Transverse spin correlation of back-to-back dihadron in unpolarized collisions

Lei Yang, ¹ Yu-Kun Song, ^{2,*} and Shu-Yi Wei^{1,†}

¹ Institute of Frontier and Interdisciplinary Science,
Key Laboratory of Particle Physics and Particle Irradiation (MOE),
Shandong University, Qingdao, Shandong 266237, China

² School of Physics and Technology, University of Jinan, Jinan, Shandong 250022, China

The spin correlation of back-to-back dihadron emerges in unpolarized high-energy collision, empowering unpolarized experiments to shed light on the spin-dependent fragmentation functions. This work investigates the transverse spin correlation of back-to-back dihadron in unpolarized e^+e^- , pp, and γp collisions, which serves as a novel probe of the chiral-odd fragmentation function $H_{1T}(z)$. We compute the transverse spin correlation at the partonic level and establish a connection with helicity amplitudes. Measuring this observable in future experiments can reveal valuable information on the hadronization of transversely polarized quarks.

I. INTRODUCTION

The hadronization of a high energy parton, described by fragmentation functions in QCD factorization theorem, is a fundamental element in understanding color confinement. The study of fragmentation functions consists of various interesting topics [1–3], and each topic amounts to a tree in the forest. For instance, the unpolarized fragmentation function, $D_1(z)$, describes the momentum distribution of produced hadron, and the longitudinal spin transfer $G_{1L}(z)$ and the transverse spin transfer $H_{1T}(z)$ deliver complementary information on the transition of spin polarization from the fragmenting parton to the produced hadron. Piecing those parts together eventually could achieve a key milestone in establishing a complete dynamic picture of hadronization mechanism.

The unpolarized fragmentation function has been extensively investigated in the last decades [4–21]. As it happens, recent progress [22, 23] even makes precision studies possible. In contrast, the spin-dependent fragmentation functions was mainly investigated in polarized experiments [24–26], such as LEP and RHIC. Very few colliders in the world are capable of conducting such research. Therefore, model calculations [27–33] have played an important role in theoretical studies. The perspective of understanding hadronization from the spin degree of freedom has been largely arrested mainly due to the difficulties in experimental measurements. On top of this, Ref. [35] also demonstrated that the decay contribution often contaminates the quantitative study of spin-dependent fragmentation functions, and therefore makes precision measurements even more challenging.

A pioneering study [36] proposed to measure the longitudinal spin transfer $G_{1L}(z)$ at unpolarized electron-positron colliders utilizing the helicity correlation of back-to-back dihadron. Established on a cascade of recent studies [37–39], it comes to light that helicity correlation is a direct consequence of partonic hard interaction and exhibits in all high energy collisions. Therefore, they also explored the opportunity of investigating the longitudinal spin transfer in unpolarized pp, AA, and ep collisions. In light of the amazing progress of experimental measurements on dihadron spin-spin correlation [40, 41], this novel observable conveys hope to study spin-dependent fragmentation functions in unpolarized colliders. A global analysis in the future could significantly broaden our knowledge of the hadronization mechanism. Moreover, the spin correlation has recently inspired more proposals [42–48] in various contexts and emerges as a new frontier whose potential still has not been fully unleashed yet.

Besides the helicity correlation, the back-to-back partons also evince transverse spin correlation, which translates into that of dihadron through hadronization. The hadronization of a transversely polarized quark is described by chiral-odd fragmentation functions. In the collinear factorization, only the transverse spin transfer H_{1T} contributes for the spin-1/2 baryon production, akin to the transversity $h_{1T}(x)$ of parton distribution function [49–53]. However, it is important to note that the helicity and transverse spin correlations are two distinctly different quantities which are only loosely related to each other by the positivity constrain. They are *not* different projections of the same quantity, and therefore we cannot derive one from the other. The reasons are listed in the following.

First, the helicity correlation of two partons exists as long as they are interacting with each other. On the contrary, as demonstrated later, the transverse spin correlation of two interacting partons only manifests in a selection of partonic channels. The unconnected diagrams cannot contribute to transverse spin correlation. In the language of helicity

^{*} sps_songyk@ujn.edu.cn

[†] shuvi@sdu.edu.cn

amplitude approach, the helicity correlation measures the difference between the magnitudes of two amplitudes, while the transverse spin correlation quantifies their interference.

Furthermore, it is also not feasible to establish a naive connection between the transverse spin transfer H_{1T} and its longitudinal counterpart G_{1L} . For instance, both quark and gluon contribute to the longitudinal spin transfer. (The partner of the longitudinally polarized quark is the *circularly polarized gluon*.) However, for the transverse spin transfer, only the quark sector contributes. The linearly polarized gluon, which is the counterpart of the transversely polarized quark, cannot contribute to the transverse polarization of produced hadrons [3, 54–56] in the collinear factorization. As a result, the DGLAP evolution [57] of H_{1T} becomes a diagonal one [58, 59]. The gluon sector vanishes. This feature grants H_{1T} fragmentation function a unique advantage in understanding hadronization mechanism: the perfect separation between quark and gluon contributions. Therefore, the aim of this paper is to investigate the transverse spin correlation in unpolarized high energy collisions and to improve our quantitative understanding of the chiral-odd H_{1T} fragmentation function.

The rest of this paper is organized as follows. In Sec. II, we first study the most simple case: the transverse spin correlation in unpolarized electron positron annihilation process. In Sec. III, we present the transverse spin correlation in unpolarized hadronic collisions. In Sec. IV, we present that in photon-nucleus collisions, which can be directly applied to the ultra-peripheral nucleus-nucleus collision or the electron-ion collisions. We present the relation between the transverse spin correlation and helicity amplitudes in Sec. V and give a summary in Sec. VI.

II. TRANSVERSE SPIN CORRELATION IN e^+e^- ANNIHILATION

We first consider the simple back-to-back Λ - $\bar{\Lambda}$ pair production in e^+e^- annihilations to demonstrate the origin of the transverse spin correlation in this section, and then extend this approach to the unpolarized pp collisions in the next section.

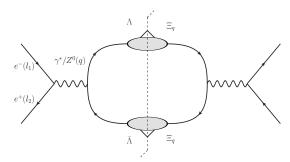


FIG. 1. The leading order Feynman diagram for the Λ - $\bar{\Lambda}$ pair production in e^+e^- annihilation.

As illustrated in Fig. 1, the LO contribution comes from $e^-(l_1) + e^+(l_2) \to q(\to \Lambda) + \bar{q}(\to \bar{\Lambda})$. In principle, we also need to consider the contribution from $q(\bar{\Lambda}) + \bar{q}(\to \Lambda)$ as well, which is rather straightforward. We will evaluate this part of the contribution when presenting the final result.

Furthermore, due to the parton shower, the final state Λ and $\bar{\Lambda}$ hyperons are not exactly back-to-back, whose momenta span the hadron production plane. The transverse spin correlation of the final state dihadron is investigated along the normal direction of this hadron production plane, i.e., $n_T = \hat{p}_{\Lambda} \times \hat{p}_{\bar{\Lambda}}$ with $\hat{p}_{\Lambda/\bar{\Lambda}}$ the unit vector along the $\Lambda/\bar{\Lambda}$ momentum direction respectively. Similar to the helicity correlation defined in Refs. [37–39], the transverse spin correlation is defined as

$$C_{TT} = \frac{\mathcal{P}(\boldsymbol{n}_T, \boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, -\boldsymbol{n}_T) - \mathcal{P}(\boldsymbol{n}_T, -\boldsymbol{n}_T) - \mathcal{P}(-\boldsymbol{n}_T, \boldsymbol{n}_T)}{\mathcal{P}(\boldsymbol{n}_T, \boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, -\boldsymbol{n}_T) + \mathcal{P}(\boldsymbol{n}_T, -\boldsymbol{n}_T) + \mathcal{P}(-\boldsymbol{n}_T, \boldsymbol{n}_T)},$$
(1)

where $\mathcal{P}(\boldsymbol{a}_T, \boldsymbol{b}_T)$ is the probability for Λ being transversely polarized along the \boldsymbol{a}_T direction and $\bar{\Lambda}$ being transversely polarized along the \boldsymbol{b}_T direction. Since the transverse spin correlation is $\boldsymbol{n}_T \leftrightarrow -\boldsymbol{n}_T$ symmetric, it does not matter if we define \boldsymbol{n}_T as $\hat{\boldsymbol{p}}_{\Lambda} \times \hat{\boldsymbol{p}}_{\bar{\Lambda}}$ or $\hat{\boldsymbol{p}}_{\bar{\Lambda}} \times \hat{\boldsymbol{p}}_{\Lambda}$. The only relevant thing is the hadron production plane.

In the parton model, the LO differential cross section reads

$$\frac{d\sigma}{dydz_1dz_2d^2\mathbf{P}_{\perp}} = \frac{2\pi N_c \alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu},\tag{2}$$

where Q is the center-of-mass energy of e^+e^- , $y=(1+\cos\theta)/2$ with θ the angle between e^- and Λ , $z_{1,2}$ is the light-cone momentum fraction of the final state $\Lambda/\bar{\Lambda}$, and \mathbf{P}_{\perp} is the relative transverse momentum of Λ with respect

to the $\bar{\Lambda}$ momentum. Here, we have only consider the electromagnetic interaction for simplicity. The weak interaction will be taken into account eventually when presenting our final result. The leptonic tensor $L_{\mu\nu}$ and the hadronic tensor $W^{\mu\nu}$ are given by

$$L_{\mu\nu} = l_{1\mu}l_{2\nu} + l_{1\nu}l_{2\mu} - g_{\mu\nu}l_1 \cdot l_2, \tag{3}$$

$$W^{\mu\nu} = \sum_{q} e_{q}^{2} \int d^{2} \boldsymbol{p}_{T1} d^{2} \boldsymbol{p}_{T2} \delta^{2} (\boldsymbol{P}_{\perp} - \boldsymbol{p}_{T2} - \frac{z_{2}}{z_{1}} \boldsymbol{p}_{T1}) \operatorname{Tr} \left[2\Xi_{q}^{\Lambda}(z_{1}, \boldsymbol{p}_{T1}) \gamma^{\mu} 2\Xi_{\bar{q}}^{\bar{\Lambda}}(z_{2}, \boldsymbol{p}_{T2}) \gamma^{\nu} \right], \tag{4}$$

with e_q being the electric change of quark q. Here Ξ_q^{Λ} and $\Xi_{\bar{q}}^{\bar{\Lambda}}$ are four by four matrices which can further be decomposed in terms of Gamma matrices. The leading twist decomposition for spin-1/2 baryon production contains eight structures which can be found in Refs. [1–3, 60, 61]. In this work, we only investigate the transverse spin correlation of final state hadrons. Therefore, we only keep the following three structures

$$\Xi_{q}^{\Lambda}(z_{1},\boldsymbol{p}_{T1}) = \frac{1}{4} \not n_{+} D_{1,q}^{\Lambda}(z_{1},\boldsymbol{p}_{T1}) + \frac{1}{8} [\not s_{T1},\not n_{+}] \gamma_{5} H_{1T,q}^{\Lambda}(z_{1},\boldsymbol{p}_{T1}) + \frac{p_{T1} \cdot S_{T1}}{8M_{1}^{2}} [\not p_{T1},\not n_{+}] \gamma_{5} H_{1T,q}^{\perp,\Lambda}(z_{1},\boldsymbol{p}_{T1}), \tag{5}$$

$$\Xi_{\bar{q}}^{\bar{\Lambda}}(z_{2},\boldsymbol{p}_{T2}) = \frac{1}{4} \not\!\! h_{-} D_{1,\bar{q}}^{\bar{\Lambda}}(z_{2},\boldsymbol{p}_{T2}) + \frac{1}{8} [\not\!\! s_{T2},\not\!\! h_{-}] \gamma_{5} H_{1T,\bar{q}}^{\bar{\Lambda}}(z_{2},\boldsymbol{p}_{T2}) + \frac{p_{T2} \cdot S_{T2}}{8 M_{2}^{2}} [\not\!\! p_{T2},\not\!\! h_{-}] \gamma_{5} H_{1T,\bar{q}}^{\perp,\bar{\Lambda}}(z_{2},\boldsymbol{p}_{T2}), \tag{6}$$

where D_1 is the unpolarized TMD fragmentation function and H_{1T} and H_{1T}^{\perp} are chiral-odd TMD fragmentation functions describing the hadronization of transversely polarized partons. Notice that this decomposition is convention-dependent. In this work, we follow the convention from Refs. [1, 2]. However, one can always allocate some contribution from H_{1T}^{\perp} to H_{1T} . For instance, Ref. [3] offers a rather complicated Lorentz structure in the TMD factorization. However, as shown later, our *simple* TMD structure [1, 2] leads to an *involved* relation between TMD fragmentation functions and the collinear ones. On the contrary, the *involved* TMD structure in Ref. [3] leads to a *simple* relation. The physical interpretations of chiral-odd fragmentation functions in different conventions are also different. They also follow different evolution equations. Nonetheless, the final results of observables remain convention-independent, since we have simply shuffled some contribution from one part to another. The sum remains the same.

Furthermore, integrating over the relative transverse momentum between two hadrons P_{\perp} , we recover the expression in collinear factorization. The cross section is then given by

$$\frac{d\sigma}{dydz_1dz_2} = \frac{2\pi N_c \alpha^2}{Q^2} L_{\mu\nu} \hat{W}^{\mu\nu},\tag{7}$$

with the P_{\perp} -integrated hadronic tensor being given by

$$\hat{W}^{\mu\nu} = \sum_{q} e_q^2 \text{Tr} \left[2\hat{\Xi}_q^{\Lambda}(z_1) \gamma^{\mu} 2\hat{\Xi}_{\bar{q}}^{\bar{\Lambda}}(z_2) \gamma^{\nu} \right]. \tag{8}$$

Here, $\hat{\Xi}_{q,\bar{q}}(z)$ is the p_T -integrated version of $\Xi_{q,\bar{q}}(z,p_T)$. The decomposition in the collinear factorization reads

$$\hat{\Xi}_{q}^{\Lambda}(z_{1}) = \frac{1}{4} \not h_{+} D_{1,q}^{\Lambda}(z_{1}) + \frac{1}{8} [\not S_{T1}, \not h_{+}] \gamma_{5} H_{1T,q}^{\Lambda}(z_{1}), \tag{9}$$

$$\hat{\Xi}_{\bar{q}}^{\bar{\Lambda}}(z_2) = \frac{1}{4} \not n_- D_{1,\bar{q}}^{\bar{\Lambda}}(z_2) + \frac{1}{8} [\not s_{T2}, \not n_-] \gamma_5 H_{1T,\bar{q}}^{\bar{\Lambda}}(z_2), \tag{10}$$

with D_1 , H_{1T} being collinear fragmentation functions which can be related to TMD ones by

$$D_1(z) = \int d^2 \boldsymbol{p}_T D_1(z, \boldsymbol{p}_T), \tag{11}$$

$$H_{1T}(z) = \int d^2 \mathbf{p}_T \left[H_{1T}(z, \mathbf{p}_T) + \frac{p_T^2}{M^2} H_{1T}^{\perp}(z, \mathbf{p}_T) \right].$$
 (12)

While $D_1(z)$ is the unpolarized cross section, $H_{1T}(z)$ is the transverse spin transfer representing the number density of producing transversely polarized hadron from transversely polarized quark. As discussed above, the contribution from H_{1T}^{\perp} term does not vanish in our TMD convention. This leads to the strange relation between the collinear function $H_{1T}(z)$ with the TMD functions $H_{1T}(z, \mathbf{p}_T)$ and $H_{1T}^{\perp}(z, \mathbf{p}_T)$. Nonetheless, if we adopt the convention from Ref. [3], we will find that the contribution from H_{1T}^{\perp} disappears integrating over the transverse momentum. Thus, we can establish a simple relation between $H_{1T}(z)$ and $H_{1T}(z, \mathbf{p}_T)$.

Inserting the leptonic tensor and the hadronic tensor into Eq. (7) and taking into account both electromagnetic and weak interactions, we obtain

$$\frac{d\sigma}{dydz_1dz_2} = \frac{2\pi N_c \alpha^2}{Q^2} \left\{ \sum_{q} \left[\omega_q(y) D_{1,q}^{\Lambda}(z_1) D_{1,\bar{q}}^{\bar{\Lambda}}(z_2) + (\boldsymbol{S}_{T1} \cdot \boldsymbol{S}_{T2}) \omega_q^T(y) H_{1T,q}^{\Lambda}(z_1) H_{1T,\bar{q}}^{\bar{\Lambda}}(z_2) \right] + \sum_{q} \left[\omega_q(1-y) D_{1,\bar{q}}^{\Lambda}(z_1) D_{1,q}^{\bar{\Lambda}}(z_2) + (\boldsymbol{S}_{T1} \cdot \boldsymbol{S}_{T2}) \omega_q^T(1-y) H_{1T,\bar{q}}^{\Lambda}(z_1) H_{1T,q}^{\bar{\Lambda}}(z_2) \right] \right\}, \tag{13}$$

where the coefficient functions are consistent with those in Refs. [60–64] and also are listed in Appendix A for self-sufficient. Both D_1 and H_{1T} are scale dependent, which can be obtained by solving the DGLAP evolution equation. However, the polarized splitting functions differ from the unpolarized ones. We explicitly lay out the evolution equation of $H_{1T}(z,\mu^2)$ and investigate the impact on the transverse spin transfer in Appendix B. Moreover, we have used the product of three-vectors in the above expression, $S_{T1} \cdot S_{T2}$. Since we are only interested in the correlation of transverse polarization along the normal direction of the hadron production plane, $S_{T1} \cdot S_{T2}$ could either be +1 for same sign polarizations or -1 for opposite sign polarizations. Moreover, the second line computes the contribution from the less important $q \to \bar{\Lambda}, \bar{q} \to \Lambda$ channel. At the end of the day, the dihadron transverse spin correlation C_{TT} is evaluated by

$$C_{TT}(y, z_1, z_2) = \frac{\sum_{q} \left[\omega_q^T(y) H_{1T, q}^{\Lambda}(z_1) H_{1T, \bar{q}}^{\bar{\Lambda}}(z_2) + \omega_q^T(1 - y) H_{1T, \bar{q}}^{\Lambda}(z_1) H_{1T, q}^{\bar{\Lambda}}(z_2) \right]}{\sum_{q} \left[\omega_q(y) D_{1, q}^{\Lambda}(z_1) D_{1, \bar{q}}^{\bar{\Lambda}}(z_2) + \omega_q(1 - y) D_{1, \bar{q}}^{\Lambda}(z_1) D_{1, q}^{\bar{\Lambda}}(z_2) \right]}.$$
(14)

Here, we have already taken into account contribution from $(q \to \bar{\Lambda}, \bar{q} \to \Lambda)$. The physical interpretation of this result is also clear. ω_q^T/ω_q is the transverse spin correlation of $q\bar{q}$ pair. The partonic correlation can be inherited by final state hadrons by convoluting the numerator and the denominator with the corresponding fragmentation functions. By measuring the transverse spin correlation at the Belle/LEP experiments, we can investigate the chiral-odd $H_{1T}(z)$ fragmentation function.

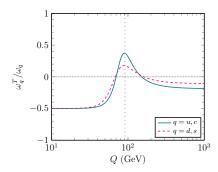


FIG. 2. Transverse spin correlation of final state $q\bar{q}$ pair in unpolarized e^+e^- collisions as a function of Q.

Since our quantitative knowledge on $H_{1T}(z)$ is next to nothing, we only present our numerical results at the partonic level. The transverse spin correlation of the final state $q\bar{q}$ pair in unpolarized electron-positron annihilation experiments is shown in Fig. 2. In the numerical evaluation, we have integrated over y in the allowed kinematic region. There is a sign flip between low-energy experiments, such as Belle, and high energy experiments, such as LEP. The sign flip stems from the competition between the electromagnetic γ^{μ} vertex and the weak $c_V\gamma_{\mu} - c_A\gamma_{\mu}\gamma_5$ vertex.

Thanks to the positivity constrain $|\mathcal{C}_{TT}| \leq \sqrt{1 - |\mathcal{P}_L|^2}$ with $|\mathcal{P}_L|$ the magnitude of the helicity of quark/antiquark, the transverse spin correlation at the Z^0 pole is a bit smaller than that at lower energy. Furthermore, since the down-type-quark helicity is larger than that of the up-type quark at the Z^0 pole [65–67], the transverse spin correlation of the down-type quark-antiquark pair is smaller than that of the up-type pair. Furthermore, the spin-dependent fragmentation functions also satisfy a positivity constrain as presented in Refs. [58, 68].

It is also intriguing to note that if the quark- Z^0 -boson vertex is assumed to be $\gamma_{\mu}(1\pm\gamma_5)$, the transverse spin correlation between quark and antiquark reduces to zero. The reason for this phenomenon can be better appreciated in the language of the helicity amplitude approach. The transverse spin correlation requires a helicity flip between the amplitude and the conjugate amplitude [69]. However, this cannot happen if the coupling vertex takes the $\gamma_{\mu}(1\pm\gamma_5)$ form. Furthermore, the vanishing transverse spin correlation can happen in other circumstances as well, which will be elaborated in Sec. V.

TRANSVERSE SPIN CORRELATION IN UNPOLARIZED pp COLLISIONS

In this section, we extend our research of the transverse spin correlation to unpolarized pp collisions and explore the opportunity of studying the transverse spin transfer in unpolarized hadron collider experiments such as LHC, RHIC, and Tevatron. Similar to the case in e^+e^- annihilation, the momenta of Λ and Λ produced in the hadronic collisions also span a hadron production plane. We thus can investigate the transverse spin correlation of Λ - $\bar{\Lambda}$ pair along the normal direction of the hadron production plane.

We would like to discuss the difference between the helicity correlation and the transverse spin correlation in ppcollisions. All partonic channels contribute to the helicity correlation, albeit the sign varies with channels. To be more specific, the helicities of final state partons take precisely the opposite sign for the $q_i\bar{q}_i \to q_j\bar{q}_j$, $gg \to q_i\bar{q}_i$ and $q_i\bar{q}_i \to gg$ channels. Therefore, they contribute to the negative helicity correlation, while the other channels prefer the same sign correlations. The partial cancellation among different channels results in a tiny helicity correlation at the hadronic level. Per contra, the transverse spin correlation arises from the chiral-odd H_{1T} fragmentation function. Only connected channels (i.e., the final state quark and/or antiquark are connected by the same trace line) contribute, while the others amount to the total production rate.

Notice that, in principle, the linear polarizations of gluons produced from the $q_i\bar{q}_i \to gg$ channel are also correlated. Vis-à-vis the transverse spin correlation of the quark-antiquark pair, the linear polarization correlation of gluons indicates that the probabilities for the parallel and perpendicular polarizations are different. However, the collinear H_{1T} fragmentation function of gluon does not exist. They can contribute to other observables, such as the tensor polarization of vector mesons [70–76] etc. In this paper, we only investigate the transverse spin correlation of backto-back diquark and leave that of gluons for future work.

After integrating over the relative transverse momenta, we arrive at the cross section in the collinear factorization framework given by

$$\frac{d\sigma_{pp\to\Lambda\bar{\Lambda}+X}}{dy_1d^2\boldsymbol{p}_{T1}dy_2d^2\boldsymbol{p}_{T2}} = \int \frac{dz_1}{z_1^2} \frac{dz_2}{z_2^2} \sum_{ab\to cd} x_a f_{1,a}(x_a) x_b f_{2,b}(x_b) \delta^2 \left(\frac{\boldsymbol{p}_{T1}}{z_1} + \frac{\boldsymbol{p}_{T2}}{z_2}\right) \\
\times \frac{1}{\pi} \left[\frac{d\hat{\sigma}_{ab\to cd}}{dt} D_{1,c}^{\Lambda}(z_1) D_{1,d}^{\bar{\Lambda}}(z_2) + (\boldsymbol{S}_{T1} \cdot \boldsymbol{S}_{T2}) \frac{d\hat{\sigma}_{ab\to cd}^T}{dt} H_{1T,c}^{\Lambda}(z_1) H_{1T,d}^{\bar{\Lambda}}(z_2)\right]. \tag{15}$$

Here, $d\hat{\sigma}_{ab\to cd}/dt$ is the unpolarized cross section of $ab\to cd$ scattering which can be found in Refs. [77], and $d\hat{\sigma}_{ab\to cd}^T/dt$ is the transversely polarized cross section. The exchange between $(c \to \Lambda, d \to \bar{\Lambda})$ and $(c \to \bar{\Lambda}, d \to \Lambda)$ is implicit.

First, it is straight forward to find that the $q_i\bar{q}_i \to q_j\bar{q}_j$, $gg \to q_i\bar{q}_i$, and $q_i\bar{q}_i \to q_i\bar{q}_i$ channels satisfy the aforementioned criteria. While the unpolarized cross sections of these channels are well known, the transversely polarized cross sections read

$$\frac{d\hat{\sigma}_{q_i\bar{q}_i \to q_j\bar{q}_j}^T}{dt} = -\frac{2\pi\alpha_s^2}{9s^2} \frac{4ut}{s^2},$$

$$\frac{d\hat{\sigma}_{q_i\bar{q}_i \to q_i\bar{q}_i}^T}{dt} = \frac{2\pi\alpha_s^2}{9s^2} \frac{4u(s-3t)}{3s^2},$$
(16)

$$\frac{d\hat{\sigma}_{q_i\bar{q}_i \to q_i\bar{q}_i}^T}{dt} = \frac{2\pi\alpha_s^2}{9s^2} \frac{4u(s-3t)}{3s^2},\tag{17}$$

$$\frac{dt}{dt} \frac{9s^2}{gg \to q_i \bar{q}_i} = \frac{\pi \alpha_s^2}{12s^2} \frac{ut - 4u^2 - 4t^2}{s^2},\tag{18}$$

with s, t, u Maldanstan variables.

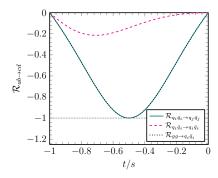


FIG. 3. The partonic transverse spin correlation \mathcal{R} as a function of t/s for $q\bar{q}$ production channels in pp collisions.

All three channels are negative indicating that the final state partons prefer to be be transversely polarized along opposite directions. We define $\mathcal{R}_{ab\to cd}$ as the ratio between the polarized cross section and the unpolarized one, i.e., $\mathcal{R}_{ab\to cd} \equiv d\hat{\sigma}_{ab\to cd}^T/d\hat{\sigma}_{ab\to cd}$, which quantifies the partonic transverse spin correlation. The numerical results are shown in Fig. 3 as a function of t/s. The transverse spin correlation of the $q_i\bar{q}_i \to q_i\bar{q}_i$ channel is much smaller than those of $q_i\bar{q}_i/gg \to q_j\bar{q}_j$ channels which even reach unity at t=-s/2. This is because that the $q_i\bar{q}_i/gg \to q_j\bar{q}_j$ channel contains only connected diagrams where final state partons are connected by the same Fermion line. On the other hand, the $q_i\bar{q}_i \to q_i\bar{q}_i$ channel contains both s channel and t channel contributions. The unconnected t channel diagram does not contribute to the transverse spin correlation, and therefore reduces the magnitude of the correlation.

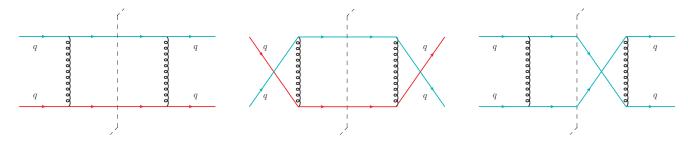


FIG. 4. Leading order Feynman diagrams for the $qq \rightarrow qq$ scatting. Left: t-channel diagram; Middle: u-channel diagram; Right: the interference diagram. The color of the quark line represents a closed trace line. While the left and middle diagrams do not contribute to the transverse spin correlation, the right one is a connected diagram that contributes.

Furthermore, the $2 \to 2$ process with identical quarks, i.e., $q_i q_i \to q_i q_i$, also contributes. The correlation arises from the interference between t-channel and u channel scatterings. As illustrated in Fig. 4, the left and the middle plots represent the t and u channel diagrams which do not contribute to the transverse spin correlation. The right plot represents the interference between u and t channel scatterings. We have utilized different colors to represent different trace lines. The final state identical quarks in the interference diagram are connected by the same Fermion line. Therefore, it contributes to the transverse spin correlation. We obtain the partonic transverse spin correlation of this channel as

$$\frac{d\hat{\sigma}_{q_i q_i \to q_i q_i}^T}{dt} = -\frac{2\pi\alpha_s^2}{9s^2} \frac{4}{3}.$$
(19)

We show the numerical result of $\mathcal{R}_{q_iq_i\to q_iq_i}$ as a function in Fig. 5. The magnitude becomes much smaller compared with the other unconnected channels, since it only arises from the interference diagrams. Nonetheless, this channel can become important at the threshold regime with $x_{a,b} \to 1$.

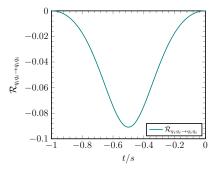


FIG. 5. The partonic transverse spin correlation \mathcal{R} as a function of t/s for the identical-parton-scattering channel in pp collision.

The transverse spin correlation of final state hadrons is given by convoluting σ^T with the transverse spin transfers. We obtain

$$C_{TT} = \frac{\int d\mathcal{P}.\mathcal{S}. \sum_{ab \to cd} x_a f_{1,a}(x_a) x_b f_{1,b}(x_b) \frac{1}{\pi} \frac{d\hat{\sigma}_{ab \to cd}^T}{dt} H_{1T,c}^{\Lambda}(z_1) H_{1T,d}^{\bar{\Lambda}}(z_2)}{\int d\mathcal{P}.\mathcal{S}. \sum_{ab \to cd} x_a f_{1,a}(x_a) x_b f_{1,b}(x_b) \frac{1}{\pi} \frac{d\hat{\sigma}_{ab \to cd}}{dt} D_{1,c}^{\Lambda}(z_1) D_{1,d}^{\bar{\Lambda}}(z_2)},$$
(20)

where $d\mathcal{P}.\mathcal{S}$. represents the phase space to be integrated over and $f_1(x_{a,b})$ is the collinear parton distribution function with $x_{a,b}$ the momentum fraction. Since the vast majority of partonic channels do not contribute to the transverse spin correlation, the experimental signal is expected to be small.

IV. TRANSVERSE SPIN CORRELATION IN PHOTON-NUCLEUS COLLISIONS

The high energy large nucleus is accompanied by enormous coherent quasireal photons. In the ultra-peripheral relativistic nucleus-nucleus collisions (UPC), we can have photon-nucleus collisions as discussed in Ref. [39]. At the leading order in QCD, the partonic hard scattering consists of $\gamma g \to q\bar{q}$ and $\gamma q \to qg$ channels. As discussed in the previous section, the final state gluon does not contribute to the transverse spin transfer. Therefore, only the γg channel contributes to the transverse spin correlation. The γq channel only contributes to the denominator. In the end, the transverse spin correlation in UPC is given by

$$C_{TT} = \frac{\int d\mathcal{P}.\mathcal{S}. \sum_{q} x_{\gamma} f_{\gamma}(x_{\gamma}) x_{g} f_{1,g}(x_{g}) \frac{1}{\pi} \frac{d\hat{\sigma}_{\gamma g \to q\bar{q}}^{T}}{dt} H_{1T,q}^{\Lambda}(z_{1}) H_{1T,\bar{q}}^{\bar{\Lambda}}(z_{2})}{\int d\mathcal{P}.\mathcal{S}. \sum_{b,c,d} x_{\gamma} f_{\gamma}(x_{\gamma}) x_{b} f_{1,b}(x_{b}) \frac{1}{\pi} \frac{d\hat{\sigma}_{\gamma b \to cd}}{dt} D_{1,c}^{\Lambda}(z_{1}) D_{1,d}^{\bar{\Lambda}}(z_{2})},$$
(21)

where x_{γ} is the per-nucleon momentum fraction carried by the quasireal photon and $f_{\gamma}(x_{\gamma})$ is the collinear photon distribution [78] which can be expressed as

$$x_{\gamma} f_{\gamma}(x_{\gamma}) = \frac{2Z^{2}\alpha}{\pi} \left[\zeta K_{0}(\zeta) K_{1}(\zeta) - \frac{\zeta^{2}}{2} [K_{1}^{2}(\zeta) - K_{0}^{2}(\zeta)] \right], \tag{22}$$

with α the electromagnetic coupling constant, Z the atomic number, $\zeta = 2x_{\gamma}M_{p}R_{A}$, M_{p} the proton mass, and R_{A} the nucleus radius. The transversely polarized cross section reads

$$\frac{d\hat{\sigma}_{\gamma g \to q\bar{q}}^T}{dt} = -\frac{2\pi\alpha\alpha_s e_q^2}{s^2}.$$
 (23)

Although both γg and γq contributes to the back-to-back dihadron production in photon-nucleus collisions, the dominant contribution arises from the connected γg channel [39]. This is because that the photon flux drops exponentially at slight large x_{γ} , forcing x_b to be very small as well. At small- x_b , the gluon distribution function is expected to be much larger than that of quark. Therefore, we expect that the experimental signal of dihadron transverse spin correlation in the UPC experiments is sizable.

Furthermore, in the future Electron Ion Collider (EIC) experiment, we can also perform the photon-nucleus scattering with a spacelike virtual photon. As presented in Ref. [79], the unphysical longitudinal photon also contributes, besides the physical transverse photon. Again, the transverse spin correlation only arises from the $\gamma^* g \to q\bar{q}$ channel, which is given by

$$C_{TT} = \frac{\int d\mathcal{P}.\mathcal{S}. \sum_{\mathcal{S}=\mathcal{L},\mathcal{T}} \sum_{q} G_{\gamma_{\mathcal{S}}^{*}}(x_{\text{Bj}}, Q^{2}) x_{g} f_{1,g}(x_{g}) \frac{1}{\pi} \frac{d\hat{\sigma}_{\gamma_{\mathcal{S}}^{*}g \to q\bar{q}}^{\mathcal{T}}}{dt} H_{1T,q}^{\Lambda}(z_{1}) H_{1T,\bar{q}}^{\bar{\Lambda}}(z_{2})}{\int d\mathcal{P}.\mathcal{S}. \sum_{\mathcal{S}=\mathcal{L},\mathcal{T}} \sum_{b,c,d} G_{\gamma_{\mathcal{S}}^{*}}(x_{\text{Bj}}, Q^{2}) x_{b} f_{1,b}(x_{b}) \frac{1}{\pi} \frac{d\hat{\sigma}_{\gamma_{\mathcal{S}}^{*}b \to cd}^{\Lambda}}{dt} D_{1,c}^{\Lambda}(z_{1}) D_{1,d}^{\bar{\Lambda}}(z_{2})}.$$
(24)

Here $G_{\gamma_s^*}(x_{\text{Bj}}, Q^2)$ is the photon flux in the DIS process [39] with $\mathcal{S} = \mathcal{L}, \mathcal{T}$ representing the photon polarization. For the self-sufficient of this paper, we list the photon flux in Appendix A. x_{Bj} is the Bjorken variable and Q is the virtuality of the virtual photon. The transversely polarized partonic cross section can be expressed as

$$\frac{d\hat{\sigma}_{\gamma_{\mathcal{L}}^* g \to q\bar{q}}^T}{dt} = \frac{2\pi\alpha\alpha_s e_q^2}{(s+Q^2)^2} \frac{4Q^2s}{(s+Q^2)^2},\tag{25}$$

$$\frac{d\hat{\sigma}_{\gamma_{\mathcal{T}}^*g \to q\bar{q}}^T}{dt} = -\frac{2\pi\alpha\alpha_s e_q^2}{(s+Q^2)^2} \left[1 - \frac{2Q^2s}{(s+Q^2)^2} \right]. \tag{26}$$

It is interesting to note that $d\sigma^T_{\gamma^*_L g \to q\bar{q}}/d\sigma_{\gamma^*_L g \to q\bar{q}} = +1$ indicating the unitary transverse spin correlation at arbitrary kinematics. The longitudinal photon leads to the positive correlation while the transverse photon leads to the negative correlation. Furthermore, when plotting $d\sigma^T_{\gamma^*_T g \to q\bar{q}}/d\sigma_{\gamma^*_T g \to q\bar{q}}$ as a function of $t/(s+Q^2)$, the ratio is Q^2 irrelevant. We obtain again the valley-shape correlation which is on a par with that of $gg \to q_i \bar{q}_i$ as a function of t/s.

V. TRANSVERSE SPIN CORRELATION IN THE HELICITY AMPLITUDE APPROACH

The spin correlation can be better appreciated in the helicity amplitude approach. While it is quite obvious for the helicity correlation, it is a bit subtle for the transverse spin correlation. Nonetheless, it has been well presented in Refs. [80–82]. In this section, we summarize the essential idea on how to relate helicity amplitudes with the transversely polarized cross section.

Let us consider the simple LO partonic scattering $a + b \rightarrow c + d$ with unpolarized beams. The helicity amplitude is thus denoted as $\mathcal{M}_{\lambda_a,\lambda_b}(\lambda_c,\lambda_d)$ with $\lambda_{a,b,c,d}=\pm 1$ denoting the helicity of the corresponding parton. The unpolarized cross section can simply be related to the helicity amplitudes [83] by

$$\frac{d\hat{\sigma}_{ab\to cd}}{dt} = \frac{1}{16\pi s^2} \frac{1}{4} \sum_{\lambda_a, \lambda_b, \lambda_c, \lambda_d} \mathcal{M}_{\lambda_a, \lambda_b}(\lambda_c, \lambda_d) \mathcal{M}_{\lambda_a, \lambda_b}^*(\lambda_c, \lambda_d). \tag{27}$$

The factor of 1/4 averages the spin degree-of-freedom of the initial partons a and b. The relation between the cross section and scattering amplitudes is taken from Ref. [83]. When employing the above normalization, one needs to adopt the same convention for the scattering amplitude as that in Ref. [83]. The transversely polarized cross section is expressed as

$$\frac{d\hat{\sigma}_{ab\to cd}^T}{dt} = \frac{1}{16\pi s^2} \frac{1}{4} \sum_{\lambda_a, \lambda_b, \lambda_c, \lambda_d} \mathcal{M}_{\lambda_a, \lambda_b}(\lambda_c, \lambda_d) \mathcal{M}_{\lambda_a, \lambda_b}^*(-\lambda_c, -\lambda_d). \tag{28}$$

The transverse spin correlation in unpolarized collisions demands a sign flip for final-state-parton helicities in the amplitude and the conjugate amplitude, while maintaining the same helicities for initial state partons. If this requisite cannot be achieved, the transverse spin correlation disappears. Moreover, the positivity constrain $|d\sigma^T/d\sigma| < 1$ is automatically satisfied, since the unpolarized cross section takes the form of $|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2$ and σ^T is evaluated from $2\text{Re}[\mathcal{M}_1\mathcal{M}_2^*] \leq 2|\mathcal{M}_1||\mathcal{M}_2|$. This is thus fully consistent with the probability interpretation of polarized and unpolarized cross sections. Moreover, one can also derive $|\mathcal{C}_{TT}| \leq \sqrt{1 - |\mathcal{P}_L|^2}$ with $|\mathcal{P}_L|$ the magnitude of quark/antiquark helicity from the above relation. This positivity constrain reduces the transverse spin correlation at LEP experiment.

First, we use $e^+e^- \to \gamma^* \to q\bar{q}$ to demonstrate the emergence of the transverse spin correlation. As presented in Ref. [83], the nonvanishing helicity amplitudes read

$$\mathcal{M}_{+-}(+,-) = e^2 e_q \sin \theta \sqrt{\frac{1+\cos \theta}{1-\cos \theta}}, \qquad \mathcal{M}_{+-}(-,+) = -e^2 e_q \sin \theta \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}, \qquad (29)$$

$$\mathcal{M}_{-+}(+,-) = -e^2 e_q \sin \theta \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}, \qquad \mathcal{M}_{-+}(-,+) = e^2 e_q \sin \theta \sqrt{\frac{1+\cos \theta}{1-\cos \theta}}, \qquad (30)$$

$$\mathcal{M}_{-+}(+,-) = -e^2 e_q \sin \theta \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}, \qquad \mathcal{M}_{-+}(-,+) = e^2 e_q \sin \theta \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}, \qquad (30)$$

with θ the angle between momenta of e^+ and q. Substituting Eqs. (29-29) into Eq. (28), the transversely polarized cross section is

$$\frac{d\hat{\sigma}_{e^{+}e^{-}\to\gamma^{*}\to q\bar{q}}^{T}}{dt} = \frac{N_{c}}{4} \frac{2\text{Re}[\mathcal{M}_{+-}(+,-)\mathcal{M}_{+-}^{*}(-,+) + \mathcal{M}_{-+}(+,-)\mathcal{M}_{-+}^{*}(-,+)]}{16\pi s^{2}} = -\frac{\pi\alpha^{2}N_{c}e_{q}^{2}\sin^{2}\theta}{Q^{4}}.$$
 (31)

Utilizing $\sin^2\theta = 4y(1-y) = 2C(y)$ with $y = (1+\cos\theta)/2$ and $d\sigma/dy = Q^2d\hat{\sigma}/dt$, we can recognize that it contributes to the electromagnetic term of ω_q^T in Eq. (A2).

Second, we use $q_iq_j \rightarrow q_iq_j$ channel as an example to demonstrate the disappearance of transverse spin correlation. According to Ref. [83], the scattering amplitudes in the helicity basis are given by

$$\mathcal{M}_{++}(+,+) \neq 0,$$
 $\mathcal{M}_{++}(-,-) = 0,$ (32)

$$\mathcal{M}_{+-}(+,-) \neq 0,$$
 $\mathcal{M}_{+-}(-,+) = 0,$ (33)

$$\mathcal{M}_{-+}(-,+) \neq 0,$$
 $\mathcal{M}_{-+}(+,-) = 0,$ (34)

$$\mathcal{M}_{--}(-,-) \neq 0,$$
 $\mathcal{M}_{--}(+,+) = 0.$ (35)

Since the sign flip cannot happen, the transverse spin correlation vanishes. Furthermore, another special case has already been discussed in Sec. II. When the coupling vertex takes the form of $\gamma^{\mu}(1\pm\gamma_5)$, the final state quark and antiquark are 100% polarized along the helicity direction, leaving no phase space for a helicity flip. Therefore, the transverse spin correlation vanishes as required by the positivity constrain.

VI. SUMMARY

The quantitative research of spin-dependent fragmentation functions is way below satisfactory. Based on a series of studies [37–39], the spin correlation of back-to-back dihadron provides a complementary platform to understanding spin-dependent fragmentation functions in unpolarized high energy collisions. Therefore, the unpolarized colliders, currently available worldwide, have great potential to improve the status quo.

In this paper, we focus on the transverse spin correlation of back-to-back hadrons in unpolarized e^+e^- , pp, and $\gamma^{(*)}p$ collisions and investigate the opportunity of understanding the transverse spin transfer H_{1T} in unpolarized high energy colliders. Unlike the case for the helicity correlation, the transverse spin correlation only resurfaces in the $q\bar{q}$ production channels and the identical-quark-scattering channel. Therefore, the magnitude of the transverse spin correlation in e^+e^- and photon-nucleus collisions is expected to be sizable. In contrast, since pp collisions contain substantial contributions from the unconnected diagrams, the magnitude of the transverse spin correlation is expected to be significantly reduced. To summarize, we have demonstrated the proof-of-concept that the unpolarized colliders can provide valuable information on the transverse spin transfer $H_{1T}(z)$. Future measurements of this observable can cast more light on the spin dependence of fragmentation functions.

ACKNOWLEDGMENTS

We thank Jian Zhou, Marco Zaccheddu, and Ya-Jin Zhou for fruitful discussion. This work is supported by Natural Science Foundation of China under grant No. 12405156, the Shandong Province Natural Science Foundation under grant No. 2023HWYQ-011 and No. ZFJH202303, and the Taishan fellowship of Shandong Province for junior scientists.

Appendix A: Coefficient functions

In this appendix, we list all the coefficient functions used in this paper for the completeness of this paper. Some of those function can also be found in other references.

For the electron positron annihilation process, those coefficient functions [60–63] are given by

$$\omega_q(y) = e_q^2 A(y) + \chi_{\text{int}}^q I_0^q(y) + \chi T_0^q(y), \tag{A1}$$

$$\omega_q^T(y) = -\left\{e_q^2 + \chi_{\text{int}}^q c_V^e c_V^q + \chi c_1^e [(c_V^q)^2 - (c_A^q)^2]\right\} C(y), \tag{A2}$$

$$T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y), \tag{A3}$$

$$I_0^q(y) = c_V^e c_V^q A(y) - c_A^e c_A^q B(y), \tag{A4}$$

$$A(y) = y^2 + (1 - y)^2, (A5)$$

$$B(y) = 1 - 2y, (A6)$$

$$C(y) = 2y(1-y),\tag{A7}$$

$$\chi = \frac{Q^4}{[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W},\tag{A8}$$

$$\chi_{\text{int}}^{q} = -\frac{2e_{q}Q^{2}(Q^{2} - M_{Z}^{2})}{[(Q^{2} - M_{Z}^{2})^{2} + \Gamma_{Z}^{2}M_{Z}^{2}]\sin^{2}2\theta_{W}},$$
(A9)

with θ_W the Weinberg angle, $c_1^e = (c_V^e)^2 + (c_A^e)^2$, $c_3^e = 2c_V^e c_A^e$, $c_1^q = (c_V^q)^2 + (c_A^q)^2$, $c_3^q = 2c_V^q c_A^q$, and $c_{V/A}$ the weak coupling constant [84]. For completeness, we list them in Table I.

	c_V	c_A
e	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$
q = u, c, t	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
q = d, s, b	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

TABLE I. Table for the coupling constants of weak interaction. Here, $\sin^2 \theta_W = 0.231$.

For the deep-inelastic scattering, the photon flux [85] is given by

$$G_{\gamma^*,\mathcal{L}}(x_{\rm Bj},Q^2) = \frac{\alpha}{\pi Q^2 x_{\rm Bj}} (1-y),$$
 (A10)

$$G_{\gamma^*,\mathcal{T}}(x_{\rm Bj},Q^2) = \frac{\alpha}{2\pi Q^2 x_{\rm Bj}} [1 + (1-y)^2],$$
 (A11)

with $y = Q^2/(x_{\rm Bj}s)$.

Appendix B: The DGLAP evolution of $H_{1T}(z, \mu_f^2)$

The collinear $H_{1T}(z, \mu_f^2)$ fragmentation function follows the DGLAP evolution equation [57] with μ_f the factorization scale. Only quark/antiquark contributes, since the gluon fragmentation function disappears. The DGLAP evolution equation becomes diagonal, which is given by

$$\frac{\partial H_{1T,q}(z,\mu_f^2)}{\partial \ln \mu_f^2} = \frac{\alpha_s(\mu_f^2)}{2\pi} \int_z^1 \frac{d\xi}{\xi} P_{qq}^T(\xi) H_{1T,q}(\frac{z}{\xi},\mu_f^2). \tag{B1}$$

Here, P_{qq}^T is the transverse splitting function [50, 58, 59] which, at the leading order, reads

$$P_{qq}^{T}(\xi) = P_{qq}(\xi) - C_F(1 - \xi) = C_F \frac{1 + \xi^2}{(1 - \xi)_+} + 2\delta(1 - \xi) - C_F(1 - \xi).$$
 (B2)

The next-to-leading order result has also been derived in Ref. [58].

Since the transverse splitting function, P_{qq}^T , is smaller than the unpolarized one, P_{qq} , it is obvious that the transverse polarization inherited by final state hadron becomes smaller after each splitting. This transverse spin loss due to the QCD evolution guarantees that the positivity constraint of spin dependent fragmentation function [58, 68] is automatically attained is as long as it is satisfied at the initial condition. We investigate the loss of transverse spin due to the QCD evolution in a toy model in the following.

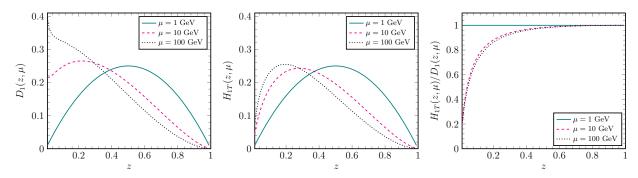


FIG. 6. Numerical results for the unpolarized and polarized fragmentation functions $D_1(z, \mu^2)$ and $H_{1T}(z, \mu^2)$ at $\mu = 1, 10$, and 100 GeV.

First, we parameterize the polarized and unpolarized fragmentation functions at the initial scale μ_0 as $H_{1T,q}(z,\mu_0^2) = G_{1L}(z,\mu_0^2) = D_{1,q}(z,\mu_0^2) = z(1-z)$. The ratio H_{1T}/D_1 is unity at the initial scale, indicating that the transverse polarization of the quark q is completely inherited by the final state hadron. Second, we only consider the diagonal terms in the DGLAP evolution for both functions, i.e., the unpolarized gluon fragmentation function is also neglected. The numerical results of D_1 and H_{1T} at larger factorization scales are presented in Fig. 6. Thanks to the DGLAP evolution, both H_{1T} and D_1 decrease with increasing scale at large z, while they increase at small z. However, the increase of H_{1T} at small z is much slower than that of D_1 because of the transverse spin loss effect.

The QCD evolution always suppresses the ratio H_{1T}/D_1 . As shown in the right panel of Fig. 6, it rapidly decreases at small z after a few steps of QCD evolution and eventually becomes stable at a large factorization scale. Furthermore, the transverse spin loss becomes negligible at the large z region indicating the optimal kinematic region to measure this effect in high energy colliders. Notice that in this study with a toy model, we have also neglected the gluon contribution to the DGLAP evolution of D_1 . Therefore, in reality, the loss of transverse polarization is even more severe. However, the qualitative feature remains unaltered. The polarization loss effect at small z is more significant than that at large z We leave a more sophisticated evaluation for a future study.

Furthermore, this is a toy model calculation which is employed to demonstrate the transverse-spin-loss effect due to parton shower. In reality, the transverse spin transfer is required to satisfy a positivity constrain [58, 68] which reads

$$|H_{1T,q}(z)| \le \frac{1}{2} \left[D_{1,q}(z) + G_{1L,q}(z) \right].$$
 (B3)

According to the DSV parametrization [86] and the LEP data [87, 88], $G_{1L,q}^{\Lambda}(z) \sim z^{\alpha} D_{1,q}^{\Lambda}(z)$. Therefore, the ratio H_{1T}/D_1 can only reach unity at the threshold regime and it is expected to be smaller than 1/2 at the $z \to 0$ limit. To summarize, the optimal kinematics of measuring the transverse spin correlation is the large z region.

- [1] A. Metz and A. Vossen, Prog. Part. Nucl. Phys. 91 (2016), 136-202 doi:10.1016/j.ppnp.2016.08.003 [arXiv:1607.02521 [hep-ex]].
- [2] K. B. Chen, T. Liu, Y. K. Song and S. Y. Wei, Particles 6 (2023) no.2, 515-545 doi:10.3390/particles6020029 [arXiv:2307.02874 [hep-ph]].
- [3] R. Boussarie, M. Burkardt, M. Constantinou, W. Detmold, M. Ebert, M. Engelhardt, S. Fleming, L. Gamberg, X. Ji and Z. B. Kang, et al. [arXiv:2304.03302 [hep-ph]].
- [4] J. Binnewies, B. A. Kniehl and G. Kramer, Z. Phys. C 65 (1995), 471-480 doi:10.1007/BF01556135 [arXiv:hep-ph/9407347 [hep-ph]].
- [5] B. A. Kniehl, G. Kramer and B. Potter, Nucl. Phys. B 582 (2000), 514-536 doi:10.1016/S0550-3213(00)00303-5 [arXiv:hep-ph/0010289 [hep-ph]].
- [6] T. Kneesch, B. A. Kniehl, G. Kramer and I. Schienbein, Nucl. Phys. B 799 (2008), 34-59 doi:10.1016/j.nuclphysb.2008.02.015 [arXiv:0712.0481 [hep-ph]].
- [7] S. Kretzer, Phys. Rev. D **62** (2000), 054001 doi:10.1103/PhysRevD.62.054001 [arXiv:hep-ph/0003177 [hep-ph]]
- [8] S. Albino, B. A. Kniehl and G. Kramer, Nucl. Phys. B 725 (2005), 181-206 doi:10.1016/j.nuclphysb.2005.07.010 [arXiv:hep-ph/0502188 [hep-ph]].
- [9] S. Albino, B. A. Kniehl and G. Kramer, Nucl. Phys. B 734 (2006), 50-61 doi:10.1016/j.nuclphysb.2005.11.006 [arXiv:hep-ph/0510173 [hep-ph]].
- [10] S. Albino, B. A. Kniehl and G. Kramer, Nucl. Phys. B 803 (2008), 42-104 doi:10.1016/j.nuclphysb.2008.05.017 [arXiv:0803.2768 [hep-ph]].
- [11] D. de Florian, R. Sassot and M. Stratmann, Phys. Rev. D 75 (2007), 114010 doi:10.1103/PhysRevD.75.114010 [arXiv:hep-ph/0703242 [hep-ph]].
- [12] D. de Florian, R. Sassot and M. Stratmann, Phys. Rev. D **76** (2007), 074033 doi:10.1103/PhysRevD.76.074033 [arXiv:0707.1506 [hep-ph]].
- [13] M. Hirai, S. Kumano, T. H. Nagai and K. Sudoh, Phys. Rev. D 75 (2007), 094009 doi:10.1103/PhysRevD.75.094009 [arXiv:hep-ph/0702250 [hep-ph]].
- [14] C. A. Aidala, F. Ellinghaus, R. Sassot, J. P. Seele and M. Stratmann, Phys. Rev. D 83 (2011), 034002 doi:10.1103/PhysRevD.83.034002 [arXiv:1009.6145 [hep-ph]].
- [15] D. de Florian, R. Sassot, M. Epele, R. J. Hernández-Pinto and M. Stratmann, Phys. Rev. D 91 (2015) no.1, 014035 doi:10.1103/PhysRevD.91.014035 [arXiv:1410.6027 [hep-ph]].
- [16] D. de Florian, M. Epele, R. J. Hernandez-Pinto, R. Sassot and M. Stratmann, Phys. Rev. D 95 (2017) no.9, 094019 doi:10.1103/PhysRevD.95.094019 [arXiv:1702.06353 [hep-ph]].
- [17] V. Bertone et al. [NNPDF], Eur. Phys. J. C 77 (2017) no.8, 516 doi:10.1140/epjc/s10052-017-5088-y [arXiv:1706.07049 [hep-ph]].
- [18] R. A. Khalek *et al.* [MAP (Multi-dimensional Analyses of Partonic distributions)], Phys. Rev. D **104** (2021) no.3, 034007 doi:10.1103/PhysRevD.104.034007 [arXiv:2105.08725 [hep-ph]].
- [19] N. Sato et al. [JAM], Phys. Rev. D 101 (2020) no.7, 074020 doi:10.1103/PhysRevD.101.074020 [arXiv:1905.03788 [hep-ph]].
- [20] E. Moffat et al. [Jefferson Lab Angular Momentum (JAM)], Phys. Rev. D 104 (2021) no.1, 016015 doi:10.1103/PhysRevD.104.016015 [arXiv:2101.04664 [hep-ph]].
- [21] M. Czakon, T. Generet, A. Mitov and R. Poncelet, JHEP 03 (2023), 251 doi:10.1007/JHEP03(2023)251 [arXiv:2210.06078 [hep-ph]].
- [22] J. Gao, C. Liu, X. Shen, H. Xing and Y. Zhao, Phys. Rev. Lett. 132 (2024) no.26, 26 doi:10.1103/PhysRevLett.132.261903 [arXiv:2401.02781 [hep-ph]].
- [23] J. Gao, C. Liu, X. Shen, H. Xing and Y. Zhao, [arXiv:2407.04422 [hep-ph]].
- [24] B. Q. Ma, I. Schmidt and J. J. Yang, Phys. Rev. D 61 (2000), 034017 doi:10.1103/PhysRevD.61.034017 [arXiv:hep-ph/9907224 [hep-ph]].
- [25] B. Q. Ma, I. Schmidt, J. Soffer and J. J. Yang, Eur. Phys. J. C 16 (2000), 657-664 doi:10.1007/s100520000447 [arXiv:hep-ph/0001259 [hep-ph]].
- [26] B. Q. Ma, I. Schmidt, J. Soffer and J. J. Yang, Nucl. Phys. A 703 (2002), 346-364 doi:10.1016/S0375-9474(01)01460-9 [arXiv:hep-ph/0107157 [hep-ph]].
- [27] M. Nzar and P. Hoodbhoy, Phys. Rev. D 51 (1995), 32-36 doi:10.1103/PhysRevD.51.32 [arXiv:hep-ph/9502349 [hep-ph]].
- [28] A. Metz, Phys. Lett. B **549** (2002), 139-145 doi:10.1016/S0370-2693(02)02899-X [arXiv:hep-ph/0209054 [hep-ph]].
- [29] A. Bacchetta, L. P. Gamberg, G. R. Goldstein and A. Mukherjee, Phys. Lett. B **659** (2008), 234-243 doi:10.1016/j.physletb.2007.09.076 [arXiv:0707.3372 [hep-ph]].
- [30] L. P. Gamberg, A. Mukherjee and P. J. Mulders, Phys. Rev. D 77 (2008), 114026 doi:10.1103/PhysRevD.77.114026 [arXiv:0803.2632 [hep-ph]].
- [31] Z. Lu and I. Schmidt, Phys. Lett. B **747** (2015), 357-364 doi:10.1016/j.physletb.2015.06.011 [arXiv:1501.04379 [hep-ph]].

- $[32] \ Y. \ Yang, \ Z. \ Lu \ and \ I. \ Schmidt, \ Phys. \ Lett. \ B \ \textbf{761} \ (2016), \ 333-339 \ doi:10.1016/j.physletb.2016.08.053 \ [arXiv:1607.01638 \ [hep-ph]].$
- [33] Y. Yang, Z. Lu and I. Schmidt, Phys. Rev. D 96 (2017) no.3, 034010 doi:10.1103/PhysRevD.96.034010 [arXiv:1706.03355 [hep-ph]].
- [34] X. Xie and Z. Lu, Phys. Lett. B 842 (2023), 137973 doi:10.1016/j.physletb.2023.137973 [arXiv:2210.16532 [hep-ph]].
- [35] Y. L. Pan, K. B. Chen, Y. K. Song and S. Y. Wei, Phys. Lett. B 850 (2024), 138509 doi:10.1016/j.physletb.2024.138509 [arXiv:2311.04462 [hep-ph]].
- [36] K. Chen, G. R. Goldstein, R. L. Jaffe and X. D. Ji, Nucl. Phys. B 445 (1995), 380-398 doi:10.1016/0550-3213(95)00193-V [arXiv:hep-ph/9410337 [hep-ph]].
- [37] H. C. Zhang and S. Y. Wei, Phys. Lett. B 839 (2023), 137821 doi:10.1016/j.physletb.2023.137821 [arXiv:2301.04096 [hep-ph]].
- [38] X. Li, Z. X. Chen, S. Cao and S. Y. Wei, Phys. Rev. D 109 (2024) no.1, 014035 doi:10.1103/PhysRevD.109.014035 [arXiv:2309.09487 [hep-ph]].
- [39] Z. X. Chen, H. Dong and S. Y. Wei, Phys. Rev. D 110 (2024) no.5, 056040 doi:10.1103/PhysRevD.110.056040 [arXiv:2404.19202 [hep-ph]].
- [40] W. Gong, G. Parida, Z. Tu and R. Venugopalan, Phys. Rev. D 106 (2022) no.3, L031501 doi:10.1103/PhysRevD.106.L031501 [arXiv:2107.13007 [hep-ph]].
- [41] J. Vanek [STAR], [arXiv:2307.07373 [nucl-ex]].
- [42] Z. Tu, Phys. Rev. C 109 (2024) no.5, 055205 doi:10.1103/PhysRevC.109.055205 [arXiv:2308.09127 [hep-ph]].
- [43] J. Barata, W. Gong and R. Venugopalan, Phys. Rev. D 109 (2024) no.11, 116003 doi:10.1103/PhysRevD.109.116003 [arXiv:2308.13596 [hep-ph]].
- [44] D. Shao, B. Yan, S. R. Yuan and C. Zhang, Sci. China Phys. Mech. Astron. 67 (2024) no.8, 281062 doi:10.1007/s11433-024-2389-y [arXiv:2310.14153 [hep-ph]].
- [45] J. p. Lv, Z. h. Yu, Z. t. Liang, Q. Wang and X. N. Wang, Phys. Rev. D 109 (2024) no.11, 114003 doi:10.1103/PhysRevD.109.114003 [arXiv:2402.13721 [hep-ph]].
- [46] S. Wu, C. Qian, Y. G. Yang and Q. Wang, [arXiv:2402.16574 [hep-ph]].
- [47] S. Wu, C. Qian, Q. Wang and X. R. Zhou, Phys. Rev. D 110 (2024) no.5, 054012 doi:10.1103/PhysRevD.110.054012 [arXiv:2406.16298 [hep-ph]].
- [48] D. Shen, J. Chen and A. Tang, [arXiv:2407.21291 [nucl-th]].
- [49] J. P. Ralston and D. E. Soper, Nucl. Phys. B 152 (1979), 109 doi:10.1016/0550-3213(79)90082-8
- [50] X. Artru and M. Mekhfi, Z. Phys. C 45 (1990), 669 doi:10.1007/BF01556280
- [51] R. L. Jaffe and X. D. Ji, Phys. Rev. Lett. **67** (1991), 552-555 doi:10.1103/PhysRevLett.67.552
- [52] R. L. Jaffe and X. D. Ji, Nucl. Phys. B 375 (1992), 527-560 doi:10.1016/0550-3213(92)90110-W
- [53] J. L. Cortes, B. Pire and J. P. Ralston, Z. Phys. C 55 (1992), 409-416 doi:10.1007/BF01565099
- [54] R. L. Jaffe and A. Manohar, Phys. Lett. B 223 (1989), 218-224 doi:10.1016/0370-2693(89)90242-6
- [55] X. D. Ji, Phys. Lett. B **289** (1992), 137-142 doi:10.1016/0370-2693(92)91375-J
- [56] J. Soffer and O. V. Teryaev, Phys. Rev. D 56 (1997), R1353-R1356 doi:10.1103/PhysRevD.56.R1353 [arXiv:hep-ph/9702352 [hep-ph]].
- [57] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977), 298-318 doi:10.1016/0550-3213(77)90384-4
- [58] W. Vogelsang, Phys. Rev. D 57 (1998), 1886-1894 doi:10.1103/PhysRevD.57.1886 [arXiv:hep-ph/9706511 [hep-ph]].
- [59] V. Barone, A. Drago and P. G. Ratcliffe, Phys. Rept. 359 (2002), 1-168 doi:10.1016/S0370-1573(01)00051-5 [arXiv:hep-ph/0104283 [hep-ph]].
- [60] S. y. Wei, Y. k. Song and Z. t. Liang, Phys. Rev. D 89 (2014) no.1, 014024 doi:10.1103/PhysRevD.89.014024 [arXiv:1309.4191 [hep-ph]].
- [61] S. Y. Wei, K. b. Chen, Y. k. Song and Z. t. Liang, Phys. Rev. D 91 (2015) no.3, 034015 doi:10.1103/PhysRevD.91.034015 [arXiv:1410.4314 [hep-ph]].
- [62] K. b. Chen, W. h. Yang, S. y. Wei and Z. t. Liang, Phys. Rev. D 94 (2016) no.3, 034003 doi:10.1103/PhysRevD.94.034003 [arXiv:1605.07790 [hep-ph]].
- [63] K. b. Chen, Z. t. Liang, Y. k. Song and S. y. Wei, Phys. Rev. D 105 (2022) no.3, 034027 doi:10.1103/PhysRevD.105.034027 [arXiv:2108.07740 [hep-ph]].
- [64] D. Boer, R. Jakob and P. J. Mulders, Nucl. Phys. B 504 (1997), 345-380 doi:10.1016/S0550-3213(97)00456-2 [arXiv:hep-ph/9702281 [hep-ph]].
- [65] J. E. Augustin and F. M. Renard, Nucl. Phys. B 162 (1980), 341 doi:10.1016/0550-3213(80)90269-2
- [66] G. Gustafson and J. Hakkinen, Phys. Lett. B 303 (1993), 350-354 doi:10.1016/0370-2693(93)91444-R
- [67] K. b. Chen, W. h. Yang, Y. j. Zhou and Z. t. Liang, Phys. Rev. D 95 (2017) no.3, 034009 doi:10.1103/PhysRevD.95.034009 [arXiv:1609.07001 [hep-ph]].
- $[68] \ J. \ Soffer, \ Phys. \ Rev. \ Lett. \ \textbf{74} \ (1995), \ 1292-1294 \ doi: 10.1103/PhysRevLett. \\ \textbf{74.1292} \ [arXiv:hep-ph/9409254 \ [hep-ph]].$
- [69] P. Hoodbhoy and X. D. Ji, Phys. Rev. D 58 (1998), 054006 doi:10.1103/PhysRevD.58.054006 [arXiv:hep-ph/9801369 [hep-ph]].
- [70] D. Boer, S. Cotogno, T. van Daal, P. J. Mulders, A. Signori and Y. J. Zhou, JHEP 10 (2016), 013 doi:10.1007/JHEP10(2016)013 [arXiv:1607.01654 [hep-ph]].
- [71] D. Boer, P. J. Mulders, J. Zhou and Y. j. Zhou, JHEP 10 (2017), 196 doi:10.1007/JHEP10(2017)196 [arXiv:1702.08195 [hep-ph]].
- [72] S. Kumano and Q. T. Song, Phys. Rev. D 101 (2020) no.5, 054011 doi:10.1103/PhysRevD.101.054011 [arXiv:1910.12523

- [hep-ph]].
- [73] S. Kumano and Q. T. Song, Phys. Rev. D 101 (2020) no.9, 094013 doi:10.1103/PhysRevD.101.094013 [arXiv:2003.06623 [hep-ph]].
- [74] S. Kumano and Q. T. Song, Phys. Rev. D 103 (2021) no.1, 014025 doi:10.1103/PhysRevD.103.014025 [arXiv:2011.08583 [hep-ph]].
- [75] S. Kumano and Q. T. Song, JHEP **09** (2021), 141 doi:10.1007/JHEP09(2021)141 [arXiv:2106.15849 [hep-ph]].
- $[76] \ Q. \ T. \ Song, \ Phys. \ Rev. \ D \ \textbf{108} \ (2023) \ no.9, \ 094041 \ doi: 10.1103/PhysRevD. 108.094041 \ [arXiv: 2309.06757 \ [hep-ph]].$
- [77] J. F. Owens, Rev. Mod. Phys. **59** (1987), 465 doi:10.1103/RevModPhys.59.465
- [78] J. D. Jackson, "Classical Electrodynamics", Wiley, 1998, ISBN 978-0-471-30932-1
- [79] U. Jezuita-Dabrowska and M. Krawczyk, [arXiv:hep-ph/0211112 [hep-ph]].
- [80] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis and F. Murgia, Phys. Rev. D 73 (2006), 014020 doi:10.1103/PhysRevD.73.014020 [arXiv:hep-ph/0509035 [hep-ph]].
- [81] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, E. R. Nocera and A. Prokudin, Phys. Rev. D 83 (2011), 114019 doi:10.1103/PhysRevD.83.114019 [arXiv:1101.1011 [hep-ph]].
- [82] U. D'Alesio, F. Murgia and M. Zaccheddu, JHEP 10 (2021), 078 doi:10.1007/JHEP10(2021)078 [arXiv:2108.05632 [hep-ph]].
- [83] R. Gastmans and T. T. Wu, Int. Ser. Monogr. Phys. 80 (1990), 1-648
- [84] D. Griffiths, "Introduction to elementary particles", Wiley, 2008.
- [85] P. Caucal, F. Salazar, B. Schenke, T. Stebel and R. Venugopalan, JHEP **08** (2023), 062 doi:10.1007/JHEP08(2023)062 [arXiv:2304.03304 [hep-ph]].
- [86] D. de Florian, M. Stratmann and W. Vogelsang, Phys. Rev. D 57 (1998), 5811-5824 doi:10.1103/PhysRevD.57.5811 [arXiv:hep-ph/9711387 [hep-ph]].
- [87] D. Buskulic et al. [ALEPH], Phys. Lett. B 374 (1996), 319-330 doi:10.1016/0370-2693(96)00300-0
- [88] K. Ackerstaff et al. [OPAL], Eur. Phys. J. C 2 (1998), 49-59 doi:10.1007/s100520050123 [arXiv:hep-ex/9708027 [hep-ex]].