

# Polarization of an electron scattered by static potentials

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We study the polarization of an electron scattered by different static potentials. The initial state of the electron is chosen as a wavepacket to construct the definite orbital angular momentum, and the final polarization of the electron, scattered by different static potentials such as vector, pseudovector, scalar and pseudoscalar potentials, is calculated. Numerical results show that, the sign of the polarization of the electron scattered by the vector potential is opposite to the other three cases, and the magnitude order of the polarization value is consistent with recent experimental result in the collision parameter range  $0 < b < 2$  fm.

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## I. INTRODUCTION

It has long been known that particles with nonzero spin quantum number can polarise in a rotating system [1, 2]. It can be qualitatively explained by spin-orbit coupling through which the orbital angular momentum (OAM) of the macroscopic rotating system transfers into the spin angular momentum of microscopic particles [3]. In high energy peripheral heavy-ion collisions at STAR in BNL and at LHC in CERN, an extremely hot and dense matter (named quark gluon plasma (QGP)) with huge OAM is created [3–6], hence a huge vorticity configuration is formed in this matter [7–11]. It is expected that the final hadrons with nonzero spin quantum number will polarise globally along the direction of the initial OAM at the freeze-out stage of QGP [3, 12–14]. After more than ten years' effort, the global polarization of  $\Lambda$  hyperon had been firstly observed at STAR [15, 16]. The collisional energy dependence of the global polarization of  $\Lambda$  hyperon can be well recovered by the numerical simulations of the relativistic hydrodynamics model and the multi-phase transport model [11, 17]. The spin alignment of vector mesons in heavy-ion collisions are also measured these years [14, 18].

There are many methods to theoretically study the polarization of particles in a fluid with relativistic vorticity. Based on the assumption of local equilibrium of spin, the authors in [7, 19, 20] derived the relation between the 4-dimensional spin vector (Pauli-Lubanski pseudovector) and vorticity in relativistic case through Wigner function approach. Recently the 4-dimensional spin vector induced by vorticity is also obtained from the method of spin density matrix but without the factor of Fermi-Dirac thermal distribution [21]. The first theoretical calculation of the polarization based on microscopic scattering by a static potential is performed in [3], in which the concept of global polarization of particles in high energy heavy-ion collisions is firstly proposed. Based on [3], the theoretical calculation of quark polarization through two-body scattering [12] is performed, and the transport property of polarization in a fluid is also discussed in [22]. The statistical model and quark coalescence model of hydron polarization are carried out in recent studies [13, 20, 23–25]. Recently one of us and his collaborators put forward a systematic formulism to calculate the particle polarization in QGP from the spin-orbit coupling [26], in which all  $2 \rightarrow 2$  processes are considered and one can see clearly how the macroscopic vorticity in the fluid induces the microscopic polarization of a particle.

In order to carefully understand the polarization due to spin-orbit coupling, in this article we consider a simple model to calculate the polarization of a Dirac fermion scattered by four types of static potential. We choose the wavepacket as the initial state of the electron which is the same as [26] and sum over the initial polarization states, so that the initial total angular momentum is just the OAM of the incident electron. The final state of the electron is chosen as the common eigenstate of momentum and spin, then the scattering probability of different final spin can be calculated.

This article is organised as follows. In Sec. II, we set up the theoretical formulism for the scattering probability of different final spin. In Sec. III, the polarization of the scattered electron by four types of static potential is studied. We briefly summarise this article in Sec. IV.

In this article, the natural unit where  $\hbar = c = 1$  is adopted. We choose the metric tensor as  $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ . Greek indices, such as  $\mu, \nu, \rho, \sigma$ , run over 0, 1, 2, 3, or  $t, x, y, z$ , while Roman indices, such as  $i, j, k$ , run over 1, 2, 3 or  $x, y, z$ . The Heaviside-Lorentz convention is chosen for electromagnetism which is consistent with Peskin and Schroeder [27].

## II. A WAVEPACKET IS SCATTERED BY A STATIC POTENTIAL

In this section we consider an electron wavepacket which is scattered by a classical static potential (time independent). The classical static potential is denoted as  $\mathcal{A}$ , the center of which is located at the origin. A very convenient choice for the static potential is the screened potential model [28], whose explicit form in position space is  $V(\mathbf{x}) = Qe^{-d|\mathbf{x}|}/(4\pi|\mathbf{x}|)$  with electric charge source  $Q$  and force distance  $1/d$ . The form of this screened potential in momentum space is  $V(\mathbf{q}) = Q/(\mathbf{q}^2 + d^2)$ . The electron wavepacket is denoted as  $\mathcal{B}$ . The in-state  $|\phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b}\rangle_{\text{in}}$  of the single particle wavepacket for this electron in the remote past with impact parameter  $\mathbf{b} = (b, 0, 0)$  can be represented as

$$|\phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b}\rangle_{\text{in}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \phi(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{b}} |\mathbf{k}, \lambda_{\mathcal{B}}\rangle_{\text{in}}, \quad (1)$$

where  $|\mathbf{k}, \lambda_{\mathcal{B}}\rangle_{\text{in}}$  represents the in-state of a plane wave state of an electron with momentum  $\mathbf{k}$  and spin projection component  $\lambda_{\mathcal{B}}$  along the direction of positive  $y$ -axis, and  $\phi(\mathbf{k})$  is a normalised Gaussian wavepacket with momentum center  $\mathbf{p}_i = (0, 0, c)$  and momentum width

$a$  whose explicit form is

$$\phi(\mathbf{k}) = \left( \frac{(8\pi)^3}{a^6} \right)^{\frac{1}{4}} \exp \left( - \frac{(\mathbf{k} - \mathbf{p}_i)^2}{a^2} \right). \quad (2)$$

Taking use of  ${}_{\text{in}}\langle \mathbf{k}', \lambda_{\mathcal{B}} | \mathbf{k}, \lambda_{\mathcal{B}} \rangle_{\text{in}} = 2E_k (2\pi)^3 \delta^{(3)}(\mathbf{k}' - \mathbf{k})$ , we can see that the in-state in Eq. (1) can be normalised to 1 as

$${}_{\text{in}}\langle \phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b} | \phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b} \rangle_{\text{in}} = \int \frac{d^3k}{(2\pi)^3} |\phi(\mathbf{k})|^2 = 1. \quad (3)$$

Since the electron moves from  $-z$ -axis to  $+z$ -axis, we set  $c > 0$ . We also set  $b > 0$ , then the initial orbital angular momentum of the electron is along  $-y$ -axis. When this electron moves toward the static potential, it will be scattered by some probability. The scattering probability with spin component  $\lambda_{\mathcal{B}}$  summed over in the initial state and with spin projection component  $\lambda$  along  $+y$ -axis in the final state is

$$\begin{aligned} \mathcal{P}(\lambda, b) &= \frac{1}{2} \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{\lambda_{\mathcal{B}}} \left| {}_{\text{out}}\langle \mathbf{p}, \lambda | \phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b} \rangle_{\text{in}} \right|^2 \\ &= \frac{1}{2} \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{\lambda_{\mathcal{B}}} \left| \langle \mathbf{p}, \lambda | \mathcal{S} | \phi_{\mathcal{B}}, \lambda_{\mathcal{B}}; \mathbf{b} \rangle \right|^2 \\ &= \frac{1}{2} \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{\lambda_{\mathcal{B}}} \left| \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \phi(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{b}} \langle \mathbf{p}, \lambda | \mathcal{S} | \mathbf{k}, \lambda_{\mathcal{B}} \rangle \right|^2 \end{aligned} \quad (4)$$

where  $\mathcal{S}$  denotes the  $S$ -matrix for the interaction and we choose the plane wave state  $|\mathbf{p}, \lambda\rangle$  as the final state with normalization:  $\langle \mathbf{p}', \lambda | \mathbf{p}, \lambda \rangle = 2E_p (2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p})$ . Then we can obtain the polarization  $\chi(b)$  of the final electron along  $+y$ -axis as

$$\chi(b) = \frac{\mathcal{P}(+, b) - \mathcal{P}(-, b)}{\mathcal{P}(+, b) + \mathcal{P}(-, b)}. \quad (5)$$

The denominator  $\mathcal{P}(+, b) + \mathcal{P}(-, b)$  in Eq. (5) is the normalization factor. In fact, due to the unitarity of the  $S$ -matrix,  $\mathcal{S}^\dagger \mathcal{S} = 1$ , the sum  $\sum_{\lambda} \mathcal{P}(\lambda, b)$  automatically equal to one,

i.e.

$$\begin{aligned}
\sum_{\lambda} \mathcal{P}(\lambda, b) &= \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3 2E_p} \sum_{\lambda} \sum_{\lambda_B} \left| \langle \mathbf{p}, \lambda | \mathcal{S} | \phi_B, \lambda_B; \mathbf{b} \rangle \right|^2 \\
&= \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3 2E_p} \sum_{\lambda} \sum_{\lambda_B} \langle \phi_B, \lambda_B; \mathbf{b} | \mathcal{S}^\dagger | \mathbf{p}, \lambda \rangle \langle \mathbf{p}, \lambda | \mathcal{S} | \phi_B, \lambda_B; \mathbf{b} \rangle \\
&= \frac{1}{2} \sum_{\lambda_B} \langle \phi_B, \lambda_B; \mathbf{b} | \mathcal{S}^\dagger \mathcal{S} | \phi_B, \lambda_B; \mathbf{b} \rangle \\
&= \frac{1}{2} \sum_{\lambda_B} \langle \phi_B, \lambda_B; \mathbf{b} | \phi_B, \lambda_B; \mathbf{b} \rangle \\
&= 1.
\end{aligned} \tag{6}$$

But in actual calculation, we only consider the process of tree level and ignore the high order processes, so the sum  $\sum_{\lambda} \mathcal{P}(\lambda, b)$  no longer equal to one and the normalization factor in the denominator in Eq. (5) is necessary.

### III. POLARIZATION OF THE ELECTRON SCATTERED BY DIFFERENT STATIC POTENTIALS

In this section, we will calculate the polarization of an electron scattered by different static potentials. The electron is described by Dirac field. Firstly, we consider the coupling between Dirac field and static vector field, i.e. the interaction part of the Lagrangian is  $\mathcal{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu$  with  $A^\mu(t, \mathbf{x}) = (V(\mathbf{x}), \mathbf{0})$ . Ignoring the identity operator in  $S$ -matrix, the term  $\langle \mathbf{p}, \lambda | \mathcal{S} | \mathbf{k}, \lambda_B \rangle$  in Eq. (4) at first order in coupling  $e$  is

$$\begin{aligned}
\langle \mathbf{p}, \lambda | \mathcal{S} | \mathbf{k}, \lambda_B \rangle &= \langle \mathbf{p}, \lambda | (-ie) \int d^4 x \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu(x) | \mathbf{k}, \lambda_B \rangle \\
&\equiv (-ie) \bar{u}(p, \lambda) \gamma \cdot A(\mathbf{p} - \mathbf{k}) u(k, \lambda_B) (2\pi) \delta(E_p - E_k),
\end{aligned} \tag{7}$$

where  $A^\mu(\mathbf{p} - \mathbf{k}) = (V(\mathbf{p} - \mathbf{k}), \mathbf{0})$  and  $u(k, \lambda_B), u(p, \lambda)$  are solved in chiral representation of Dirac matrix for free Dirac equation. In the following calculation, we will choose  $u(k, \lambda_B), u(p, \lambda)$  as

$$\begin{aligned}
u(k, \lambda_B) &= \frac{1}{\sqrt{2(m + E_k)}} \begin{pmatrix} (m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k}) \xi_{\lambda_B} \\ (m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k}) \xi_{\lambda_B} \end{pmatrix} \\
u(p, \lambda) &= \frac{1}{\sqrt{2(m + E_p)}} \begin{pmatrix} (m + E_p - \boldsymbol{\sigma} \cdot \mathbf{p}) \xi_\lambda \\ (m + E_p + \boldsymbol{\sigma} \cdot \mathbf{p}) \xi_\lambda \end{pmatrix},
\end{aligned} \tag{8}$$

where  $\xi_\lambda$  is the eigenstate of spin operator  $\boldsymbol{\sigma}$  along  $+y$ -axis with eigenvalue  $\lambda$ , i.e.  $\xi_\lambda^\dagger \boldsymbol{\sigma} \xi_\lambda = \lambda \hat{\mathbf{y}}$  with  $\lambda = \pm 1$ . Since  $\lambda_{\mathcal{B}}$  will be summed over later, the explicit form of  $\xi_{\lambda_{\mathcal{B}}}$  is irrelevant.

From Eq. (4), we can obtain the scattering probability  $\mathcal{P}^V(\lambda, b)$  with the final spin projection component  $\lambda$  and collision parameter  $b$  for static vector potential, which is calculated in detail in Appendix A. We list the explicit form of  $\mathcal{P}^V(\lambda, b)$  as follows,

$$\mathcal{P}^V(\lambda, b) = \mathcal{P}_0^V(b) + \lambda \mathcal{P}_1^V(b), \quad (9)$$

with unpolarised probability  $\mathcal{P}_0^V(b)$  and polarised probability  $\mathcal{P}_1^V(b)$  defined as 7-dimensional integrals,

$$\begin{aligned} \mathcal{P}_0^V(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \cos[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times [(E_p + m)^2 + (E_p - m)^2 \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' + p^2 \hat{\mathbf{p}} \cdot (\hat{\mathbf{k}} + \hat{\mathbf{k}}')] \\ \mathcal{P}_1^V(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \sin[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times \left( (E_p - m)^2 (2\hat{\mathbf{y}} \cdot \hat{\mathbf{p}}\hat{\mathbf{p}} - \hat{\mathbf{y}}) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{k}}') + p^2 \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \right) \end{aligned}$$

where  $d\Omega_p$ ,  $d\Omega$ ,  $d\Omega'$  represent the differential solid angle of  $\hat{\mathbf{p}}$ ,  $\hat{\mathbf{k}}$ ,  $\hat{\mathbf{k}}'$  respectively, and  $\hat{\mathbf{x}} = (1, 0, 0)$ ,  $\hat{\mathbf{y}} = (0, 1, 0)$ .

Then the polarization  $\chi^V(b)$  of the electron scattered by the static vector potential in Eq. (5) becomes

$$\chi^V(b) = \frac{\mathcal{P}_1^V(b)}{\mathcal{P}_0^V(b)}. \quad (10)$$

If the electron is scattered by the static pseudovector, scalar and pseudoscalar potentials respectively, then the interaction part of the Lagrangian  $\mathcal{L}_{\text{int}}$  becomes  $\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu$ ,  $\bar{\psi}\psi\phi$ ,  $\bar{\psi}\gamma^5\psi\phi$ , where  $\phi$  is a scalar or pseudoscalar field. In Appendix A, we also calculate the polarization function  $\chi(b)$  for these three types of static potential.

Figure 1 shows the numerical results of the polarization  $\chi(b)$  as a function of collision parameter  $b$  for four types of static potential with the parameters, where we take following parameters: initial energy of the incident electron  $c = 1$  GeV, the width of wavepacket  $a = 0.1$  GeV, Debye screen mass  $d = 0.1$  GeV, and electron mass  $m = 0.000511$  GeV. Since  $b > 0$ ,  $c > 0$ , i.e. the initial OAM of the electron is along negative  $y$ -axis, we may expect that the final electron scattered by the static potentials polarises more preferentially along

negative  $y$ -axis, which is consistent with the results for pseudovector, scalar and pseudoscalar potentials as shown in Figure 1. However, for vector potential, the final electron polarises more preferentially along positive  $y$ -axis, which is opposite to the direction of the initial OAM of the electron. This inconsistency may result from the fact that the virtual photon exchanged by the electron and the static potential also carries spin angular momentum of  $1\hbar$ , leading to the opposite polarization of the final electron due to the conservation of angular momentum. In Figure 1, we only plot the curves in the range  $0 < b < 6$  fm, where the the absolute value of polarization becomes larger as  $b$  increases. Especially in the range  $0 < b < 2$  fm, the magnitude order of the polarization value is the same as the recent experimental result [16]. For the range  $b > 6$  fm, the numerical result becomes unstable, which is not shown in the plot. It is expected that the polarization  $\chi(b)$  becomes zero as  $b$  tends to be very large, since in this case the influence of the static potential on the electron is very weak.

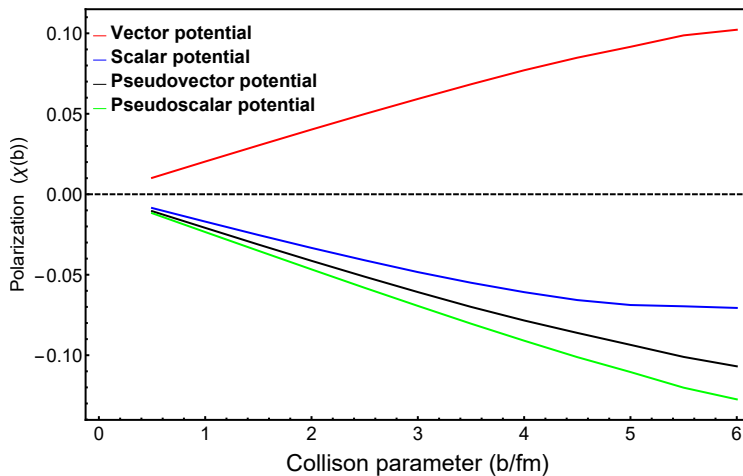


Figure 1. Polarization  $\chi(b)$  as a function of collision parameter  $b$

#### IV. SUMMARY

In this article, we calculate the polarization of an electron scattered by four different types of static potential. Through the scattering of static potentials, it is expected that the initial OAM of the incident electron can transfer into the final spin angular momentum, which is consistent with the numerical results for pseudovector, scalar and pseudoscalar potentials. However the electron polarises more preferentially opposite to the initial OAM

of the electron, which may result from the spin of the virtual photon exchanged. For the range of the collision parameter  $0 < b < 2$  fm, the magnitude order of the polarization value is consistent with the recent experimental result.

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### Appendix A: Calculation for $\mathcal{P}(\lambda, b)$

From Eq. (7) and Eq. (1), we obtain

$$\begin{aligned}
& \langle \mathbf{p}, \lambda | \mathcal{S} | \phi_{\mathcal{B}}, \lambda_{\mathcal{B}} \rangle \\
&= \int \frac{d^3k}{(2\pi)^3} \frac{\phi(\mathbf{k})}{\sqrt{2E_k}} e^{-i\mathbf{k}\cdot\mathbf{b}} (-ie) \bar{u}(p, \lambda) \gamma \cdot A(\mathbf{p} - \mathbf{k}) u(k, \lambda_{\mathcal{B}}) (2\pi) \delta(E_p - E_k) \\
&= \frac{1}{(2\pi)^2} \int_0^\infty d|\mathbf{k}| k^2 \int d\Omega \frac{\phi(\mathbf{k})}{\sqrt{2E_k}} e^{-i\mathbf{k}\cdot\mathbf{b}} (-ie) \bar{u}(p, \lambda) \gamma \cdot A(\mathbf{p} - \mathbf{k}) u(k, \lambda_{\mathcal{B}}) \frac{E_p}{|\mathbf{p}|} \delta(|\mathbf{k}| - |\mathbf{p}|) \\
&= \frac{-ie|\mathbf{p}|}{(2\pi)^2} \sqrt{\frac{E_p}{2}} \int d\Omega A_0(\mathbf{p} - \mathbf{k}) \phi(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{b}} u^\dagger(p, \lambda) u(k, \lambda_{\mathcal{B}}), \tag{A1}
\end{aligned}$$

where  $d\Omega$  represents the solid angle of  $\hat{\mathbf{k}}$ . Taking square of Eq. (A1) and summing over  $\lambda_{\mathcal{B}}$  gives

$$\begin{aligned}
& \sum_{\lambda_{\mathcal{B}}} \left| \langle \mathbf{p}, \lambda | \mathcal{S} | \phi_{\mathcal{B}}, \lambda_{\mathcal{B}} \rangle \right|^2 \\
&= \frac{\alpha \mathbf{p}^2 E_p}{(2\pi)^3} \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') e^{-i(\mathbf{k} - \mathbf{k}')\cdot\mathbf{b}} \\
& \quad \times \sum_{\lambda_{\mathcal{B}}} [u^\dagger(p, \lambda) u(k, \lambda_{\mathcal{B}}) u^\dagger(k', \lambda_{\mathcal{B}}) u(p, \lambda)] \tag{A2}
\end{aligned}$$

where  $d\Omega'$  represents the solid angle of  $\hat{\mathbf{k}}'$ . In the following, we will calculate the third



line of Eq. (A2) in detail. We can see

$$\sum_{\lambda_{\mathcal{B}}} [u^\dagger(p, \lambda)u(k, \lambda_{\mathcal{B}})u^\dagger(k', \lambda_{\mathcal{B}})u(p, \lambda)] = \sum_{\lambda_{\mathcal{B}}} \text{tr}[u(p, \lambda)u^\dagger(p, \lambda)u(k, \lambda_{\mathcal{B}})u^\dagger(k', \lambda_{\mathcal{B}})] \quad (\text{A3})$$

For  $u(k, \lambda_{\mathcal{B}})u^\dagger(k', \lambda_{\mathcal{B}})$  part, we have

$$\begin{aligned} M_{\mathcal{B}} &= \sum_{\lambda_{\mathcal{B}}} u(k, \lambda_{\mathcal{B}})u^\dagger(k', \lambda_{\mathcal{B}}) \\ &= \frac{1}{2\sqrt{(m + E_k)(m + E_{k'})}} \\ &\quad \times \sum_{\lambda_{\mathcal{B}}} \begin{pmatrix} (m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k})\xi_{\mathcal{B}} \\ (m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k})\xi_{\mathcal{B}} \end{pmatrix} \begin{pmatrix} \xi_{\mathcal{B}}^\dagger(m + E_{k'} - \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \xi_{\mathcal{B}}^\dagger(m + E_{k'} + \boldsymbol{\sigma} \cdot \mathbf{k}') \end{pmatrix} \\ &= \frac{1}{2\sqrt{(m + E_k)(m + E_{k'})}} \\ &\quad \times \begin{pmatrix} (m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} - \boldsymbol{\sigma} \cdot \mathbf{k}'), & (m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} + \boldsymbol{\sigma} \cdot \mathbf{k}') \\ (m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} - \boldsymbol{\sigma} \cdot \mathbf{k}'), & (m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} + \boldsymbol{\sigma} \cdot \mathbf{k}') \end{pmatrix} \end{aligned}$$

For  $u(p, \lambda)u^\dagger(p, \lambda)$  part, we have

$$u(p, \lambda)u^\dagger(p, \lambda) = u(p, \lambda)\bar{u}(p, \lambda)\gamma^0 = \frac{1}{2}(1 + \lambda\gamma^5\boldsymbol{\gamma} \cdot \mathbf{S})(\boldsymbol{\gamma} \cdot \mathbf{p} + m)\gamma^0 \quad (\text{A4})$$

where  $S^\mu$  is the 4-dimensional spin vector

$$S^\mu = (S^0, \mathbf{S}) = \left( \frac{\mathbf{p} \cdot \hat{\mathbf{y}}}{m}, \hat{\mathbf{y}} + \frac{(\hat{\mathbf{y}} \cdot \mathbf{p})\mathbf{p}}{m(m + E_p)} \right) \quad (\text{A5})$$

We can see

$$\begin{aligned} \boldsymbol{\gamma} \cdot \mathbf{p} + m &= \begin{pmatrix} m & E_p - \boldsymbol{\sigma} \cdot \mathbf{p} \\ E_p + \boldsymbol{\sigma} \cdot \mathbf{p} & m \end{pmatrix} \\ \gamma^5\boldsymbol{\gamma} \cdot \mathbf{S} &= \begin{pmatrix} 0 & -S^0 + \boldsymbol{\sigma} \cdot \mathbf{S} \\ S^0 + \boldsymbol{\sigma} \cdot \mathbf{S} & 0 \end{pmatrix} \end{aligned}$$

The term related to  $\lambda$  in  $u(p, \lambda)u^\dagger(p, \lambda)$  is

$$\begin{aligned} M_1 &= \frac{\lambda}{2}\gamma^5\boldsymbol{\gamma} \cdot \mathbf{S}(\boldsymbol{\gamma} \cdot \mathbf{p} + m)\gamma^0 \\ &= \frac{\lambda}{2} \begin{pmatrix} 0 & -S^0 + \boldsymbol{\sigma} \cdot \mathbf{S} \\ S^0 + \boldsymbol{\sigma} \cdot \mathbf{S} & 0 \end{pmatrix} \begin{pmatrix} m & E_p - \boldsymbol{\sigma} \cdot \mathbf{p} \\ E_p + \boldsymbol{\sigma} \cdot \mathbf{p} & m \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \frac{\lambda}{2} \begin{pmatrix} (-S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})m & (-S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})(E_p + \boldsymbol{\sigma} \cdot \mathbf{p}) \\ (S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})(E_p - \boldsymbol{\sigma} \cdot \mathbf{p}) & (S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})m \end{pmatrix} \end{aligned}$$

The term without  $\lambda$  in  $u(p, \lambda)u^\dagger(p, \lambda)$  is

$$M_0 = \frac{1}{2}(\gamma \cdot p + m)\gamma^0 = \frac{1}{2} \begin{pmatrix} m & E_p - \boldsymbol{\sigma} \cdot \mathbf{p} \\ E_p + \boldsymbol{\sigma} \cdot \mathbf{p} & m \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} E_p - \boldsymbol{\sigma} \cdot \mathbf{p} & m \\ m & E_p + \boldsymbol{\sigma} \cdot \mathbf{p} \end{pmatrix}$$

The term related to  $\lambda$  in  $\sum_{\lambda_{\mathcal{B}}} [u^\dagger(p, \lambda)u(k, \lambda_{\mathcal{B}})u^\dagger(k', \lambda_{\mathcal{B}})u(p, \lambda)]$  is

$$\begin{aligned} \text{tr}(M_1 M_{\mathcal{B}}) &= (M_1 M_{\mathcal{B}})_{11} + (M_1 M_{\mathcal{B}})_{22} \\ &= (M_1)_{11}(M_{\mathcal{B}})_{11} + (M_1)_{12}(M_{\mathcal{B}})_{21} + (M_1)_{21}(M_{\mathcal{B}})_{12} + (M_1)_{22}(M_{\mathcal{B}})_{22} \\ &\equiv \frac{\lambda}{\sqrt{(m + E_k)(m + E_{k'})}} \times \frac{1}{4}(\text{I}_1 + \text{II}_1 + \text{III}_1 + \text{IV}_1) \end{aligned}$$

where

$$\begin{aligned} \text{I}_1 &= \text{tr}(-S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})m(m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} - \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{II}_1 &= \text{tr}(-S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})(E_p + \boldsymbol{\sigma} \cdot \mathbf{p})(m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} - \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{III}_1 &= \text{tr}(S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})(E_p - \boldsymbol{\sigma} \cdot \mathbf{p})(m + E_k - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} + \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{IV}_1 &= \text{tr}(S^0 + \boldsymbol{\sigma} \cdot \mathbf{S})m(m + E_k + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_{k'} + \boldsymbol{\sigma} \cdot \mathbf{k}') \end{aligned}$$

Taking use of

$$\begin{aligned} \frac{1}{2}\text{tr}(\boldsymbol{\sigma} \cdot \mathbf{a})(\boldsymbol{\sigma} \cdot \mathbf{b}) &= \mathbf{a} \cdot \mathbf{b} \\ \frac{1}{2}\text{tr}(\boldsymbol{\sigma} \cdot \mathbf{a})(\boldsymbol{\sigma} \cdot \mathbf{b})(\boldsymbol{\sigma} \cdot \mathbf{c}) &= i\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \\ \frac{1}{2}\text{tr}(\boldsymbol{\sigma} \cdot \mathbf{a})(\boldsymbol{\sigma} \cdot \mathbf{b})(\boldsymbol{\sigma} \cdot \mathbf{c})(\boldsymbol{\sigma} \cdot \mathbf{d}) &= (\mathbf{a} \cdot \mathbf{b})(\mathbf{c} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) + (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) \end{aligned}$$

we have

$$\begin{aligned} \frac{1}{4}(\text{I}_1 + \text{IV}_1) &= im\mathbf{S} \cdot (\mathbf{k} \times \mathbf{k}') \\ \frac{1}{4}(\text{II}_1 + \text{III}_1) &= i(S^0 \mathbf{p} - E_p \mathbf{S}) \cdot (\mathbf{k} \times \mathbf{k}') + i(m + E_p)\mathbf{S} \cdot [\mathbf{p} \times (\mathbf{k} - \mathbf{k}')] \end{aligned}$$

Note that  $E_k = E_{k'} = E_p$  due to the delta function  $\delta(E_p - E_k)$  in Eq. (7). Finally we have

$$\begin{aligned} \text{tr}(M_1 M_{\mathcal{B}}) &= \frac{\lambda}{\sqrt{(m + E_k)(m + E_{k'})}} \times \frac{1}{4}(\text{I}_1 + \text{II}_1 + \text{III}_1 + \text{IV}_1) \\ &= i\lambda \left( (E_p - m)^2 (2\hat{\mathbf{y}} \cdot \hat{\mathbf{p}}\hat{\mathbf{p}} - \hat{\mathbf{y}}) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{k}}') + p^2 \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \right) \end{aligned}$$

Then we can obtain the coefficient of  $\lambda$  in Eq. (4) as

$$\begin{aligned} \mathcal{P}_1(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \sin[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times \left( (E_p - m)^2 (2\hat{\mathbf{y}} \cdot \hat{\mathbf{p}}\hat{\mathbf{p}} - \hat{\mathbf{y}}) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{k}}') + p^2 \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \right) \end{aligned}$$

where  $\alpha = e^2/4\pi$ ,  $p = |\mathbf{p}| = |\mathbf{k}| = |\mathbf{k}'|$  and  $d\Omega_p$  represents the solid angle of  $\hat{\mathbf{p}}$ . The term irrelevant to  $\lambda$  in  $\sum_{\lambda_{\mathcal{B}}} [u^\dagger(p, \lambda) u(k, \lambda_{\mathcal{B}}) u^\dagger(k', \lambda_{\mathcal{B}}) u(p, \lambda)]$  is

$$\begin{aligned} \text{tr}(\mathbf{M}_0 \mathbf{M}_{\mathcal{B}}) &= (\mathbf{M}_0 \mathbf{M}_{\mathcal{B}})_{11} + (\mathbf{M}_0 \mathbf{M}_{\mathcal{B}})_{22} \\ &= (\mathbf{M}_0)_{11} (\mathbf{M}_{\mathcal{B}})_{11} + (\mathbf{M}_0)_{12} (\mathbf{M}_{\mathcal{B}})_{21} + (\mathbf{M}_0)_{21} (\mathbf{M}_{\mathcal{B}})_{12} + (\mathbf{M}_0)_{22} (\mathbf{M}_{\mathcal{B}})_{22} \\ &\equiv \frac{1}{m + E_p} \times \frac{1}{4} (\text{I}_0 + \text{II}_0 + \text{III}_0 + \text{IV}_0) \end{aligned}$$

where

$$\begin{aligned} \text{I}_0 &= \text{tr} (E_p - \boldsymbol{\sigma} \cdot \mathbf{p})(m + E_p - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_p - \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{II}_0 &= \text{tr} m(m + E_p + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_p - \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{III}_0 &= \text{tr} m(m + E_p - \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_p + \boldsymbol{\sigma} \cdot \mathbf{k}') \\ \text{IV}_0 &= \text{tr} (E_p + \boldsymbol{\sigma} \cdot \mathbf{p})(m + E_p + \boldsymbol{\sigma} \cdot \mathbf{k})(m + E_p + \boldsymbol{\sigma} \cdot \mathbf{k}') \end{aligned}$$

We have

$$\begin{aligned} \frac{1}{4} (\text{I}_0 + \text{IV}_0) &= E_p(m + E_p)^2 + E_p \mathbf{k} \cdot \mathbf{k}' + (m + E_p) \mathbf{p} \cdot \mathbf{k} + (m + E_p) \mathbf{p} \cdot \mathbf{k}' \\ \frac{1}{4} (\text{II}_0 + \text{III}_0) &= m(m + E_p)^2 - m \mathbf{k} \cdot \mathbf{k}' \end{aligned}$$

then

$$\frac{1}{4} (\text{I}_0 + \text{II}_0 + \text{III}_0 + \text{IV}_0) = (m + E_p)^3 + (E_p - m) \mathbf{k} \cdot \mathbf{k}' + (m + E_p) \mathbf{p} \cdot (\mathbf{k} + \mathbf{k}')$$

Finally we have

$$\begin{aligned} \text{tr}(\mathbf{M}_0 \mathbf{M}_{\mathcal{B}}) &= \frac{1}{m + E_p} \times \frac{1}{4} (\text{I}_0 + \text{II}_0 + \text{III}_0 + \text{IV}_0) \\ &= (E_p + m)^2 + (E_p - m)^2 \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' + p^2 \hat{\mathbf{p}} \cdot (\hat{\mathbf{k}} + \hat{\mathbf{k}}') \end{aligned}$$

Then we can obtain the term without  $\lambda$  in Eq. (4) as

$$\begin{aligned} \mathcal{P}_0(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \cos[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times [(E_p + m)^2 + (E_p - m)^2 \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' + p^2 \hat{\mathbf{p}} \cdot (\hat{\mathbf{k}} + \hat{\mathbf{k}}')] \end{aligned}$$

The probability  $\mathcal{P}(\lambda, b)$  at impact parameter  $b$  becomes

$$\mathcal{P}(\lambda, b) = \mathcal{P}_0(b) + \lambda \mathcal{P}_1(b)$$

For the potentials of pseudovector, scalar and pseudoscalar, we can replace the  $\gamma^\mu A_\mu$  factor in the interaction part of the Lagrangian  $\mathcal{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu$  by  $\gamma^\mu\gamma^5 A_\mu$ ,  $\phi$ ,  $\gamma^5\phi$ , where  $\phi$  is a scalar or pseudoscalar field. The calculations for the scattering probability by this three types of interaction are similar to the vector case, and we list the results as follows.

For pseudovector potential, we have

$$\begin{aligned} \mathcal{P}_1^{PV}(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^6 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \cos[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times \{ \hat{\mathbf{y}} \cdot (\mathbf{k} \times \mathbf{k}') - \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \} \end{aligned}$$

$$\begin{aligned} \mathcal{P}_0^{PV}(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^6 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \sin[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times (1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' + \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}' + \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \end{aligned}$$

For scalar potential, we have

$$\begin{aligned} \mathcal{P}_1^S(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \sin[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times \{ (E_p - m)^2 (2\hat{\mathbf{y}} \cdot \hat{\mathbf{p}}\hat{\mathbf{p}} - \hat{\mathbf{y}}) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{k}}') - p^2 \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \} \end{aligned}$$

$$\begin{aligned} \mathcal{P}_0^S(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^4 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \cos[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times [(E_p + m)^2 + (E_p - m)^2 \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' - p^2 \hat{\mathbf{p}} \cdot (\hat{\mathbf{k}} + \hat{\mathbf{k}}')] \end{aligned}$$

For pseudoscalar potential, we have

$$\begin{aligned} \mathcal{P}_1^{PS}(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^6 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \cos[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times \{ \hat{\mathbf{y}} \cdot (\mathbf{k} \times \mathbf{k}') + \hat{\mathbf{y}} \cdot [\hat{\mathbf{p}} \times (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \} \end{aligned}$$

$$\begin{aligned} \mathcal{P}_0^{PS}(b) &= \frac{\alpha}{4(2\pi)^6} \int dpp^6 \int d\Omega_p \int d\Omega d\Omega' A_0(\mathbf{p} - \mathbf{k}) A_0(\mathbf{p} - \mathbf{k}') \phi(\mathbf{k}) \phi(\mathbf{k}') \sin[pb\hat{\mathbf{x}} \cdot (\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \\ &\quad \times (1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' - \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}' - \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \end{aligned}$$

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