

Optimal Sensitivity of Anomalous Charged Triple Gauge Couplings through W boson helicity at the e^+e^- colliders

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Abstract

We study the estimation of anomalous charged triple gauge couplings (cTGCs) parameterized in a model-independent Standard Model effective field theory (SMEFT) framework via WW production followed by semi-leptonic decay at the e^+e^- colliders. The anomalous (WWV ($V = \gamma, Z$)) couplings are given in terms of Wilson coefficients of three CP-conserving and two CP-violating dimension-6 operators in the HISZ basis. We adopt the optimal observable technique (OOT) to extract the sensitivity of these anomalous couplings and compare it with the latest experimental limits on anomalous couplings studied at the LHC. The limits on the anomalous couplings obtained via OOT are significantly tighter than the ones obtained using standard χ^2 analysis. The impact of different helicity combinations of the W boson pair in determining optimal sensitivity is analyzed. The constraints on CP-violating operators from the electron electric dipole moment (EDM) are also discussed.

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1 Introduction

After the discovery of the Higgs boson at the large hadron collider (LHC) [1,2] which proved the Standard Model (SM) as the most credible theory to describe nature, the immediate question is whether there is any physics beyond the Standard Model (BSM). In the absence of any decisive signal of new physics (NP), it is essential in the upcoming years to pin down the electroweak symmetry breaking (EWSB) scenario by measuring the SM couplings to unprecedented precision. The gauge interactions within the electroweak sector are an integral part of this scenario. The non-abelian $SU(2)_L \times U(1)_Y$ group suggests that self-interaction among gauge bosons exists within the EW sector. Therefore, the measurement of self-interactions of gauge bosons such as charged triple gauge couplings (cTGCs) WWV ($V = \gamma, Z$) and/or quartic gauge couplings (QGCs) have to be very precise, which is yet to be done. Any deviation in the measurement of gauge boson self-interactions from the SM predictions would be a clear indication of NP. NP search via WWV couplings may also provide an additional probe of CP violation, which can explain electroweak baryogenesis. The precise measurement of gauge boson self-interactions also becomes important to test whether the gauge symmetry is realized linearly or non-linearly in the low energy effective field theory.

The precision measurement of anomalous cTGCs points us toward a lepton collider. This choice is driven by two key advantages: first, the elimination of QCD backgrounds and the reduction of the parton distribution function (PDF) uncertainties, both of which are inherent challenges at hadron colliders; second, the ability to utilize initial beam polarization, which can enhance the NP signal and/or suppress the SM backgrounds. Therefore, we consider a proposed electron-positron collider, *e.g.*, compact linear collider (CLIC) [3] to determine the sensitivity of cTGCs at the baseline design of the CLIC and compare existing bounds given by the CMS experiments [4–6] at the LHC.

Di-boson production at colliders has been receiving a lot of attention since the past as this is sensitive to modification of triple gauge couplings (TGC). In this regard, W -boson pair production plays a crucial role to estimate the sensitivity of anomalous cTGCs. The phenomenology of cTGCs through W boson pair production has been studied at the hadron colliders [7–17], lepton colliders [18–31], and electron-ion colliders [32–37]. Pair production of W -boson at the e^+e^- colliders is governed by neutrino-mediated t -channel diagram and γ and Z mediated s -channel diagram. In this work, we will study the estimation of anomalous cTGCs through WW production at the CLIC with center-of-mass (CM) energy $(\sqrt{s}) = 3$ TeV and integrated luminosity $(\mathcal{L}_{\text{int}}) = 1000 \text{ fb}^{-1}$, using different choice of beam polarization for different WW helicity resolutions. The analysis follows the mathematical framework described in [38], which we describe in the next section. The modification of cTGC vertices appears via s -channel mediation. The estimation of future sensitivity of cTGCs is done using the optimal observable technique (OOT) [39–42], a statistical method to determine the statistical sensitivity in an economical way. The OOT has been used to perform statistical analysis in the context of Higgs couplings [43,44], top-quark couplings [45–50], neutral triple gauge couplings [51,52], and Z couplings with vector-like leptons [53–55] at the e^+e^- colliders. Using the OOT, search for NP through $t\bar{t}$ production has been performed at the $\gamma\gamma$ colliders [56–58]. This statistical method is also applied to explore the CP properties of the Higgs boson at the LHC [59], the $e\gamma$ collider [60], the muon collider [61], and in the field of flavor physics [62–66]. Determination of future sensitivity of anomalous cTGCs using different

helicity combination of W boson pair has not been done yet, to the best of our knowledge. Most of the analyses have focused on the unresolved helicity case. We consider different helicity combinations of W boson pairs to determine the sensitivity of anomalous cTGCs using the OOT and compare them with standard χ^2 analysis at the CLIC.

This paper is organized as follows: in Sec. 2, we discuss the anomalous cTGCs on the basis of 14 anomalous parameters within the SMEFT framework. The SM and SMEFT contributions to the WW production are discussed in Sec. 3. We then give a brief description of the OOT in Sec. 4 and discuss the limits obtained on five independent dimension-6 effective couplings via OOT for different helicity combinations of two W bosons. In Sec. 5, we discuss the limits on the CP-odd couplings through the electric dipole moment (EDM), and finally, we conclude in Sec. 6.

2 Anomalous charged triple gauge couplings

The most general couplings of two charged vector bosons with a neutral vector boson, $WW\gamma/Z$, can be derived from the following effective Lagrangian [20]

$$\begin{aligned} \mathcal{L}_{WWV} = & ig_{WWV}(g_1^V(W_{\mu\nu}^+W^{-\mu} - W^{+\mu}W_{\mu\nu}^-)V^\nu + ig_4^V W_\mu^+W_\nu^-(\partial^\mu V^\nu + \partial^\nu V^\mu) \\ & - ig_5^V \epsilon^{\mu\nu\rho\sigma}(W_\mu^+\partial_\rho W_\nu^- - \partial_\rho W_\mu^+W_\nu^-)V_\sigma + \frac{\lambda^V}{m_W^2}W_\mu^{+\nu}W_\nu^{-\rho}V_\rho^\mu + \frac{\tilde{\lambda}^V}{m_W^2}W_\mu^{+\nu}W_\nu^{-\rho}\tilde{V}_\rho^\mu \\ & + \kappa^V W_\mu^+W_\nu^-V^{\mu\nu} + \tilde{\kappa}^V W_\mu^+W_\nu^-\tilde{V}^{\mu\nu}), \end{aligned} \quad (1)$$

where $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ and the dual field is defined as $\tilde{V}_{\mu\nu} = 1/2\epsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$, with the Levi-Civita tensor following the standard convention, *i.e.*, $\epsilon_{0123} = 1$. The $SU(2)_L$ coupling constants related to photon and Z boson are, $g_{WW\gamma} = -g \sin \theta_W$ and $g_{WWZ} = -g \cos \theta_W$, where θ_W is the weak mixing angle. The couplings g_1^V , λ^V and κ^V are CP-even, while $\tilde{\lambda}^V$ and $\tilde{\kappa}^V$ are CP-odd. Within the SM, $g_1^\gamma = g_1^Z = 1$ and all other couplings are explicitly zero. The conservation of charge of W boson implies that any contribution to g_1^γ should vanish. In the momentum space, $W(q_1)W(q_2)V(p)$ coupling is written as [20]

$$\begin{aligned} \Gamma_V^{\alpha\beta\mu}(q_1, q_2, p) = & f_1^V(q_1 - q_2)^\mu g^{\alpha\beta} - \frac{f_2^V}{m_W^2}(q_1 - q_2)^\mu p^\alpha p^\beta + f_3^V(p^\alpha g^{\mu\beta} - p^\beta g^{\mu\alpha}) \\ & + if_4^V(p^\alpha g^{\mu\beta} + p^\beta g^{\mu\alpha}) + if_5^V \epsilon^{\mu\alpha\beta\rho}(q_1 - q_2)_\rho \\ & - f_6^V \epsilon^{\mu\alpha\beta\rho} p_\rho - \frac{f_7^V}{m_W^2}(q_1 - q_2)^\mu \epsilon^{\alpha\beta\rho\sigma} p_\rho (q_1 - q_2)_\sigma, \end{aligned} \quad (2)$$

with f_i^V being the dimensionless functions of p^2 . At the lowest order, the form factors are related to the anomalous parameters of Lagrangian given in Eq. (1) as [38]

$$\begin{aligned}
f_1^V &= g_1^V + \frac{s}{2m_W^2} \lambda_V, \\
f_2^V &= \lambda_V, \\
f_3^V &= g_1^V + \kappa_V + \lambda_V, \\
f_i^V &= g_i^V \in i = 4, 5, \\
f_6^V &= \tilde{\kappa}_V - \tilde{\lambda}_V, \\
f_7^V &= -\frac{\tilde{\lambda}_V}{2}.
\end{aligned} \tag{3}$$

The above formalism was probed extensively in experiments like LEP to precisely test the predictions of SM with much success. The experiments were able to constrain the anomalous parameters to high accuracy with comparatively low energy and datasets. Nevertheless, the formalism suffers from subtle problems [38]; one can construct an infinite number of additional terms in Eq. (1) by adding derivatives. Each derivative would be accompanied by a factor of m_W^{-1} since it is only mass in the theory. These terms are not suppressed at higher energies, and thus, there is no principle to neglect them. These questions, along with the null results from experiments searching for new states, motivate the development of a model-independent framework to address the theory, either in whole or in part.

Up to now, the collider experiments have not observed any new state required to explain the shortcomings of the SM. In such scenarios, one assumes that the new states are too heavy to be produced at the CM energy of current colliders, and the effects of such states are observed at the electroweak scale in terms of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant higher dimensional operators. This framework of probing any NP deviation in observables considering higher dimensional operators is usually known as Standard Model effective field theory (SMEFT). The effect of NP set at some characteristic scale, Λ , is encoded in the Wilson coefficient associated with effective operators. The effective Lagrangian in presence of higher Lorentz terms is given as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \tag{4}$$

where $\mathcal{O}_i^{(d)}$ are the set of effective operators at dimension, d , and C_i , are the associated Wilson coefficients of operators. Assuming baryon and lepton number conservation, we consider the lowest order, *i.e.*, $d = 6$, affecting WWV couplings. In the case of HISZ basis, there are five $d = 6$ operators that contribute to WWV couplings. In terms of five $d = 6$ operators, the effective Lagrangian becomes [38]

$$\begin{aligned}
\mathcal{L}^{d=6} &= \frac{C_B}{\Lambda^2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi) + \frac{C_W}{\Lambda^2} (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi) + \frac{C_{WWW}}{\Lambda^2} \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_\rho^\mu] \\
&+ \frac{C_{\widetilde{WWW}}}{\Lambda^2} \text{Tr}[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_\rho^\mu] + \frac{C_{\widetilde{W}}}{\Lambda^2} (D_\mu \Phi)^\dagger \widetilde{W}^{\mu\nu} (D_\nu \Phi),
\end{aligned} \tag{5}$$

where $C_i \in \{C_{WWW}, C_W, C_B\}$ are three CP-even and $C_i \in \{C_{\widetilde{W}}, C_{\widetilde{WWW}}\}$ are two CP-odd effective couplings. These couplings encode the effect of any BSM physics at the electroweak

scale. Here, Φ is the SM Higgs doublet, D_μ is the covariant derivative, $B_{\mu\nu}$ and $W_{\mu\nu}$ are the $U(1)$ and $SU(2)$ field strength tensors defined as

$$\begin{aligned} D_\mu &= \partial_\mu + i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2}B_\mu, \\ W_{\mu\nu} &= i\frac{g}{2}\tau^i (\partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon_{ijk}W_\mu^j W_\nu^k), \\ B_{\mu\nu} &= i\frac{g'}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu), \end{aligned}$$

with g and g' are $SU(2)$ and $U(1)$ couplings constant, respectively. After electroweak symmetry breaking, the anomalous contribution to the $W^-W^+\gamma/Z$ couplings can be parametrized in terms of generic effective Lagrangian [20] The anomalous part of Eq. (1) can be written in terms of effective couplings as [38]

$$\begin{aligned} \Delta g_1^Z &= C_W \frac{m_Z^2}{2\Lambda^2}, \\ \Delta g_1^\gamma &= g_4^V = g_5^V = 0, \\ \lambda^\gamma &= \lambda^Z = C_{WWW} \frac{3g^2 m_W^2}{2\Lambda^2}, \\ \Delta \kappa^\gamma &= (C_W + C_B) \frac{m_W^2}{2\Lambda^2}, \\ \Delta \kappa^Z &= (C_W - C_B \tan^2 \theta_W) \frac{m_W^2}{2\Lambda^2}, \\ \tilde{\kappa}^\gamma &= C_{\widetilde{W}} \frac{m_W^2}{2\Lambda^2}, \\ \tilde{\kappa}^Z &= -C_{\widetilde{W}} \tan^2 \theta_W \frac{m_W^2}{2\Lambda^2}, \\ \tilde{\lambda}^\gamma &= \tilde{\lambda}^Z = C_{\widetilde{WWW}} \frac{3g^2 m_W^2}{2\Lambda^2}. \end{aligned} \tag{6}$$

Thus, from the above equation, it becomes clear that five dimension-6 operators become a complete set to describe the effective Lagrangian describing general WWV couplings. Also, the number of independent anomalous couplings can be reduced by writing down two more relations from above Eq. (6)

$$\Delta g_1^Z = \Delta \kappa^Z + \tan^2 \theta_W \Delta \kappa^\gamma, \quad \tilde{\kappa}^Z + \tan^2 \theta_W \tilde{\kappa}^\gamma = 0. \tag{7}$$

Search for NP through dimension-6 cTGCs has been performed via different di-boson productions at the CMS experiment in the LHC. The most stringent limit on each dimension-6 cTCGs at 95% CL is listed in Table 1. In the subsequent analysis, we will determine the optimal sensitivity of dimension-6 cTGCs through WW production and compare the sensitivity with the existing experimental limit. We focus our analysis at the CLIC running with $\sqrt{s} = 3$ TeV, $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$, and polarized electron beam.

NP couplings	Limits (TeV ⁻²)	References
C_B/Λ^2	[-8.78, +8.54]	CMS [4]
C_W/Λ^2	[-3.10, +0.30]	CMS [5]
C_{WW}/Λ^2	[-0.90, +0.91]	CMS [6]
$C_{\widetilde{WW}}/\Lambda^2$	[-0.45, +0.45]	CMS [6]
$C_{\widetilde{W}}/\Lambda^2$	[-20, +20]	CMS [6]

Table 1: Experimental constraints (95% CL) on dimension-6 cTGCs from the CMS experiments at the LHC.

3 WW production at the e^+e^- colliders

In e^+e^- colliders, at tree level, W -boson pair production is predominantly driven by s -channel processes mediated by γ and Z bosons, along with a t -channel contribution mediated by neutrinos within the SM (top panel of Fig. 1). For the BSM scenario considered here, dimension-6 operators contribute to WW production through γ - and Z -mediated s -channel mediation shown in the bottom of Fig. 1. The helicity amplitude for the di-boson pair production at the e^+e^- collider can be written as [20]

$$M_{\sigma,\bar{\sigma};\lambda,\lambda'} = \sqrt{2}e^2\widetilde{M}_{\sigma,\bar{\sigma};\lambda,\lambda'}(\Theta)d_{\Delta\sigma,\Delta\lambda}^{\max(|\Delta\sigma|,|\Delta\lambda|)}(\Theta), \quad (8)$$

where σ and $\bar{\sigma}$ ($= \pm 1$) are the electron and positron helicities, λ , and λ' are the W^- and W^+ helicities, respectively. The angle Θ is the scattering angle of W^- with respect to the e^- direction in the CM frame. Here, $\Delta\sigma = (\sigma - \bar{\sigma})/2$ and $\Delta\lambda = \lambda - \lambda'$. Due to the conservation of angular momentum, the amplitude is non-zero only when $\bar{\sigma} = -\sigma$ in the high energy limit $\sqrt{s} \gg m_e$ implying the relevance of $\Delta\sigma = \pm 1$ states only. Thus for $\Delta\sigma = \pm 1$, total amplitude becomes [20],

$$\begin{aligned} \widetilde{M}_{\sigma,-\sigma;\lambda,\lambda'} &= \frac{\beta}{\sin^2\theta_W} \left(-\frac{1}{2}\delta_{\sigma,-1} + \sin^2\theta_W \right) A_{\lambda,\lambda'}^Z \frac{s}{s - m_Z^2} - \beta A_{\lambda,\lambda'}^\gamma \\ &+ \delta_{\sigma,-1} \frac{1}{2\beta \sin^2\theta_W} \left[B_{\lambda,\lambda'} - \frac{C_{\lambda,\lambda'}}{1 + \beta^2 - 2\beta \cos\Theta} \right], \end{aligned} \quad (9)$$

where $\beta^2 = 1 - \gamma^{-2}$, $\gamma = 1/2\sqrt{s}/m_W$ and θ_W is the weak mixing angle. The coefficients A^γ and A^Z corresponds to s -channel amplitude mediated by photon (γ) and Z boson, respectively. And B and C are related to t -channel neutrino exchange amplitude. Focusing on the corrections to the amplitude due to CP-violating terms given in Eq. 2, it has been found in [21] that CP-violating terms would only contribute to certain combination of helicity combination. In particular, the CP-violation in triple gauge boson vertex affects terms with only $\lambda + \lambda' \neq 0$ states. We obtain the similar helicity structure for ‘LL’ state of W boson pair¹. Therefore, the CP-violating dimension-6 operators do not exist for this particular state. Using ward identity, it can be easily shown that the C_B/Λ^2 contribution to WW production

¹L(T) denotes longitudinal(transverse) helicity mode of W boson.

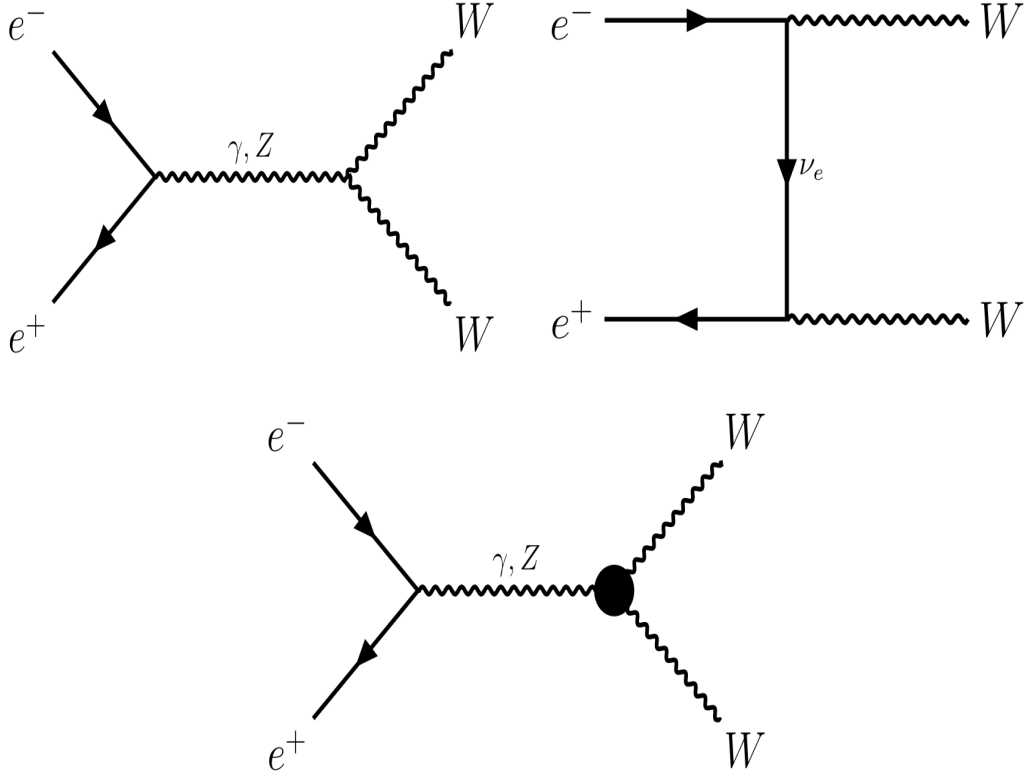


Figure 1: Schematic Feynman diagrams for WW production at the e^+e^- colliders. Top panel: SM contribution, bottom: BSM contribution is highlighted in the dark blob appearing through the cTGCs vertex.

is zero for ‘TT’ helicity state. The contribution of NP couplings to different helicity combinations to the WW production are listed in Table 2. One of the important advantages that the future lepton colliders offer, is the possibility to use initial polarized beams. These could lead to suppression of background, eventually leading to higher sensitivity. The differential

Couplings	Helicity combinations			
	LL	LT+TL	TT	Unresolved
C_B/Λ^2	✓	✓	✗	✓
C_W/Λ^2	✓	✓	✓	✓
C_{WWW}/Λ^2	✓	✓	✓	✓
$C_{\widetilde{WWW}}/\Lambda^2$	✗	✓	✓	✓
$C_{\widetilde{W}}/\Lambda^2$	✗	✓	✓	✓

Table 2: Contributions of different NP couplings to the total cross-section for different helicity combinations. ✓ (✗) indicates that particular couplings are present (absent) for a specific helicity combination.

cross-section for partially polarized initial beams ($-1 \leq P_{e^\pm} \leq +1$) can be expressed as

$$\frac{d\sigma(P_{e^+}, P_{e^-})}{d\Omega} = \frac{(1 - P_{e^-})(1 + P_{e^+})}{4} \left(\frac{d\sigma}{d\Omega} \right)_{LR} + \frac{(1 + P_{e^-})(1 - P_{e^+})}{4} \left(\frac{d\sigma}{d\Omega} \right)_{RL}. \quad (10)$$

Here, $(d\sigma/d\Omega)_{ij}$ represents the differential cross-sections for electron and positron beams with helicities $i \in \{L, R\}$ and $j \in \{L, R\}$, where L and R denote left- and right-handed helicities of the initial beams, respectively².

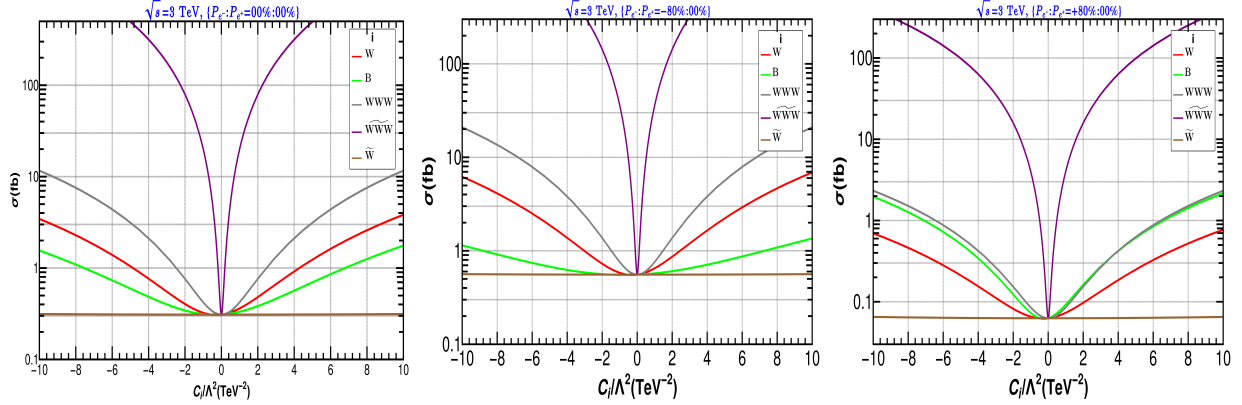


Figure 2: Variation of total cross-section with different dimension-6 effective couplings at $\sqrt{s} = 3$ TeV CLIC. Left: Unpolarized beam; middle: $\{P_{e^-} : P_{e^+} = -80\% : 00\%\}$, right: $\{P_{e^-} : P_{e^+} = +80\% : 00\%\}$. Here we consider the unresolved helicity W boson pair.

The dependence of the total cross-section on the dimension-6 cTGCs for unpolarized beams and two different beam polarization settings is displayed in Fig. 2. The primary CP-even BSM contributions to the WW cross-section originate from the interference term between the SM and dim-6 operators when $|C_i/\Lambda^2| < 1$. BSM contribution starts to dominate when $|C_i/\Lambda^2| > 1$. For the CP-odd scenario, there is no interference between SM and BSM. Therefore, the BSM contribution is directly proportional to C_i^2/Λ^4 for the whole range of the effective coupling. As a result, dimension-8 SMEFT contribution may be comparable to the $C_{\widetilde{WWW}}^2/\Lambda^4$ term. For simplicity, we neglect the effect of dimension-8 contribution and stick to the dimension-6 SMEFT. In Fig. 2, we show the variation total cross-section with the dimension-6 effective couplings for three different polarization combinations considering unresolved helicity of WW production. $C_{\widetilde{WWW}}/\Lambda^2$ has the most dominant contribution to the WW production while $C_{\widetilde{W}}/\Lambda^2$ has the least. We consider the semi-leptonic process $WW \rightarrow qq'\ell\nu_\ell$ as our final state signal, which is a background-free process at the e^+e^- colliders.

4 Optimal Observable Technique

The Optimal Observable Technique (OOT) is a powerful tool for optimally determining the statistical sensitivity of any NP coupling. In this section, we provide a brief overview of the

²In general, left- and right-handed helicities are denoted by the symbols ‘-’ and ‘+’, respectively.

mathematical framework underlying OOT, which has been thoroughly detailed in previous studies [41, 42, 53]. Typically, any observable *e.g.*, differential cross-section that incorporates contributions from both the SM and BSM can be represented as

$$\mathcal{O}(\phi) = \frac{d\sigma}{d\phi} = g_i f_i(\phi), \quad (11)$$

Here, g_i s represent functions of the NP coefficients, while $f_i(\phi)$ depends on the phase-space variable ϕ . Our analysis focuses on the process $e^+e^- \rightarrow W^+W^-$, therefore, the phase-space variable of interest is the cosine of the angle of the outgoing particle ($\cos\theta$) in the CM frame. Depending on the specific observable or process under study, other variables can be altered in place of ϕ .

Our objective is to estimate g_i , which can be accomplished by applying a suitable weighting function, $w_i(\phi)$:

$$g_i = \int w_i(\phi) \mathcal{O}(\phi) d\phi, \quad (12)$$

While various options for $w_i(\phi)$ are possible, there is a specific choice that optimizes the covariance matrix, V_{ij} , minimizing statistical uncertainties in NP couplings. For this optimal choice, V_{ij} is given by

$$V_{ij} \propto \int w_i(\phi) w_j(\phi) \mathcal{O}(\phi) d\phi, \quad (13)$$

Consequently, the weighting functions that meet the optimal condition $\delta V_{ij} = 0$ are

$$w_i(\phi) = \frac{M_{ij}^{-1} f_j(\phi)}{\mathcal{O}(\phi)}, \quad (14)$$

where

$$M_{ij} = \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}(\phi)} d\phi. \quad (15)$$

The optimal covariance matrix is formulated as follows:

$$V_{ij} = \frac{M_{ij}^{-1}}{\mathfrak{L}_{\text{int}}}. \quad (16)$$

Here, $\sigma_T = \int \mathcal{O}(\phi) d\phi$. N signifies the total number of events which is expressed as $N = \sigma_T \mathfrak{L}_{\text{int}}$.

The function χ^2 , which determines the optimal constraint for NP couplings, is defined as

$$\chi^2 = \sum_{ij} (g_i - g_i^0)(g_j - g_j^0) V_{ij}^{-1}, \quad (17)$$

where g_0 represents ‘seed values’ that depend on the specific NP scenario. The limit defined by $\chi^2 \leq n^2$ corresponds to $n\sigma$ standard deviations from these seed values g_0 , setting the optimal constraint for NP couplings under the fact that the covariance matrix V_{ij} is minimized. Using the definition of the χ^2 function provided in Eq. (17), the optimal constraints on NP couplings are explored in the following sections.

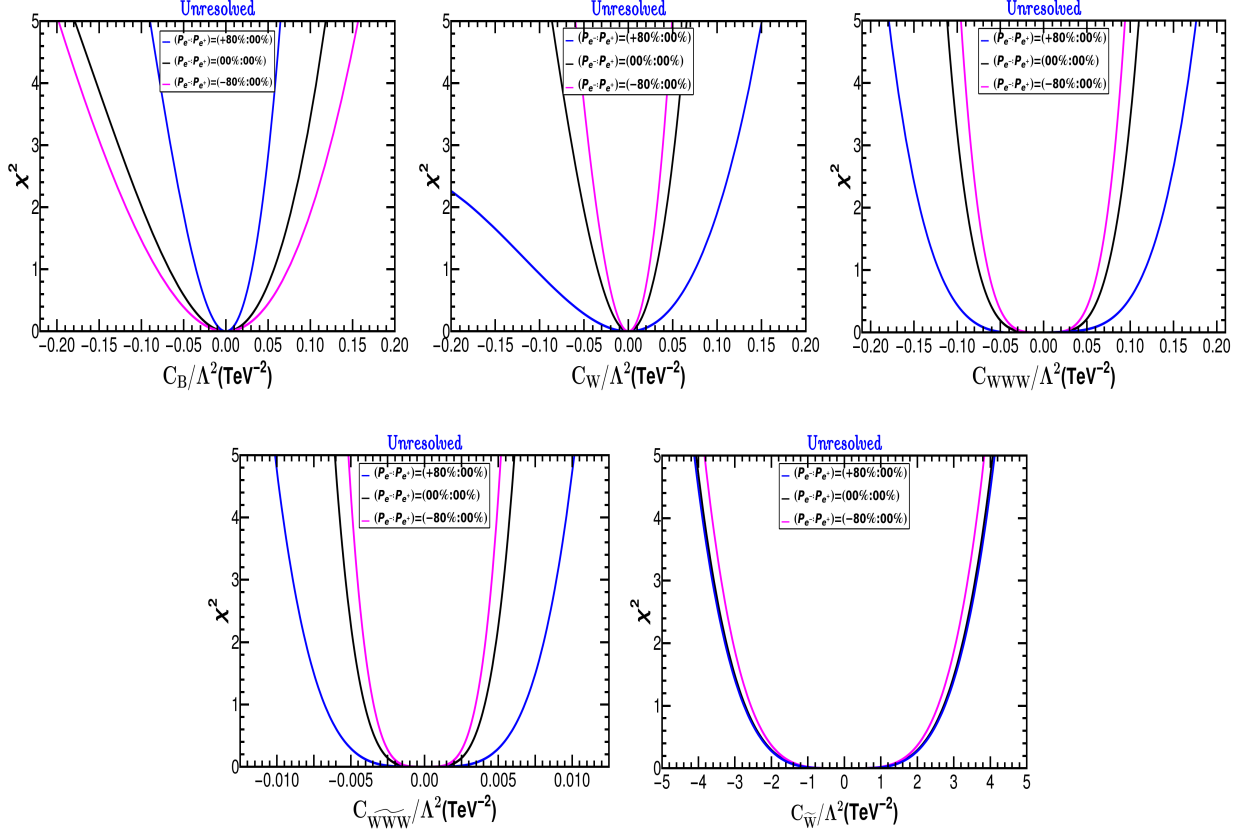


Figure 3: Optimal χ^2 variation with anomalous dimension-6 cTGCs for unresolved helicity combination of W pair. Polarization information is written inside the plots.

4.1 Optimal Sensitivity

In this section, we explore the optimal sensitivity of different dimension-6 effective couplings contributing to the cTGCs with $\sqrt{s} = 3 \text{ TeV}$ and $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$. Thanks to the distinct contribution to the cross-section, different helicity states of final state WW provide different sensitivity to the NP couplings. Judicious choice of initial beam polarization is also important to enhance the sensitivity of NP couplings by a few factors. Using Eq. (17), the optimal χ^2 variation with NP couplings for different final state helicity combinations considering three different polarization combinations are shown in Figs. 3-6. The optimal sensitivity to NP couplings is influenced by the combined effect of two factors: the relative change in the contributions of the SM and NP to WW production and the relative variation in the cross-section due to beam polarization.

For the unresolved helicity combination in Fig. 3, the optimal sensitivity for C_W/Λ^2 is highest with the polarization combination $\{P_{e^-} : P_{e^+} = -80\% : 00\%\}$. This is because, under this polarization choice, the NP contribution to WW production is enhanced. Similarly, the strongest constraint on C_{WWW}/Λ^2 is observed for the ‘TT’ helicity combination (Fig. 4), while the most stringent constraint on $C_{\widetilde{WWW}}/\Lambda^2$ occur for the ‘LT’ helicity combination (Fig. 5). On the other hand, the opposite-sign polarization combination offers the best sensi-

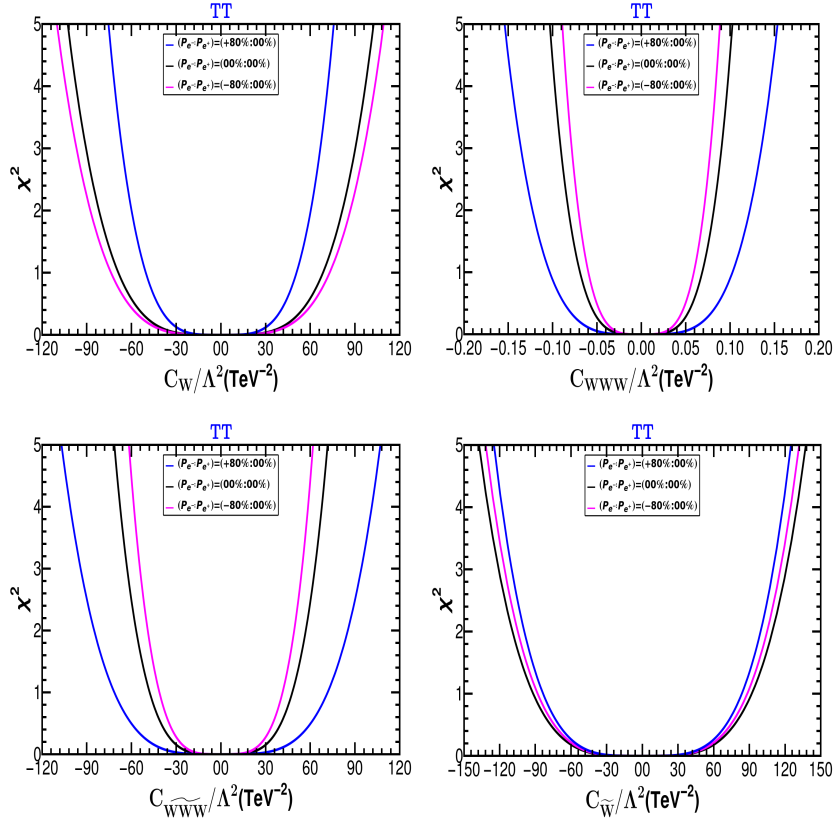


Figure 4: Same as Fig. 3 but for TT helicity combination of W pair.

tivity for C_W couplings in the case of the ‘LL’ helicity combination (Fig. 6). In this scenario, SM contribution to the WW cross-section is reduced due to the opposite-sign polarization choice. It is notable that, in the ‘TT’ combination, apart from the C_{WWW}/Λ^2 case, the contribution from other couplings to WW production at the same order are negligible. As a result, the constraints on these couplings are less stringent. The most stringent optimal limit (95% CL) for a particular operator, corresponding helicity of W boson pair, and initial beam polarization are summarized in Table 3. In comparison with the experimental limit listed in Table 1, we find that optimal sensitivity of C_B/Λ^2 , C_W/Λ^2 , $C_{\widetilde{WWW}}/\Lambda^2$ are improved by two order. Meanwhile, for C_{WWW}/Λ^2 and $C_{\widetilde{W}}/\Lambda^2$, an enhancement of one order of magnitude is achieved.

4.2 Sensitivity comparison: OOT vs standard χ^2 analysis

We turn to discuss the utility of the OOT over standard χ^2 analysis in estimating the sensitivity of dim-6 effective couplings. To this end, the definition of the standard χ^2 function is given by

$$\chi^2 = \sum_j^{\text{bins}} \left(\frac{N_j^{\text{obs}} - N_j^{\text{theo}}(g_i)}{\Delta N_j} \right)^2, \quad (18)$$

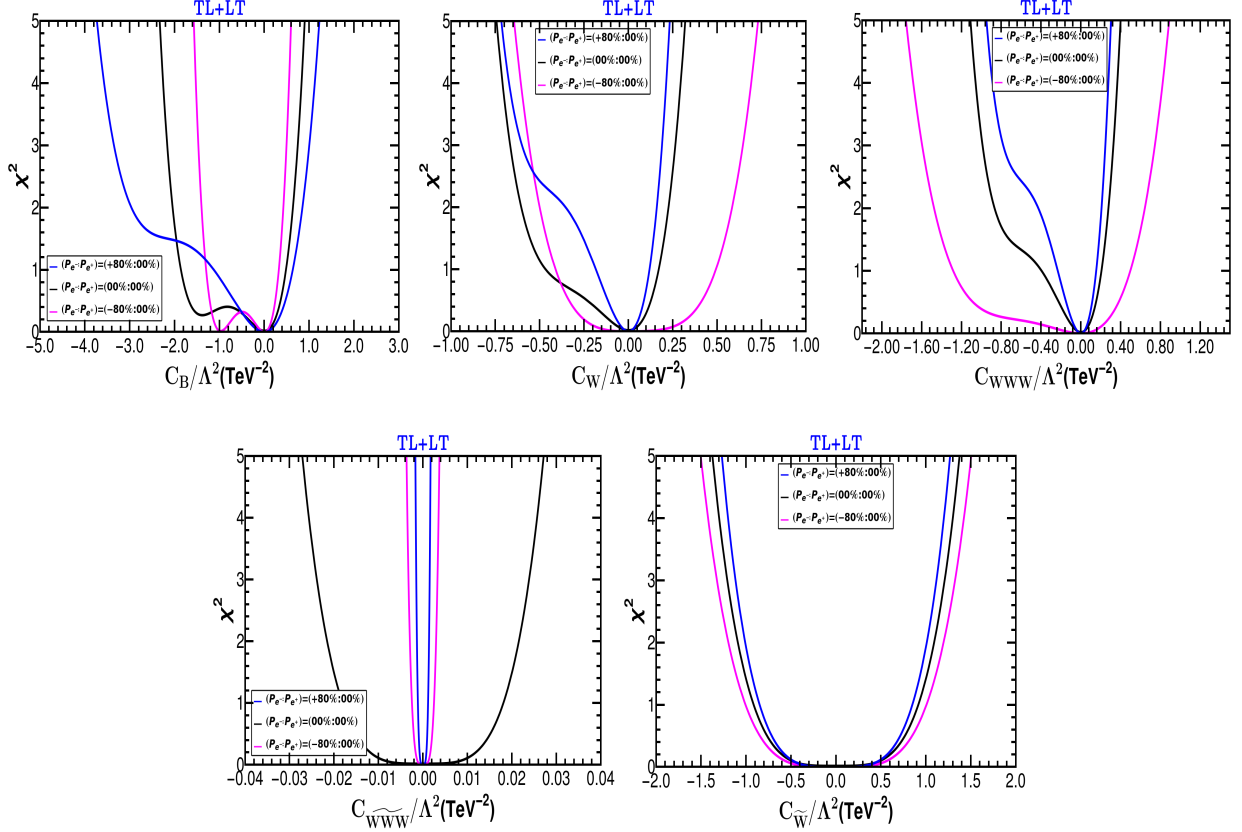


Figure 5: Same as Fig. 3 but for TL helicity combination of W pair.

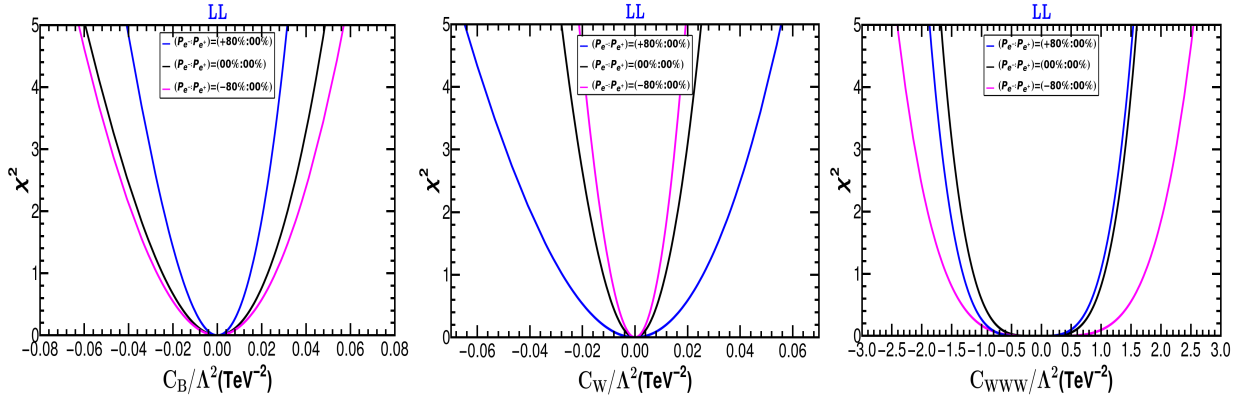


Figure 6: Same as Fig. 3 but for LL helicity combination of W pair.

where N_j^{obs} and N_j^{theo} represent the number of observed and theoretical events, respectively, in the j^{th} bin of the differential cross-section distribution. The uncertainty in the j^{th} bin, denoted by ΔN_j , is defined as $\Delta N_j = \sqrt{N_j^{\text{obs}}}$. Using the specified CLIC machine parameters mentioned above, we show the standard χ^2 variation using Eq. (18), shown in Fig. 7 in cyan color for the unresolved helicity state of W boson pair with unpolarized beam. We

NP couplings	Optimal limits (TeV^{-2})	WW helicity	beam polarization
C_B/Λ^2	$[-0.036, +0.028]$	LL	$\{P_{e^-} : P_{e^+} = +80\% : 0\%\}$
C_W/Λ^2	$[-0.019, +0.017]$	LL	$\{P_{e^-} : P_{e^+} = -80\% : 0\%\}$
C_{WWW}/Λ^2	$[-0.084, +0.084]$	TT	$\{P_{e^-} : P_{e^+} = -80\% : 0\%\}$
$C_{\widetilde{WWW}}/\Lambda^2$	$[-0.002, +0.002]$	LT	$\{P_{e^-} : P_{e^+} = +80\% : 0\%\}$
$C_{\widetilde{W}}/\Lambda^2$	$[-1.20, +1.20]$	LT	$\{P_{e^-} : P_{e^+} = +80\% : 0\%\}$

Table 3: Most stringent optimal sensitivity (95% CL) fo dimension-6 cTGCs at the CLIC with $\sqrt{s} = 3 \text{ TeV}$ and $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$.

also display the optimal χ^2 variation in the same plot with pink color for comparison. A significant improvement is achieved in estimating the sensitivity of NP couplings in the case of the OOT. We find that the optimal sensitivity for C_B/Λ^2 , C_W/Λ^2 , C_{WWW}/Λ^2 , improves over the traditional sensitivity by factors of 20, 25, and 9, respectively, whereas for both CP-violating scenarios, optimal sensitivity is better for both cases by a factor of 12.

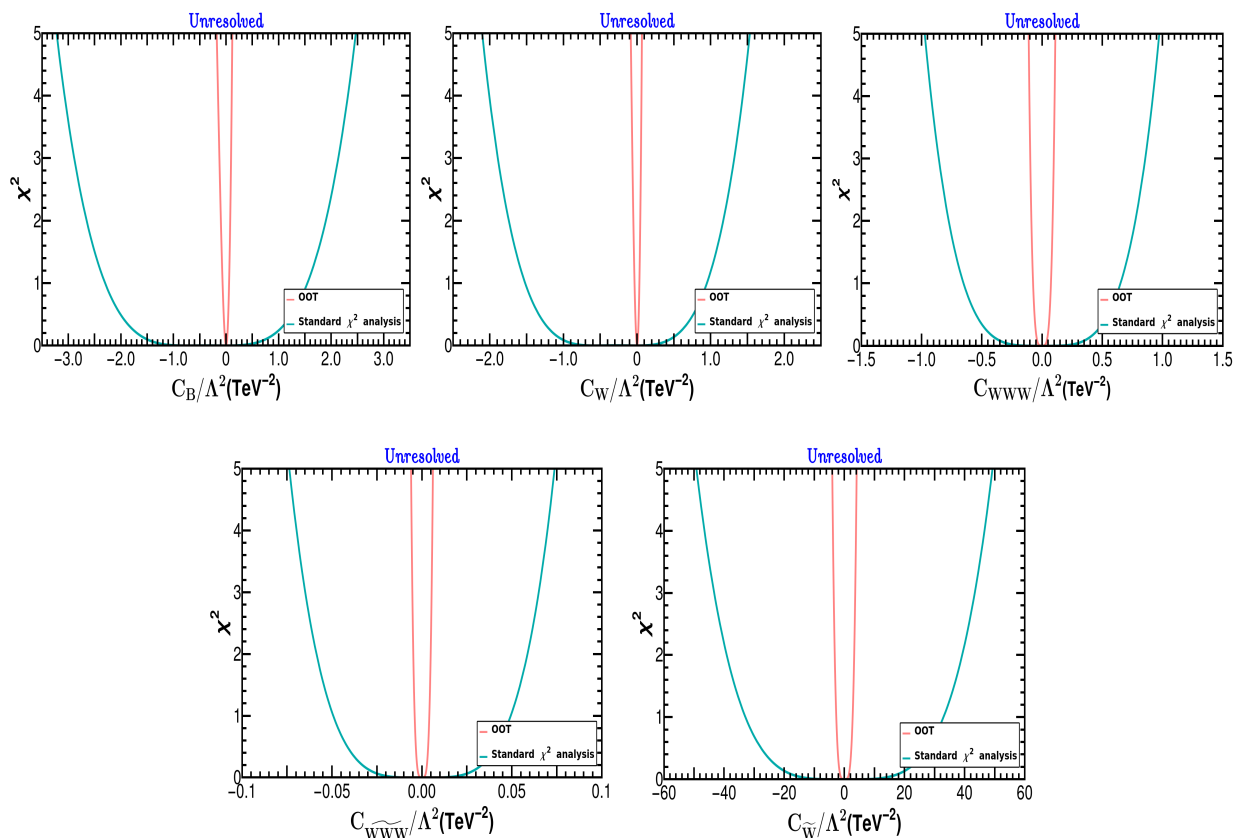


Figure 7: Comparison of χ^2 variation between OOT and standard scenario for different dimension-6 effective couplings. We consider the unresolved helicity state of W and unpolarized initial beam.

5 Constraints from Electric Dipole Moment

Electric dipole moment (EDM) proves to be one of the cleanest probes of NP signal. A non-zero EDM arises from CP-violation, and the contribution from the SM is highly suppressed. This makes the EDM an exceptionally sensitive probe for uncovering BSM physics. NP at the TeV scale, accompanied by CP-violating interactions, could produce sizable EDM that are within reach of near-future observations. The most recent ACME bound on electron EDM turns out as follows [67]:

$$|d_e| < 1.1 \times 10^{-29} \text{ e.cm} \quad \text{or} \quad |d_e| < 1.7 \times 10^{-16} \text{ GeV}^{-1}. \quad (19)$$

Since the EDM measurement has been performed at unprecedented precision, they are competitive with collider measurements in constraining CP-violating higher dimensional effective operators.

From Eq. (5), we identify two dimension-6 CP-violating SMEFT operators that contribute to cTCGs and, consequently, can induce electron EDM. Therefore, it is essential in this context to examine the constraints on these operators derived from the aforementioned EDM experiment. The contributions of $\mathcal{O}_{\widetilde{W}\widetilde{W}}$ and $\mathcal{O}_{\widetilde{W}}$ operators to electron EDM can be written as follows [68]

$$\frac{d_e}{e} = \frac{y_e g m_w}{32\sqrt{2}\pi^2 \Lambda^2} \left(3s_w g^2 C_{\widetilde{W}\widetilde{W}} + \ln \left[\frac{m_h}{\mu} \right] C_{\widetilde{W}} \right), \quad (20)$$

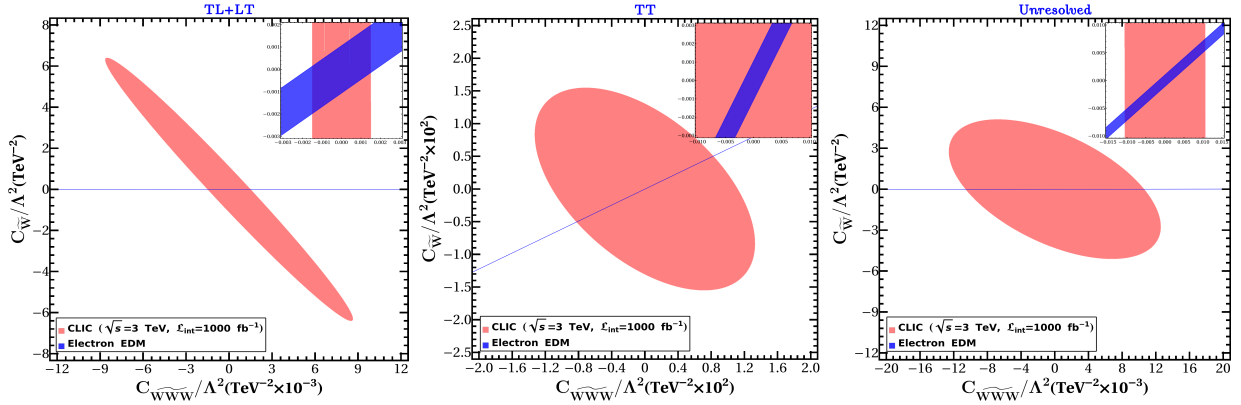


Figure 8: Electron EDM (Blue color) and CLIC sensitivity (Pink color) in $C_{\widetilde{W}\widetilde{W}}/\Lambda^2 - C_{\widetilde{W}}/\Lambda^2$ plane. For CLIC, we consider $\{P_{e^-} : P_{e^+} = +80\% : 0\%\}$ polarization combination. The blue regions in all the plots are allowed by the electron EDM.

where y_e , g , s_w , m_w , m_h , and μ are the electron-Yukawa coupling, $SU(2)_L$ coupling, sine of the weak mixing angle, W -boson mass, Higgs boson mass, and renormalization scale, respectively. Using Eqs. (19) and (20), we obtain the following limit on CP-violating couplings as

$$\begin{aligned} C_{\widetilde{W}\widetilde{W}}/\Lambda^2 (\text{TeV}^{-2}) &\lesssim 0.002, \\ C_{\widetilde{W}}/\Lambda^2 (\text{TeV}^{-2}) &\lesssim 0.001. \end{aligned} \quad (21)$$

Comparing the bounds on CP-violating operators from the electron EDM ((21)) with the most stringent sensitivity in Table 1 obtainable at the CLIC, we find that the limits are similar for $C_{\widetilde{WWW}}/\Lambda^2$. However, in case of $C_{\widetilde{W}}/\Lambda^2$ the EDM constraint is two orders of magnitude stronger than the corresponding limit achievable at CLIC with $\sqrt{s} = 3$ TeV and $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$. In Fig. 8, we present the electron EDM bound and CLIC sensitivity in $C_{\widetilde{WWW}}/\Lambda^2 - C_{\widetilde{W}}/\Lambda^2$ plane. The dominance of the operator $\mathcal{O}_{\widetilde{WWW}}$ in the collider scenario is evident from the figure. In contrast, for the electron EDM scenario, both operators contribute at the similar order of magnitude.

6 Conclusion

Precise measurement of charged triple gauge couplings (cTGCs) is essential to test the SM and look for potential signals beyond the Standard Model (BSM) physics. In this paper, we have analyzed the precision measurement of cTGCs through WW production process followed by semi-leptonic decay in the upcoming electron-positron collider with the initial polarized beams. Due to the significant cross section, the WW process benefits from a sufficient rate to perform precision measurement of cTGCs at $\sqrt{s} = 3$ TeV and $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$. We have parameterized the deviation of cTGCs in the SMEFT framework with three CP-even and two CP-odd dimension-6 SMEFT operators. We have employed the optimal observable technique (OOT) to estimate the sensitivity of dimension-6 cTGCs considering differential distribution as an observable with different helicity combinations of W boson pair. We have established that optimal sensitivities of NP couplings are improved by two orders of magnitude for the CP-conserving case and one order of magnitude for the CP-violating case with respect to the existing LHC bounds on the respective couplings. Furthermore, we compared the projected sensitivity at CLIC using standard χ^2 -analysis and OOT. We found that compared to the standard χ^2 -square analysis, the optimal sensitivities are significantly stringent, highlighting the effectiveness of the OOT. Different helicity combinations of W boson pairs exhibit sensitivity to distinct dimension-6 cTGCs, offering a pathway to probe specific NP couplings. Initial beam polarizations play a critical role in enhancing the sensitivity of NP couplings, influencing their estimation by several factors. Furthermore, the two CP-odd operators can give rise to contributions to the electric dipole moment (EDM) at the one-loop level. Our analysis reveals that, given the center-of-mass (CM) energy and luminosity parameters mentioned earlier, the sensitivity obtained at the future electron-positron collider experiments for the operator $\mathcal{O}_{\widetilde{WWW}}$ is comparable to that of the electron EDM. However, for $\mathcal{O}_{\widetilde{W}}$, current EDM experiments impose more stringent constraints than those achievable at colliders.

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