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$\tilde{\xi}$-attractors in metric-affine gravity

A. Racioppi[a]

[a]National Institute of Chemical Physics and Biophysics, Rävala 10, 10143 Tallinn, Estonia

E-mail: antonio.racioppi@kbfi.ee

Abstract. We propose a new class of inflationary attractors in metric-affine gravity. Such class features a non-minimal coupling $\tilde{\xi}\,\Omega(\phi)$ with the Holst invariant $\tilde{\mathcal{R}}$ and an inflaton potential proportional to $\Omega(\phi)^2$. The attractor behaviour of the class takes place with two combined strong coupling limits. The first limit is realized at large $\tilde{\xi}$, which makes the theory equivalent to a $\tilde{\mathcal{R}}^2$ model. Then, the second limit considers a very small Barbero-Immirzi parameter which leads the inflationary predictions of the $\tilde{\mathcal{R}}^2$ model towards the ones of Starobinsky inflation. Because of the analogy with the renown ξ-attractors, we label this new class as $\tilde{\xi}$-attractors.

Keywords: inflation, attractors, metric-affine gravity

Contents

1 Introduction

Cosmic inflation, i.e. an accelerated expansion during the very early Universe, is the current paradigm for explaining the flatness and homogeneity of the Universe at large scales [1–4]. Moreover, it also provides an origin for the small inhomogeneities observed in the Cosmic Microwave Background radiation. In its minimal version, inflation is usually formulated by adding to the Einstein-Hilbert action one scalar field, the inflaton, whose energy density induces a near-exponential expansion.

The latest combination of Planck, BICEP/Keck and BAO data [5] has sensibly reduced the allowed parameters space, strongly favouring nearly-flat concave inflaton potentials and already ruling out many proposed models. Nevertheless, the most popular inflationary realizations, the Starobinsky model [1] and Higgs-inflation [6], still sit in the allowed region. Both models can be described by a scalar field non-minimally coupled to gravity (e.g. [7] and references therein).

However, when theories are non-minimally coupled to gravity there is more than one *choice* of the dynamical degrees of freedom. In the more popular metric gravity, the metric tensor is the only dynamical degree of freedom, while the connection is fixed to be the Levi-Civita one. On the other hand, in metric-affine gravity (MAG), both the metric and the connection are dynamical variables and their corresponding equations of motion will dictate the eventual relation between them. When the gravity action contains only the term linear in the curvature scalar, the two approaches lead to equivalent theories (e.g. [8] and refs. therein), otherwise the theories are completely different [8–10] and lead to different phenomenological predictions, as recently investigated in e.g. [11–63]. Moreover, MAG admits two, rather than just one, two-derivative curvature invariants: the usual Ricci-like scalar and the Holst invariant [64–66], which can be used to construct new models (e.g. [67–78]).

The purpose of this article is to study a new model in MAG, where the Jordan frame inflaton scalar potential is proportional to the square of the non-minimal coupling function involving the Holst invariant and/or the Ricci-like curvature scalar. As we will see later, this kind of setup will induce a new class of inflationary attractors.

The discussion is organized as follows. In Section 2 we introduce the action for our inflationary model in metric-affine gravity. For the sake of minimality, we consider only constructions where the physical degrees of freedom are only the graviton and the inflaton.

In Section 3 we present an analytical study of the inflaton potential and describe how the attractor configuration is reached. Then, in Section 4 we present a detailed numerical study with the corresponding inflationary predictions. Finally, in Section 5 we summarize our conclusions. In addition, in Appendix A, we show the full analytical equations of the slow-roll parameters and the inflationary observables.

2 Model

We start with the Jordan frame action for a real scalar ϕ non-minimally coupled to gravity

$$S_{\rm J} = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\left(f(\phi)\mathcal{R} + \tilde{f}(\phi)\tilde{\mathcal{R}}\right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right], \tag{2.1}$$

where M_P is the reduced Planck mass, $V(\phi)$ the inflaton potential, $f(\phi)$ and $\tilde{f}(\phi)$ are non-minimal coupling functions, $\mathcal{R}$ and $\tilde{\mathcal{R}}$ respectively, a scalar and pseudoscalar contraction of the curvature (the latter also known as the Holst invariant [64–66]),

$$\mathcal{R} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}, \qquad \tilde{\mathcal{R}} \equiv \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu\rho\sigma}, \tag{2.2}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric Levi-Civita symbol with $\epsilon^{0123} = 1$. $\mathcal{F}_{\mu\nu}{}^\rho{}_\sigma$ is the curvature associated with the connection $\mathcal{A}_\mu{}^\rho{}_\sigma$,

$$\mathcal{F}_{\mu\nu}{}^\rho{}_\sigma \equiv \partial_\mu\mathcal{A}_\nu{}^\rho{}_\sigma - \partial_\nu\mathcal{A}_\mu{}^\rho{}_\sigma + \mathcal{A}_\mu{}^\rho{}_\lambda\mathcal{A}_\nu{}^\lambda{}_\sigma - \mathcal{A}_\nu{}^\rho{}_\lambda\mathcal{A}_\mu{}^\lambda{}_\sigma\ . \tag{2.3}$$

As mentioned before, we do not consider any other term in action (2.1) in order to keep the model as minimal as possible, with only the massless graviton and the inflaton as physical degrees of freedom and without terms that feature more than two derivatives.

We remind that in MAG, the connection $\mathcal{A}_\mu{}^\rho{}_\sigma$ is not assumed to be the Levi-Civita one, but it is computed from the corresponding equation of motion. We also remind that, if $\mathcal{A}_\mu{}^\rho{}_\sigma$ is the Levi-Civita connection, $\tilde{\mathcal{R}}$ vanishes[1] and $\mathcal{R}$ equals the Ricci scalar R. After some manipulations (e.g. [78] and refs. therein), the action (2.1) can be written in the Einstein frame as

$$S_{\rm E} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - U(\chi)\right], \tag{2.4}$$

where the Einstein frame scalar potential is

$$U(\chi) = \frac{V(\phi(\chi))}{f^2(\phi(\chi))}, \tag{2.5}$$

and the canonical normalized scalar is defined by solving

$$\frac{d\chi}{d\phi} = \sqrt{k(\phi)}\,, \qquad k(\phi) = \frac{1}{f} + \frac{12\left[f(\phi)'\tilde{f}(\phi) - f(\phi)\tilde{f}(\phi)'\right]^2}{f(\phi)^2\left[f(\phi)^2 + 4\tilde{f}(\phi)^2\right]}, \tag{2.6}$$

where a prime represents a derivative with respect to argument of the function. Since $f(\phi)$ must be positive in order to avoid repulsive gravity, $k(\phi)$ is always positive and ϕ never a ghost.

[1] The careful reader might notice that $\tilde{\mathcal{R}}$ actually vanishes for any $\mathcal{A}_\mu{}^\rho{}_\sigma$ that satisfies $\mathcal{A}_\mu{}^\rho{}_\sigma = \mathcal{A}_\sigma{}^\rho{}_\mu$.

In the present article we are interested in the ξ-attractors-inspired [79, 80] configuration

$$V(\phi) = \Lambda^4\,\Omega(\phi)^2, \qquad f(\phi) = 1 + \xi\,\Omega(\phi), \qquad \tilde{f}(\phi) = \tilde{f}_0^2 + \tilde{\xi}\,\Omega(\phi)\,, \tag{2.7}$$

where Ω is a positive continuous and differentiable function of ϕ. The quantity $1/(4\tilde{f}_0^2)$ is known as the Barbero-Immirzi parameter [81, 82]. Similar setups have been already studied in the literature. For instance, the configuration with $\tilde{f}_0 = \tilde{\xi} = 0$ (i.e. without the Holst invariant contribution) has been already studied in (e.g. [10, 17] and refs. therein), while the choice with all the contributions active and $\Omega \propto \phi^2$ has been studied in [69, 70]. According to our knowledge, for what concerns the last configuration, the setup with a generic Ω has not been studied yet. In particular, in this article we study the setup where $\xi = 0$ (i.e. only a non-minimal coupling between ϕ and the Holst invariant), leaving the most general study for a future work.

3 General features of the scalar potential

In the case of $f(\phi) = 1$ i.e. $\xi = 0$ (see eq. (2.7)), the scalar potential in eq. (2.5) just becomes $U(\chi) = V(\phi(\chi))$. Therefore, the eventual change of shape in the potential is all due to the field redefinition in eq. (2.6). When $\xi = 0$, the non-minimal kinetic function simply becomes

$$k(\phi) = 1 + \frac{12\left[\tilde{f}(\phi)'\right]^2}{\left[1 + 4\tilde{f}(\phi)^2\right]} = 1 + \frac{12\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{2\left[1 + 4\left(\tilde{f}_0{}^2 + \tilde{\xi}\Omega(\phi)\right)^2\right]}\,. \tag{3.1}$$

It is well known that when $k(\phi)$ presents a pole (or just a pronounced peak), $U(\chi)$ exhibits a flat region that might be suitable for inflation. Since the denominator in eq. (3.1) is strictly positive, the chance of a pole is excluded and we are left only with the possibility of an eventual local maximum with $k(\phi) \gg 1$. This should naively happen when $|\tilde{\xi}| \gg 1$ (see eq. (3.1)). In such a case, then we can easily approximate the behaviour of $k(\phi)$ nearby the maximum by neglecting the "1+" term before the fraction in (3.1), obtaining

$$k(\phi) \simeq \frac{12\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{2\left[1 + 4\left(\tilde{f}_0{}^2 + \tilde{\xi}\Omega(\phi)\right)^2\right]}\,. \tag{3.2}$$

Using such an expression, we can provide an approximated solution for (2.6)

$$\chi \simeq \sqrt{\frac{3}{2}}M_P\left\{\operatorname{arcsinh}\left[2\left(\tilde{f}_0{}^2 + \tilde{\xi}\Omega(\phi)\right)\right] - \operatorname{arcsinh}\left(2\tilde{f}_0{}^2\right)\right\}\,, \tag{3.3}$$

which can be inverted in function of $\Omega(\phi)$, allowing us to write explicitly the Einstein frame potential as

$$U(\chi)_{\tilde{\mathcal{R}}^2} \simeq \frac{\Lambda^4}{4\tilde{\xi}^2}\left\{\sinh\left[\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}\chi}{M_P} + 2\tilde{f}_0{}^2\right)\right] - 2\tilde{f}_0{}^2\right\}^2\,. \tag{3.4}$$

Note that the potential (3.4) is completely independent on $\Omega(\phi)$ and can be generated by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\left(\mathcal{R} + \tilde{f}_0^2\tilde{\mathcal{R}}\right) + c\tilde{\mathcal{R}}^2\right], \qquad c = \tilde{\xi}^2\left(\frac{M_P}{4\Lambda}\right)^4\,, \tag{3.5}$$

which has been already studied in [72]. Using the properties of the hyperbolic functions, it can be proven that eq. (3.4) is equivalent to the expression of the inflaton potential used in [72]. Moreover, taking the $\tilde{f}_0 \to \infty$ limit of (3.4), we obtain the Starobinsky potential [1]

$$U(\chi)_{R^2} \simeq \frac{\Lambda^4 \tilde{f_0}^4}{\tilde{\xi}^2} \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right)^2 . \tag{3.6}$$

Therefore, because of such an universal strong coupling limit and the analogy with the ξ-attractors, we decide to label the class of models defined by eqs. (2.7) as $\tilde{\xi}$-*attractors*. It is reasonable to expect that the $\tilde{\xi}$-attractors will show two asymptotic behaviours in two different steps, the first one approaching the predictions of the $\tilde{\mathcal{R}}^2$ model in (3.5) for $|\tilde{\xi}| \gg 1$ and then a second one approaching the results of Starobinsky inflation when also $|\tilde{f}_0| \gg 1$. In order to test numerically such a behaviour, from now on we study the specific choice

$$\Omega(\phi)^2 = \left(\frac{\phi}{M_P}\right)^n , \tag{3.7}$$

with $n > 0$. We note that for even n's $V(\phi)$ is positive for any ϕ values and all the functions in (2.7) are symmetric under the transformation $\phi \to -\phi$. On the other hand, for odd n's, the inflaton potential is positive only for $\phi > 0$. Therefore from now on we only work in the positive quadrant for ϕ. Moreover, the absolute signs of $\tilde{f}$ and $\tilde{f}_0$ are irrelevant (see eqs. (2.6) and (2.7)), therefore from now on we choose the convention where $\tilde{f}_0 > 0$ while $\tilde{\xi}$ changes sign. Inserting (3.7) into (3.1), the kinetic function becomes

$$k(\phi) = 1 + \frac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f_0}^2 + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]} . \tag{3.8}$$

The position of the corresponding local maximum can be computed to be

$$\phi_{\text{peak}} = \left(\frac{\Delta}{\tilde{\xi}}\right)^{2/n} M_P \qquad \Delta = \frac{\tilde{f_0}^2}{4}\left(n - 4 - \sqrt{n^2 + \frac{2(n-2)}{\tilde{f_0}^4}}\right) . \tag{3.9}$$

and the value of the maximum of the kinetic function is

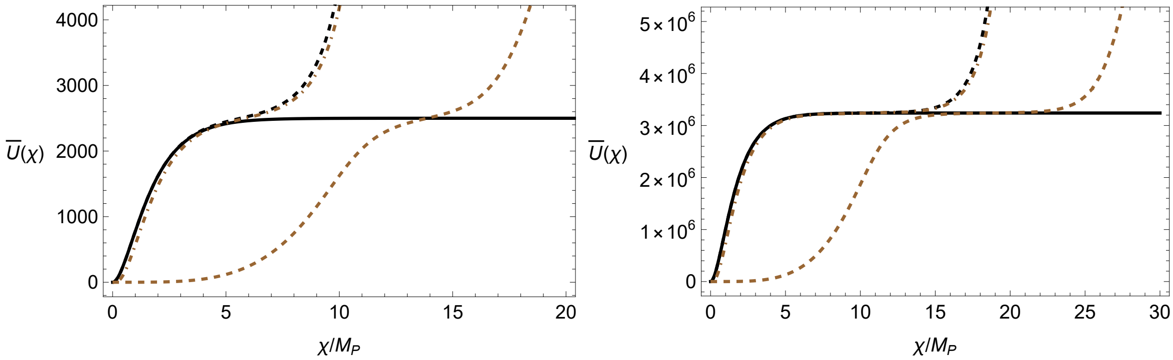
$$k(\phi_{\text{peak}}) = 1 + \tilde{\xi}^{4/n} \frac{3n^2 \Delta^{2-\frac{4}{n}}}{2\left(4\left(\tilde{f_0}^2 + \Delta\right)^2 + 1\right)} . \tag{3.10}$$

Since Δ is always negative, in order to ensure that ϕ_{peak} is real and positive, $\tilde{\xi}$ must be negative as well. Therefore, from now on we will only consider $\tilde{\xi} < 0$. Moreover, if $0 < n < 2$, then $\tilde{f}_0$ has the lower bound $\tilde{f_0}^2 \geq \sqrt{2}\sqrt{\frac{2-n}{n^2}}$. Now let us see how the peak in $k(\phi)$ generates a flat region in $U(\chi)$. This happens via an inflection point, whose equation is:

$$U''(\chi_{\text{flex}}) = 0 . \tag{3.11}$$

In terms of ϕ, eq. (3.11) can be rewritten as

$$\frac{1}{2}\frac{k'(\phi_{\text{flex}})}{k(\phi_{\text{flex}})} = \frac{V''(\phi_{\text{flex}})}{V'(\phi_{\text{flex}})} , \tag{3.12}$$



(a) (b)

Figure 1: $\bar{U}(\chi)$ vs. χ/M_P for $n=4$. (a) $\tilde{f}_0=5$ with $|\tilde{\xi}|\simeq 0.22$ (brown, dashed) and $|\tilde{\xi}|\simeq 46.8$ (brown, dot-dashed). (b) $\tilde{f}_0=30$ with $|\tilde{\xi}|\simeq 7.24$ (brown, dashed) and $|\tilde{\xi}|\simeq 1.66\times10^3$ (brown, dot-dashed). For reference the corresponding $\bar{U}(\chi)_{\tilde{\mathcal{R}}^2}$ (black, dashed) and $\bar{U}(\chi)_{R^2}$ (black, continuous).

where we have used $\xi=0$. Now, using eqs. (2.7) and (3.7), eq. (3.12) becomes

$$\frac{M_P}{2}\frac{k'(\phi_{\rm flex})}{k(\phi_{\rm flex})}=(n-1)\frac{M_P}{\phi_{\rm flex}}\,. \tag{3.13}$$

Then, it is easy to see that for $n=1$, eq. (3.13) just becomes $k'(\phi_{\rm flex})=0$ and $\phi_{\rm flex}=\phi_{\rm peak}$. For any other n, if $\phi_{\rm flex}\gg M_P$, then $\phi_{\rm flex}\simeq\phi_{\rm peak}$. This happens when $\tilde{f}_0\gg1$ (see eq. (3.9)). Let us now study the behaviour of $U(\chi)$ in the vicinity of the pronounced peak in $k(\phi)$. From eq. (3.10) it is easy to check that if $|\tilde{\xi}|\gg1$ and/or $\tilde{f}_0\gg1$, then the value of $k(\phi)$ at the maximum behaves like

$$k(\phi_{\rm peak})\approx\frac{3}{2}n^2\,\tilde{\xi}^{4/n}\,\tilde{f}_0{}^{4-\frac{8}{n}}\gg1\,. \tag{3.14}$$

Moreover, using the result of (3.3) combined with eq. (3.7), we can provide an approximated solution for the field redefinition as

$$\chi\simeq\sqrt{\frac{3}{2}}M_P\left\{\operatorname{arcsinh}\left[2\left(\tilde{f}_0{}^2+\tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)\right]-\operatorname{arcsinh}\left(2\tilde{f}_0{}^2\right)\right\}\,, \tag{3.15}$$

which leads to the same Einstein frame potential shown in eq. (3.4). In order to have a better understanding of the behaviour of the inflaton potential at big $\tilde{f}_0$, we show in Fig. 1, the plot of $\bar{U}(\chi)=\frac{4\tilde{\xi}^2}{\Lambda^4}U(\chi)$ (brown) for $n=4$, $\tilde{f}_0=5$ (a), $\tilde{f}_0=30$ (b) with respectively $|\tilde{\xi}|\simeq0.22$ (dashed), 46.8 (dot-dashed) and $|\tilde{\xi}|\simeq7.24$ (dashed), 1.66×10^3 (dot-dashed). For reference we will also show the plots of the corresponding $\bar{U}(\chi)$ potentials for the $\tilde{\mathcal{R}}^2$ model in eq. (3.4) (black, dashed) and for the Starobinsky model in eq. (3.6) (black, continuous). We can see that all the brown lines in both plots exhibit a quite flat inflection point and that the corresponding concave knee if moving to smaller χ's by increasing $|\tilde{\xi}|$. Moreover when $|\tilde{\xi}|$ is large, the plateau region of $\bar{U}(\chi)$ is essentially indistinguishable from the one of $\bar{U}(\chi)_{\tilde{\mathcal{R}}^2}$. Finally, when both $|\tilde{\xi}|$ and $\tilde{f}_0$ are big, the plot of $\bar{U}(\chi)_{\tilde{\mathcal{R}}^2}$ overlaps the one of $\bar{U}(\chi)_{R^2}$ until it passes the inflection point. Then $\bar{U}(\chi)_{\tilde{\mathcal{R}}^2}$ turns upwards into a convex behaviour diverging to infinity while $\bar{U}(\chi)_{R^2}$ remains concave while approaching its horizontal asymptote.

4 Inflationary results

In this section we discuss the inflationary predictions of the model. As well known, in the slow-roll approximation, all the inflationary observables can be computed from the potential slow-roll parameters:

$$\epsilon_U(\chi) = \frac{M_P^2}{2}\left(\frac{U'(\chi)}{U(\chi)}\right)^2 , \tag{4.1}$$

$$\eta_U(\chi) = M_P^2 \frac{U''(\chi)}{U(\chi)} . \tag{4.2}$$

The expansion of the Universe is estimated in number of e-folds, which is

$$N_e = \frac{1}{M_P^2}\int_{\chi_{\rm end}}^{\chi_N} {\rm d}\chi \, \frac{U(\chi)}{U'(\chi)}, \tag{4.3}$$

where the field value at the end of inflation is given by $\epsilon(\chi_{\rm end}) = 1$, while the field value χ_N at the time a given scale left the horizon is given by the corresponding N_e. The tensor-to-scalar ratio r and the scalar spectral index $n_{\rm s}$ are:

$$r = 16\epsilon_U(\chi_N) , \tag{4.4}$$

$$n_{\rm s} = 1 + 2\eta_U(\chi_N) - 6\epsilon_U(\chi_N) . \tag{4.5}$$

Finally, the amplitude of the scalar power spectrum is

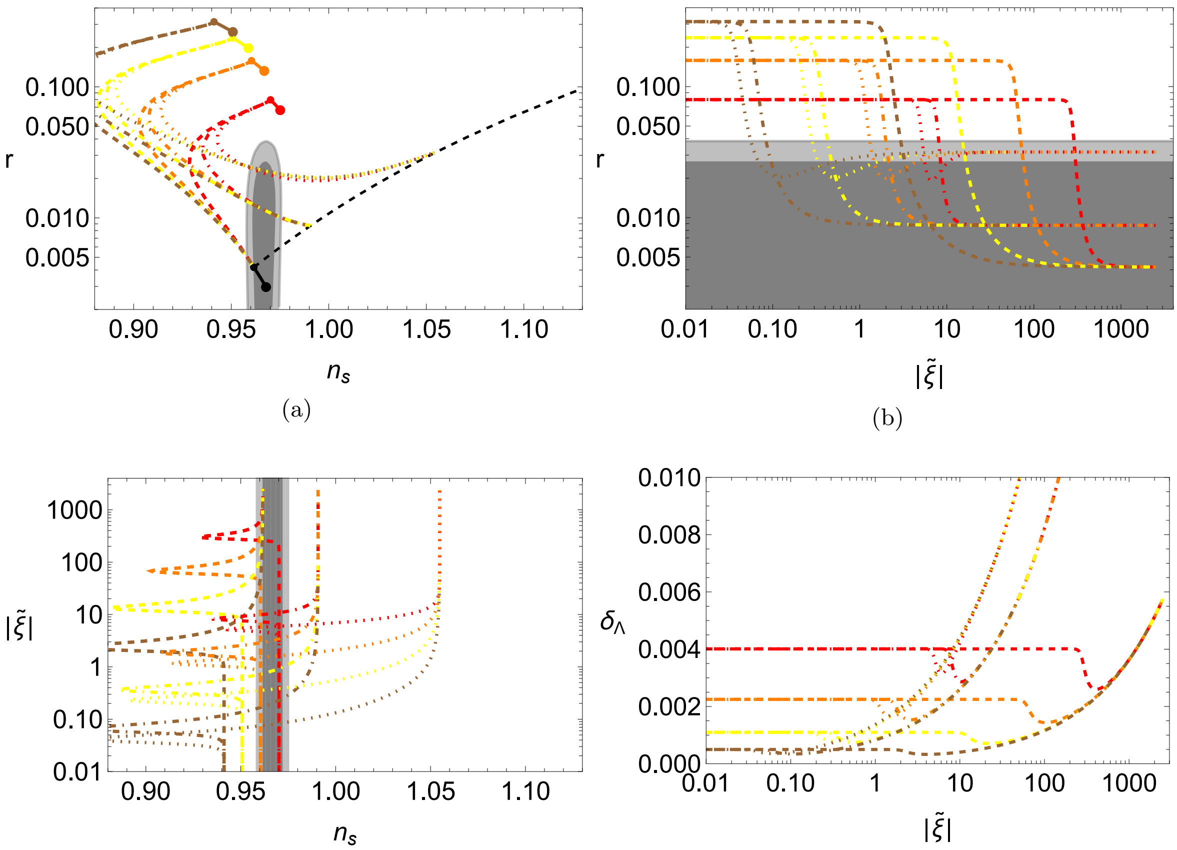
$$A_{\rm s} = \frac{1}{24\pi^2 M_P^4}\frac{U(\chi_N)}{\epsilon_U(\chi_N)} \simeq 2.1 \times 10^{-9} , \tag{4.6}$$

whose experimental constraint [83] usually fixes the energy scale of inflation. The explicit analytical expression for the inflationary observables are too cumbersome to provide any useful information at first glance, therefore we postpone them into a separate Appendix A.

The corresponding numerical results are instead illustrated in Fig. 2 for $N_e = 50$ and Fig. 3 for $N_e = 60$, where we show r vs. $n_{\rm s}$ (a), r vs. $|\tilde{\xi}|$ (b), $|\tilde{\xi}|$ vs. $n_{\rm s}$ (c), δ_Λ vs. $|\tilde{\xi}|$ (d) with $\delta_\Lambda = \frac{\Lambda}{M_P}$ counting the prefactor of the inflaton potential (2.7) in Planck units. We considered the following values for n: 1 (red), 2 (orange), 3 (yellow) and 4 (brown). In both Figs. 2 and 3 we used $\tilde{f}_0 = 4$ (dotted), $\tilde{f}_0 = 5$ (dot-dashed) and $\tilde{f}_0 = 30$ (dashed). For reference we also added the 1,2σ allowed regions coming from the latest combination of Planck, BICEP/Keck and BAO data [5] (gray areas) and the predictions of $\tilde{\mathcal{R}}^2$ inflation (black, dashed) at $N_e = 50, 60$ (according to the figure) and of standard ϕ^n inflation (continuous, same colors as before) and Starobinsky inflation (black, continuous) for $N_e \in [50, 60]$.

First all we notice that, as usual, moving from $N_e = 50$ (Fig. 2) to $N_e = 60$ (Fig. 3), the predictions move towards lower (higher) r (n_s) but the generic behaviour remains unaffected. However, the lower $\tilde{f}_0$, the higher is the increase in n_s at a given $\tilde{\xi}$ (cf. Figs. 2(c) and 3(c)). As expected from the analytical study of the inflaton potential and the limits in eqs. (3.4) and (3.6), we see that for $|\tilde{\xi}| \gg 1$ we predictions are aligned with the ones of the $\tilde{\mathcal{R}}^2$ model. When also $\tilde{f}_0 \gg 1$ (specifically $\tilde{f}_0 = 30$ in our case), then the results match the ones of Starobinsky inflation[2]. Even though it is possible to get predictions in agreement with the

[2]This is in agreement with the results of [72], where, using our notation, the author just stopped the numerical analysis at $\tilde{f}_0 = 10\sqrt{3} \simeq 17.3$.



(a) (b) (c) (d)

Figure 2: r vs. n_s (a), r vs. $|\tilde{\xi}|$ (b), $|\tilde{\xi}|$ vs. n_s (c), δ_Λ vs. $|\tilde{\xi}|$ (d) for $N_e = 50$ and $n = 1$ (red), 2 (orange), 3 (yellow) and 4 (brown) with $\tilde{f}_0 = 4$ (dotted), $\tilde{f}_0 = 5$ (dot-dashed) and $\tilde{f}_0 = 30$ (dashed). The gray areas represent the 1,2σ allowed regions coming from the latest combination of Planck, BICEP/Keck and BAO data [5]. For reference the predictions of $\tilde{\mathcal{R}}^2$ inflation (black, dashed) at $N_e = 50$ and of standard ϕ^n inflation (continuous, same colors as before) and Starobinsky inflation (black, continuous) for $N_e \in [50, 60]$.

latest constraints [5], most of the predictions actually fall out of the 2σ allowed region. On the other hand, it is possible to reach the 1σ region without using very large $\tilde{f}_0$ but just $\tilde{f}_0 = 4$.

From Figs. 2(b) and 3(b), we see that r is insensitive to the value of $\tilde{\xi}$ when $|\tilde{\xi}| < 1$, for all the considered values of $\tilde{f}_0$. By increasing $|\tilde{\xi}|$, r decreases until it reaches its asymptotic value corresponding to $\tilde{\mathcal{R}}^2$ inflation, for all the studied values of $\tilde{f}_0$ but $\tilde{f}_0 = 4$, where instead, it first reaches a minimum and then increases towards the aforementioned limit. Finally we notice that the generic shape of the results is not affected by changing n, but the asymptotic configuration is reached at larger $|\tilde{\xi}|$ values with n increasing.

From Figs. 2(c) and 3(c), we see that behaviour of n_s is the same for all the considered values of $\tilde{f}_0$. First, it is insensitive to the value of $\tilde{\xi}$ when $|\tilde{\xi}| < 1$. By increasing $|\tilde{\xi}|$, n_s decreases until it reaches a minimum and then it increases reaching its asymptotic value corresponding to $\tilde{\mathcal{R}}^2$ inflation. As before, the generic shape of the results is not affected by changing n, but the asymptotic configuration is reached at larger $|\tilde{\xi}|$ values with n increasing.

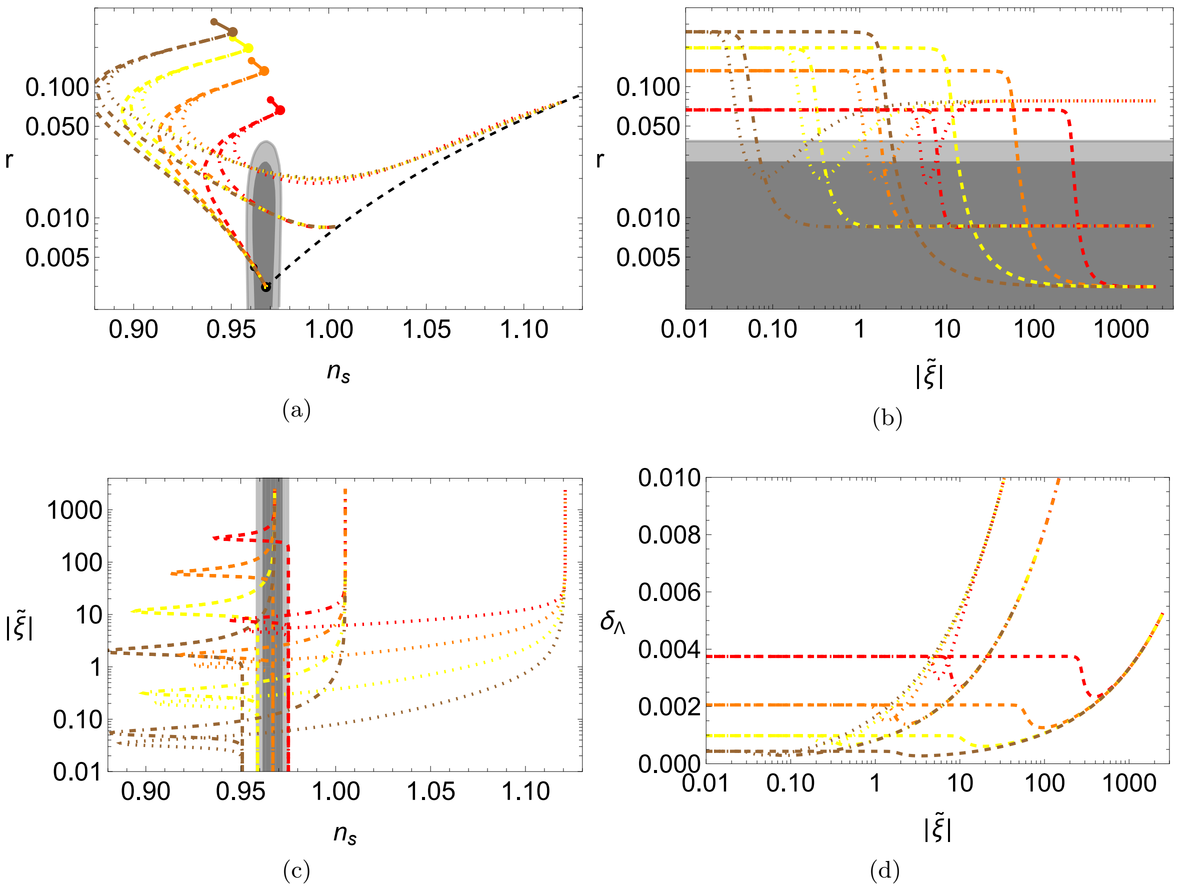


Figure 3: r vs. n_s (a), r vs. $|\tilde{\xi}|$ (b), $|\tilde{\xi}|$ vs. n_s (c), δ_Λ vs. $|\tilde{\xi}|$ (d) for $N_e = 60$ and $n = 1$ (red), 2 (orange), 3 (yellow) and 4 (brown) with $\tilde{f}_0 = 4$ (dotted), $\tilde{f}_0 = 5$ (dot-dashed) and $\tilde{f}_0 = 30$ (dashed). The gray areas represent the 1,2σ allowed regions coming from the latest combination of Planck, BICEP/Keck and BAO data [5]. For reference the predictions of $\tilde{\mathcal{R}}^2$ inflation (black, dashed) at $N_e = 60$ and of standard ϕ^n inflation (continuous, same colors as before) and Starobinsky inflation (black, continuous) for $N_e \in [50, 60]$.

Finally, from Figs. 2(d) and 3(d), we see that behaviour of δ_Λ is the same for all the considered values of $\tilde{f}_0$. First, it is insensitive to the value of $\tilde{\xi}$ when $|\tilde{\xi}| < 1$. By increasing $|\tilde{\xi}|$, δ_Λ decreases until it reaches a minimum and then it increases reaching its asymptotic configuration corresponding to $\tilde{\mathcal{R}}^2$ inflation. Again, the generic shape of the results is not affected by changing n, but the asymptotic configuration is reached at larger $|\tilde{\xi}|$ values with n increasing.

To conclude we note that our results agree with the ones of [69] in the corner of the parameters space where the two models are comparable. However, no direct comparison can be done with [70] because they do not study the case of a negative $\tilde{\xi}$.

5 Conclusions

We studied a new class of inflationary attractors in metric-affine gravity. Such class exhibits a non-minimal coupling function, $\Omega(\phi)$, with the Holst invariant $\tilde{\mathcal{R}}$ and an inflaton potential proportional to $\Omega(\phi)^2$. Because of the analogy with the renown ξ-attractors, we decided to label this new class as $\tilde{\xi}$-attractors. The attractor behaviour of the class takes place with two combined strong coupling limits. The first limit is the obvious $\tilde{\xi} \gg 1$, which makes the theory equivalent to a $\tilde{\mathcal{R}}^2$ model. Then, the second limit considers a very small Barbero-Immirzi parameter (i.e. $\tilde{f}_0 \gg 1$), driving the inflationary predictions of the model into the ones of Starobinsky inflation. We also performed a detailed numerical study for $\Omega(\phi)^2 = (\phi/M_P)^n$ with $n = 1, 2, 3, 4$. The two-steps attractor behaviour has been confirmed for all the considered values of n. The Starobinsky limit has been reached for $\tilde{f}_0 = 30$. On the other hand, compatibility with experimental data [5] at 1σ level, is already possible for $\tilde{f}_0 = 4$ and $\tilde{\xi} \simeq 6, 1.5, 0.3, 0.08$, respectively for $n = 1, 2, 3, 4$, but far away from the Starobinsky solution. The forthcoming experiments with a precision of $\delta r \sim 10^{-3}$, such as Simons Observatory [84], CMB-S4 [85] and LITEBIRD [86], will be capable to confirm or rule out our scenario, in particular for the cases away from the Starobinsky limit.

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A Equations for inflationary parameters

In this Appendix we give the analytical expressions of the slow-roll parameters and the corresponding inflationary observables. Since the field redefinition (2.6) cannot be solved analytically, as customary, we apply the chain rule of derivatives and express the parameters in function of ϕ. Therefore, using eqs. (2.5), (2.6), (4.1) and (4.2), we get

$$\epsilon_U(x_\phi) = \frac{n^2}{2x_\phi^2} \frac{1}{1 + \frac{3n^2\tilde{\xi}^2 x_\phi^{n-2}}{2\left[1+4\left(\tilde{f}_0{}^2+\tilde{\xi}x_\phi^{n/2}\right)^2\right]}}, \tag{A.1}$$

$$\begin{aligned}\eta_U(x_\phi) = & \frac{1}{\left(16\tilde{f}_0{}^2\tilde{\xi}x_\phi^{\frac{n}{2}+2} + 2\left(4\tilde{f}_0{}^4+1\right)x_\phi^2 + \tilde{\xi}^2\left(3n^2+8x_\phi^2\right)x_\phi^n\right)^2} \times \\ & \times\Big[12n^4\tilde{f}_0{}^4\tilde{\xi}^2x_\phi^n + 36n^4\tilde{f}_0{}^2\tilde{\xi}^3x_\phi^{3n/2} + 3n^4\tilde{\xi}^2x_\phi^n + 24n^4\tilde{\xi}^4x_\phi^{2n} + \\ & \quad +4(n-1)nx_\phi^2\left(8\tilde{f}_0{}^2\tilde{\xi}x_\phi^{n/2} + 4\tilde{f}_0{}^4 + 4\tilde{\xi}^2x_\phi^n + 1\right)^2\Big],\end{aligned} \tag{A.2}$$

where we have defined $x_\phi = \phi/M_P$. Hence, the number of e-folds is computed using (4.3) as

$$N_e = \left[\frac{x_\phi^2}{2n} + \frac{3}{8}\ln\left(1+4\left(\tilde{f}_0{}^2+\tilde{\xi}x_\phi^{n/2}\right)^2\right) - \frac{3}{2}\tilde{f}_0{}^2\arctan\left(2\left(\tilde{f}_0{}^2+\tilde{\xi}x_\phi^{n/2}\right)\right)\right]_{x_{\phi_{\rm end}}}^{x_{\phi_N}}, \tag{A.3}$$

while the tensor-to-scalar ratio (4.4) and the scalar spectral index (4.5) become respectively

$$r = \frac{8n^2}{x_{\phi_N}^2} \frac{1}{1 + \frac{3n^2\tilde{\xi}^2 x_{\phi_N}^{n-2}}{2\left[1+4\left(\tilde{f_0}^2+\tilde{\xi}x_{\phi_N}^{n/2}\right)^2\right]}}\,, \tag{A.4}$$

$$n_s = 1 - \frac{n(n+2)}{x_{\phi_N}^2} + \frac{3n^4\tilde{\xi}^2 x_{\phi_N}^{n-2}\left(\tilde{\xi}^2 x_{\phi_N}^n\left(3(n-4)n+8x_{\phi_N}^2\right) - 2\left(4\tilde{f_0}^4+1\right)x_{\phi_N}^2\right)}{2\left(16\tilde{f_0}^2\tilde{\xi}x_{\phi_N}^{\frac{n}{2}+2} + 2\left(4\tilde{f_0}^4+1\right)x_{\phi_N}^2 + \tilde{\xi}^2\left(3n^2+8x_{\phi_N}^2\right)x_{\phi_N}^n\right)^2}$$
$$+ \frac{3n^3(n+8)\tilde{\xi}^2 x_{\phi_N}^n}{4x_{\phi_N}^4\left(4\left(\tilde{f_0}^2+\tilde{\xi}x_{\phi_N}^{n/2}\right)^2+1\right) + 6n^2\tilde{\xi}^2 x_{\phi_N}^{n+2}}\,. \tag{A.5}$$

To conclude, the amplitude of scalar perturbations (4.6) is given by

$$A_s = \frac{x_{\phi_N}^n}{24\pi^2}\frac{\Lambda^4}{M_P^4}\left[\frac{2x_{\phi_N}^2}{n^2} + \frac{3\tilde{\xi}^2 x_{\phi_N}^n}{1+4\left(\tilde{f_0}^2+\tilde{\xi}x_{\phi_N}^{n/2}\right)^2}\right]\,. \tag{A.6}$$

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