than expected, ATLAS [137] observes a stronger limit than expected but does not provide a bound on the total cross section. Since both bounds deviate from the expected limit by $\approx 1\,\sigma$ level, but in opposite directions, we will thus use the expected limit on the cross section provided by CMS [136] of $0.34^{+0.24}_{-0.12}$ pb. Using once more `MadGraph5aMC@NLO` at LO, we find a production cross section of 0.86 pb which we again multiply by a factor 1.15 [120, 138] to include NLO QCD effects. Taking into account that CMS and ATLAS assume a 100% branching ratio of the stau to tau and neutralino, while we have Br$[H^\pm\to\tau^\pm\nu_\tau]\approx0.66\pm0.03$ [122, 139–153][6], a cross section of $\approx0.44\pm0.03$ pb is predicted. This is in slight tension with the 95% exclusion limit.

Let us therefore consider the option to reduce Br$[H^\pm\to\tau\nu]$ by increasing the mass splitting $\Delta m$ such that Br$[H^\pm\to HW^*]$ becomes sizable:[7]

$$\Gamma(H^\pm\to HW^*)=\frac{9g^4m_{H^\pm}}{512\pi^3}\lambda^2_{HH^\pm W}G\left(\frac{m_H^2}{m_{H^\pm}^2},\frac{m_W^2}{m_{H^\pm}^2}\right), \tag{13}$$

where $\lambda_{HH^\pm W}=2\cos\alpha\cos\beta-\sin\alpha\sin\beta$, and the loop function $G(x,y)$ is given in the Appendix.

As one can see in Sec. 3 in the Appendix, choosing $v_\Delta=0.86$ GeV as a benchmark point, allows for a small region in parameter space with sizable mass splitting, that is allowed by the vacuum stability and perturbative unitarity[8] as well as compatible with $h\to\gamma\gamma, ZZ^*$, $H\to\gamma\gamma$ and $Zbb$. Note that this scenario predicts a small positive shift in the $W$ mass.

## III. CONCLUSIONS AND OUTLOOK

In summary, the predictions if the neutral component of the $SU(2)_L$ triplet with hypercharge 0 is the origin of the 95 GeV excess are:

- LHC Run 3 shows a stau-like excess.
- Positive shift in the $W$ mass.
- $H$ is produced in association with jets and $\tau$ leptons.
- A charged Higgs with a mass below $\approx 100$ GeV which could be very well studied at future $e^+e^-$ colliders [154–157].

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[6] Since in our case the branching ratio is dominated by $\tau\nu$ and $cs$, the error on Br$[H^\pm\to cs]$ is dominating the error of Br$[H^\pm\to\tau^\pm\nu_\tau]$.

[7] Note that Br$[H^\pm\to H^*W]$ is much smaller such that it can be neglected.

[8] Note that, for sizeable $\alpha$, the requirements of vacuum stability and perturbative unitarity dictate that $\Delta_m\approx 22\alpha-3.75$ GeV.

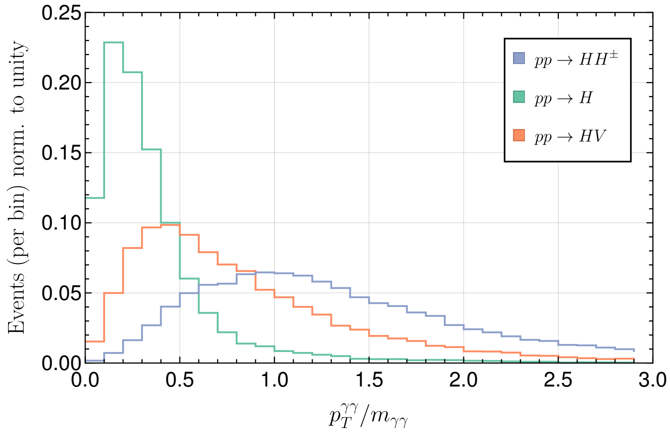


FIG. 3. Transverse momentum normalized to the invariant mass of the photon pair system for different production mechanisms of a 95 GeV scalar $H$: VH (orange), ggF (green), DY production in the triplet model (blue).

- A significantly broader $p_T$ spectrum of the diphoton system compared to ggF, as shown in Fig. 3.[9]

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## Appendix A: The Model

The SM supplemented by a $SU(2)_L$ triplet scalar with hypercharge 0, is commonly referred to as the $\Delta$SM [96–103]. The scalar sector consists of the SM doublet $\Phi$, and the triplet $\Delta$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \Delta = \frac{1}{2}\begin{pmatrix} \delta^0 & \sqrt{2}\delta^+ \\ \sqrt{2}\delta^- & -\delta^0 \end{pmatrix}, \tag{A1}$$

with $\delta^0$ being real and $\delta^- = (\delta^+)^*$. The covariant derivative for $SU(2)_L$ is fixed, in the usual conventions, by the generators, *i.e.* $T_k = \frac{\sigma_k}{2}$ for the doublet, with $\sigma_k$ being the Pauli matrices. This then fixes the structure constants $f_{ijk} = i\epsilon_{ijk}$ and the covariant derivative in the adjoint representation for the triplet is

$$D_\mu \Delta = \partial_\mu \Delta - i g_2 \left[\frac{\sigma_k}{2}\, W_\mu^k,\ \Delta\right], \tag{A2}$$

where the square bracket stands for the commutator. Note that therefore the $SU(2)_L$ gauge boson interactions with the triplet are a factor of 2 higher than for a doublet.

Since the triplet cannot have direct couplings to quarks or leptons, the scalar potential

$$\begin{aligned} V = &-\mu_\Phi^2 \Phi^\dagger\Phi + \frac{\lambda_\Phi}{4}\left(\Phi^\dagger\Phi\right)^2 - \mu_\Delta^2 \mathrm{Tr}\left(\Delta^\dagger\Delta\right) \\ &+ \frac{\lambda_\Delta}{4}\left[\mathrm{Tr}\left(\Delta^\dagger\Delta\right)\right]^2 + \mu\Phi^\dagger\Delta\Phi + \lambda_{\Phi\Delta}\Phi^\dagger\Phi\mathrm{Tr}\left(\Delta^\dagger\Delta\right), \end{aligned} \tag{A3}$$

describes its remaining interactions. Note that Eq. (A3) has a global $O(4)_H \times O(3)_\Delta$ symmetry softly broken by the $\mu$-term. After electroweak symmetry breaking, $\phi^0$ and $\delta^0$ acquire the VEVs $\langle\sqrt{2}\phi^0\rangle = v \approx 246\,\mathrm{GeV}$ and $\langle\delta^0\rangle = v_\Delta$. The minimization conditions

$$\begin{aligned} \mu_\Phi^2 &= -\mu\frac{v_\Delta}{2} + \frac{1}{4}v^2\lambda_\Phi + \frac{1}{2}\lambda_{\Phi\Delta}v_\Delta^2, \\ \mu_\Delta^2 &= -\mu\frac{v^2}{4v_\Delta} + \frac{1}{2}v^2\lambda_{\Phi\Delta} + \frac{1}{4}\lambda_\Delta v_\Delta^2, \end{aligned} \tag{A4}$$

can then be used to eliminate $\mu_\Phi^2$ and $\mu_\Delta^2$ in terms of the other parameters of Eq. (A3). The scalar mass matrices, in the bases $(\phi^+, \delta^+)$ and $(\mathrm{Re}(\phi^0), \delta^0)$, are

$$\begin{aligned} M_\pm^2 &= \mu\begin{pmatrix} v_\Delta & \frac{v}{2} \\ \frac{v}{2} & \frac{v^2}{4v_\Delta} \end{pmatrix}, \\ M_0^2 &= \begin{pmatrix} \lambda_\Phi\frac{v^2}{2} & \lambda_{\Phi\Delta}vv_\Delta - \mu\frac{v}{2} \\ \lambda_{\Phi\Delta}vv_\Delta - \mu\frac{v}{2} & \lambda_\Delta\frac{v_\Delta^2}{2} + \mu\frac{v^2}{4v_\Delta} \end{pmatrix}. \end{aligned} \tag{A5}$$

The resulting mass eigenstates, in addition to $\mathrm{Im}(\phi^0)$, are

$$\begin{aligned} G^\pm &= \cos\zeta\ \phi^\pm + \sin\zeta\ \delta^\pm, \\ H^\pm &= -\sin\zeta\ \phi^\pm + \cos\zeta\ \delta^\pm, \\ h &= \cos\alpha\ \mathrm{Re}(\phi^0) + \sin\alpha\ \delta^0, \\ H &= -\sin\alpha\ \mathrm{Re}(\phi^0) + \cos\alpha\ \delta^0, \end{aligned} \tag{A6}$$

with the CP-even and charged Higgs mixing angles

$$\tan 2\alpha = \frac{4vv_\Delta\left(2\lambda_{\Phi\Delta}v_\Delta - \mu\right)}{2\lambda_\Phi v^2 v_\Delta - 2\lambda_\Delta v_\Delta^3 - \mu v^2},\ \tan\zeta = -2\frac{v_\Delta}{v}. \tag{A7}$$

[9] While this information is currently not available, it can be used in future analyses as a discriminator. To compare the $p_T$ of the diphoton system of the $\Delta$SM to the SM, we generated 100k events at NLO using `MadGraph5aMC@NLO` with the parton shower performed by `Pythia8.3` [158] and the detector simulation for the CMS detector [34], carried out with `Delphes` [159]. The UFO model file at NLO of the $\Delta$SM was built using `FeynRules` [160–162] and to increase the efficiency of the simulation, the decay of $H$ to a photon pair was forced using `MadSpin` [163].

The massless states $G^\pm$ and $\mathrm{Im}(\phi^0)$ are the *would-be* Goldstone bosons, eaten by the $W^+$ and $Z$. Among the massive states, $h$ is identified as the 125 GeV (SM-like) Higgs, and $H$ and $H^\pm$ are the triplet-like neutral and charged scalars with masses

$$\begin{aligned} m_H^2 &= \lambda_\Delta \frac{v_\Delta^2}{2} + \mu \frac{v^2}{4 v_\Delta} - \tan\alpha \left( \lambda_{\Phi\Delta} v_\Delta - \frac{\mu}{2} \right) v \,, \\ m_{H^\pm}^2 &= \mu \frac{v^2 + 4 v_\Delta^2}{4 v_\Delta} \,. \end{aligned} \tag{A8}$$

For vanishing $\alpha$, $m_{H^\pm}^2 - m_H^2 \simeq \mu v_\Delta - \lambda_\Delta v_\Delta^2/2$, and thus the components are nearly mass-degenerate for $v_\Delta \ll v$. However, for large $\alpha$ and $v_\Delta$, vacuum stability and perturbative unitarity (see Sec. III of the main text) allow a mass-splitting $\Delta_m = m_{H^\pm} - m_H$ of a few GeV. In addition, the EW radiative correction induces a mass-splitting of 160 MeV–170 MeV [164]. However, such a small splitting is of little consequence as far as the LHC phenomenology and their contribution to the electroweak oblique parameters are concerned [82, 165].

Note that in the end, all parameters of the scalar potential can be expressed in terms of the physical masses and mixing angles and the two VEVs $v$ and $v_\Delta$. In particular, the (dimension-full) couplings of the neutral to the charged Higgses can be written as

$$\begin{aligned} A_{hH^\pm H^\mp} &\approx \frac{1}{2} \lambda_\Delta v_\Delta \sin\alpha + \lambda_{\Phi\Delta} v \cos\alpha \,, \\ A_{HH^\pm H^\mp} &\approx \frac{1}{2} \lambda_\Delta v_\Delta \cos\alpha - \lambda_{\Phi\Delta} v \sin\alpha \,, \end{aligned} \tag{A9}$$

in the limit of small $v_\Delta$ and $m_H \approx m_{H^\pm}$.

## Appendix B: Vacuum stability and perturbative unitarity

In the following we provide the condition necessary to respect vacuum stability and perturbative unitarity (at tree level). The first can be derived requiring the potential to be bounded from below, while the latter, limiting the size of the quartic interactions, can be obtained from $2 \to 2$ scalar-scalar scattering[10]. The vacuum stability conditions read [80, 166, 167]

$$\lambda_\Phi > 0, \quad \lambda_\Delta > 0, \quad \sqrt{2}\lambda_{\Phi\Delta} + \sqrt{\lambda_\Phi \lambda_\Delta} > 0, \tag{B1}$$

and from perturbative unitarity we obtain

$$\begin{aligned} & |\lambda_\Phi| \le 2\kappa\pi, \quad |\lambda_\Delta| \le 2\kappa\pi, \quad |\lambda_{\Phi\Delta}| \le \kappa\pi, \\ & |6\lambda_\Phi + 5\lambda_\Delta \pm \sqrt{(6\lambda_\Phi - 5\lambda_\Delta)^2 + 192\lambda_{\Phi\Delta}^2}| \le 8\kappa\pi, \end{aligned} \tag{B2}$$

where $\kappa = 16$ or 8 depending on whether one chooses $|(a_0)| \le 1$ or $|\mathrm{Re}\,(a_0)| \le \frac{1}{2}$ with $a_0$ denoting the leading partial wave amplitude [168]. In order to be conservative, *i.e.* not to exclude any potentially allowed parameter space, we opt for $\kappa = 16$. Further, to ensure perturbativity at all higher orders, we require all the trilinear and quartic scalar couplings in (A3) to be smaller than $4\pi$.[11]

## Appendix C: One loop functions for di-photon decay

The loop functions for di-photon decay in Eq. (11) of the main text are given as

$$\begin{aligned} \beta_H^0(x) &= -x\left[1 - x f(x)\right], \\ \beta_H^{1/2}(x) &= 2x\left[1 + (1-x) f(x)\right], \\ \beta_H^1(x) &= -\left[2 + 3x + 3x(2-x) f(x)\right]. \end{aligned} \tag{C1}$$

The loop function for $H^\pm \to W H^0$ (in Eq. (20) of main text) is given by

$$\begin{aligned} G(x,y) = \frac{1}{12y} \Bigg[ & 2(-1+x)^3 - 9(-1+x^2)y + 6(-1+x)y^2 \\ & - 6(1+x-y)y\sqrt{-\lambda(x,y)} \Bigg\{ \tan^{-1}\left( \frac{1-x+y}{\sqrt{-\lambda(x,y)}} \right) \\ & + \tan^{-1}\left( \frac{1-x-y}{\sqrt{-\lambda(x,y)}} \right) \Bigg\} - 3\left(1 + (x-y)^2 - 2y\right) y \log x \Bigg], \end{aligned}$$

with $\lambda(x,y) = (1-x-y)^2 - 4xy$.

## Appendix D: $H^\pm$ in stau searches

As to account for the slight tension with the 95% exclusion limit of stau searches at the LHC, one can reduce $\mathrm{Br}[H^\pm \to \tau\nu]$ by increasing the mass splitting $\Delta m$ such that $\mathrm{Br}[H^\pm \to HW^*]$ becomes sizable. In Fig. 4, we choose $v_\Delta = 0.86$ GeV as a benchmark point, which allows for a small region in parameter space with sizable mass splitting, that is allowed by the vacuum stability and perturbative unitarity as well as compatible with $h \to \gamma\gamma, ZZ^*$, $H \to \gamma\gamma$ and $Zbb$. Note that this scenario predicts a small positive shift in the $W$ mass.

[10] Note that this is valid as long as the $\mu$ parameter is not very large, which is satisfied for a small VEV since $\mu \sim v_\Delta \ll v$ for $m_{H^\pm} \sim v/2$.

[11] We checked numerically using Vevacious [169, 170], SPheno [171, 172] and BSMArt [173] that the inclusion of the one-loop effective potential and meta stability has only a marginal effect on vacuum stability and perturbative unitarity.

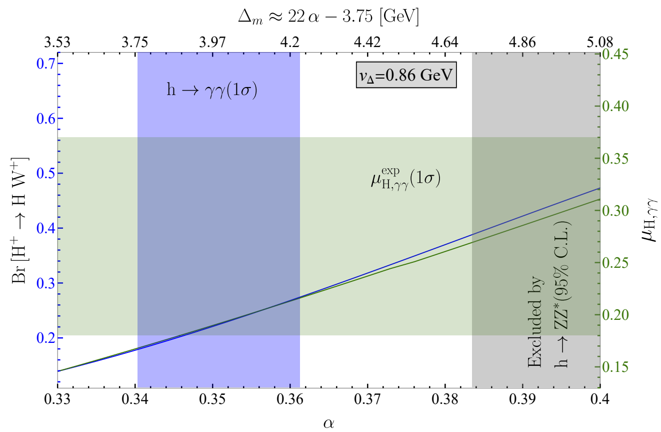


FIG. 4. Scenario with sizable $\Delta_m$ that is allowed by vacuum stability due to $\Delta_m \approx 22\alpha - 3.75$ [GeV] that suppresses Br$[H^{\pm} \to \tau\nu]$ by enhancing Br$[H^{\pm} \to HW^{*}]$

## REFERENCES


[1] **ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group** Collaboration, S. Schael *et al.*, "Precision electroweak measurements on the $Z$ resonance," *Phys. Rept.* **427** (2006) 257–454, arXiv:hep-ex/0509008.

[2] **Heavy Flavor Averaging Group, HFLAV** Collaboration, Y. S. Amhis *et al.*, "Averages of b-hadron, c-hadron, and $\tau$-lepton properties as of 2021," *Phys. Rev. D* **107** no. 5, (2023) 052008, arXiv:2206.07501 [hep-ex].

[3] **Particle Data Group** Collaboration, R. L. Workman *et al.*, "Review of Particle Physics," *PTEP* **2022** (2022) 083C01.

[4] P. W. Higgs, "Broken symmetries, massless particles and gauge fields," *Phys. Lett.* **12** (1964) 132–133.

[5] F. Englert and R. Brout, "Broken Symmetry and the Mass of Gauge Vector Mesons," *Phys. Rev. Lett.* **13** (1964) 321–323.

[6] P. W. Higgs, "Broken Symmetries and the Masses of Gauge Bosons," *Phys. Rev. Lett.* **13** (1964) 508–509.

[7] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, "Global Conservation Laws and Massless Particles," *Phys. Rev. Lett.* **13** (1964) 585–587.

[8] **ATLAS** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys. Lett. B* **716** (2012) 1–29, arXiv:1207.7214 [hep-ex].

[9] **CMS** Collaboration, S. Chatrchyan *et al.*, "Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC," *Phys. Lett. B* **716** (2012) 30–61, arXiv:1207.7235 [hep-ex].

[10] **CDF, D0** Collaboration, T. Aaltonen *et al.*, "Evidence for a particle produced in association with weak bosons and decaying to a bottom-antibottom quark pair in Higgs boson searches at the Tevatron," *Phys. Rev. Lett.* **109** (2012) 071804, arXiv:1207.6436 [hep-ex].

[11] **CMS** Collaboration, S. Chatrchyan *et al.*, "Study of the Mass and Spin-Parity of the Higgs Boson Candidate Via Its Decays to Z Boson Pairs," *Phys. Rev. Lett.* **110** no. 8, (2013) 081803, arXiv:1212.6639 [hep-ex].

[12] **ATLAS** Collaboration, G. Aad *et al.*, "Evidence for the spin-0 nature of the Higgs boson using ATLAS data," *Phys. Lett. B* **726** (2013) 120–144, arXiv:1307.1432 [hep-ex].

[13] **ATLAS, CMS** Collaboration, G. Aad *et al.*, "Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV," *JHEP* **08** (2016) 045, arXiv:1606.02266 [hep-ex].

[14] **ATLAS, CMS** Collaboration, J. M. Langford, "Combination of Higgs measurements from ATLAS and CMS: couplings and $\kappa$- framework," *PoS* **LHCP2020** (2021) 136.

[15] **ATLAS** Collaboration, "Combined measurements of Higgs boson production and decay using up to 139 fb$^{-1}$ of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment,".

[16] **CMS** Collaboration, G. Kole, "Search for Additional Higgs Boson with CMS Detector at LHC," *Moscow Univ. Phys. Bull.* **77** no. 2, (2022) 223–225.

[17] **ATLAS, CMS** Collaboration, C. Palmer, "Higgs boson experimental overview for ICHEP 2020," *PoS* **ICHEP2020** (2021) 034.

[18] V. Silveira and A. Zee, "SCALAR PHANTOMS," *Phys. Lett. B* **161** (1985) 136–140.

[19] M. Pietroni, "The Electroweak phase transition in a nonminimal supersymmetric model," *Nucl. Phys. B* **402** (1993) 27–45, arXiv:hep-ph/9207227.

[20] J. McDonald, "Gauge singlet scalars as cold dark matter," *Phys. Rev. D* **50** (1994) 3637–3649, arXiv:hep-ph/0702143.

[21] T. D. Lee, "A Theory of Spontaneous T Violation," *Phys. Rev. D* **8** (1973) 1226–1239.

[22] H. E. Haber and G. L. Kane, "The Search for Supersymmetry: Probing Physics Beyond the Standard Model," *Phys. Rept.* **117** (1985) 75–263.

[23] J. E. Kim, "Light Pseudoscalars, Particle Physics and Cosmology," *Phys. Rept.* **150** (1987) 1–177.

[24] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," Phys. Rev. Lett. **38** (1977) 1440–1443.
[25] N. Turok and J. Zadrozny, "Electroweak baryogenesis in the two doublet model," Nucl. Phys. B **358** (1991) 471–493.
[26] W. Konetschny and W. Kummer, "Nonconservation of Total Lepton Number with Scalar Bosons," Phys. Lett. B **70** (1977) 433–435.
[27] T. P. Cheng and L.-F. Li, "Neutrino Masses, Mixings and Oscillations in $SU(2) \times U(1)$ Models of Electroweak Interactions," Phys. Rev. D **22** (1980) 2860.
[28] G. Lazarides, Q. Shafi, and C. Wetterich, "Proton Lifetime and Fermion Masses in an $SO(10)$ Model," Nucl. Phys. B **181** (1981) 287–300.
[29] J. Schechter and J. W. F. Valle, "Neutrino Masses in $SU(2) \times U(1)$ Theories," Phys. Rev. D **22** (1980) 2227.
[30] M. Magg and C. Wetterich, "Neutrino Mass Problem and Gauge Hierarchy," Phys. Lett. B **94** (1980) 61–64.
[31] R. N. Mohapatra and G. Senjanovic, "Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation," Phys. Rev. D **23** (1981) 165.
[32] O. Fischer *et al.*, "Unveiling hidden physics at the LHC," Eur. Phys. J. C **82** no. 8, (2022) 665, arXiv:2109.06065 [hep-ph].
[33] **CMS** Collaboration, A. M. Sirunyan *et al.*, "Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton-proton collisions at $\sqrt{s} = 8$ and 13 TeV," Phys. Lett. B **793** (2019) 320–347, arXiv:1811.08459 [hep-ex].
[34] **CMS** Collaboration, "Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton-proton collisions at $\sqrt{s} = 13$ TeV,".
[35] **CMS** Collaboration, "Searches for additional Higgs bosons and for vector leptoquarks in $\tau\tau$ final states in proton-proton collisions at $\sqrt{s} = 13$ TeV," arXiv:2208.02717 [hep-ex].
[36] **ATLAS** Collaboration, "Search for diphoton resonances in the 66 to 110 GeV mass range using 140 fb$^{-1}$ of 13 TeV $pp$ collisions collected with the ATLAS detector,".
[37] **LEP Working Group for Higgs boson searches, ALEPH, DELPHI, L3, OPAL** Collaboration, R. Barate *et al.*, "Search for the standard model Higgs boson at LEP," Phys. Lett. B **565** (2003) 61–75, arXiv:hep-ex/0306033.
[38] **CMS** Collaboration, "Measurements of the Higgs boson production cross section and couplings in the $W$ boson pair decay channel in proton-proton collisions at $\sqrt{s} = 13$ TeV," arXiv:2206.09466 [hep-ex].
[39] **ATLAS** Collaboration, "Measurements of Higgs boson production by gluon−gluon fusion and vector-boson fusion using $H \to WW^* \to e\nu\mu\nu$ decays in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector," arXiv:2207.00338 [hep-ex].
[40] G. Coloretti, A. Crivellin, S. Bhattacharya, and B. Mellado, "Searching for Low-Mass Resonances Decaying into $W$ Bosons," arXiv:2302.07276 [hep-ph].
[41] S. Bhattacharya, G. Coloretti, A. Crivellin, S.-E. Dahbi, Y. Fang, M. Kumar, and B. Mellado, "Growing Excesses of New Scalars at the Electroweak Scale," arXiv:2306.17209 [hep-ph].
[42] J. Cao, X. Guo, Y. He, P. Wu, and Y. Zhang, "Diphoton signal of the light Higgs boson in natural NMSSM," Phys. Rev. D **95** no. 11, (2017) 116001, arXiv:1612.08522 [hep-ph].
[43] A. Crivellin, J. Heeck, and D. Müller, "Large $h \to bs$ in generic two-Higgs-doublet models," Phys. Rev. D **97** no. 3, (2018) 035008, arXiv:1710.04663 [hep-ph].
[44] U. Haisch and A. Malinauskas, "Let there be light from a second light Higgs doublet," JHEP **03** (2018) 135, arXiv:1712.06599 [hep-ph].
[45] P. J. Fox and N. Weiner, "Light Signals from a Lighter Higgs," JHEP **08** (2018) 025, arXiv:1710.07649 [hep-ph].
[46] D. Liu, J. Liu, C. E. M. Wagner, and X.-P. Wang, "A Light Higgs at the LHC and the $B$-Anomalies," JHEP **06** (2018) 150, arXiv:1805.01476 [hep-ph].
[47] K. Wang, F. Wang, J. Zhu, and Q. Jie, "The semi-constrained NMSSM in light of muon $g-2$, LHC, and dark matter constraints," Chin. Phys. C **42** no. 10, (2018) 103109–103109, arXiv:1811.04435 [hep-ph].
[48] L. Liu, H. Qiao, K. Wang, and J. Zhu, "A Light Scalar in the Minimal Dilaton Model in Light of LHC Constraints," Chin. Phys. C **43** no. 2, (2019) 023104, arXiv:1812.00107 [hep-ph].
[49] T. Biekötter, M. Chakraborti, and S. Heinemeyer, "A 96 GeV Higgs boson in the N2HDM," Eur. Phys. J. C **80** no. 1, (2020) 2, arXiv:1903.11661 [hep-ph].
[50] J. M. Cline and T. Toma, "Pseudo-Goldstone dark matter confronts cosmic ray and collider anomalies," Phys. Rev. D **100** no. 3, (2019) 035023, arXiv:1906.02175 [hep-ph].
[51] K. Choi, S. H. Im, K. S. Jeong, and C. B. Park, "Light Higgs bosons in the general NMSSM," Eur. Phys. J. C **79** no. 11, (2019) 956, arXiv:1906.03389 [hep-ph].
[52] A. Kundu, S. Maharana, and P. Mondal, "A 96 GeV scalar tagged to dark matter models," Nucl. Phys. B **955** (2020) 115057, arXiv:1907.12808 [hep-ph].
[53] J. Cao, X. Jia, Y. Yue, H. Zhou, and P. Zhu, "96 GeV diphoton excess in seesaw extensions of the natural NMSSM," Phys. Rev. D **101** no. 5, (2020) 055008, arXiv:1908.07206 [hep-ph].
[54] T. Biekötter, M. Chakraborti, and S. Heinemeyer, "The "96 GeV excess" at the LHC," Int. J. Mod. Phys. A **36** no. 22, (2021) 2142018, arXiv:2003.05422 [hep-ph].
[55] A. A. Abdelalim, B. Das, S. Khalil, and S. Moretti, "Di-photon decay of a light Higgs state in the BLSSM," Nucl. Phys. B **985** (2022) 116013, arXiv:2012.04952 [hep-ph].
[56] S. Heinemeyer, C. Li, F. Lika, G. Moortgat-Pick, and S. Paasch, "Phenomenology of a 96 GeV Higgs boson in the 2HDM with an additional singlet," Phys. Rev. D **106** no. 7, (2022) 075003, arXiv:2112.11958 [hep-ph].
[57] T. Biekötter, S. Heinemeyer, and G. Weiglein, "Mounting evidence for a 95 GeV Higgs boson," JHEP **08** (2022) 201, arXiv:2203.13180 [hep-ph].
[58] S. Iguro, T. Kitahara, and Y. Omura, "Scrutinizing the 95–100 GeV di-tau excess in the top associated process," Eur. Phys. J. C **82** no. 11, (2022) 1053, arXiv:2205.03187 [hep-ph].

[59] W. Li, J. Zhu, K. Wang, S. Ma, P. Tian, and H. Qiao, "A light Higgs boson in the NMSSM confronted with the CMS di-photon and di-tau excesses," arXiv:2212.11739 [hep-ph].

[60] S. Iguro, T. Kitahara, Y. Omura, and H. Zhang, "Chasing the two-Higgs doublet model in the di-Higgs boson production," Phys. Rev. D **107** no. 7, (2023) 075017, arXiv:2211.00011 [hep-ph].

[61] T. Biekötter, S. Heinemeyer, and G. Weiglein, "The CMS di-photon excess at 95 GeV in view of the LHC Run 2 results," arXiv:2303.12018 [hep-ph].

[62] C. Bonilla, A. E. Carcamo Hernandez, S. Kovalenko, H. Lee, R. Pasechnik, and I. Schmidt, "Fermion mass hierarchy in an extended left-right symmetric model," arXiv:2305.11967 [hep-ph].

[63] D. Azevedo, T. Biekötter, and P. M. Ferreira, "2HDM interpretations of the CMS diphoton excess at 95 GeV," arXiv:2305.19716 [hep-ph].

[64] S. Banik, A. Crivellin, S. Iguro, and T. Kitahara, "Asymmetric Di-Higgs Signals of the N2HDM-$U(1)$," arXiv:2303.11351 [hep-ph].

[65] T. Biekötter, S. Heinemeyer, and G. Weiglein, "The 95.4 GeV di-photon excess at ATLAS and CMS," arXiv:2306.03889 [hep-ph].

[66] P. Escribano, V. M. Lozano, and A. Vicente, "A Scotogenic explanation for the 95 GeV excesses," arXiv:2306.03735 [hep-ph].

[67] A. Belyaev, R. Benbrik, M. Boukidi, M. Chakraborti, S. Moretti, and S. Semlali, "Explanation of the Hints for a 95 GeV Higgs Boson within a 2-Higgs Doublet Model," arXiv:2306.09029 [hep-ph].

[68] **D0** Collaboration, V. M. Abazov *et al.*, "Search for doubly-charged Higgs boson pair production in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," Phys. Rev. Lett. **108** (2012) 021801, arXiv:1106.4250 [hep-ex].

[69] **CDF** Collaboration, T. Aaltonen *et al.*, "Search for new physics in high $p_T$ like-sign dilepton events at CDF˜II," Phys. Rev. Lett. **107** (2011) 181801, arXiv:1108.0101 [hep-ex].

[70] **ATLAS** Collaboration, G. Aad *et al.*, "Search for anomalous production of prompt same-sign lepton pairs and pair-produced doubly charged Higgs bosons with $\sqrt{s} = 8$ TeV $pp$ collisions using the ATLAS detector," JHEP **03** (2015) 041, arXiv:1412.0237 [hep-ex].

[71] **CMS** Collaboration, "Search for a doubly-charged Higgs boson with $\sqrt{s} = 8$ TeV $pp$ collisions at the CMS experiment,".

[72] J. Heeck, "W-boson mass in the triplet seesaw model," Phys. Rev. D **106** no. 1, (2022) 015004, arXiv:2204.10274 [hep-ph].

[73] S. Ashanujjaman, K. Ghosh, and R. Sahu, "Low-mass doubly charged Higgs bosons at the LHC," Phys. Rev. D **107** no. 1, (2023) 015018, arXiv:2211.00632 [hep-ph].

[74] S. Chakrabarti, D. Choudhury, R. M. Godbole, and B. Mukhopadhyaya, "Observing doubly charged Higgs bosons in photon-photon collisions," Phys. Lett. B **434** (1998) 347–353, arXiv:hep-ph/9804297.

[75] A. G. Akeroyd and M. Aoki, "Single and pair production of doubly charged Higgs bosons at hadron colliders," Phys. Rev. D **72** (2005) 035011, arXiv:hep-ph/0506176.

[76] M. Aoki, S. Kanemura, and K. Yagyu, "Testing the Higgs triplet model with the mass difference at the LHC," Phys. Rev. D **85** (2012) 055007, arXiv:1110.4625 [hep-ph].

[77] S. Ashanujjaman and K. Ghosh, "Revisiting type-II see-saw: present limits and future prospects at LHC," JHEP **03** (2022) 195, arXiv:2108.10952 [hep-ph].

[78] S. Ashanujjaman, K. Ghosh, and K. Huitu, "Type-II see-saw: searching the LHC elusive low-mass triplet-like Higgses at $e^-e^+$ colliders," Phys. Rev. D **106** no. 7, (2022) 075028, arXiv:2205.14983 [hep-ph].

[79] S. Ashanujjaman and S. P. Maharathy, "Probing compressed mass spectra in the type-II seesaw model at the LHC," Phys. Rev. D **107** no. 11, (2023) 115026, arXiv:2305.06889 [hep-ph].

[80] M. Chabab, M. C. Peyranère, and L. Rahili, "Probing the Higgs sector of $Y = 0$ Higgs Triplet Model at LHC," Eur. Phys. J. C **78** no. 10, (2018) 873, arXiv:1805.00286 [hep-ph].

[81] P. Fileviez Perez, H. H. Patel, and A. D. Plascencia, "On the $W$ mass and new Higgs bosons," Phys. Lett. B **833** (2022) 137371, arXiv:2204.07144 [hep-ph].

[82] Y. Cheng, X.-G. He, F. Huang, J. Sun, and Z.-P. Xing, "Electroweak precision tests for triplet scalars," Nucl. Phys. B **989** (2023) 116118, arXiv:2208.06760 [hep-ph].

[83] T.-K. Chen, C.-W. Chiang, and K. Yagyu, "Explanation of the $W$ mass shift at CDF II in the extended Georgi-Machacek model," Phys. Rev. D **106** no. 5, (2022) 055035, arXiv:2204.12898 [hep-ph].

[84] T. G. Rizzo, "Kinetic mixing, dark Higgs triplets, and $M_W$," Phys. Rev. D **106** no. 3, (2022) 035024, arXiv:2206.09814 [hep-ph].

[85] W. Chao, M. Jin, H.-J. Li, and Y.-Q. Peng, "Axion-like Dark Matter from the Type-II Seesaw Mechanism," arXiv:2210.13233 [hep-ph].

[86] J.-W. Wang, X.-J. Bi, P.-F. Yin, and Z.-H. Yu, "Electroweak dark matter model accounting for the CDF W-mass anomaly," Phys. Rev. D **106** no. 5, (2022) 055001, arXiv:2205.00783 [hep-ph].

[87] Y. Shimizu and S. Takeshita, "$W$ Boson Mass and Grand Unification via the Type-II Seesaw-like Mechanism," arXiv:2303.11070 [hep-ph].

[88] G. Lazarides, R. Maji, R. Roshan, and Q. Shafi, "Heavier $W$ boson, dark matter, and gravitational waves from strings in an SO(10) axion model," Phys. Rev. D **106** no. 5, (2022) 055009, arXiv:2205.04824 [hep-ph].

[89] G. Senjanović and M. Zantedeschi, "$SU(5)$ grand unification and W-boson mass," Phys. Lett. B **837** (2023) 137653, arXiv:2205.05022 [hep-ph].

[90] A. Crivellin, M. Kirk, and A. Thapa, "$Y = 0$ Scalar Triplet Beyond the $W$ Mass: $(g-2)_\mu$, $h \to \mu^+\mu^-$ and CKM Unitarity," arXiv:2305.03081 [hep-ph].

[91] T.-K. Chen, C.-W. Chiang, and K. Yagyu, "CP Violation in a Model with Higgs Triplets," JHEP **06** (2023) 069, arXiv:2303.09294 [hep-ph].

[92] J. de Blas, M. Pierini, L. Reina, and L. Silvestrini, "Impact of the Recent Measurements of the Top-Quark and $W$-Boson Masses on Electroweak Precision Fits," Phys. Rev. Lett. **129** no. 27, (2022) 271801, arXiv:2204.04204 [hep-ph].

[93] P. Athron, M. Bach, D. H. J. Jacob, W. Kotlarski, D. Stöckinger, and A. Voigt, "Precise calculation of the W boson pole mass beyond the standard model with FlexibleSUSY," Phys. Rev. D **106** no. 9, (2022) 095023, arXiv:2204.05285 [hep-ph].

[94] E. Bagnaschi, J. Ellis, M. Madigan, K. Mimasu, V. Sanz, and T. You, "SMEFT analysis of $m_W$," JHEP **08** (2022) 308, arXiv:2204.05260 [hep-ph].

[95] **CDF** Collaboration, T. Aaltonen *et al.*, "High-precision measurement of the $W$ boson mass with the CDF II detector," Science **376** no. 6589, (2022) 170–176.

[96] D. A. Ross and M. J. G. Veltman, "Neutral Currents in Neutrino Experiments," Nucl. Phys. B **95** (1975) 135–147.

[97] J. F. Gunion, R. Vega, and J. Wudka, "Higgs triplets in the standard model," Phys. Rev. D **42** (1990) 1673–1691.

[98] P. H. Chankowski, S. Pokorski, and J. Wagner, "(Non)decoupling of the Higgs triplet effects," Eur. Phys. J. C **50** (2007) 919–933, arXiv:hep-ph/0605302.

[99] T. Blank and W. Hollik, "Precision observables in $SU(2) \times U(1)$ models with an additional Higgs triplet," Nucl. Phys. B **514** (1998) 113–134, arXiv:hep-ph/9703392.

[100] J. R. Forshaw, A. Sabio Vera, and B. E. White, "Mass bounds in a model with a triplet Higgs," JHEP **06** (2003) 059, arXiv:hep-ph/0302256.

[101] M.-C. Chen, S. Dawson, and T. Krupovnickas, "Higgs triplets and limits from precision measurements," Phys. Rev. D **74** (2006) 035001, arXiv:hep-ph/0604102.

[102] R. S. Chivukula, N. D. Christensen, and E. H. Simmons, "Low-energy effective theory, unitarity, and non-decoupling behavior in a model with heavy Higgs-triplet fields," Phys. Rev. D **77** (2008) 035001, arXiv:0712.0546 [hep-ph].

[103] P. Bandyopadhyay and A. Costantini, "Obscure Higgs boson at Colliders," Phys. Rev. D **103** no. 1, (2021) 015025, arXiv:2010.02597 [hep-ph].

[104] **ATLAS** Collaboration, "Improved $W$ boson Mass Measurement using 7 TeV Proton-Proton Collisions with the ATLAS Detector,".

[105] **ATLAS** Collaboration, M. Aaboud *et al.*, "Measurement of the $W$-boson mass in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector," Eur. Phys. J. C **78** no. 2, (2018) 110, arXiv:1701.07240 [hep-ex]. [Erratum: Eur.Phys.J.C 78, 898 (2018)].

[106] **LHCb** Collaboration, R. Aaij *et al.*, "Measurement of the $W$ boson mass," JHEP **01** (2022) 036, arXiv:2109.01113 [hep-ex].

[107] **ALEPH, DELPHI, L3, OPAL, LEP Electroweak** Collaboration, S. Schael *et al.*, "Electroweak Measurements in Electron-Positron Collisions at W-Boson-Pair Energies at LEP," Phys. Rept. **532** (2013) 119–244, arXiv:1302.3415 [hep-ex].

[108] A. Sirlin, "On the $O(\alpha^2)$ Corrections to $\tau$ $(\mu)$, $m_W$, $m_Z$ in the $SU(2)_L \times U(1)$ Theory," Phys. Rev. D **29** (1984) 89.

[109] A. Djouadi and C. Verzegnassi, "Virtual Very Heavy Top Effects in LEP / SLC Precision Measurements," Phys. Lett. B **195** (1987) 265–271.

[110] L. Avdeev, J. Fleischer, S. Mikhailov, and O. Tarasov, "$0(\alpha\alpha_s^2)$ correction to the electroweak $\rho$ parameter," Phys. Lett. B **336** (1994) 560–566, arXiv:hep-ph/9406363. [Erratum: Phys.Lett.B 349, 597–598 (1995)].

[111] K. G. Chetyrkin, J. H. Kuhn, and M. Steinhauser, "QCD corrections from top quark to relations between electroweak parameters to order $\alpha_s^2$," Phys. Rev. Lett. **75** (1995) 3394–3397, arXiv:hep-ph/9504413.

[112] K. G. Chetyrkin, J. H. Kuhn, and M. Steinhauser, "Corrections of order $\mathcal{O}(G_F M_t^2 \alpha_s^2)$ to the $\rho$ parameter," Phys. Lett. B **351** (1995) 331–338, arXiv:hep-ph/9502291.

[113] M. Awramik, M. Czakon, A. Freitas, and G. Weiglein, "Precise prediction for the $W$ boson mass in the standard model," Phys. Rev. D **69** (2004) 053006, arXiv:hep-ph/0311148.

[114] G. Degrassi, P. Gambino, and P. P. Giardino, "The $m_W - m_Z$ interdependence in the Standard Model: a new scrutiny," JHEP **05** (2015) 154, arXiv:1411.7040 [hep-ph].

[115] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide*, vol. 80. 2000.

[116] **CMS** Collaboration, A. M. Sirunyan *et al.*, "Measurements of Higgs boson production cross sections and couplings in the diphoton decay channel at $\sqrt{s} = 13$ TeV," JHEP **07** (2021) 027, arXiv:2103.06956 [hep-ex].

[117] **ATLAS** Collaboration, "Measurement of the properties of Higgs boson production at $\sqrt{s} = 13$ TeV in the $H \to \gamma\gamma$ channel using 139 fb$^{-1}$ of $pp$ collision data with the ATLAS experiment," arXiv:2207.00348 [hep-ex].

[118] **CMS** Collaboration, A. Tumasyan *et al.*, "A portrait of the Higgs boson by the CMS experiment ten years after the discovery," Nature **607** no. 7917, (2022) 60–68, arXiv:2207.00043 [hep-ex].

[119] **ATLAS** Collaboration, G. Aad *et al.*, "Higgs boson production cross-section measurements and their EFT interpretation in the $4\ell$ decay channel at $\sqrt{s}$ =13 TeV with the ATLAS detector," Eur. Phys. J. C **80** no. 10, (2020) 957, arXiv:2004.03447 [hep-ex]. [Erratum: Eur.Phys.J.C 81, 29 (2021), Erratum: Eur.Phys.J.C 81, 398 (2021)].

[120] A. H. Ajjath, B. Fuks, H.-S. Shao, and Y. Simon, "Precision predictions for exotic lepton production at the Large Hadron Collider," Phys. Rev. D **107** no. 7, (2023) 075011, arXiv:2301.03640 [hep-ph].

[121] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, and T. Stelzer, "MadGraph 5 : Going Beyond," JHEP **06** (2011) 128, arXiv:1106.0522 [hep-ph].

[122] **LHC Higgs Cross Section Working Group** Collaboration, D. de Florian *et al.*, "Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector," arXiv:1610.07922 [hep-ph].

[123] D. Graudenz, M. Spira, and P. M. Zerwas, "QCD corrections to Higgs boson production at proton proton colliders," Phys. Rev. Lett. **70** (1993) 1372–1375.

[124] M. Spira, A. Djouadi, D. Graudenz, and P. M. Zerwas, "Higgs boson production at the LHC," Nucl. Phys. B **453** (1995) 17–82, arXiv:hep-ph/9504378.

[125] C. Anastasiou and K. Melnikov, "Higgs boson production at hadron colliders in NNLO QCD," Nucl. Phys. B **646** (2002) 220–256, arXiv:hep-ph/0207004.

[126] R. V. Harlander and W. B. Kilgore, "Next-to-next-to-leading order Higgs production at hadron colliders," Phys. Rev. Lett. **88** (2002) 201801, arXiv:hep-ph/0201206.

[127] R. V. Harlander and W. B. Kilgore, "Soft and virtual corrections to proton proton → H + x at NNLO," Phys. Rev. D **64** (2001) 013015, arXiv:hep-ph/0102241.

[128] U. Aglietti, R. Bonciani, G. Degrassi, and A. Vicini, "Analytic Results for Virtual QCD Corrections to Higgs Production and Decay," JHEP **01** (2007) 021, arXiv:hep-ph/0611266.

[129] H.-L. Li, P.-C. Lu, Z.-G. Si, and Y. Wang, "Associated Production of Higgs Boson and $t\bar{t}$ at LHC," Chin. Phys. C **40** no. 6, (2016) 063102, arXiv:1508.06416 [hep-ph].

[130] C. Anastasiou, S. Beerli, S. Bucherer, A. Daleo, and Z. Kunszt, "Two-loop amplitudes and master integrals for the production of a Higgs boson via a massive quark and a scalar-quark loop," JHEP **01** (2007) 082, arXiv:hep-ph/0611236.

[131] R. Harlander and P. Kant, "Higgs production and decay: Analytic results at next-to-leading order QCD," JHEP **12** (2005) 015, arXiv:hep-ph/0509189.

[132] V. Ravindran, J. Smith, and W. L. van Neerven, "NNLO corrections to the total cross-section for Higgs boson production in hadron hadron collisions," Nucl. Phys. B **665** (2003) 325–366, arXiv:hep-ph/0302135.

[133] A. Delgado, M. Garcia-Pepin, M. Quiros, J. Santiago, and R. Vega-Morales, "Diphoton and Diboson Probes of Fermiophobic Higgs Bosons at the LHC," JHEP **06** (2016) 042, arXiv:1603.00962 [hep-ph].

[134] **ATLAS** Collaboration, G. Aad *et al.*, "Search for direct stau production in events with two hadronic $\tau$-leptons in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector," Phys. Rev. D **101** no. 3, (2020) 032009, arXiv:1911.06660 [hep-ex].

[135] **CMS** Collaboration, A. M. Sirunyan *et al.*, "Search for direct pair production of supersymmetric partners to the $\tau$ lepton in proton-proton collisions at $\sqrt{s} = 13$ TeV," Eur. Phys. J. C **80** no. 3, (2020) 189, arXiv:1907.13179 [hep-ex].

[136] **CMS** Collaboration, "Search for direct pair production of supersymmetric partners of $\tau$ leptons in the final state with two hadronically decaying $\tau$ leptons and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV," arXiv:2207.02254 [hep-ex].

[137] **ATLAS** Collaboration, "Search for electroweak SUSY production in final states with two $\tau$-leptons in $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector,".

[138] R. Ruiz, "QCD Corrections to Pair Production of Type III Seesaw Leptons at Hadron Colliders," JHEP **12** (2015) 165, arXiv:1509.05416 [hep-ph].

[139] E. Braaten and J. P. Leveille, "Higgs Boson Decay and the Running Mass," Phys. Rev. D **22** (1980) 715.

[140] N. Sakai, "Perturbative QCD Corrections to the Hadronic Decay Width of the Higgs Boson," Phys. Rev. D **22** (1980) 2220.

[141] T. Inami and T. Kubota, "Renormalization Group Estimate of the Hadronic Decay Width of the Higgs Boson," Nucl. Phys. B **179** (1981) 171–188.

[142] S. G. Gorishnii, A. L. Kataev, and S. A. Larin, "The Width of Higgs Boson Decay Into Hadrons: Three Loop Corrections of Strong Interactions," Sov. J. Nucl. Phys. **40** (1984) 329–334.

[143] S. G. Gorishnii, A. L. Kataev, S. A. Larin, and L. R. Surguladze, "The Analytical four loop corrections to the QED Beta function in the MS scheme and to the QED psi function: Total reevaluation," Phys. Lett. B **256** (1991) 81–86.

[144] S. G. Gorishnii, A. L. Kataev, S. A. Larin, and L. R. Surguladze, "Corrected Three Loop QCD Correction to the Correlator of the Quark Scalar Currents and $\gamma$ (Tot) ($H^0 \to$ Hadrons)," Mod. Phys. Lett. A **5** (1990) 2703–2712.

[145] S. G. Gorishnii, A. L. Kataev, S. A. Larin, and L. R. Surguladze, "Scheme dependence of the next to next-to-leading QCD corrections to Gamma(tot) (H0 → hadrons) and the spurious QCD infrared fixed point," Phys. Rev. D **43** (1991) 1633–1640.

[146] A. Djouadi, P. Gambino, and B. A. Kniehl, "Two loop electroweak heavy fermion corrections to Higgs boson production and decay," Nucl. Phys. B **523** (1998) 17–39, arXiv:hep-ph/9712330.

[147] G. Degrassi and F. Maltoni, "Two-loop electroweak corrections to the Higgs-boson decay $H \to \gamma\gamma$," Nucl. Phys. B **724** (2005) 183–196, arXiv:hep-ph/0504137.

[148] G. Passarino, C. Sturm, and S. Uccirati, "Complete Two-Loop Corrections to $H \to \gamma\gamma$," Phys. Lett. B **655** (2007) 298–306, arXiv:0707.1401 [hep-ph].

[149] S. Actis, G. Passarino, C. Sturm, and S. Uccirati, "NNLO Computational Techniques: The Cases $H \to \gamma\gamma$ and $H \to gg$," Nucl. Phys. B **811** (2009) 182–273, arXiv:0809.3667 [hep-ph].

[150] A. Djouadi, M. Spira, J. J. van der Bij, and P. M. Zerwas, "QCD corrections to gamma gamma decays of Higgs particles in the intermediate mass range," Phys. Lett. B **257** (1991) 187–190.

[151] K. G. Chetyrkin, "Correlator of the quark scalar currents and Gamma(tot) (H → hadrons) at $O(\alpha_s^3)$ in pQCD," Phys. Lett. B **390** (1997) 309–317, arXiv:hep-ph/9608318.

[152] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, "Scalar correlator at $O(alpha_s^4)$, Higgs decay into b-quarks and bounds on the light quark masses," Phys. Rev. Lett. **96** (2006) 012003, arXiv:hep-ph/0511063.

[153] M. Spira, A. Djouadi, and P. M. Zerwas, "QCD corrections to the H Z gamma coupling," Phys. Lett. B **276** (1992) 350–353.

[154] **FCC** Collaboration, A. Abada *et al.*, "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2," Eur. Phys. J. ST **228** no. 2, (2019) 261–623.

[155] "A Multi-TeV Linear Collider Based on CLIC Technology: CLIC Conceptual Design Report,".

[156] H. Abramowicz *et al.*, "The International Linear Collider Technical Design Report - Volume 4: Detectors," arXiv:1306.6329 [physics.ins-det].

[157] F. An *et al.*, "Precision Higgs physics at the CEPC," Chin. Phys. C **43** no. 4, (2019) 043002, arXiv:1810.09037 [hep-ex].

[158] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, "An introduction to PYTHIA 8.2" Comput. Phys. Commun. **191** (2015) 159–177, arXiv:1410.3012 [hep-ph].

[159] **DELPHES 3** Collaboration, J. de Favereau, C. Delaere, P. Demin, A. Giammanco, V. Lemaître, A. Mertens, and M. Selvaggi, "DELPHES 3, A modular framework for fast simulation of a generic collider experiment," *JHEP* **02** (2014) 057, arXiv:1307.6346 [hep-ex].

[160] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, and T. Reiter, "UFO - The Universal FeynRules Output," *Comput. Phys. Commun.* **183** (2012) 1201–1214, arXiv:1108.2040 [hep-ph].

[161] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, "FeynRules 2.0 - A complete toolbox for tree-level phenomenology," *Comput. Phys. Commun.* **185** (2014) 2250–2300, arXiv:1310.1921 [hep-ph].

[162] C. Degrande, "Automatic evaluation of UV and R2 terms for beyond the Standard Model Lagrangians: a proof-of-principle," *Comput. Phys. Commun.* **197** (2015) 239–262, arXiv:1406.3030 [hep-ph].

[163] D. Buarque Franzosi, O. Mattelaer, R. Ruiz, and S. Shil, "Automated predictions from polarized matrix elements," *JHEP* **04** (2020) 082, arXiv:1912.01725 [hep-ph].

[164] M. Cirelli, N. Fornengo, and A. Strumia, "Minimal dark matter," *Nucl. Phys. B* **753** (2006) 178–194, arXiv:hep-ph/0512090.

[165] S. Kanemura and K. Yagyu, "Radiative corrections to electroweak parameters in the Higgs triplet model and implication with the recent Higgs boson searches," *Phys. Rev. D* **85** (2012) 115009, arXiv:1201.6287 [hep-ph].

[166] N. Khan, "Exploring the hyperchargeless Higgs triplet model up to the Planck scale," *Eur. Phys. J. C* **78** no. 4, (2018) 341, arXiv:1610.03178 [hep-ph].

[167] P. Fileviez Perez, H. H. Patel, M. J. Ramsey-Musolf, and K. Wang, "Triplet Scalars and Dark Matter at the LHC," *Phys. Rev. D* **79** (2009) 055024, arXiv:0811.3957 [hep-ph].

[168] H. E. Logan, "Lectures on perturbative unitarity and decoupling in Higgs physics," arXiv:2207.01064 [hep-ph].

[169] J. E. Camargo-Molina, B. Garbrecht, B. O'Leary, W. Porod, and F. Staub, "Constraining the Natural MSSM through tunneling to color-breaking vacua at zero and non-zero temperature," *Phys. Lett. B* **737** (2014) 156–161, arXiv:1405.7376 [hep-ph].

[170] J. E. Camargo-Molina, B. O'Leary, W. Porod, and F. Staub, "**Vevacious**: A Tool For Finding The Global Minima Of One-Loop Effective Potentials With Many Scalars," *Eur. Phys. J. C* **73** no. 10, (2013) 2588, arXiv:1307.1477 [hep-ph].

[171] W. Porod, "SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders," *Comput. Phys. Commun.* **153** (2003) 275–315, arXiv:hep-ph/0301101.

[172] W. Porod and F. Staub, "SPheno 3.1: Extensions including flavour, CP-phases and models beyond the MSSM," *Comput. Phys. Commun.* **183** (2012) 2458–2469, arXiv:1104.1573 [hep-ph].

[173] M. D. Goodsell and A. Joury, "BSMArt: simple and fast parameter space scans," arXiv:2301.01154 [hep-ph].