Observational constraints on α-Starobinsky inflation

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Abstract

In this work we revisit α-Starobinsky inflation, also know as E-model, in the light of current CMB and LSS observations. The inflaton potential in the Einstein frame for this model contains a parameter α in the exponential, which alters the predictions for the scalar and tensor power spectra of Starobinsky inflation. We obtain these power spectra numerically without using slow-roll approximation and perform MCMC analysis to put constraints on parameters M and α from Planck-2018, BICEP/Keck (BK18) and other LSS observations. We consider general reheating scenario by varying the number of e-foldings during inflation, N_{pivot}, along with the other parameters. We find $\log_{10}\alpha = 0.0^{+1.6}_{-5.6}$, $\log_{10} M = -4.91^{+0.69}_{-2.7}$ and $N_{pivot} = 53.2^{+3.9}_{-5}$ with 95% C. L.. This implies that the present CMB and LSS observations are insufficient to constrain the parameter α. We also find that there is no correlation between N_{pivot} and α.

1. INTRODUCTION

Inflation [1] is a theoretical framework in cosmology that proposes a brief and extremely rapid expansion of the early universe before big bang nucleosynthesis. It provides solution to some key puzzles of the big bang model such as the horizon problem, which is related to the uniformity of the cosmic microwave background radiation, and the flatness problem, which concerns the geometry of the universe. The driving force behind inflation is a hypothetical scalar field named as inflaton, whose potential energy dominates the energy density of the universe causing quasi-exponential expansion of the universe for a very short period of time. During this period the quantum fluctuations in the inflaton, which are coupled to the metric perturbations, give rise to the primordial density perturbations. The quantum fluctuations in the spacetime geometry during inflation are responsible for the primordial gravitational waves (tensor perturbations). These primordial perturbations generated during inflation provide seeds for cosmic microwave background anisotropy and structures in the universe. Observations of CMB anisotropy and polarization by COBE [2], WMAP [3], Planck [4, 5], and BICEP/Keck array offer robust experimental support for the predictions of inflation, i.e. adiabatic, nearly scale invariant and Gaussian perturbations. Since its inception, several models of inflation based on particle physics and string theory have been explored [6, 7]. The most popular models of inflation, where inflaton has quadratic or quartic potentials, have been ruled out by Planck observations [5].

One of the well suited model of inflation from recent Planck and BICEP/Keck array observations is Starobinsky inflation [8, 9] where inflation is achieved by $\frac{1}{M^2}R^2$ interaction, R being the Ricci scalar , in the Einstein Hilbert action without additional scalar field. In the Einstein frame the R^2 term gives rise to plateau potential of the inflaton, named as scalaron in this case. Starobinsky inflation is of great importance in the light of recent observations as it predicts a scalar tilt value of [10] $n_s \simeq 0.965$ and a smaller value of tensor-to-scalar ratio $r \simeq 0.003$. Besides this, it also incorporates a graceful exit to the radiation dominated epoch via reheating [11–13], where the standard model particles were produced by the oscillatory decay of scalaron.

Another interesting feature of Starobinsky inflation is that the scalaron potential in the Einstein frame can be easily realized in the framework of no-scale supergravity [14] with noncompact $SU(2,1)/SU(2) \times U(1)$ symmetry. In this case we have a modulus field that can be fixed by the other dynamics [15], and the inflaton field is a part of chiral superfield with a minimal Wess-Zumino superpotential. No-scale supergravity [16–18], where the supersymmetry breaking scale is undetermined in a first approximation and the energy scale of the effective potential can be much smaller than the Planck scale, provides a framework to connect inflation to a viable quantum theory of gravity at high scales and the standard model of particle physics at lower scale. Attempts have been made to incorporate standard model of particle physics in no-scale supergravity models of inflation [19–22]. In [15, 23] various possible examples in no-scale supergravity framework, which can reproduce the effective potential of the Starobinsky model and other related models, have been explored.

In this work we consider a variant of Starobinsky model known as $\alpha-$Starobinsky model of inflation [15], where the Einstein frame potential of the scalaron is modified by a parameter α in the exponential. This potential is obtained by generalizing the coefficient of the logarithm of the volume modulus field in the Kähler potenial in $SU(2,1)/SU(2) \times U(1)$

no-scale supergravity framework [24, 25] with a suitable choice of the superpotential having both the volume modulus field and the chiral superfield; the inflaton being the scalar part of the chiral superfield. Similar potentials can also be obtained in supergravity models where the inflaton is a part of the vector multiplet rather than the chiral multiplet [26, 27]. In this case the $\alpha-$Starobinsky model belongs to a class of superconformal inflationary models known as $\alpha-$attractors. The parameter α is crucial as it is connected to various physical aspects, such as the geometry of the *kähler* manifold, hyperbolic geometry [28–30], the behavior of the boundary of moduli space [31], modified gravity [32], maximal supersymmetry [33], and string theory [34, 35]. In [36] α-attractor models have been studied in the dynamical system framework to find inflationary attractor solutions in the light of observational viability of these models. A two-field inflation model where both the fields have α-attractor potentials is studied in [37]. The E-model potential in brane inflation along with its various observational aspects has been studied in [38]. The primordial black hole formation in α-attractor inflation models has been studied is [39],[40]. The predictions for tensor-to-scalar ratio r are modified in the α-Starobinsky model by a factor of α [15, 27]. To obtain constraints on α, this model was analyzed with Planck-2015 observations in [41], where the power spectra was calculated using public code ASPIC [6] based on HFF's ϵ_i. By choosing a flat prior over [0,4] for $\log_{10}(\alpha^2)$ it was found that $\log_{10}(\alpha^2) < 1.7(2.0)$ for E model and $\log_{10}(\alpha^2) < 2.3(2.5)$ at 95% CL for T model of α attractors. Further, the Planck 2018 results have also placed an upper limit on the parameter α [5], $\log_{10}\alpha < 1.3$ ($\alpha < 19$) for the E-model and $\log_{10}\alpha < 1.0$ ($\alpha < 10$) for the T-model at 95% CL by choosing the parameter range $-2 < \log_{10}\alpha < 4$. The constraints on α from Planck-2018 observations for few choices of e-folding along with various phenomenological aspects of this model were studied in [42], and it was found that the data was insufficient to constrain α. However, an upper limit on α ($\alpha \leq 46(88)$ for e-folds $N = 50(60)$) was obtained from the Planck upper limit on r in [43]. In [44] the joint constraints on $n_s - r$ from Planck-2018 and BICEP/Keck observations were used to put constraints on the parameter α and the bounds on reheating temperature were used to put constraints on the e-folds N. Both these analysis were based on slow-roll approximation. Reheating constraints on α-attractor models were also studied in [43] and it was found that the parameter α is roughly constrained as $\alpha \geq 0.01$ along a broad resonance preheating scenario. By assuming a two-phase reheating with a preheating for specific duration, constraint on α has been obtained in [45] and it is shown that small values of α give good results on r. The constraint on parameter α has also been obtained in [46] by solving the cosmological perturbation equations in k-space.

Here, we analyze α-Starobinsky inflation model in the light of Planck-2018, BICEP/Keck (BK18) and BAO observations to obtain constraints on parameter α. To obtain the power spectra of primordial perturbations we use ModeChord, an updated version of ModeCode [47], which solves the background and perturbation equations for inflaton numerically without the usual slow-roll approximation. ModeCode is coupled with CAMB [48], which computes the angular power spectra for CMB anisotropy and polarization. The theoretical angular power spectra obtained with CAMB are used to compute constraints on inflationary parameters from CMB observations with the help of CosmoMC [49]. CosmoMC performs the Markov Chain Monte Carlo (MCMC) analysis, which is a statistical technique used to sample probability distributions of model parameters. By using ModeCode one can directly constrain the parameters of inflaton potential and N_{pivot} (the number of e-foldings between horizon exit of the CMB pivot mode and the end of inflation) from CMB observations. To find the best-fit parameters of the α-Starobinsky model, we vary $log_{10}\alpha$ between -8.0 to 4.0 along with M and N_{pivot}.

The format of this paper is as follows: A brief discussion of α - Starobinsky model is given in Sec.2. In section 3, we obtain the background equations for Hubble parameter and inflaton in terms of e-foldings N. We also find perturbation equations for Mukhanov-Sasaki variable and tensor mode in terms of N. These equations are used in ModeCode. The details of MCMC analysis performed using CosmoMC along with observational constraints are described in section 4. In Section 5, we provide our conclusions and a summary of our findings.

2. α-STAROBINSKY MODEL(E-MODEL)

The action for the Starobinsky inflation [8, 9] contains R^2 term due to quantum effects, and is given as

$$S_J = \frac{-M_{Pl}^2}{2}\int \sqrt{-g}(R+\mu R^2)d^4x. \tag{1}$$

Here $\mu = \frac{1}{6M^2}$, M is a parameter having a mass dimension of one and $M << M_{Pl}$. M_{Pl} is the reduced Planck mass ($M_{Pl}^2 = (8\pi G)^{-1}$) and g is the determinant of the metric $g_{\mu\nu}$. After this point, we will work in units where $M_{Pl} = 1$. After Weyl transformation $\tilde{g}_{\mu\nu} = (1+2\mu\phi)g_{\mu\nu}(x)$ and using the field redefinition $\chi \equiv \sqrt{\frac{3}{2}}\ln(1+\frac{\phi}{3M^2})$, the action Eq. (1) gets transformed to an Einstein Hilbert form:

$$S_E = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left(-\tilde{R}+\partial_\mu\chi\partial^\mu\chi+\frac{3}{2}M^2(1-e^{-\sqrt{\frac{2}{3}}\chi})^2\right). \tag{2}$$

It is evident from the above equation that the inflaton potential for Starobinsky inflation is given by:

$$V(\chi) = \frac{3}{4}M^2(1-e^{-\sqrt{\frac{2}{3}}\chi})^2 \tag{3}$$

This model's predictions for n_s and r can be expressed as a function of the number of e-foldings N_e. In the limit of large N_e, one discovers

$$n_s = 1-\frac{2}{N_e} \tag{4}$$

$$r = \frac{12}{N_e^2} \tag{5}$$

It is shown in [14] that in the framework of no-scale supergravity with noncompact $SU(2,1)/SU(2)\times U(1)$ symmetry, one can obtain the inflaton potential (3), which can be generalized with the introduction of a new parameter α [15]. This generalization is based on the Kähler potential [24, 25]

$$K = -3\alpha\ln\left(T+T^\dagger-\frac{\phi\phi^\dagger}{2}\right) \tag{6}$$

Here the field T corresponds to a generic compactification volume modulus and ϕ is a part of generic chiral mater field. The parameter α is known as Kähler curvature parameter as it is inversely related to the curvature of the Kähler manifold ($R = \frac{2}{3\alpha}$). In [14] the Wess-Zumino superpotential $W = M\left(\frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}}\right)$ was used to obtain the Starobinsky potential (3) in Einstein frame, along with the Kähler potential (6) for $\alpha = 1$ yields

$$V(\phi) = 3M^2\left(\frac{\phi}{\phi + \sqrt{3}}\right)^2, \tag{7}$$

which with field redefinition $\phi = \sqrt{3}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$ gives the Starobinsky potential (3).

For $\alpha \neq 1$ the field redefinition can be generalized as [51]

$$\phi = \sqrt{3}\tanh\left(\frac{\chi}{\sqrt{6\alpha}}\right). \tag{8}$$

This, on substitution in (7), yields the potential for α-Starobinsky inflation as

$$V(\chi) = \frac{3}{4}M^2(1 - e^{-\sqrt{\frac{2}{3\alpha}}\chi})^2. \tag{9}$$

For any value of α the potential (9) can also be generated by considering the superpotential [51]

$$W = \sqrt{\alpha}f(\phi)\left(2T - \frac{\phi^2}{3}\right)^{\frac{3}{2}(\alpha-\sqrt{\alpha})}. \tag{10}$$

To determine the function $f(\phi)$, ϕ and T are considered to be real along with $\langle T\rangle = \frac{1}{2}$; and the potential obtained from the superpotential (10) is equated to (7). The superpotential (10) can be successfully combined with the dark energy and supersymmetry breaking [51].

The potential (9) can also be derived from supergravity models where the inflaton belongs to the vector superfield [26, 27], and it belongs to a class of attractor models of inflation known as α - attractors. This potential, for $\alpha \leq \mathcal{O}(1)$ and large N, leads to a general prediction for inflationary observables [15, 27],

$$n_s = 1 - \frac{2}{N_e}, \tag{11}$$

$$r = \frac{12\alpha}{N_e^2}. \tag{12}$$

For $\alpha = 1$, the potential (9) corresponds to the Starobinsky-Whitt potential (3). On the other hand, for $\sqrt{\frac{2}{3\alpha}} << 1$, this potential is quadratic.

$$V(\chi) = \frac{3}{4}M^2(1 - e^{-\sqrt{\frac{2}{3\alpha}}\chi})^2 = \frac{m^2}{2}\phi^2, \tag{13}$$

where $m^2 = \frac{M^2}{2}$. The α - Starobinsky potential (9) for different value of α is plotted in Fig. 1. It is evident from the Fig. that smaller the values of α, narrower the potential minima. The potential has a wall for tiny negative χ and has an inflationary plateau at large positive χ. It can be seen from the figure that increasing the value of curvature parameter α stretches the Starobinsky potential horizontally, which reduces the flatness of the plateau at any fixed value of χ.

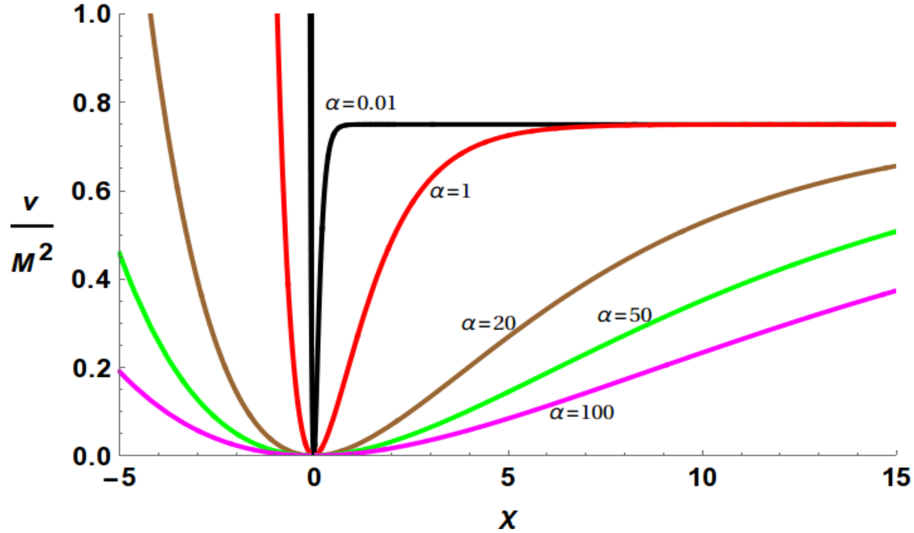


FIG. 1: The variation of potential (9) with $\alpha = 0.01, 1, 20, 50, 100$. The red line corresponds to the original Starobinsky inflationary potential with $\alpha = 1$. The values of potential and field being in $M_{Pl} = 1$ units.

3. INFLATIONARY DYNAMICS AND POWER SPECTRA

As mentioned earlier, we use ModeCode to obtain the power spectra of scalar and tensor perturbations generated during inflation. ModeCode solves both the background and perturbation equations in terms of e-folds (N) as an independent variable numerically without slow-roll approximation. In this section we obtain the necessary equations describing the background dynamics during inflation and the perturbation equations in terms of e-folds. The evolution of the Hubble parameter during inflation is governed by the Friedmann equations:

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\chi}^2 + V(\chi)\right]. \tag{14}$$

$$\dot{H} = -\frac{1}{2}\dot{\chi}^2, \tag{15}$$

where $V(\chi)$ is the potential (9) for the inflaton χ in the Einstein frame. The dynamics of the scalar field χ is governed by its equation of motion, which is the Klein-Gordon equation in spacetime:

$$\ddot{\chi} + 3H\dot{\chi} + \frac{dV(\chi)}{d\chi} = 0. \tag{16}$$

Here, the overdot denotes derivative with respect to cosmic time. As we use the number of e-folding, $N = \ln a$ as an independent variable to solve the equations numerically, we can rewrite the Friedmann equations (14) and (15) and the equation of motion of inflaton (16) in terms of N as

$$H^2 = \frac{\frac{1}{3}V(\chi)}{1-\frac{1}{6}\chi'^2}, \tag{17}$$

$$H' = -\frac{1}{2}H\chi'^2, \tag{18}$$

and

$$\chi'' + \left(\frac{H'}{H}+3\right)\chi' + \frac{1}{H^2}\frac{dV(\chi)}{d\chi} = 0, \tag{19}$$

where prime denotes the differentiation with respect to N. The numerical solution of these equations provides various background quantities that are used in perturbations equations. We describe the scalar perturbation generated during inflation by the gauge invariant Mukhanov-Sasaki variable u_k [52, 53], which is related to the curvature perturbations $\mathcal{R}$ as $u_k = -z\mathcal{R}$, where $z = \frac{1}{H}\frac{d\chi}{d\tau}$ and τ denotes the conformal time. The quantity z can be determined from background equations Eqs. (18) and (19). The Fourier modes

$$u_k(\tau) = \int d^3x e^{-ik.x}u(\tau, x) \tag{20}$$

satisfies the Mukhanov-Sasaki equation

$$\frac{d^2u_k}{d\tau^2} + \left(k^2 - \frac{1}{z}\frac{d^2z}{d\tau^2}\right)u_k = 0, \tag{21}$$

The primordial power spectrum of scalar perturbations is given in terms of the two point correlation function of comoving curvature and its relation with Mukhanov-Sasaki variable u_k is given by Eq. (22)and Eq. (23) respectively:

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\langle\mathcal{R}_k\mathcal{R}^*_{k'}\rangle\delta^3(k-k'), \tag{22}$$

which can be expressed in terms of Mukhanov-Sasaki variable u_k as

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2. \tag{23}$$

Similarly, for tensor perturbations, the mode equation and the primordial tensor power spectrum is given by following equations:

$$\frac{d^2v_k}{d\tau^2} + \left(k^2 - \frac{1}{a}\frac{d^2a}{d\tau^2}\right)v_k = 0, \tag{24}$$

$$\mathcal{P}_t(k) = \frac{4k^3}{\pi^2}\left|\frac{v_k}{a}\right|^2. \tag{25}$$

To obtain the scalar and tensor power spectra, the mode equations (21) and (24) are solved numerically along with background equations (18) and (19). For this we can rewrite these equations in terms of e-foldings $N = \ln a$ as

$$u_k'' + \left(\frac{H'}{H} + 1\right) u_k' + \left\{ \frac{k^2}{a^2H^2} - \left[2 - 4\frac{H'}{H}\frac{\chi''}{\chi'} - 2\left(\frac{H'}{H}\right)^2 - 5\frac{H'}{H} - \frac{1}{H^2}\frac{d^2V}{d\chi^2} \right] \right\} u_k = 0, \quad (26)$$

$$v_k'' + \left(\frac{H'}{H} + 1\right) v_k' + \left[\frac{k^2}{a^2H^2} - \left(\frac{H'}{H} + 2\right) \right] v_k = 0. \quad (27)$$

In this approach the scalar spectral index n_s and the tensor spectral index n_t are not the parameter of the power spectra, and hence they are determined from the power spectra obtained numerically using their definitions [54]:

$$n_s = 1 + \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}, \quad (28)$$

$$n_t = \frac{d \ln \mathcal{P}_t}{d \ln k}, \quad (29)$$

Similarly, the tensor-to-scalar ratio r, defined by [54]

$$r = \frac{\mathcal{P}_t}{\mathcal{P}_{\mathcal{R}}}, \quad (30)$$

is also a derived parameter and is obtained from the numerical solution of the power spectra. Both n_s and r are determined at the pivot scale $k = 0.05\text{Mpc}^{-1}$. By obtaining the power spectra numerically the parameters of the inflaton potential (9), M and α can be directly constrained from the CMB and LSS observations.

4. OBSERVATIONAL CONSTRAINTS

The background and perturbations equations, described in the previous section, are solved by ModeCode [47] numerically without using slow roll approximation. Thus, slow-roll violating effects, which can be significant for confronting models with precision data, are automatically captured. We modify ModeChord, un updated version of ModeCode, to compute the scalar and tensor power spectra for α-Starobinsky inflation in the Einstein frame. Inflationary models are described in ModeCode using an array of parameters for the potential. We incorporate the parameters M and α of the potential (9) in this array. Without depending on the slow roll approximation, we consider the general reheating scenario, where the parameter N_{pivot} representing the number of e-foldings from the end of inflation to the time when length scales corresponding to the Fourier mode k_{pivot} leave the horizon during inflation, is also varied along with other potential parameters. The numerically computed primordial power spectra with ModeChord can be used in CAMB [48], which solves Boltzmann equation and computes the two-point correlation function for CMB temperature anisotropy and polarization for a given set of cosmological parameters. CosmoMC uses these theoretical angular power spectra to put constraints on the parameters of the inflaton potential, N_{pivot}, and the other parameters of the λCDM model from various CMB and large-scale structure observations. We use Planck-2018, BICEP (BK18) [55], BAO and Pantheon data to determine the constraints on the parameters M and α of inflaton potential (9) and N_{pivot} . We have used flat priors for these parameters, which are given in Table I. To cover a wide range, we sample M and α on a logarithmic scale. We also vary the parameters of the ΛCDM model with their priors provided in [56]. To ensure MCMC convergence, we analyze four chains using the Gelman and Rubin $R-1$ statistics, which evaluates the "variance of the mean" against the "mean of the chain variance."

Parameter	Prior range
$\mathbf{N}_{\text{pivot}}$	[20, 90]
$\log_{\mathbf{10}}\mathbf{M}$	[-10.0, -1.0]
$\log_{\mathbf{10}}\alpha$	[-8.0, 4.0]

TABLE I: Priors on N_{pivot} and model parameters.

Table II shows the constraints obtained from MCMC analysis for parameters of potential (9), the e-foldings N_{pivot} and the derived parameters, r and n_s.

Parameter	68% limits	95% limits	99% limits
$\boldsymbol{N}_{\text{pivot}}$	$53.2^{+2.9}_{-1.5}$	$53.2^{+3.9}_{-5.0}$	$53.2^{+4.3}_{-7.2}$
$\mathbf{log_{10}}\,\boldsymbol{M}$	$-4.91^{+0.54}_{-0.043}$	$-4.91^{+0.69}_{-2.7}$	$-4.91^{+0.72}_{-3.3}$
$\mathbf{log_{10}}\,\boldsymbol{\alpha}$	$0.022^{+1.2}_{+0.023}$	$0.0^{+1.6}_{-5.6}$	$0.0^{+1.4}_{-7.3}$
$\boldsymbol{n_s}$	$0.9644^{+0.0027}_{-0.0017}$	$0.9644^{+0.0038}_{-0.0044}$	$0.9644^{+0.0045}_{-0.0064}$
$\boldsymbol{r}$	$0.0121^{+0.0073}_{-0.012}$	$0.012^{+0.020}_{-0.016}$	$0.012^{+0.028}_{-0.019}$

TABLE II: Constraints on parameters of potential, r and n_s using Planck-2018, BICEP/Keck (BK18) and BAO observations.

It can be seen from the Table II that the mean value of α along with 95% $C.L.$ is

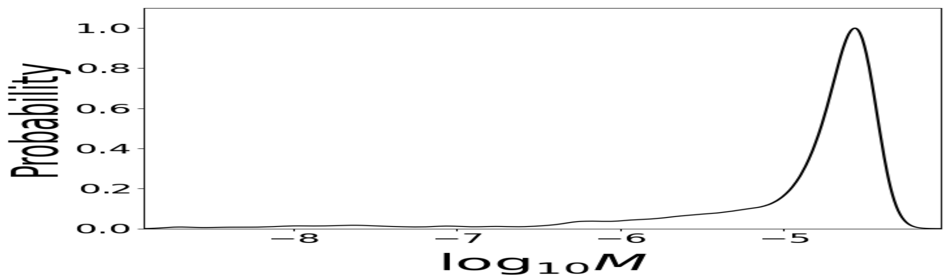
$$\log_{10}\alpha = 0.0^{+1.6}_{-5.6}. \tag{31}$$

As the limits are larger than the mean value, $\alpha = 1$ is consistent with the Planck-2018 and BICEP/Keck (BK18) observations. So the present data is not sufficient to constrain the value of α; a similar result was obtained in [42]. The 95% upper limit obtained on α in our analysis, i.e. $\log_{10}\alpha \leq 1.6$ is slightly larger than that obtained by Planck-2018 [5]. This difference arises as we have varied M and N_{pivot} along with α. However, the upper limit on α obtained in our analysis, i.e, $\alpha \leq 39.81$ with 95% C.L., is smaller than the one obtained in [44], and the lower limit, $\alpha \geq 2.51^{-6}$ with 95% C.L., is also smaller than the lower limit obtained in [43] using a broad resonance preheating. The number of e-folding obtained for α-Starobinsky model with 95% $C.L.$ is

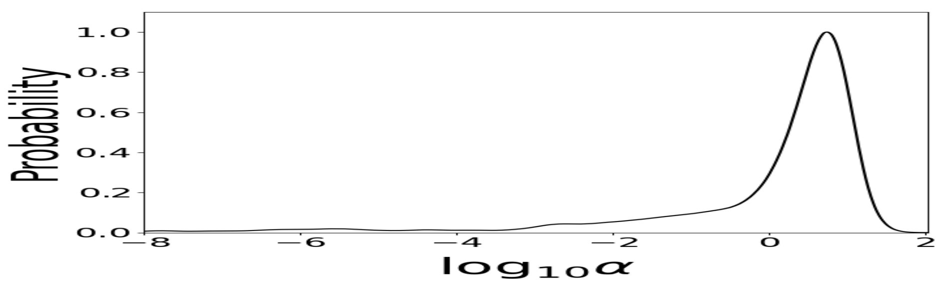
$$N_{pivot} = 53.2^{+3.9}_{-5.0}, \tag{32}$$

which is sufficient to solve the horizon problem.

The best fit values of scalar spectral index n_s and tensor-to-scalar ratio r for α-Starobinsky model, derived from the potential parameters, lie within the limits obtained by Planck-2018 and BICEP/Keck (BK18) observations as shown in Table II. Fig.2 shows the marginalized



(a)



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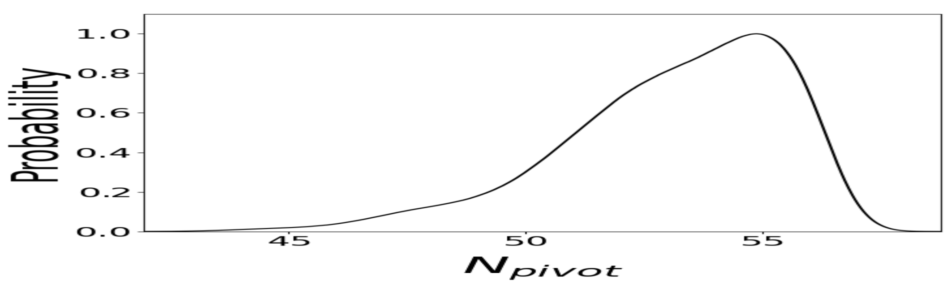
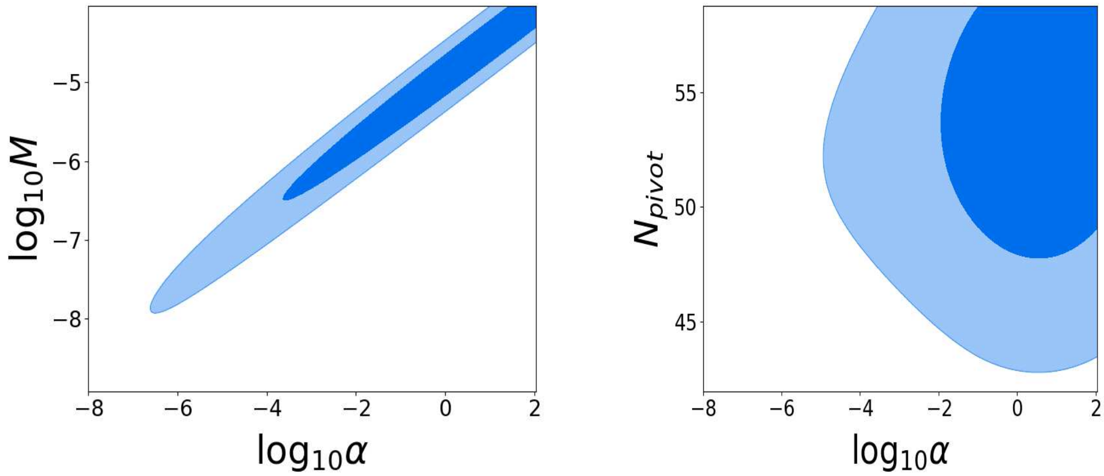


(c)

FIG. 2: Marginalized probability constraints on the potential parameters and N_{pivot} from Planck-2018, BICEP/Keck (BK18) and BAO data

probability distributions for various inflationary parameters, while Figs. 3 and 4 illustrate the marginalized joint 68% and 95% constraints on the potential parameters α and M, as well as the e-folds N_{pivot}. It can be seen from Fig. 2(b) that the most probable value of $\log_{10}\alpha$ is around 0.81, however, it does not have any statistical significance as the limits are quite large compared to mean value. As depicted in Fig. 3(a) the potential parameters α and M are strongly correlated. However, Fig.3(b) and Fig.4 show that there is no correlation between the e-folds N_{pivot} and potential parameters α and M respectively. The joint 68% and 95% C.L. constraints on scalar spectral index n_s and tensor-to-scalar ratio r are shown in Fig. 5. As these two parameters are derived parameters, the constraints on them are obtained from the constraints on potential parameters and N_{pivot}.



(a) (b)

FIG. 3: Marginalized joint two-dimensional 68% C.L. and 95% C.L. constraints on parameters of potential (a) and N_{pivot} (b) using Planck-2018, BICEP/Keck (BK18) and BAO data

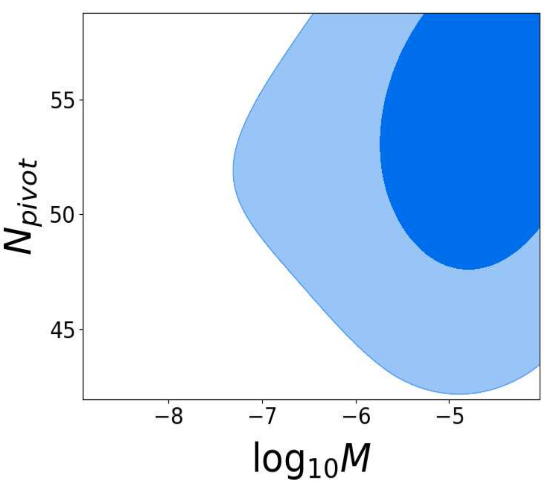


FIG. 4: 1σ and 2σ confidence contours in the potential parameter $\log_{10} M$ and N_{pivot} parameter space

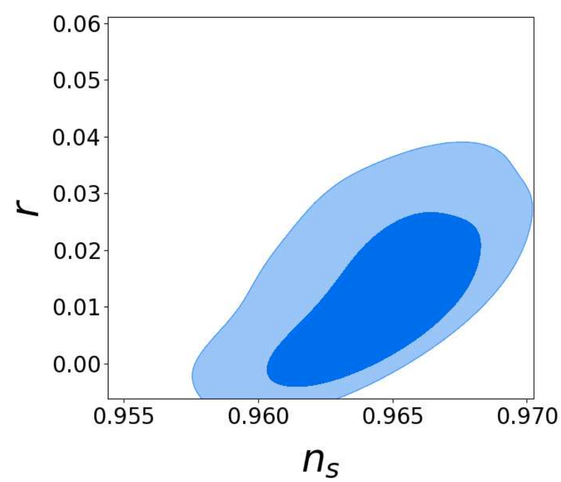


FIG. 5: 1σ and 2σ confidence contours in the n_s-r parameter space

5. CONCLUSIONS

Starobinsky inflation [8, 9] is one of the best suited model of inflation from the Planck-2018 [5] and BICEP/Keck (BK18) [55] observations as it predicts smaller value for tensor-to-scalar ratio r. The inflaton potential of this model in the Einstein frame can be derived from no-scale supergravity with an underlying noncompact $SU(2,1)/SU(2) \times U(1)$ symmetry

[14], where the modulus field T of the Kähler potential is fixed by other dynamics and inflaton field ϕ is a part of the chiral superfield with a minimum Wess-Zumino superpotential. By generalizing the coefficient of the logarithm of the volume modulus field in the Kähler potential that parameterizes $SU(2,1)/SU(2) \times U(1)$ coset Kähler manifold, and considering a superpotential having both the volume modulus field and inflaton field ϕ, one can obtain a potential for inflaton that is similar to Starobinsky model with a parameter α in the exponential [15], known as α-Starobinsky inflation. One can also obtain similar potential in supergravity models with inflaton as a part of vector multiplet rather than chiral multiplet [26, 27], and the α-Starobinsky model belongs to a class of superconformal inflationary models known as E-models of α-attractors.

In this work we have used the inflation potential (9) to analyze $\alpha-$ attractor E-model, in the light of Planck-2018 and BICEP/Keck (BK18) [55] CMB observations and other LSS observations. We have used ModeChord, an updated version of ModeCode [47] to numerically compute the power spectra of both scalar and tensor perturbations without slow-roll approximation. This helps us to constrain the number of e-foldings N_{pivot} and the model parameter α and M using CMB and LSS observations by MCMC analysis using CosmoMC [49]. To explore the parameter range we vary $log_{10}\alpha$ between -8 to 4. In our analysis we find that $log_{10}\alpha = 0.0^{+1.6}_{-5.6}$, 95% C.L. from Planck-2018, BICEP/Keck (BK18) and BAO data. The larger limits than the mean value on α indicates that the current observations are consistent with $\alpha = 1$, which highlights the challenge in precisely determining α from current observational data, The 95% upper limit on α, $\log_{10}\alpha \leq 1.6$ or $\alpha \leq 39.81$, obtained in our analysis is slightly larger than the one obtained by Planck-2018 [5] i.e. $\log_{10}\alpha \leq 1.3$, but, smaller than the one obtained by [44], which is $\alpha \leq 46$. By considering general reheating scenario we find that the number of e-foldings from the end of inflation to the time when the length scales corresponding to k_{pivot} leave the Hubble radius during inflation $N_{pivot} = 53.2^{+3.9}_{-5.0}$ with 95% C. L., which is sufficient to solve horizon problem. We also find that the potential parameters M and α are strongly correlated, Fig. 3(a); and there is no correlation between these two parameters and N_{pivot} as shown in Fig. 3(b) and Fig. 4.

The parameter α of the α-Starobinsky model is inversely related to the curvature of the Kähler manifold. As the parameter α affects the amplitude of tensor perturbations, it will be possible with the future observations like CMB-S4 [57] and LiteBIRD [58] to obtain its precise value. The α-Starobinsky inflation can be incorporated in a flipped $SU(5) \times U(1)$ GUT [59, 60], where inflaton decay during reheating can produce cold dark matter and also can have implications for neutrinos masses and leptogenesis. The precise determination of the parameter α will help us in connecting the models of particle physics phenomenology with inflation in the framework of supergravity and string theory.

6. ACKNOWLEDGEMENTS

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