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Quantum transport in strongly correlated Fermi gases

Transport quantique dans des gaz de fermions fortement corrélés

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Abstract. Transport in strongly correlated fermions cannot be understood by fermionic quasiparticles alone. We present a theoretical framework for quantum transport that incorporates strong local correlations of fermion pairs. These contact correlations add essential contributions to viscous, thermal and sound transport coefficients. The bulk viscosity, in particular, receives its dominant contribution from pair excitations. Moreover, it can be measured elegantly by observing the response to a time-dependent scattering length even when the fluid is not moving. Rapid changes of the scattering length drive the system far out of local equilibrium, and we show how it relaxes back to equilibrium following a hydrodynamic attractor before a Navier-Stokes description becomes valid. This paper summarizes a talk given at the Symposium "Open questions in the quantum many-body problem" at the Institut Henry Poincaré, Paris, in July 2024.

Résumé. Le transport dans les fermions fortement corrélés ne peut être compris par les seules quasiparticules fermioniques. Nous présentons un cadre théorique pour le transport quantique qui incorpore les fortes corrélations locales des paires de fermions. Ces corrélations de contact ajoutent des contributions essentielles aux coefficients de transport visqueux, thermique et sonore. La viscosité de volume, en particulier, reçoit sa contribution dominante des excitations de paires. En outre, elle peut être mesurée de manière élégante en observant la réponse à une longueur de diffusion dépendant du temps, même lorsque le fluide n'est pas en mouvement. Des changements rapides de la longueur de diffusion éloignent le système de l'équilibre local, et nous montrons comment il retourne à l'équilibre en suivant un attracteur hydrodynamique avant qu'une description de Navier-Stokes ne devienne correcte. Cet article résume un exposé donné au symposium «Open questions in the quantum many-body problem» à l'Institut Henri Poincaré, Paris, en juillet 2024.

Keywords. Strongly correlated fermions, quantum transport, bulk viscosity, hydrodynamics, attractors.

Mots-clés. fermions fortement corrélés, transport quantique, viscosité de volume, hydrodynamique, attracteur.

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1. Introduction

Resonantly interacting fermions are characterized by strong short-range correlations between ↑ and ↓ fermions (see Yvan Castin's presentation in this volume). These correlations give rise to remarkable transport properties that have been observed in experiments with ultracold Fermi gases in recent years. Noteworthy examples include (i) dilute clouds of opposite spin bounce off one another and create shock waves before they eventually merge diffusively [1]; (ii) the unitary Fermi gas exhibits extremely low friction, given by the ratio of shear viscosity to entropy density $\eta/s \gtrsim 0.5\hbar/k_B$, and thereby constitutes a nearly perfect fluid [2–4]; (iii) a quantum lower bound on diffusivity $D \gtrsim \hbar/m$ is observed for spin diffusion (i) [1, 5–9] and momentum diffusion (ii) but also for thermal and sound diffusion [10–17]; (iv) several transport relaxation rates $\tau^{-1} \sim k_B T/\hbar$ scale proportional to temperature in the normal state above the superfluid critical temperature T_c, reminiscent of quantum critical scaling [9, 13, 18, 19].

Important questions include how this collective behavior arises from the microscopic Hamiltonian and how to derive an effective description at large scales. Near equilibrium, hydrodynamics works well as an effective description in the strongly correlated regime that is dominated by frequent collisions. However, dissipative hydrodynamics requires the equation of state and the transport cofficients as input, and their computation from first principles remains a challenging task. Explicit computations have shown quantum limited diffusion in many instances, but a universal many-body mechanism for different microscopic models has not yet emerged. Beyond hydrodynamics, the short-time behavior and the approach to equilibrium can exhibit relaxation phenomena on different scales, for instance attractor behavior beyond a Navier-Stokes description [20].

2. Boltzmann kinetic theory

The dilute two-component Fermi gas is described by the Hamiltonian [21]

$$\hat{H} = \int d^d x \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger(\boldsymbol{x}) \left(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\right) \psi_\sigma(\boldsymbol{x}) + g_0 \int d^d x\, \psi_\uparrow^\dagger(\boldsymbol{x}) \psi_\downarrow^\dagger(\boldsymbol{x}) \psi_\downarrow(\boldsymbol{x}) \psi_\uparrow(\boldsymbol{x}) \qquad (1)$$

for nonrelativistic fermions of mass m with an attractive short-range (contact) interaction. The bare coupling strength $g_0 = [(4\pi\hbar^2 a/m)^{-1} - m\Lambda/(2\pi^2\hbar^2)]^{-1}$ in three dimensions is given in terms of the low-energy scattering length a and a large-wavenumber cutoff Λ. In the following we set $\hbar = 1$.

The first approach to transport in a Fermi gas is by Boltzmann kinetic theory [22]. The single-particle distribution function $f(\boldsymbol{r}, \boldsymbol{p}, t)$ evolves according to the Boltzmann equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v}_{\boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \qquad (2)$$

where the left-hand side is the streaming term that includes mean-field interactions, while the right-hand side denotes the collision integral

$$\left(\frac{\partial f_1}{\partial t}\right)_{\text{coll}} \simeq -\int d\boldsymbol{p}_2\, d\Omega \frac{d\sigma}{d\Omega} |\boldsymbol{v}_1 - \boldsymbol{v}_2| \left[f_1 f_2 (1-f_{1'})(1-f_{2'}) - (1-f_1)(1-f_2) f_{1'} f_{2'}\right]. \qquad (3)$$

The collision integral describes how scattering between two particles 1, 2 into new states $1'$, $2'$ leads to a loss (first term) or gain (second term) of particles in state 1. At high temperatures above the Fermi temperature ($T \gg T_F$) the resonant cross section $d\sigma/d\Omega = 4\hbar^2/|\boldsymbol{p}_1 - \boldsymbol{p}_2|^2$ is so simple that the collision integral can be computed analytically. In the degenerate Fermi gas ($T \lesssim T_F$) Pauli blocking of final states reduces the Fermi distribution factors in the collision integral. At the same time, however, Pauli blocking of the intermediate virtual states between scatterings enhances the cross section $d\sigma/d\Omega$ [19, 23]. Near the scattering resonance, remarkably these two competing effects cancel almost perfectly and the resulting collision rate τ^{-1} follows nearly classical scaling [13, 24]. The relaxation time τ is then combined with thermodynamics (in the case of shear viscosity, the pressure p) to yield the frequency dependent transport coefficient, for instance the complex shear viscosity $\eta(\omega) = p\tau_\eta/(1 - i\omega\tau_\eta)$ as follows from the memory function formalism [13]. The Boltzmann prediction for sound attenuation is found to agree with experimental data [11] for the degenerate unitary gas down to $T \simeq 2T_c$, which constitutes a remarkable success of kinetic theory in the strongly correlated regime.

In the collision integral (3) the two-particle distribution function has been factorized into the product of two separate distribution functions for particles 1 and 2. This factorization based on the assumption of molecular chaos does not capture the strong local pair correlations $g^{(2)}_{\uparrow\downarrow}(r) \sim \mathscr{C}/r^2 + \mathscr{O}(1/r)$ at short distance, where $\mathscr{C}$ denotes the expectation value of the contact operator (see below). In the following we will see how these short-range correlations affect transport.

3. Kubo formula and bulk viscosity

A more general approach to transport, which makes no quasiparticle assumption, is derived in linear response theory. The transport coefficients are related by Kubo formulas to equilibrium expectation values; for instance the frequency dependent shear viscosity is given in terms of the transverse stress response function [4, 25],

$$\eta(\omega) = \int d^d x\, dt\, \frac{e^{i(\omega+i0)t} - 1}{i(\omega + i0)}\, i\theta(t)\langle[\hat{\Pi}_{xy}(\boldsymbol{x}, t), \hat{\Pi}_{xy}(0, 0)]\rangle, \tag{4}$$

where $\hat{\Pi}_{xy}(\boldsymbol{x}, t)$ denotes the shear stress operator. The bulk viscosity ζ, furthermore, characterizes friction during isotropic expansion and contributes to sound attenuation. In constrast to the shear viscosity, however, the bulk viscosity is constrained by symmetry and vanishes identically for a scale invariant gas such as the ideal gas but also the unitary Fermi gas [26, 27]. It can be computed by the Kubo formula [28]

$$\zeta(\omega) = \int d^d x\, dt\, \frac{e^{i(\omega+i0)t} - 1}{i(\omega + i0)}\, i\theta(t)\langle[\delta\hat{p}(\boldsymbol{x}, t), \delta\hat{p}(0, 0)]\rangle \tag{5}$$

in terms of the operator $\delta\hat{p}$ that measures pressure fluctuations. The pressure $p = -\partial E/\partial V$ is obtained by performing a scale transformation, and specifically for the dilute gas in three dimensions one obtains the pressure operator

$$\hat{p} = \frac{2}{3}\hat{\mathscr{H}} + \frac{\hat{\mathscr{C}}}{12\pi m a}, \tag{6}$$

where $\hat{\mathscr{H}}$ denotes the Hamiltonian density and a the scattering length. The last term involves on the contact operator

$$\hat{\mathscr{C}} = m^2 g_0^2\, \hat{n}_\uparrow(\boldsymbol{x})\hat{n}_\downarrow(\boldsymbol{x}) = \hat{\Delta}^\dagger(\boldsymbol{x})\hat{\Delta}(\boldsymbol{x}), \tag{7}$$

which is the continuum version of the doublon or pair density regularized by the bare coupling $g_0 \sim -r_0$ such that its zero-range limit $r_0 \to 0$ is well defined [29]. Equivalently, the contact operator can be expressed in terms of the local pair operator $\hat{\Delta} = m g_0 \hat{\psi}_\downarrow \hat{\psi}_\uparrow$, such that the contact measures the density of local pairs. At the scattering resonance $1/a = 0$ the scale invariant pressure relation $p = (2/3)\mathscr{E}$ is recovered, while the contact term quantifies the deviation from scale invariance due to pairing fluctuations. The pressure fluctuations are now given as the component of the pressure orthogonal to density and energy fluctuations [28],

$$\delta\hat{p} = \hat{p} - (\partial p/\partial n)_{\mathscr{E}}\, \hat{n} - (\partial p/\partial \mathscr{E})_n \hat{\mathscr{H}}. \tag{8}$$

Because conserved quantities do not contribute to dissipation, in the dynamical response the dissipation at $\omega > 0$ can only arise from the response function of the contact operator, $\delta\hat{p} =$

$\hat{\mathscr{C}}/(12\pi ma)$, which is not conserved. One thus finds that the bulk viscosity at nonzero frequency is given by [28, 30–32]

$$\zeta(\omega > 0) = \frac{1}{(12\pi ma)^2} \int d^d x\, dt\, \frac{e^{i(\omega+i0)t} - 1}{i(\omega + i0)}\, i\theta(t)\langle[\hat{\mathscr{C}}(\boldsymbol{x}, t), \hat{\mathscr{C}}(0,0)]\rangle. \tag{9}$$

Hence, bulk viscosity is a pure interaction effect that arises from fluctuations of the pair density, not of single fermions. These contributions are not easy to capture in a fermionic kinetic theory, even if the interaction functional is included [33]. Instead, the bulk viscosity can be computed using self-consistent conserving approaches (Luttinger-Ward), which are formulated in terms of coupled fermion and pair degrees of freedom [4, 5, 31, 34]. Explicit microscopic computations for the contact correlations and bulk viscosity in the degenerate, strongly correlated gas [31] show a low-frequency Drude peak in the complex bulk viscosity $\zeta(\omega) \simeq \chi\tau_\zeta/(1 - i\omega\tau_\zeta)$ followed by an anomalous contact tail $\zeta(\omega \to \infty) \sim C/\omega^{3/2}$ at large frequency. Remarkably, in the unitary gas the bulk scattering rate $\tau_\zeta^{-1} \propto T$ exhibits a T-linear scaling in a wide temperature range from slightly above T_c to high temperatures above T_F, in distinction to other transport relaxation times that decay at high temperatures. This unusual scaling arises from scattering between pairs, not individual fermions, and is specific to the bulk viscosity.

Open questions concern the response in the low-temperature, superfluid state, where a superfluid of fermion pairs can behave differently from a bosonic superfluid due to the additional pair-breaking excitations [35, 36]. In particular for the bulk viscosity, but also for the other transport coefficients it is desirable to derive a kinetic theory that captures the strong fermion correlations. A kinetic formulation in terms of coupled fermions and pairs has been derived in the high-temperature virial expansion [37]: the fermionic contribution to the total bulk viscosity agrees with previous Boltzmann calculations [33], but the pair contribution is found to be much larger near unitarity. Efforts are underway to extend this to the quantum degenerate regime.

3.1. *Measurement of the bulk viscosity*

Often transport measurements observe the damping of fluid motion: elliptic flow or quadrupole motion for shear viscosity, and isotropic flow or radial breathing for the bulk viscosity. The measurement of sound attenuation [11, 12, 14–17, 38] gives access to a combination of several transport coefficients, as sound decays by both momentum and thermal relaxation processes. For the bulk viscosity, however, there is another way of measurement in a dilute gas that works even if the fluid is homogeneous and at rest [39]. In linear response the contact correlation, and thereby the bulk viscosity, is given by the response of the contact to an earlier change of scattering length [31],

$$i\theta(t-t')\langle[\hat{\mathscr{C}}(\boldsymbol{x}, t), \hat{\mathscr{C}}(0, t')]\rangle = -4\pi m \left.\frac{\partial\langle\hat{\mathscr{C}}(\boldsymbol{x}, t)\rangle}{\partial a^{-1}(0, t')}\right|_{S,N} \tag{10}$$

at fixed entropy and particle number. Experimentally the spatially integrated contact has been measured with a high temporal resolution [6, 8], and one can modulate the scattering length in time via the applied magnetic field to measure the response. In this way, the frequency dependence of the bulk viscosity can be mapped out.

4. Attractors to hydrodynamics

When a system is brought far from equilibrium, one might expect that the approach to equilibrium at long times is governed by hydrodynamics (Navier-Stokes equation). In heavy-ion collisions, however, fluid behavior is found already at short times after a collision, earlier than hydrodynamics is expected to be valid, in a so-called hydrodynamic attractor [40]. In general, one can ask which equations describe the approach to hydrodynamics and whether they are universal. Furthermore, if hydrodynamics is viewed as a "derivative expansion" in powers of $\omega\tau$, what determines the higher orders? Some answers may be provided by experiments with ultracold quantum gases, where the time-resolved evolution toward equilibrium can be observed starting from defined initial conditions or subject to a particular driving.

Hydrodynamic attractors can arise in many forms of fluid motion, but there is a particularly simple case where it can be studied in a uniform ultracold atomic gas at rest, with no moving parts, when the scattering length $a(t)$ is ramped at time $t > 0$ to bring the system out of local equilibrium [20, 39]. The relaxation back to equilibrium can be observed in the equation of state, most directly in the contact density expectation value, which is given in linear response as

$$\mathscr{C}(t) = \mathscr{C}_{\text{eq}} + \int_0^\infty dt' \, \frac{\partial \mathscr{C}(t)}{\partial a^{-1}(t')} \delta a^{-1}(t'). \tag{11}$$

Using Eq. (10) this can be expressed in terms of the contact correlation, and one finds that the approach to equilibrium occurs via local dissipation, with the dissipation rate set by the bulk viscosity [39]. The Drude peak of the bulk viscosity [31] $\zeta(\omega) \simeq \chi\tau_\zeta/(1 - i\omega\tau_\zeta)$ (see above) corresponds in the time domain to an exponential decay of the contact response within the bulk relaxation time τ_ζ:

$$\frac{\partial \mathscr{C}(t)}{\partial a^{-1}(t')} \simeq \theta(t - t') \left(\frac{\partial \mathscr{C}}{\partial a^{-1}} \right)_{S,N} \frac{\exp[-(t - t')/\tau_\zeta]}{\tau_\zeta}. \tag{12}$$

By inserting this form into Eq. (11) one can predict the time evolution of the contact following arbitrary drives $\delta a^{-1}(t)$ as long as the drive amplitude is small enough to remain in the linear response regime. But does this time evolution agree with the prediction of Navier-Stokes hydrodynamics? When the scattering length is varied, the local pressure also changes in time, and the nonequilibrium component of the pressure is quantified by the dissipative bulk pressure

$$\pi(t) = \frac{\mathscr{C}(t) - \mathscr{C}_{\text{eq}}[a(t)]}{12\pi m a(t)}, \tag{13}$$

which for a dilute gas is given in terms of the difference of the instantaneous contact and the equilibrium contact for the instantaneous scattering length [39]. In Navier-Stokes hydrodynamics the bulk pressure $\pi = -\zeta V_a$ is driven by the local expansion of the fluid, $V_a = \nabla \cdot \boldsymbol{v}$, times the bulk viscosity ζ. On the other hand, when the scattering length is changed, the local scale variation arises equally from the rate of change of the scattering length, $V_a(t) = -3\dot{a}(t)/a(t)$. Therefore, both expansion and variations of the scattering length are equivalent ways to probe local bulk dissipation. In contrast to Navier-Stokes hydrodynamics, we obtain the equation of motion for $\pi(t)$ from the time derivative of Eq. (12), which we have derived microscopically [20]:

$$\tau\dot{\pi}(t) + \pi(t) = -\zeta[a(t)] V_a(t). \tag{14}$$

This differs from Navier-Stokes by the relaxation term on the left-hand side, which has the same form as in a Müller-Israel-Stewart formulation. For a given external drive $a(t)$ the bulk pressure is obtained by integrating this differential equation, and the result can be compared to the Navier-Stokes prediction $\pi_{\text{NS}}(t) = -\zeta[a(t)] V_a(t)$. For slow drive frequencies $\omega\tau_\zeta \ll 1$ the dissipative term $\tau\dot{\pi}$ has little effect and the bulk pressure follows the drive almost instantaneously. For fast drives $\omega\tau_\zeta \gtrsim 1$, instead, $\pi(t)$ follows the drive with a time delay and a deviation from Navier-Stokes hydrodynamics is predicted. This is exemplified by a power-law drive $a^{-1}(t > t_{\text{ini}}) \equiv a_{\text{ini}}^{-1}(t/t_{\text{ini}})^{-\alpha}$, which starts at a finite scattering length and sweeps at first fast, then slower toward unitarity $a^{-1} = 0$. In this case the bulk pressure is found analytically as [20]

$$\pi(t) = \pi_{\text{ini}} \, e^{-(t - t_{\text{ini}})/\tau_\zeta} + \pi_{\text{att}}(t), \quad \pi_{\text{att}}(t) = c_\alpha \chi \, e^{-t/\tau_\zeta} \Gamma(-2\alpha, -t/\tau_\zeta) \tag{15}$$

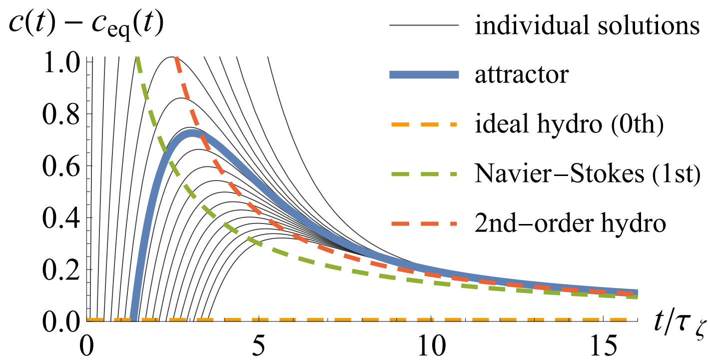

Figure 1. Hydrodynamic attractor. The normalized bulk pressure $c(t) - c_{eq}(t) = \pi(t)/\chi$ exhibits different time evolutions for different initial conditions (thin lines), which quickly converge toward the attractor solution (thick blue line) and only later approach Navier-Stokes hydrodynamics (green dashed line). Adapted from [20].

in terms of the sum rule $\chi = \zeta/\tau_\zeta$ and the incomplete Gamma function $\Gamma(s,z)$. The first term describes the exponential decay of initial conditions on time scale τ_ζ, while the so-called hydrodynamic attractor solution $\pi_{att}(t)$ is the same for different initial conditions and depends only on the transport properties ζ, τ_ζ as well as the drive parameter α. For different initial conditions the bulk pressure is found to first converge toward the attractor solution π_{att} before the attractor itself approaches the Navier-Stokes prediction at longer times, cf. Fig 1.

Standard hydrodynamics is recovered in the solution $\pi(t)$ in the long-time limit. When expanding in "temporal gradients" $\tau_\zeta/t \ll 1$, the leading order reproduces $\pi_{NS}(t)$, but the subsequent orders have factorially growing coefficients $a_n \sim (n+2\alpha)!$ and form an asymptotic series. The initial condition, furthermore, is nonperturbative in τ_ζ/t and is therefore a nonhydrodynamic mode. Even though the gradient expansion does not converge, the solution obtained from the equation of motion is physical and accessible with current experiments [20].

5. Conclusion

The short-time attractor behavior in a driven system is an example of a microscopically motivated extension of hydrodynamics beyond Navier-Stokes. Cold atom experiments can observe these attractors in real time by measuring the response of the contact to variations in the scattering length, thus probing isotropic expansion and local dissipation by an external drive with no moving parts. A remarkable prediction of quantum transport theory is that the bulk relaxation rate $\tau_\zeta^{-1} \propto T$ scales approximately linearly in temperature but is largely independent of density [31]; this is because pressure fluctuations couple predominantly to pairs rather than individual fermions. The computation of frequency dependent transport coefficients remains a challenge, also in the superfluid state. Recently, the accurate computation of fermion and pair spectra was achieved by solving the self-consistent Luttinger-Ward equations directly in real frequency [41–43], which match recent experiments [44]. It will be interesting to extend these methods and compute dynamical response functions in real frequency, such as Eqs. (4) and (5), which determine the transport coefficients near equilibrium. Interesting questions arise also in the far-from-equilibrium response, which can be computed using the Keldysh formulation [45].

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