Dark plasmas in the nonlinear regime: constraints from particle-in-cell simulations

William DeRocco

Maryland Center for Fundamental Physics, University of Maryland, College Park, 4296 Stadium Drive, College Park, MD 20742, USA and Department of Physics & Astronomy, The Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA

Pierce Giffin

Department of Physics, University of California Santa Cruz, 1156 High St., Santa Cruz, CA 95064, USA and Santa Cruz Institute for Particle Physics, 1156 High St., Santa Cruz, CA 95064, USA

If the dark sector possesses long-range self-interactions, these interactions can source dramatic collective instabilities even in astrophysical settings where the collisional mean free path is long. Here, we focus on the specific case of dark matter halos composed of a dark U(1) gauge sector undergoing a dissociative cluster merger. We study this by performing the first dedicated particle-in-cell plasma simulations of interacting dark matter streams, tracking the growth, formation, and saturation of instabilities through both the linear and nonlinear regimes. We find that these instabilities give rise to local (dark) electromagnetic inhomogeneities that serve as scattering sites, inducing an effective dynamic collisional cross-section. Mapping this effective cross-section onto existing results from large-scale simulations of the Bullet Cluster, we extend the limit on the dark charge-to-mass ratio by over ten orders of magnitude. Our results serve as a simple example of the rich phenomenology that may arise in a dark sector with long-range interactions and motivate future dedicated study of such "dark plasmas."

I. INTRODUCTION

There are multiple lines of compelling evidence that indicate the existence of an unknown matter content termed "dark matter" (DM) [1-4], the microphysical nature of which remains unknown. Vast theoretical and experimental effort has been devoted to discovering the nature of dark matter, often under the assumption that dark matter consists of a new fundamental particle with a low interaction cross-section with the visible sector. As a result, the phenomenology of dark matter has relied almost exclusively on single-particle interactions. This is in direct contrast to the visible sector of the Universe, in which the vast majority of visible matter exists in the form of plasma, the behavior of which is governed not by single-particle interactions but by collective effects mediated by long-range interactions. The most dramatic of these effects arise from *plasma instabilities*, in which small initial perturbations grow exponentially and produce nonlinear structures even on scales far smaller than the particle scattering mean free path. If, in analogy to the visible sector, self-interactions in the dark sector are dominated by collective effects, then the associated instabilities can lead to observable effects in astrophysical systems.

Despite this, little work has been done on characterizing dark sectors governed by collective interactions. Under the assumption that collective effects would make dark matter effectively collisional, Heinkinheimo et al. [5– 7] performed hydrodynamic simulations of dissociative cluster mergers with an artificial viscosity that was tuned to best reproduce observed lensing maps. Growth rates for common instabilities have been computed in the linear regime for a variety of astrophysical settings [8, 9]. Simulations of interpenetrating electron-positron pair plasmas have also discussed possible implications for the dark sector [10], and analytic arguments for the existence of limits on particular models have also appeared in the literature [11].

Here, we perform dedicated fully-kinetic plasma simulations of self-interacting dark matter with observationally-motivated parameters. We characterize both the growth of instabilities in the linear regime as well as their nonlinear saturation and latetime behavior. The results are then connected to a model of the dissociative cluster merger 1E 0657-56, allowing us to place constraints on dark couplings roughly ten orders of magnitude below existing constraints.

II. DARK U(1) PLASMA

The Standard Model (SM) electromagnetic sector exhibits a rich phenomenology of plasma behaviors; a dark sector with a massless U(1) mediator is therefore a natural place to look for similar effects. As such, we will take as our fiducial model that of "dark electromagnetism," namely a sector composed of "dark" electrons and positrons interacting via a massless U(1) gauge boson. This model has been explored previously in the literature [12–14] and has been found to be able to produce the correct relic abundance of dark matter [14]. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \bar{\chi} (\gamma^{\mu} (i\partial_{\mu} - q_{\chi}A') - m_{\chi}) \chi \quad (1)$$



FIG. 1: Constraints on dark U(1) electromagnetism in the $q_{\chi} - m_{\chi}$ plane. Bullet Cluster constraints from two-to-two scattering are shown in blue. Constraints from the survival of elliptical galaxies are shown in red [12–14]. The results presented in this work place constraints above the black line.

where χ , $\bar{\chi}$ are dark fermions with mass m_{χ} and charge q_{χ} under a new U(1) gauge symmetry mediated by a massless "dark photon" A'. While the particle content is similar to the SM electromagnetic sector, the charge q_{χ} and mass m_{χ} of the fermions are taken to be free parameters. We restrict our attention to the case of a massless mediator and assume throughout this paper that the dark sector is completely decoupled from the Standard Model, i.e., there is no kinetic mixing term $\epsilon F_{\mu\nu}F'^{\mu\nu}$. As discussed in Ref. [12], this is a self-consistent choice, as if ϵ is set to zero at some high scale, $\epsilon = 0$ is preserved under renormalization group evolution.

Neglecting plasma effects, the strongest existing constraints on this model have been found to arise from the survival of elliptical galaxies on cosmological timescales [12, 14]. Dissociative cluster mergers also bound the selfinteractions of this model, however, when purely considering two-to-two scattering, the associated bounds are several orders of magnitude weaker than the ellipticity bound (see Fig. 1) [12–14]. Despite this, as we show in Sec. VI, taking into account *collective* effects allows such mergers to constrain couplings as low as ten orders of magnitude below the ellipticity bound.

III. PLASMA: LINEAR AND NON-LINEAR REGIMES

Plasmas are quasineutral mixtures of charged particles in which oppositely charged particles are not bound to each other. When particle collisions can be neglected, the evolution of the plasma is governed by the Vlasov-Maxwell equations [15], which can be recast in a form in which the only unitful quantity is the *plasma frequency*, defined as

$$\omega_{\mathrm{pl},s} = \sqrt{\frac{q_s^2 n_s}{m_s}} \tag{2}$$

where *n* is the number density and *s* indexes species. Here, we use units in which $\hbar = c = \epsilon_0 = \mu_0 = 1$. This allows the behavior of plasma at one set of (q_s, m_s) parameters to be generalized to other regions in parameter space at which the plasma takes on the same $\omega_{\text{pl},s}$.

The Vlasov-Maxwell system is inherently nonlinear; however, when perturbations are small, the equation can be linearized and a dispersion equation computed. When a solution to the dispersion relation contains a positive imaginary part, small perturbations grow exponentially, giving rise to an instability. The growth rate of instabilities in the linear regime can be computed analytically in certain simplified settings; however, once the perturbation has grown sufficiently large that the linear approximation breaks down, the dynamics enter the *nonlinear* regime, and numerical simulations are needed to characterize the subsequent evolution. In this regime, the exponential growth back-reacts on the background plasma, leading to a saturation of the instability and, often, the formation of non-linear structures such as collisionless shocks [16–22].

In the following subsections, we will focus on the linear and nonlinear behavior of one astrophysically-relevant system, namely counter-streaming non-magnetized pair plasmas.

A. Linear regime

A simple system in which instabilities arise consists of two beams of plasma with comparable densities streaming through one another. Though simple, this system captures the essential features of the astrophysical settings we will discuss in Section IV. Two primary instabilities dominate this system. The first is the *two-stream instability*, in which longitudinal modes grow exponentially as a result of electrostatic interactions. The second is the *Weibel instability*, in which modes transverse to the beam's direction of motion are amplified by electromagnetic interactions with the other beam.

1. Two-stream instability

The electrostatic instability arises due to the presence of an unstable longitudinal mode in the counterstreaming plasmas. The growth rate of perturbations can be calculated in the linear regime via the plasma dispersion relation [18, 19]

$$\Gamma_{\rm TS} \approx \frac{1}{2\sqrt{2}} \omega_{\rm pl}$$
 (3)

with the fastest growing mode having $k_{x,\text{TS}}^{\text{crit}} \approx \frac{\sqrt{3/2}}{v_0} \omega_{\text{pl}}$, where the subscript TS denotes "two-stream" and v_0 is the initial relative bulk velocity of the plasma beams. This growth rate has been computed under the assumption that the relative velocity of the beams (v_0) is significantly larger than their velocity dispersions (the "cold limit"). The inclusion of a non-zero temperature only serves to weaken the instability and suppress the growth rate.

The instability saturates when the potential becomes large enough that approaching particles become trapped in the troughs between regions of high potential. This increases the charge density in the trough, raising its potential and reducing the amplitude of the mode.

2. Weibel instability

While the two-stream instability is driven entirely by interactions with the electric field, hence is "electrostatic," there is an additional electromagnetic instability in the system known as the Weibel instability.¹ The growth rate of perturbations can be calculated in the linear regime and is given by

$$\Gamma_{\rm W} \approx v_0 \omega_{\rm pl} \tag{4}$$

with the instability approaching its maximum for modes with $k \gtrsim k_{y,W}^{\text{crit}} \approx \omega_{\text{pl}}$ [19] in the zero-temperature limit. Note that the Weibel instability grows at a rate suppressed by v_0 with respect to the two-stream instability, as is expected given that the two-stream instability arises due to the $q\mathbf{E}$ term in the Vlasov equation while the Weibel instability arises due to the $q\mathbf{v} \times \mathbf{B}$ term.

While the Weibel instability grows more slowly than the two-stream instability at non-relativistic velocities, it still has a significant effect on late-time dynamics. The instability saturates when the magnetic field becomes sufficiently strong such that the gyroradius of particles becomes comparable to the spatial scale of the perturbation. At this stage, deflected particles can no longer penetrate through the magnetic filaments formed during instability growth, and the instability mechanism is halted. Saturation of the Weibel instability generally occurs at a higher electromagnetic field density than the two-stream instability hence, at late times, it is the nonlinear behavior of this instability that dominantly drives the evolution of the plasma.

B. Nonlinear regime

The growth of the two-stream instability will cause the formation of "collisionless shocks" [15, 16, 18, 19, 23–26], shock fronts between the beams supported by collective electromagnetic interactions despite the mean-free-path of an individual particle being much longer than the scale of the shock. The formation of this structure halts when the exponential growth rate becomes comparable to the bounce frequency of the streaming particles [18]

$$\Gamma_{TS}^2 \approx \omega_{b,E}^2 = \frac{q_\chi E k_x}{m_\chi}.$$
(5)

This process interrupts the bulk motion of the countersteaming plasma, slowing down the relative bulk velocity while heating up the plasma [23].

While electrostatic modes initially generate collisionless shocks before dissipating quickly, Weibel filaments will continue to grow and support these shocks on much longer timescales, further contributing to the slowdown of the beams. The Weibel instability saturates when the exponential growth rate becomes comparable to the magnetic trapping frequency [18]

$$\Gamma_W^2 \approx \omega_{b,B}^2 = \frac{q_\chi B v_0 k_y}{m_\chi} \tag{6}$$

which in general corresponds to a larger electromagnetic field strength than at two-stream saturation.

After saturation, the Weibel filaments will undergo macroscopic instabilities which lead to further turbulence and slowdown. See Section A 3 for further discussion.

IV. THE BULLET CLUSTER

The Bullet Cluster (1E 0657-56) [27–30] is a wellknown dissociative cluster merger that has been used extensively to constrain dark matter self-interactions [31– 35]. The system consists of a large main galaxy cluster of mass $\approx 10^{15} M_{\odot}$ and a smaller subcluster of mass $\approx 10^{14} M_{\odot}$ that merged at a high relative velocity. The observed offset of the dark matter and shocked Standard Model gas in this system has been used to constrain the collisional cross-section of self-interacting dark matter with mass m to be $\sigma/m \lesssim 1 - 4 \text{ cm}^2/\text{g}$ [33–36].

¹ Additional electromagnetic oblique modes can occur in asymmetric beam plasmas. For beams that are approximately symmetric, as in this work, their growth rates and saturation mechanisms only vary slightly and can be determined numerically. See [18] for further discussion.

Though there is tension on the precise value of this constraint, in this paper, we adopt $4 \text{ cm}^2/\text{g}$ as a conservative limit.

Existing limits on self-interactions are derived under the assumption of individual particle scattering. However, the Bullet Cluster is also ideal for studying the effects of plasma instabilities on dark sectors charged under a long-range interaction. At small scales on which the dark matter density of each cluster varies negligibly, the merger is well-modeled by the two-stream system described in Section III. Two-stream and Weibel instabilities may arise, leading to the formation of smallscale electromagnetic fields that efficiently scatter DM particles through a wide variety of diffusion mechanisms [37–46] inducing an effective collisional cross-section despite the plasma being largely collisionless. We can compute this effective collisional cross-section via dedicated simulations of the formation and saturation of plasma microinstabilities, as discussed in the following Section. Having extracted these collision rates from our simulations, we can rescale the existing constraints on two-totwo scattering to place constraints on collective interactions as well.²

V. SIMULATIONS

We simulate two counter-streaming beams of net neutral plasma using Smilei, an open-source particle-in-cell simulation framework [47]. To connect our results to the Bullet Cluster, we adopt values taken to be consistent with the analytic Hernquist profiles of the merging clusters provided in [34], taking r = 150 kpc for both clusters, corresponding to the time at which their central densities begin to merge [33]. This choice yields velocity dispersions of $\sigma_{\text{Main}} = 1080 \text{ km/s}$ for the Main cluster and $\sigma_{\text{Bullet}} = 630 \,\text{km/s}$ in the Bullet sub-cluster and a relative number density of $n_{\text{Main}}/n_{\text{Bullet}} = 1.91$. Since both higher temperatures and larger density ratios tend to inhibit instability growth, the values we have selected are conservative. The beams are initialized with a relative bulk velocity of $v_0 = 3000 \,\mathrm{km/s}$, which is consistent with the findings of [48] that the DM relative velocity may be much less than the observed gas shock velocity (≈ 4700 km/s). Furthermore, we note that as larger relative velocities only serve to enhance the microinstabilities, this choice of a low v_0 is also conservative. All other parameters can be absorbed into the plasma frequency as discussed in Section III. Our choice of parameters places the system in the "warm" regime where the thermal velocity of each stream is comparable to the streams' relative velocity. Warm, non-relativistic pair plasmas are rare in the Standard Model, hence their evolution is less wellstudied than the cold and hot regimes, making dedicated simulations even more necessary.

Our simulation consists of a 2D3V Cartesian geometry of dimensions $L_x = 7 c / \omega_{pl,B}$ in the longitudinal direction and $L_y = 175 c/\omega_{pl,B}$ in the transverse direction with periodic boundary conditions across all boundaries. Here, $\omega_{pl,B}$ is the plasma frequency corresponding to the Bullet sub-cluster. We choose a grid spacing of $\Delta x = 0.0035 \, c/\omega_{pl,B}$ in the longitudinal direction and $\Delta y = 0.18 \, c/\omega_{pl,B}$ in the transverse direction to ensure proper resolutions of both two-stream and Weibel instabilities. Each simulation was run for $10^3 \omega_{pl,B}^{-1}$ with a temporal resolution of $\Delta t = 0.98 \Delta x$. Two species were initialized, χ and $\bar{\chi}$, with equal and opposite chargeto-mass ratios, 64 total macroparticles per cell, and bulk velocities $v_{\text{Bullet}} = -n_{\text{Main}} v_0 / (n_{\text{Bullet}} + n_{\text{Main}})$ and $v_{Main} = n_{\text{Bullet}} v_0 / (n_{\text{Bullet}} + n_{\text{Main}})$ such that the simulation is in the center of momentum frame. Additionally, five multi-pass binomial current filters [49] were applied in each dimension after each time step for increased numerical stability. We found this setup to properly resolve all of the relevant length and time scales and produced results with negligible numerical heating. See Section A 1 of the Supplemental Material for further discussion.

VI. RESULTS

Fig. 2 shows the energy densities of the longitudinal electric field (red), the transverse magnetic field (blue), and total electromagnetic energy density (purple) from our simulation normalized to the initial kinetic energy of the two-beam system as a function of time. As discussed in Section III A 1, at early times, we expect the two-stream instability to dominate; in Fig. 2, we see that the energy density of the longitudinal electric fields grows exponentially with a growth rate of $\Gamma_{TS} = 0.396 \,\omega_{pl,B}$, but saturates rapidly. This growth rate agrees well with numerical estimates (see SM). After the electrostatic instability saturates, the Weibel instability continues to grow at $0.00992 \,\omega_{pl,B}$, then saturates at ~ $750 \,\omega_{pl,B}^{-1}$ (Fig. 2).

Throughout the simulation, we record the trajectories of 10^4 test particles. At regular intervals, we calculate for each particle

$$\Delta v_t = \frac{\vec{v_0} \cdot \vec{v_t}}{|\vec{v_0}|^2} \tag{7}$$

where $\vec{v_0}$ is the particle's velocity vector at the start of the simulation and $\vec{v_t}$ is the particle's velocity vector at time t. If Δv_t changes by a factor of e, we then consider the particle to have undergone a hard scatter, similar to the study performed in [50]. We can then determine the two-to-two scattering cross-section that this fraction of

² Note that in this work, we do not explore the formation or stability of dark matter halos, instead assuming that for very weak couplings, the standard picture of structure formation is not appreciably altered. If, however, this is not the case, then stronger constraints may arise from the existence of dark matter halos themselves. We leave a thorough exploration of this to future work.



FIG. 2: Energy densities from simulation normalized to the initial kinetic energy of the two-beam system. The energy stored in the longitudinal electric fields (red), the transverse magnetic field (blue), and total electromagnetic energy (purple) are shown. Fits to the exponential growth associated with the two-stream and Weibel instabilities are shown as dashed and dashed-dotted lines respectively.

dashed-dotted lines respectively.

scattered particles would correspond to through [51]

$$\frac{\sigma}{m} = -\Sigma \log(1-p),\tag{8}$$

where p is the fraction of particles that have scattered and $\Sigma \sim 0.33 \,\text{g/cm}^2$ is the projected mass surface density of the Bullet Cluster at a radius of 150 kpc [34]. This relation allows us to connect our results to existing limits on DM self-interactions from large-scale simulations of the Bullet Cluster.

Fig. 3 shows a cumulative count of the percentage of test particles that have undergone a momentum change equivalent to a hard scatter as a function of time. We find that at the time of Weibel saturation $(750 \,\omega_{pl,B}^{-1})$, 73.3% of particles have been scattered. By Eq. 8, this corresponds to a cross-section of $\sigma/m \sim 4.4 \,\mathrm{cm^2/g}$. Previous studies agree that such a cross-section is ruled out by the centroid offsets of the Bullet Cluster's dark matter halo and galaxies [33–36]. We therefore place a constraint on all charge-to-mass ratios that achieve Weibel saturation in less than 1% of the Bullet Cluster crossing time, where we have chosen 1% of crossing as a conservative scale over which the halo density varies negligibly ($\Delta \rho/\rho < 5\%$) hence our simulations remain an appropriate treatment of the system. We estimate the crossing



FIG. 3: Fraction of tracked macroparticles that have undergone an $\mathcal{O}(1)$ momentum change as a function of time (cf. Eq. 7). The dash-dotted lines show the fraction of particles that existing simulations show undergo *two-to-two* scattering events during the merger for varying cross-sections [51]. At the time of Weibel saturation (750 $\omega_{pl,B}^{-1}$), 73.3% of particles have been scattered, corresponding to an effective cross-section of $4.4 \,\mathrm{cm}^2/\mathrm{g}$.

time $t_{\rm cross}$ as twice the time it takes for the Hernquist mass distributions described in Sec. IV to move from a centroid separation of 300 kpc to 0 kpc assuming the halos were infalling from rest at infinite separation and that the infall does not significantly disrupt the halo shape. This calculation yields $t_{\rm cross} = 1.07 \times 10^8$ yr. This leads to a constraint on the dark charge-to-mass ratio of

$$q_{\chi} < 2.8 \times 10^{-14} \left(\frac{m_{\chi}}{\text{GeV}}\right). \tag{9}$$

Our constraint is shown in Fig. 1 as a solid black line. Existing constraints from the ellipticity of dwarf galaxies [12, 14] (red) and two-to-two scattering limits from the Bullet Cluster (blue) are shown as well. Even for our conservative choice of parameters, our results extend existing limits by over ten orders of magnitude.

We further verify our results by explicitly computing the slowdown of the bulk velocity of the counterstreaming beams as a function of time. We show the evolution of the velocity distributions in Fig. 4, where the left-hand (center) panel shows the velocity distribution in the longitudinal (transverse) direction at three different times and the right-hand panel shows the bulk velocities of the two beams as a function of time. We find that the bulk velocity decreases by roughly an order of magnitude by the time of Weibel saturation, with the dominant slowdown being due to diffusion of particles by late-time Weibel modes. By the end of our simulation, the two beams have effectively isotropized in the center-of-momentum frame, as can be seen in the dotted distribution corresponding to $t = 750 \,\omega_{pl,B}^{-1}$ in the left and center panels.

funded by NSF MRI grant AST 1828315. The authors acknowledge the support of DOE grant No. DE-SC0010107.

VII. CONCLUSIONS

In this work, we have shown that long-range collective effects can have a dramatic impact on the large-scale behavior of dark matter in dissociative cluster mergers. We have performed dedicated particle-in-cell simulations of counter-propagating beams of dark plasma to characterize the growth rate, saturation, and late-time behavior of the beams. We find that instabilities produce electromagnetic inhomogeneities that act as scattering sites for dark matter. This process leads to an effective collisionality in the dark sector, which we then connect to existing limits on DM self-interactions derived from simulations of the Bullet Cluster. The associated results allow us to place new constraints at couplings orders of magnitude below existing limits.

There are many opportunities for future work. Here, we restricted ourselves to a particular model of the dark sector, namely DM charged under a new dark U(1) gauge symmetry. Other well-motivated models of DM exist (e.g. millicharged DM [8, 9, 52] and axion DM [53, 54]) that can also exhibit similar types of instabilities. Additionally, plasma instabilities may affect "dark atomic" sectors in which there are both high-mass "dark protons" and low-mass "dark electrons" [55–57].

Furthermore, it is important to note that the constraint presented in this work is likely to be very conservative. Throughout this work, parameters such as the DM thermal velocity, relative beam velocity, and constrained collisional cross-section are all taken at conservative values. Additionally, post-saturation plasma effects are likely to lead to further turbulence and disruption of the halos [37–46]; a dedicated study of these effects could place a more stringent constraint. A more sophisticated macroscopic simulation of the Bullet Cluster system is likely to yield constraints at lower charge-to-mass ratios, and would be a topic well-suited to future exploration.

Our results have only scratched the surface of the rich phenomenology that may exist in the dark sector; if it is anything like its visible counterpart, then it is time to move beyond the interactions of individual particles and instead embrace the collective.

Acknowledgements. PG and WD wish to thank Robert Lasenby for useful input during the early stages of this project, as well as Sasha Philippov, Antoine Bret, Brant Robertson, Bart Ripperda, David Curtin, Rouven Essig, Stefano Profumo, Ani Prabhu, Akaxia Cruz, Frédéric Pérez, Frederico Fiúza, and many others for insightful commentary and suggestions throughout the preparation of this manuscript. PG would also like to thank the Smilei community for their interest and assistance in running the simulations. We acknowledge use of the LUX supercomputer at UC Santa Cruz,

7



FIG. 4: Left: Center of momentum velocity distribution of particles in the longitudinal direction at $t = 0 \omega_{pl,B}^{-1}$ (solid line), $t = 5 \omega_{pl,B}^{-1}$ (dot-dashed line), and $t = 750 \omega_{pl,B}^{-1}$ (dotted line). Middle: Center of momentum velocity distribution of particles in the transverse direction at $t = 0 \omega_{pl,B}^{-1}$ (solid line), $t = 5 \omega_{pl,B}^{-1}$ (dot-dashed line), and $t = 750 \omega_{pl,B}^{-1}$ (dotted line). Right: Bulk velocity of particles in the longitudinal direction for particles originating in the Main sub-cluster beam (blue) and Bullet sub-cluster beam (orange).

- Y. Sofue and V. Rubin, Rotation curves of spiral galaxies, Ann. Rev. Astron. Astrophys. 39 (2001) 137-174, [astro-ph/0010594].
- Planck Collaboration, N. Aghanim et al., Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6, [arXiv:1807.06209].
 [Erratum: Astron.Astrophys. 652, C4 (2021)].
- [3] R. Massey, T. Kitching, and J. Richard, The dark matter of gravitational lensing, Rept. Prog. Phys. 73 (2010) 086901, [arXiv:1001.1739].
- [4] J. R. Primack, Cosmological Structure Formation, in Philosophy of Cosmology UK/US Conference, pp. 136–160, 5, 2015. arXiv:1505.02821.
- [5] M. Heikinheimo, M. Raidal, C. Spethmann, and H. Veermäe, Dark matter self-interactions via collisionless shocks in cluster mergers, Phys. Lett. B 749 (2015) 236-241, [arXiv:1504.04371].
- [6] M. Heikinheimo, M. Raidal, C. Spethmann, and H. Veermae, Collisionless shocks in self-interacting dark matter, Plasma Phys. Control. Fusion 60 (2017), no. 1 014011, [arXiv:1707.03662].
- [7] C. Spethmann, H. Veermäe, T. Sepp, M. Heikinheimo, B. Deshev, A. Hektor, and M. Raidal, Simulations of Galaxy Cluster Collisions with a Dark Plasma Component, Astron. Astrophys. 608 (2017) A125, [arXiv:1603.07324].
- [8] R. Lasenby, Long range dark matter self-interactions and plasma instabilities, JCAP 11 (2020) 034, [arXiv:2007.00667].
- [9] A. Cruz and M. McQuinn, Astrophysical Plasma Instabilities induced by Long-Range Interacting Dark Matter, arXiv: 2202.12464.
- [10] N. Shukla, K. Schoeffler, J. Vieira, R. Fonseca,

E. Boella, and L. O. Silva, Slowdown of interpenetration of two counterpropagating plasma slabs due to collective effects, Physical Review E 105 (mar, 2022).

- [11] M. V. Medvedev and A. Loeb, Plasma constraints on the millicharged dark matter, 2024.
- [12] L. Ackerman, M. R. Buckley, S. M. Carroll, and M. Kamionkowski, *Dark matter and dark radiation*, *Physical Review D* **79** (jan, 2009).
- [13] J. L. Feng, M. Kaplinghat, H. Tu, and H.-B. Yu, Hidden charged dark matter, Journal of Cosmology and Astroparticle Physics 2009 (jul, 2009) 004–004.
- [14] P. Agrawal, F.-Y. Cyr-Racine, L. Randall, and J. Scholtz, Make dark matter charged again, Journal of Cosmology and Astroparticle Physics 2017 (may, 2017) 022–022.
- [15] D. A. Gurnett and A. Bhattacharjee, Introduction to Plasma Physics: With Space, Laboratory and Astrophysical Applications. Cambridge University Press, Cambridge, 2 ed., 2017.
- [16] A. Bret, Can we trust mhd jump conditions for collisionless shocks?, The Astrophysical Journal 900 (sep, 2020) 111.
- [17] A. Bret and C. Deutsch, Hierarchy of beam plasma instabilities up to high beam densities for fast ignition scenario, Physics of Plasmas 12 (Aug., 2005) 082704.
- [18] A. Bret, L. Gremillet, and M. E. Dieckmann, Multidimensional electron beam-plasma instabilities in the relativistic regime, Physics of Plasmas 17 (Dec., 2010) 120501.
- [19] M. E. Dieckmann and A. Bret, Simulation study of the formation of a non-relativistic pair shock, Journal of Plasma Physics 83 (2017), no. 1 905830104.
- [20] A. Bret, C. C. Haggerty, and R. Narayan, Density jump

for oblique collisionless shocks in pair plasmas: physical solutions, Journal of Plasma Physics **90** (2024), no. 2 905900213.

- [21] A. Bret and R. Narayan, Density jump for oblique collisionless shocks in pair plasmas: allowed solutions, Journal of Plasma Physics 88 (2022), no. 6 905880615.
- [22] A. Bret, A. Stockem Novo, R. Narayan, C. Ruyer, M. E. Dieckmann, and L. O. Silva, *Theory of the* formation of a collisionless weibel shock: pair vs. electron/proton plasmas, Laser and Particle Beams 34 (2016), no. 2 362–367.
- [23] G. Livadiotis, Rankine-hugoniot shock conditions for space and astrophysical plasmas described by kappa distributions, The Astrophysical Journal 886 (Nov., 2019) 3.
- [24] V. Skoutnev, A. Hakim, J. Juno, and J. M. TenBarge, Temperature-dependent saturation of weibel-type instabilities in counter-streaming plasmas, The Astrophysical Journal Letters 872 (feb, 2019) L28.
- [25] S. P. Gary, *The plasma dispersion function*, p. 170–171. Cambridge Atmospheric and Space Science Series. Cambridge University Press, 1993.
- [26] A. Bret, A. Stockem, F. Fiúza, E. P. Álvaro, C. Ruyer, R. Narayan, and L. O. Silva, *The formation of a* collisionless shock, Laser and Particle Beams **31** (2013), no. 3 487–491.
- [27] W. Tucker, P. Blanco, S. Rappoport, L. David,
 D. Fabricant, E. E. Falco, W. Forman, A. Dressler, and
 M. Ramella, 1e 0657-56: A contender for the hottest known cluster of galaxies, The Astrophysical Journal
 496 (Mar., 1998) L5–L8. ADS Bibcode: 1998ApJ...496L...5T.
- [28] M. Markevitch, A. H. Gonzalez, L. David, A. Vikhlinin, S. Murray, W. Forman, C. Jones, and W. Tucker, A textbook example of a bow shock in the merging galaxy cluster 1e 0657-56, The Astrophysical Journal 567 (Mar., 2002) L27–L31. ADS Bibcode: 2002ApJ...567L..27M.
- [29] R. Barrena, A. Biviano, M. Ramella, E. E. Falco, and S. Seitz, *The dynamical status of the cluster of galaxies* 1e0657-56, Astronomy & Astrophysics **386** (May, 2002) 816–828.
- [30] D. Paraficz, J.-P. Kneib, J. Richard, A. Morandi, M. Limousin, E. Jullo, and J. Martinez, *The bullet* cluster at its best: weighing stars, gas, and dark matter, Astronomy & Astrophysics 594 (Oct., 2016) A121.
- [31] D. Clowe, M. Bradač, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, A direct empirical proof of the existence of dark matter*, The Astrophysical Journal 648 (Aug., 2006) L109.
- [32] D. Clowe, A. Gonzalez, and M. Markevitch, Weak-lensing mass reconstruction of the interacting cluster 1e 0657–558: Direct evidence for the existence of dark matter*, The Astrophysical Journal 604 (Apr., 2004) 596.
- [33] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradač, Constraints on the self interaction cross section of dark matter from numerical simulations of the merging galaxy cluster 1e 0657-56, The Astrophysical Journal 679 (June, 2008) 1173.
- [34] A. Robertson, R. Massey, and V. Eke, What does the bullet cluster tell us about self-interacting dark matter?, Monthly Notices of the Royal Astronomical Society 465 (oct, 2016) 569–587.

- [35] F. Kahlhoefer, K. Schmidt-Hoberg, M. T. Frandsen, and S. Sarkar, Colliding clusters and dark matter self-interactions, Monthly Notices of the Royal Astronomical Society 437 (Jan., 2014) 2865–2881.
- [36] S. Tulin and H.-B. Yu, Dark matter self-interactions and small scale structure, Physics Reports 730 (2018) 1–57. Dark matter self-interactions and small scale structure.
- [37] C. Ruyer and F. Fiuza, Disruption of current filaments and isotropization of the magnetic field in counterstreaming plasmas, Physical Review Letters 120 (June, 2018) 245002.
- [38] C. Ruyer, L. Gremillet, A. Debayle, and G. Bonnaud, Nonlinear dynamics of the ion weibel-filamentation instability: An analytical model for the evolution of the plasma and spectral properties, Physics of Plasmas 22 (Mar., 2015) 032102.
- [39] C. H. Jaroschek, H. Lesch, and R. A. Treumann, Self-consistent diffusive lifetimes of weibel magnetic fields in gamma-ray bursts, The Astrophysical Journal 616 (dec, 2004) 1065.
- [40] H. R. Kaufman, Explanation of bohm diffusion,
- [41] H. Takabe, Theory of magnetic turbulence and shock formation induced by a collisionless plasma instability, Physics of Plasmas 30 (Mar., 2023) 030901.
- [42] A. Vanthieghem, M. Lemoine, and L. Gremillet, Stability analysis of a periodic system of relativistic current filaments, Physics of Plasmas 25 (July, 2018) 072115.
- [43] P. Chang, A. Spitkovsky, and J. Arons, Long-term evolution of magnetic turbulence in relativistic collisionless shocks: Electron-positron plasmas, The Astrophysical Journal 674 (feb, 2008) 378.
- [44] M. D. McFarland and A. Y. Wong, Spatial, spectral, and statistical properties of the high-frequency electrostatic fluctuations in a beam-driven turbulent plasma, Physics of Plasmas 8 (Jan., 2001) 110–121.
- [45] M. D. McFarland and A. Y. Wong, Beam scattering and low-frequency fluctuations in a beam-driven strongly turbulent plasma, Physics of Plasmas 8 (Jan., 2001) 122–131.
- [46] P. H. Yoon, Weak turbulence theory for beam-plasma interaction, Physics of Plasmas 25 (Dec., 2017) 011603.
- [47] J. Derouillat, A. Beck, F. Pérez, T. Vinci, M. Chiaramello, A. Grassi, M. Fle, G. Bouchard, I. Plotnikov, N. Aunai, J. Dargent, C. Riconda, and M. Grech, Smilei: A collaborative, open-source, multi-purpose particle-in-cell code for plasma simulation, Computer Physics Communications 222 (Jan., 2018) 351–373.
- [48] V. Springel and G. R. Farrar, The speed of the 'bullet' in the merging galaxy cluster 1e0657-56, Monthly Notices of the Royal Astronomical Society 380 (Sept., 2007) 911–925.
- [49] J.-L. Vay, C. Geddes, E. Cormier-Michel, and D. Grote, Numerical methods for instability mitigation in the modeling of laser wakefield accelerators in a lorentz-boosted frame, Journal of Computational Physics 230 (2011), no. 15 5908–5929.
- [50] M. Zhou, V. Zhdankin, M. W. Kunz, N. F. Loureiro, and D. A. Uzdensky, Magnetogenesis in a collisionless plasma: From weibel instability to turbulent dynamo, The Astrophysical Journal 960 (dec, 2023) 12.
- [51] S. Adhikari, A. Banerjee, K. K. Boddy, F.-Y. Cyr-Racine, H. Desmond, C. Dvorkin, B. Jain,

F. Kahlhoefer, M. Kaplinghat, A. Nierenberg, A. H. G. Peter, A. Robertson, J. Sakstein, and J. Zavala, *Astrophysical tests of dark matter self-interactions*, 2022.

- [52] J.-T. Li and T. Lin, Dynamics of millicharged dark matter in supernova remnants, Phys. Rev. D 101 (May, 2020) 103034.
- [53] Endorsers Collaboration, A. J. Millar, S. M. Anlage, R. Balafendiev, P. Belov, K. van Bibber, J. Conrad, M. Demarteau, A. Droster, K. Dunne, A. G. Rosso, J. E. Gudmundsson, H. Jackson, G. Kaur, T. Klaesson, N. Kowitt, M. Lawson, A. Leder, A. Miyazaki, S. Morampudi, H. V. Peiris, H. S. Røising, G. Singh, D. Sun, J. H. Thomas, F. Wilczek, S. Withington, M. Wooten, J. Dilling, M. Febbraro, S. Knirck, and C. Marvinney, Searching for dark matter with plasma haloscopes, Phys. Rev. D 107 (Mar, 2023) 055013.
- [54] X. An, M. Chen, J. Liu, Z. Sheng, and J. Zhang, Modeling of axion and electromagnetic fields coupling in a particle-in-cell code, . arXiv:2406.16796 [hep-ph, physics:physics].
- [55] J. Fan, A. Katz, L. Randall, and M. Reece, *Double-Disk Dark Matter*, *Phys. Dark Univ.* 2 (2013) 139–156, [arXiv:1303.1521].
- [56] S. Roy, X. Shen, M. Lisanti, D. Curtin, N. Murray, and P. F. Hopkins, Simulating Atomic Dark Matter in Milky Way Analogues, arXiv:2304.09878.
- [57] S. Roy, X. Shen, J. Barron, M. Lisanti, D. Curtin, N. Murray, and P. F. Hopkins, Aggressively-Dissipative Dark Dwarfs: The Effects of Atomic Dark Matter on the Inner Densities of Isolated Dwarf Galaxies, arXiv:2408.15317.
- [58] S. Jubin, A. T. Powis, W. Villafana, D. Sydorenko, S. Rauf, A. V. Khrabrov, S. Sarwar, and I. D. Kaganovich, Numerical thermalization in 2d pic simulations: Practical estimates for low-temperature plasma simulations, Physics of Plasmas **31** (Feb., 2024) 023902.
- [59] P. Rambo, Finite-grid instability in quasineutral hybrid simulations, Journal of Computational Physics 118 (1995), no. 1 152–158.
- [60] M. D. Acciarri, C. Moore, and S. D. Baalrud, Artificial correlation heating in pic simulations, Physics of Plasmas 31 (Sept., 2024) 093903.
- [61] A. Grassi, M. Grech, F. Amiranoff, F. Pegoraro, A. Macchi, and C. Riconda, *Electron weibel instability* in relativistic counterstreaming plasmas with flow-aligned external magnetic fields, Phys. Rev. E 95 (Feb, 2017) 023203.
- [62] A. Bret, A. Stockem, F. Fiuza, C. Ruyer, L. Gremillet, R. Narayan, and L. O. Silva, *Collisionless shock* formation, spontaneous electromagnetic fluctuations, and streaming instabilities, *Physics of Plasmas* 20 (Apr., 2013) 042102.
- [63] M. V. Medvedev, M. Fiore, R. A. Fonseca, L. O. Silva, and W. B. Mori, Long-time evolution of magnetic fields in relativistic gamma-ray burst shocks, The Astrophysical Journal 618 (dec, 2004) L75.

Appendix A: Supplemental Material

1. Stability of Simulation

We undertook several studies to ensure the numerical stability of our simulation. In Fig. 5, we show the amount of energy increase from numerical heating, a known numerical effect arising from truncation errors and finite-sized time and space domains [58–60]. Throughout our simulation, this remained below 0.1% of the total initial kinetic and electromagnetic energy, indicating stability.

Additionally, we compared the length scales of the two-stream and Weibel instabilities in the asymmetric warm case to ensure that our choice of cell size accurately resolved the relevant instability in each direction. Fig. 6 displays the exponential growth rate for the two-stream instability for the Bullet Cluster system. We find that the maximal growth rate is $\Gamma_{TS} = 0.397 \,\omega_{pl,B}$ at $k_x = 152 \,\omega_{pl,B}/c$. This corresponds to a wavelength of $\lambda_{TS} = 0.04 \, c/\omega_{pl,B}$, which is well above our resolution in the x direction of $\Delta x = 0.0035 \, c/\omega_{pl,B}$.

Similarly, in the transverse direction, Fig. 7 shows the exponential growth rate of the Weibel instability. For the Bullet Cluster system, the instability modes reach a maximum of $0.00995 \,\omega_{pl,B}$ and display a rapid drop-off for $k < k_{y,W}^{crit} \sim 1 \,\omega_{pl,B}/c$. This corresponds to a maximal wavelength of $\lambda_W = 2\pi c/\omega_{pl,B}$, which is also well above our resolution in the y direction of $\Delta y = 0.18 \, c/\omega_{pl,B}$.

2. Saturation energies

From, Eq. 5, we calculate the approximate saturation energy of the two-stream instability, which yields

$$\frac{\varepsilon_E}{\varepsilon_K} = \frac{E_{\rm sat}^2/2}{\frac{1}{2}(4n_0)m_{\chi}(v_0/2)^2} \approx \frac{1}{96},\tag{A1}$$

where ε_K is the initial kinetic energy density of the plasma. Similarly, we use Eq. 6 to determine the saturation energy for the Weibel instability, which yields

$$\frac{\varepsilon_B}{\varepsilon_K} = \frac{B_{\text{sat}}^2/2}{\frac{1}{2}(4n_0)m_{\chi}(v_0/2)^2} \approx \left(\frac{v_W}{v_0}\right)^2, \qquad (A2)$$

where v_W is the relative bulk velocity of the plasma during saturation. This value is often less than the initial relative bulk velocity, v_0 , due to the decrease in bulk kinetic energy during the formation of the electrostatic shock by the two-stream instability. At regular intervals we sample the velocity distribution of all particles and compute the mean velocity (as shown in the right panel of Fig. 4). We find $v_W = 0.0014 c$. We explicitly measure the saturation levels of the longitudinal electric field and the transverse magnetic field in our simulation in order to compare to these analytic predictions, finding $\varepsilon_E/\varepsilon_K = 0.024$ and $\varepsilon_B/\varepsilon_K = 0.19$, respectively. We find



FIG. 5: Combined particle and field energy from simulation (blue) and energy lost due to numerical heating (orange).

these values to be within reasonable agreement with Eqs. A1 and Eq. A2.

3. Characteristics of Simulation

Here, we show some additional diagnostic plots from our simulation to highlight some of the key features of the evolution of the two-stream and Weibel instabilities.

In Fig. 8 (9), we show a plot of the (Fourier transform of the) electric field along the direction of beam propagation as a function of position for three different times. In the left-most panels, corresponding to $t = 0.3 \omega_{pl,B}^{-1}$, the simulation has properly initialized, and all electromagnetic fields are relatively small and are generated purely by thermal noise effects. At $t = 5 \omega_{pl,B}^{-1}$ (center panels), the two-stream instability has reached saturation, as is indicated by the presence of small vertical strips alternating polarity corresponding to the fastest growing longitudinal mode in Fig. 8 and the sharp peak around the critical k_x mode in Fig. 9. At $t = 200 \omega_{pl,B}^{-1}$ (right-most panels), the two-stream instability has dissipated.

In Fig. 10 (11), we show a plot of the (Fourier transform of the) magnetic field in the transverse direction as a function of position at the same three times as above. However, while the two-stream instability has saturated by $t = 750 \omega_{pl,B}^{-1}$, Fig. 10 shows that the Weibel filaments are beginning to form and are nearing saturation at this time. Fig. 11 shows that the transverse magnetic filaments peak around the critical k_y mode with increasing



FIG. 6: Exponential growth rate of longitudinal two-stream instability in the case of cold symmetric beams (blue) and for the parameters of the Bullet Cluster system (orange). The red lines show the fastest growing mode and corresponding instability rate.



FIG. 7: Exponential growth rate of transverse Weibel instability in the case of cold symmetric beams (blue) and for the parameters of the Bullet Cluster system (orange).

intensity as the fields reach saturation.

Though our simulation is 2-dimensional, it is still able to conservatively estimate the macrophysical properties of the 3-dimensional Bullet Cluster system. First, we note that the Weibel instability has previously been studied in 1 and 2-dimensional systems [61, 62], and it has been shown that the growth rates and magnetic field saturation levels agree with analytical models. After Weibel saturation, we expect our simulation to no longer capture the relevant dynamics, as at this time, the Weibel filaments should enter a late-stage merging process that requires the current filaments to move around in 3-dimensional space. Though we cannot capture this effect, it has been previously studied in the literature [18, 63], and the magnetic field energy is expected to grow approximately linearly with time. These filaments will eventually become unstable to the z-pinch instability, which generally works to slow down the flow of particles and isotropize the magnetic field [37]. Though these latetime phenomena likely only further slow the beams, we chose to set our limit at Weibel saturation, long before these merging events are expected to begin, in an effort to be as conservative as possible.



FIG. 8: Longitudinal component of the electric field at $t = 0.3 \omega_{pl,B}^{-1}$ (left), $t = 5 \omega_{pl,B}^{-1}$ (middle), and $t = 750 \omega_{pl,B}^{-1}$ (right).



FIG. 9: Power spectrum of the longitudinal electric field at $t = 0.3 \omega_{pl,B}^{-1}$ (left), $t = 5 \omega_{pl,B}^{-1}$ (middle), and $t = 750 \omega_{pl,B}^{-1}$ (right).



FIG. 10: Transverse component of the magnetic field at $t = 0.3 \,\omega_{pl,B}^{-1}$ (left), $t = 200 \,\omega_{pl,B}^{-1}$ (middle), and $t = 750 \,\omega_{pl,B}^{-1}$ (right).



FIG. 11: Power spectrum of the transverse magnetic field at $t = 0.3 \omega_{pl,B}^{-1}$ (left), $t = 200 \omega_{pl,B}^{-1}$ (middle), and $t = 750 \omega_{pl,B}^{-1}$ (right).