

Non-Allais Paradox and Context-Dependent Risk Attitudes

Edward Honda* Keh-Kuan Sun†

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Abstract

We provide and axiomatize a representation of preferences over lotteries that generalizes the expected utility model. Our representation is consistent with the violations of the independence axiom that we observe in the laboratory experiment that we conduct. The violations differ from the Allais Paradox in that they are incompatible with some of the most prominent non-expected utility models. Our representation can be interpreted as a decision-maker with context-dependent attitudes to risks and allows us to generate various types of realistic behavior. We analyze some properties of our model, including specifications that ensure preferences for first-order stochastic dominance. We test whether subjects in our experiment exhibit the type of context-dependent risk attitudes that arise in our model.

Keywords: Decision under Risk; Expected Utility Theory; Rank dependence; Allais paradox

JEL codes: C91, D81

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*Department of Economics, University of Manitoba. Email: Edward.Honda@umanitoba.ca

†Department of Economics, Fairfield University. Email: ksun@fairfield.edu

1 Introduction

“Expected utility theory has dominated the analysis of decision making under risk,” write Kahneman and Tversky (1979) as they begin their seminal prospect theory paper. To a large extent, the statement remains true. The simple idea of representing the utility of a risky prospect with the probability-weighted sum of the utilities of its possible outcomes has been the main workhorse in economic modeling in a wide range of applications. The number of alternative theories proposed and empirical evidence of violations of the expected utility theory have only accumulated over many decades. At the center of the debate is the independence axiom. The independence axiom is normatively appealing because it is the necessary and sufficient conditions for the expected utility representation, provided that the underlying preference relation is complete, transitive, and continuous. However, many counterexamples in and out of the laboratory, most notably suggested by Allais (1953), show that it is possible to systematically violate the axiom, thereby questioning its empirical validity and, thus, that of expected utility theory altogether.

Prospect theory was proposed as an alternative framework to account for decisions under uncertainty while addressing two major shortcomings of the expected utility paradigm: (1) the tendency to overweight small probabilities, and (2) the observation that wealth or money are perceived not simply as quantities but as gains and losses relative to a given reference point. This accommodation is achieved through probability weighting and a value function for gains and losses. While the original version of prospect theory encounters another conceptual challenge because it violates first-order stochastic dominance, a modified version, known as *cumulative* prospect theory (Tversky and Kahneman 1992) after incorporating the concept of rank-dependent probability weighting (Quiggin 1982), has become the leading behavioral theory of decision-making under risk as a result of its empirical appeal and psychologically intuitive assumptions.

In this paper, we propose a new middle-ground approach that splits the difference between the normative appeal of expected utility theory and the empirical strengths of prospect theory, but from

a novel angle. We first provide and axiomatize a representation of preferences over lotteries that generalizes the expected utility model. A consequence of this generalization is a more flexible utility representation than that offered by prospect theory, cumulative prospect theory, or rank-dependent utility models. To illustrate this idea, consider the following example of preferences over lotteries.

Suppose a decision-maker (DM) who faces a risky prospect in which there is a high probability of winning a low prize exhibits the following risk-loving preference,

$$(\$0, 0.9; \$100, 0.05; \$200, 0.05) \succ (\$0, 0.9; \$150, 0.1),$$

where the value listed after each monetary prize is the probability that the lottery assigns to the prize. Now, suppose we swap the zero in both lotteries with a prize value of \$90. The DM is ensured a moderate level of wealth in this case, so the DM becomes risk-averse and exhibits the following preference:

$$(\$90, 0.9; \$100, 0.05; \$200, 0.05) \prec (\$90, 0.9; \$150, 0.1).$$

Such a change in risk-taking behavior, in which the DM prefers a riskier lottery when the probability of disappointment or doom is large but prefers less risky lotteries when some wealth is guaranteed is similar to what we see in the well-known Allais Paradox. There is a significant divergence, however, from the paradox in that this violation of the independence axiom is not consistent with the aforementioned models of non-expected utility such as the rank-dependent utility of Quiggin (1982) or the more general cumulative prospect theory of Tversky and Kahneman (1992).¹

To see why it is not possible to obtain these preferences with the two models, note that the worst prize in both lotteries is \$0 at probability 0.9 in both. We merely swapped this common worst component with a prize of \$90 for both lotteries. Therefore, the probability values and the ranks of prizes within a given lottery are unaltered when viewing the \$90 as \$0. For the lottery on the left

1. Our example is not consistent with cumulative prospect theory if we let \$0 be the reference point for the model, but we can construct a similar example for any other reference point as well.

side, \$200 remains the best prize and \$100 remains the second best in the lottery after the swap, and \$0/\$90 is the worst prize before/after the swap. The probabilities associated with \$200 and \$100 do not change during this swapping, with \$0/\$90 both at probability 0.9. We have the same type of situation for the lottery on the right side, and so the preferences described in the example cannot occur for the models mentioned.

This example suggests that there could be a problem with modeling changes in risk-taking behavior using weight distortion. For such models, even if the probability weights can be distorted, the utility function, call it u , over the prizes is fixed. Having only one utility function may make it difficult to capture changes in risk attitudes because risk-loving/aversion is often captured through the curvature of utility functions.

This is exactly the case in our example. Before replacing \$0 with \$90, computing the utility of the lottery on the left side using cumulative prospect theory or rank-dependent utility would give us an expression of the form $w_1u(\$0) + w_2u(\$100) + w_3u(\$200)$, where w_1, w_2, w_3 would denote weights associated with each of the prizes and would be computed in a way specified by the model. Similarly, the utility of the lottery on the right side would be expressed as $w_4u(\$0) + w_5u(\$150)$ for weights w_4 and w_5 . The preference in the example then implies that

$$w_1u(\$0) + w_2u(\$100) + w_3u(\$200) \geq w_4u(\$0) + w_5u(\$150).$$

Given that \$0 is the worst prize in both lotteries at probability 0.9 in both, w_1 would equal w_4 . This simplifies the inequality to

$$w_2u(\$100) + w_3u(\$200) \geq w_5u(\$150). \tag{1}$$

Because the probabilities and ranks associated with a prize within each lottery do not change after we swap the \$0 for \$90 in both lotteries, we would use the same weights w_1, \dots, w_5 for the

computations of the values of the lotteries. This means that the value of the lottery on the left side becomes $w_1u(\$90) + w_2u(\$100) + w_3u(\$200)$ while the value of that on the right becomes $w_4u(\$90) + w_5u(\$150)$. The inequality from (1) and the fact that $w_1 = w_4$ gives us

$$w_1u(\$90) + w_2u(\$100) + w_3u(\$200) \geq w_4u(\$90) + w_5u(\$150).$$

This implies that it is impossible to have the types of preferences seen in the example where the DM exhibits a change in risk attitude.

The idea here is that the weights associated with the prizes when computing the values of lotteries do not change simply because we swap the common worst component with another prize that remains the worst in both lotteries and we do not alter probability values for either lottery. The utility derived from the prizes other than \$0 or \$90 also do not change because there is only one function u . As a result, we are forced to prefer the riskier lottery even in a situation where the DM should display risk aversion. This is just one example involving very specific lotteries, but the point is that a fixed utility may limit us when modeling changes in risk attitudes. There are obviously behaviors beyond those mentioned in this example that pose a similar problem.

With these considerations in mind, we aim in this study to provide and axiomatize a representation in which a decision maker can exhibit different risk attitudes through the use of different utility functions. In particular, the DM has multiple utility functions over prizes, and which of the functions the DM uses for evaluating a lottery depends on the probabilities associated with disappointing prizes. It is then possible that a DM uses a convex utility function when disappointment probabilities are high but a concave function for lotteries where the disappointment probabilities are low. This might reflect the fact that a DM will not repent taking a risk even when the outcome turns out to be unfavorable if disappointment seemed likely in any case. On the other hand, to avoid self-loathing for missing an opportunity to obtain a satisfactory prize, the DM may be risk-averse and settle for the certainty of claiming a decent prize when disappointment is unlikely. This interpretation of our

framework rephrases the idea of Friedman and Savage (1948) who provide a model that can explain a DM who purchases both lottery tickets and insurance. Instead of portraying a representative DM across different demographic groups, we take a step further to relax the independence axiom and achieve a more general representation. Therefore, while our model is similar in spirit to theirs, we are not confined to the standard expected utility model, and this freedom allows us to be consistent with behaviors like the ones we see in the Allais Paradox.

We refer to our representation as an expected contextual utility (ECU) representation because the utility function used depends on the *context*, which is the probability that disappointment will be experienced in this case. In its full generality, the model can allow for one utility function for each probability-of-disappointment value in $[0, 1]$. The model contains the standard expected utility representations as special cases, for example when the DM has only one utility function with which to evaluate lotteries. The use of multiple utility functions allows us to generate a range of realistic behaviors that seem to arise from changes in a DM's risk attitudes but may be impossible to create using existing models.

Our representation is motivated by preferences that are inconsistent with several leading non-expected utility models. This raises the question whether such preferences are prevalent among actual DMs. Hence, we conduct an experiment to check for types of preferences in the example we provide. Aside from the theoretical component of our work, the methods used in and the results of the experiment may be of independent interest. Much of the literature on experiments involving violations of the independence axiom focuses on the Allais Paradox, whereas we find violations that are distinct from those associated with the Allais Paradox. In addition, because the violations for which we check are inconsistent with cumulative prospect theory, our experimental results provide evidence against the widely accepted model. The design is extremely simple, and it is easy to understand why the observed preferences are inconsistent with cumulative prospect theory if the reasoning that applies in the example we provide above is followed. The results reported in Bernheim and Sprenger (2020) also provide evidence against cumulative prospect theory, but there are

several differences. Much as the common ratio or consequence effect can be shown to be inconsistent with the expected utility model in a very simple manner using preferences over only two pairs of lotteries, we provide evidence against the cumulative prospect theory by checking preferences over a small number of pairs of lotteries. Bernheim and Sprenger (2020) adopt a two-step approach. In the first step they provide empirical evidence suggesting that either the assumption of rank dependence of weights in cumulative prospect theory is false or the weighting function is linear, provided that rank dependence is assumed to be true. In the second step they provide evidence against the latter possibility to discount cumulative prospect theory. While Bernheim and Sprenger (2020) provide more thorough evidence against the cumulative prospect theory than we do, we propose a new framework in which to generalize the expected utility paradigm with laboratory evidence from a simple experiment.

The paper proceeds as follows. Section 2 places our paper in the related literature. In Section 3 we formally define and axiomatize the ECU representation. Section 4 introduces noteworthy special cases. In Section 5 we list a range of behaviors that our model generates. In Section 6 we describe our experimental design and report the main results.

2 Related Literature

It is no surprise that lottery-dependent utilities have been proposed because changes in the concavity/convexity of utility functions can explain changes in risk-taking behavior. Becker and Sarin (1987) propose such a model and this notion has been further explored by Schmidt (2001). Yet, several issues arise with these models, and this may explain why, unfortunately, they are not as widespread as others that model non-expected utility. Thus, another of our goals is to identify issues with previous work on lottery-dependent utilities and provide an effective solution with our ECU model.

First, as Schmidt (2001) points out, the Becker and Sarin (1987) model in its full generality has

no testable implications other than completeness and transitivity as there can be a different utility function for each lottery. Also, no axiomatic characterization is given for the special case that they consider. We are able to provide axioms that are necessary and sufficient for our model, generating testable implications and greater predictive ability.

Schmidt (2001) sharpens the results of Becker and Sarin (1987) by axiomatizing a model that falls between the two Becker and Sarin (1987) models in generality, but not without a concern. The axioms are stated in terms of a given partition of the set of lotteries, so the model assumes knowledge regarding which lotteries are evaluated using the same utility function rather than obtaining this information from the preference relation. Our axioms will not rely on the assumption that the observer has a priori knowledge of how the DM separates the set of lotteries. Another concern is that the model may violate preferences for first-order stochastic dominance, which is usually regarded as a normatively appealing feature. We consider specifications of our model that ensure preferences for first-order stochastic dominance.

Inasmuch as they do not distort the weights on prizes and incorporate multiple utility functions, the cautious expected utility of Cerreia-Vioglio, Dillenberger, and Ortoleva (2015) and the models of Maccheroni (2002) and Cerreia-Vioglio (2009) are similar to our model. There are, however, differences in terms of both the interpretation and the behaviors generated by the models. For these three models, it is as if the DM chooses among multiple utility functions and computes the expected utility of a lottery for all of those functions when evaluating this lottery. When assigning a value to any lottery after considering all of the utility functions, the DM chooses the evaluation that is the most cautious or pessimistic. By contrast, for the ECU representation that satisfies preferences for first-order stochastic dominance, the DM is pessimistic when disappointments are likely to occur but becomes optimistic when disappointments become less likely. We also note that the model can violate the negative certainty independence of Dillenberger (2010), which is a weaker condition than the independence axiom and necessary for cautious expected utility.

In Section 5.3 we discuss the possibility that our model violates the betweenness axiom. Promi-

nent models in the class of models that use this as the key axiom include those of Gul (1991), Dekel (1986), and Hong (1983). The disappointment-aversion discussed in Gul (1991) is closest to ours among these three models because the ECU also uses the notion of disappointment, but there are two significant differences. First, the threshold for disappointment can vary in the disappointment-aversion model whereas the ECU has a fixed value d . The other difference is that the utility function is fixed for the disappointment-aversion model and it is the weights placed on the prizes that can be transformed. This contrasts with our representation, where we include multiple utility functions that are used to evaluate lotteries.

3 Expected Contextual Utility Representation

3.1 Defining the Representation

We start this section by introducing the setup for the model. We let $X = [w, b] \subset \mathbb{R}$, where $w < b$, represent a set of prizes. Endow X and \mathbb{R} with a metric topology where the metric is $dist(x, y) = |x - y|$. The set of Borel probability measures with finite support on X , which we also call simple lotteries, is denoted as \mathcal{L} . For any $x \in X$, δ_x is the Dirac measure at x . We let \succsim be a binary preference relation over \mathcal{L} .

We define the threshold for what the DM considers as disappointing prizes as in Honda (2022). The value $d \in [w, b]$ is the threshold for the DM's disappointment with a prize, so prizes in $[w, d]$ are disappointing. The utility functions that the DM uses to evaluate a lottery depends on the probability associated with disappointing prizes, or prizes that are less than or equal to d .

Thus, for each $\pi \in (0, 1)$, we have a function $u_\pi : X \rightarrow \mathbb{R}$ that can be interpreted as a utility function over prizes. If a lottery $p \in \mathcal{L}$ assigned probability $\pi \in (0, 1)$ to prizes in $[w, d]$ or $p([w, d]) = \pi$, the DM uses the utility function u_π . This allows for changes in risk attitudes based on a lottery's context. We can thus generate behaviors like those mentioned in the introduction with risk-lovingness

for high probabilities of disappointment and risk-aversion for low probabilities.

Note that we have not yet defined utility functions for u_0 and u_1 because their domains differ slightly. Based on our description thus far, function u_0 is used to evaluate lotteries that place probability 0 on disappointing prizes, meaning that disappointing prizes are not in the support. That being the case, utility values for disappointing prizes do not need to be defined for u_0 , and we define function $u_0 : X \setminus [w, d] \rightarrow \mathbb{R}$.

In similar fashion, u_1 is used for lotteries that offer only disappointing prizes in the support. For this reason, the utility values $u_1(x)$ do not need to be defined for $x \in (d, b]$. As such, we define function $u_1 : [w, d] \rightarrow \mathbb{R}$. We could also adopt an alternate approach where we extend the domains of u_0 and u_1 to $[w, d]$. In this case the values of $u_0(x)$ and $u_1(y)$ are irrelevant for $x \in [w, d]$ and $y \in X \setminus [w, d]$ and would lead to the lack of any type of uniqueness of the representation because these values can be defined arbitrarily as they have no relevance when evaluating values of lotteries. We avoid using this approach as it unnecessarily complicates much of the following discussion.²

We impose several mild regularity conditions on the utility functions. For any $\pi \in [0, 1]$ such that $u_\pi(w), u_\pi(b)$ are defined³, we impose $u_\pi(w) < u_\pi(b)$. This specification simply implies that the utility the worst prize provides is lower than the utility the best prize provides. Next, for any $x \in \{w, b\}$ and $\pi, \mu \in [0, 1]$ such that $u_\pi(x), u_\mu(x)$ are defined, $u_\pi(x) = u_\mu(x)$. In simple terms, all utility functions that contain the worst prize in their domains will evaluate it identically, and, similarly, all utility functions that include the best prize in their domains evaluate it identically. For example, u_0 will not include w in the domain but all other utility functions must be defined on w , which means we must have $u_\pi(w) = u_\mu(w)$ for all $\pi, \mu \in (0, 1]$.

We sometimes refer to the common value for the utility of the worst prize as simply $u(w)$ and omit the subscript that indexes the utility functions. We do the same for the best prize and sometimes write $u(b)$. We then impose the condition that $u_\pi(x) \in [u(w), u(b)]$ for all $x \in (w, b)$ and $\pi \in [0, 1]$

2. We do, however, adopt this approach occasionally in Section 3 for convenience when we introduce special cases.

3. $u_0(x)$ is not defined for $x \in [w, d]$, and $u_1(x)$ is not defined for $x \in X \setminus [w, d]$.

such that $u_\pi(x)$ is defined.

Finally, we impose the condition that, if $d \neq b$, then for any $x \in (w, b)$ there exist $\pi, \mu \in (0, 1)$ such that $u_\pi(x) \neq u_\mu(x)$. To understand what this condition implies, note that we have a standard expected utility maximizer when the DM uses the same utility function to evaluate all lotteries. This would happen if $d = b$, as all lotteries would assign probability 1 to disappointing prizes. For other cases, the DM uses multiple utility functions, and the condition ensures that there is sufficient variation in the way that a prize is evaluated.

We say that a set of functions $\mathcal{U} = \{u_\pi | \pi \in [0, 1]\}$, consisting of real valued functions, that satisfies all these properties is **contextual**.

Definition 1. A preference relation \succsim has an expected contextual utility (ECU) representation iff there exists $d \in [w, b]$ and a set of utility functions \mathcal{U} that is contextual, such that, for any $p, q \in \mathcal{L}$, $p \succsim q$ iff $V(p) \geq V(q)$, where for any $r \in \mathcal{L}$, if $r([w, d]) = \pi$,

$$V(r) = \int u_\pi dr.$$

3.2 Representation Theorem

We now present the axioms that are necessary and sufficient for an ECU representation. In what follows, we may describe a lottery p as $p = (x_1, p_1; \dots; x_n, p_n)$ where, for any $1 \leq i \leq n$, $x_i \in X$ and p_i is the probability that x_i is obtained. In some cases, we may have $x_i = x_j$ for $i \neq j$. In this case, denoting this common prize as x , the probability that x is obtained in p is $\sum_{\{i: x_i=x\}} p_i$. The first several axioms are straightforward.

Axiom 1. \succsim is complete and transitive.

We let \succ and \sim denote the asymmetric and symmetric components of \succsim , respectively.

Axiom 2 (Monotonicity). $\forall t_1, t_2 \in [0, 1]$, $t_1\delta_b + (1-t_1)\delta_w \succsim t_2\delta_b + (1-t_2)\delta_w$ iff $t_1 \geq t_2$.

Axiom 2 is a natural restriction which says that, when we have two lotteries where both are combinations of the worst and best prizes, the one that places greater weight on the best prize is preferred.

Axiom 3 (Replacement Monotonicity). For any $x \in X$ and $\alpha \in [0, 1]$, we have

$$\alpha\delta_b + (1 - \alpha)\delta_x \succsim \alpha\delta_b + (1 - \alpha)\delta_w \succsim \alpha\delta_x + (1 - \alpha)\delta_w.$$

To understand this axiom, consider first the middle lottery in the expression of the axiom, which is a combination of the best and worst prizes. The first preference relation says that, if we replace the worst prize with any other x , then the resulting lottery is preferred over the middle lottery. Similarly, the second preference relation says that, if we replace the best prize b with any other x , then this makes the new lottery less preferable than the middle lottery. We call this the replacement monotonicity axiom for this reason.

Axiom 4 (Weak Solvability). For any $p \in \mathcal{L}$, there exists $\gamma \in [0, 1]$ such that $p \sim \gamma\delta_b + (1 - \gamma)\delta_w$.

Axiom 4 states that any lottery can be made indifferent to a combination of the worst and best prizes. It resembles the solvability of Dekel (1986), but we restrict attention to combinations of the best and worst prizes only. As such, we call this the weak solvability axiom.

Given weak solvability, we can define a value ϕ_x for each $x \in X$ where $\delta_x \sim \phi_x\delta_b + (1 - \phi_x)\delta_w$. Note that this value is unique because of Axiom 2. We use the values ϕ_x to define the set

$$\mathcal{D} = \{x \in [w, b) : \alpha\delta_x + (1 - \alpha)\delta_w \sim \alpha\phi_x\delta_b + \alpha(1 - \phi_x)\delta_w + (1 - \alpha)\delta_w \forall \alpha \in [0, 1]\}.$$

w is included in the set, so $\mathcal{D} \neq \emptyset$. Therefore, we let $\tilde{d} = \sup \mathcal{D}$. It turns out that \tilde{d} equals the threshold value d if \succsim has an ECU representation and hence can be used as the threshold value for disappointing prizes when constructing an ECU representation from preferences. This ends up being true because $\mathcal{D} = [w, d]$ when $d \neq b$ and $\mathcal{D} = [w, b)$ when $d = b$.

The intuition that explains why this is true is quite simple. First, consider the case in which $d \neq b$. If $x \in [w, d]$, then lottery $\alpha\delta_x + (1 - \alpha)\delta_w$ on the left side of \sim in the definition of \mathcal{D} would place probability 1 on disappointing prizes. As such, the lottery would be evaluated as $\alpha u_1(x) + (1 - \alpha)u_1(w)$. We can rewrite the first part as a convex combination of the utilities of the best and worst prizes to obtain

$$\alpha\phi_x u(b) + \alpha(1 - \phi_x)u(w) + (1 - \alpha)u(w).$$

As this would be the expected utility of lottery $\alpha\phi_x\delta_b + \alpha(1 - \phi_x)\delta_w + (1 - \alpha)\delta_w$, we obtain the indifference relation as in the definition of \mathcal{D} . Note that this argument is independent of the value of α and allows us to conclude $x \in \mathcal{D}$.

For any $x \in (d, b)$, it will be the case that $x \notin \mathcal{D}$. This tells us that $\mathcal{D} = [w, d]$. As a result, we can take the supremum of the set to pin down the threshold value. To see why this is the case, suppose that $x \in (d, b)$. We then have $V(\delta_x) = u_0(x)$, so the value ϕ_x such that $\delta_x \sim \phi_x\delta_b + (1 - \phi_x)\delta_w$ can be found by solving equation $u_0(x) = \phi_x u(b) + (1 - \phi_x)u(w)$. Because $d \neq b$, by assumption of an ECU representation there exists a $\alpha \in (0, 1)$ such that $u_\alpha(x) \neq u_0(x)$. Thus, if we take $p = (1 - \alpha)\delta_x + \alpha\delta_w$, we have

$$\begin{aligned} V(p) &= (1 - \alpha)u_\alpha(x) + \alpha u_\alpha(w) \\ &\neq (1 - \alpha)u_0(x) + \alpha u_\alpha(w) \\ &= (1 - \alpha)\phi_x u(b) + (1 - \alpha)(1 - \phi_x)u(w) + \alpha u(w) \\ &= V((1 - \alpha)\phi_x\delta_b + (1 - \alpha)(1 - \phi_x)\delta_w + \alpha\delta_w). \end{aligned}$$

This implies that we do not have $(1 - \alpha)\delta_x + \alpha\delta_w \sim (1 - \alpha)\phi_x\delta_b + (1 - \alpha)(1 - \phi_x)\delta_w + \alpha\delta_w$, which signifies $x \notin \mathcal{D}$.

For the case where $d = b$, if we take any $x \in [w, b)$, we can use the same argument for the case of $d \neq b$ and $x \in [w, d]$ to show that $x \in \mathcal{D}$. This gives us $\mathcal{D} = [w, b)$. We use the value \tilde{d} to define

our final axiom, which we introduce after providing a lemma.

Axioms 1-4 allow us to prove the following lemma, which states that, if we have a lottery consisting of a convex combination of some prize x with either w or b , so that we have $\alpha\delta_x + (1-\alpha)\delta_y$ where y is b or w , we can replace the component $\alpha\delta_x$ with some combination of the best and worst prize and maintain indifference. Thus, we can think of this lemma as the statement that we have solvability of the component $\alpha\delta_x$ with respect to δ_w and δ_b .

Lemma 1. *For any $x \in X, y \in \{w, b\}, \alpha \in [0, 1]$, there exists $\gamma \in [0, 1]$ such that $\alpha\delta_x + (1-\alpha)\delta_y \sim \alpha\gamma\delta_b + \alpha(1-\gamma)\delta_w + (1-\alpha)\delta_y$*

Proof. See the appendix. □

With this lemma in hand, we move on to define several values we need to formulate the final axiom. For any disappointing prize $x \in [w, \tilde{d}]$ and $\alpha \in [0, 1]$, we define the value ϕ_x^α by taking lottery $\alpha\delta_x + (1-\alpha)\delta_b$ and letting

$$\alpha\delta_x + (1-\alpha)\delta_b \sim \alpha\phi_x^\alpha\delta_b + \alpha(1-\phi_x^\alpha)\delta_w + (1-\alpha)\delta_b.$$

Notice that there is a probability α that prize x is won in the lottery on the left side. We are simply replacing that prize x with a combination of b and w , with the weight on b being ϕ_x^α to obtain the lottery on the right. Thus, the value ϕ_x^α tells us the weight we need on b to obtain indifference. These values are well-defined by Lemma 1.

To understand why we define these values as we do, it is useful to think about how the standard expected utility model works. In the standard model, if we were to take any prize $x \in [w, b]$, we could make it indifferent to the convex combination of the best and worst prizes by finding a value of ϕ_x such that $u(x) = \phi_x u(b) + (1-\phi_x)u(w)$. The value ϕ_x is the weight that needs to be placed on the best prize. Now, given any lottery p with $x \in \text{supp}(p)$, we can generate a lottery that is indifferent to p by replacing x with a convex combination of best and worst prizes with weight $\phi(x)$ on b . For

example, if $p = (w, 0.2; x; 0.8)$ then

$$\begin{aligned} V(p) &= 0.2u(w) + 0.8u(x) \\ &= 0.2u(w) + 0.8\phi_x u(b) + 0.8(1 - \phi_x)u(w) \end{aligned}$$

, so that $p \sim (w, 0.2 + 0.8(1 - \phi_x); b, 0.8\phi_x)$. This helps us pin down the utility values when constructing an expected utility representation, usually by setting $u(x) = \phi_x$.

We need a similar condition for our model. That is, we want to be able to replace a prize x with a combination of b and w . There is however a slight complication. Note that the value of ϕ_x in the argument for the standard expected utility model is independent of lottery p . More precisely, it is independent of the probability value that a lottery assigns to disappointing prizes. This is not the case for our model because the utility function used to evaluate a lottery depends on the probability that prizes have disappointing values. When $p([w, d]) = \alpha$, the utility derived in this lottery by prize x may be $u_\alpha(x)$, such that $u_\alpha(x) = \phi u(b) + (1 - \phi)u(w)$ for some ϕ in the ECU.

For $q \in \mathcal{L}$ such that $q([w, d]) = \beta \neq \alpha$, however, the utility from prize x may be $u_\beta(x) \neq u_\alpha(x)$. In this case, we will not have $u_\beta(x) = \phi u(b) + (1 - \phi)u(w)$ if we use the same ϕ as above. Therefore, we need dependence on α and require the use of ϕ_x^α . The method for finding this value depends on whether $x \in [w, \tilde{d}]$ or not.

First, for $x \in [w, \tilde{d}]$ and $\alpha \in (0, 1]$, we need lottery $\alpha\delta_x + (1 - \alpha)\delta_b$ because it places probability α on disappointing prizes (unless b is also considered disappointing, but we do not need to vary α in this case as this reduces ours to a standard expected utility model) and the value in the ECU would be computed according to

$$\alpha u_\alpha(x) + (1 - \alpha)u(b).$$

We let ϕ_x^α be the value such that

$$\alpha\delta_x + (1-\alpha)\delta_b \sim \alpha\phi_x^\alpha\delta_b + \alpha(1-\phi_x^\alpha)\delta_w + (1-\alpha)\delta_b.$$

We would then be able to use ϕ_x^α as $u_\alpha(x)$.

We need similar values for $x \notin [w, \tilde{d}]$ and $\alpha \in [0, 1)$. In these cases, we take lottery $\alpha\delta_w + (1-\alpha)\delta_x$ and let ϕ_x^α be the value such that

$$\alpha\delta_w + (1-\alpha)\delta_x \sim \alpha\delta_w + (1-\alpha)\phi_x^\alpha\delta_b + (1-\alpha)(1-\phi_x^\alpha)\delta_w.$$

The idea again is that we are replacing x with a combination of b and w with weight ϕ_x^α on b . Because x is not a disappointing prize, lottery $\alpha\delta_w + (1-\alpha)\delta_x$ places probability α on disappointing prizes.

We can use these values to state the final axiom.⁴ This axiom says simply that, if we have a lottery that places probability α on disappointing prizes, we can substitute each prize in the support with the “right combinations” of b and w to maintain indifference. We say “right combinations” because the weights depend on the lottery’s context, which is the probability placed on what seem to be disappointing prizes.

Axiom 5 (Contextual Substitutability). For any $\alpha \in [0, 1]$ and $p \in \mathcal{L}$ such that $p([w, \tilde{d}]) = \alpha$,

$$p \sim \sum_{x \in \text{supp}(p)} p(x)[\phi_x^\alpha\delta_b + (1-\phi_x^\alpha)\delta_w].$$

Theorem 1. *Preference relation \succsim has an ECU representation iff it satisfies completeness, transitivity, monotonicity, replacement monotonicity, weak solvability, and contextual substitutability.*

Proof. See the appendix. □

4. Although we do not state it as a formal axiom, we impose the condition that, if $\tilde{d} \neq b$, then for any $x \in (w, b)$ there exists $\mu, \pi \in (0, 1)$ such that $\phi_x^\pi \neq \phi_x^\mu$.

3.3 Uniqueness

Regarding the uniqueness of the parameters, first note that there are many cases in which the representation reduces to the case of an expected utility representation. If $d = b$, all lotteries are evaluated according to u_1 . That being the case, we can let u_α be any function for all $\alpha \in (0, 1)$ as long as the conditions for an ECU representation such as $u_\alpha(w) = u_\beta(w)$ for all $\alpha, \beta \in (0, 1]$ are satisfied. Therefore, d is identified in this case and u_1 is unique up to a positive linear transformation, but there is no meaningful uniqueness result for any other u_α with $\alpha \in (0, 1)$.

For all other cases with $d \neq b$, however, we can obtain a sharper uniqueness result. Namely, d is unique and the functions are unique up to positive linear transformations. The constants for the linear transformations must be the same for all utility functions. We state this result formally as Theorem 2.

Theorem 2. *Suppose preference relation \succsim has ECU representations with $\{d_1, (u_\alpha)_{\alpha \in [0,1]}\}$ where $d_1 < b$. Then $\{d_2, (v_\alpha)_{\alpha \in [0,1]}\}$ is another ECU representation of \succsim iff $d_1 = d_2$ and $v_\alpha = ku_\alpha + c$ for $k \in \mathbb{R}_{++}$ and $c \in \mathbb{R}$ for all $\alpha \in [0, 1]$.*

Proof. See the appendix. □

4 Special Cases

4.1 Preference for Stochastic Dominance

For the sake of generality, we have not imposed many restrictions on the utility functions up to this point, but we now introduce restrictions on the ECU that ensure preferences for first-order stochastic dominance. We use the standard definition for first-order stochastic dominance.

Definition 2. Lottery $p \in \mathcal{L}$ **first-order stochastically dominates** lottery $q \in \mathcal{L}$ iff for any $x \in X$, $p([w, x]) \leq q([w, x])$.

To ensure that first-order stochastically dominant lotteries are weakly preferred over those they dominate, we impose the following conditions on the set of utilities in addition to requiring them to be contextual.

Condition 1. u_π is non-decreasing for all $\pi \in [0, 1]$.

Condition 2. $u_\pi(x) \geq u_\mu(x)$ for any $x \in X$ and $0 \leq \pi \leq \mu \leq 1$ such that $u_\pi(x), u_\mu(x)$ are defined.

The second condition says that the DM becomes more pessimistic and assign lower values to prizes as disappointments become more likely. This pessimism ensures that the represented preferences satisfy preferences for first-order stochastic dominance.

Proposition 1. *If \succsim has an ECU representation with a set of contextual functions that satisfies Conditions 1 and 2, then $p \succsim q$ for any $p, q \in \mathcal{L}$ such that p first-order stochastically dominates q .*

Proof. See the appendix. □

4.2 Binary Utilities

In principle, the ECU allows a distinct utility function for each value of the probability that disappointment occurs. This means that a DM may use infinitely many utility functions. It could also be the case that the same utility function is used for many values of disappointment probability and the DM uses only a small and finite number of utility functions. This may be more descriptively appealing for those who may worry that it is unrealistic to allow a DM to evaluate lotteries with so many utility functions.

We provide a very simple example in which the DM only has two distinct utility functions, as though the DM uses one utility function for all lotteries that are associated with “low” probabilities of disappointment and another for those that are associated with “high” probabilities of disappointment. We provide this special case because this simple example turns out to be sufficiently flexible

to generate much of the behavior that we want to capture. We discuss this point further in Section 4. The example will even satisfy Conditions 1 and 2 so that it satisfies preferences for first-order stochastic dominance.

We introduce a value $\tau \in [0, 1]$ that serves as a threshold level of probability for what the DM considers to be a low level of disappointment. If a lottery assigns a probability that is less than or equal to τ to prizes whose values are less than or equal to d , the DM evaluates the lottery using one strictly increasing utility function $u : X \rightarrow \mathbb{R}$. When the probability that disappointing prizes are awarded exceeds τ , this becomes a concern for the DM and causes a change in risk attitude. Formally, there is a strictly increasing function $v : X \rightarrow \mathbb{R}$, and the DM evaluates a lottery using this utility function when $p([w, d]) > \tau$. Condition 1 is satisfied because the two functions are strictly increasing.

To the extent that we have an ECU, we need $v(w) = u(w)$ and $v(b) = u(b)$. We also impose $v(x) < u(x)$ for $x \in (w, b)$ to ensure that the two utility functions constitute a contextual set and that Condition 2 is satisfied. Note that u plays the role of u_α for all $\alpha \in [0, \tau]$. We call this a **binary ECU**.

4.3 Disappointment Probabilities as Utility Parameters

Here, we offer one simple example that allows us to define an ECU with an infinite number of utility functions using just the parameter value π of the probability placed on disappointing prizes. The example will satisfy Conditions 1 and 2 so that preferences for first-order stochastic dominance is ensured.

For any $\pi \in [0, 1]$ and $x \in X$, we let

$$u_\pi(x) = \left(\frac{x-w}{b-w} \right)^{0.5+\pi}.$$

To see that this set of utility functions is contextual, note that $u_\pi(w) = 0$ and $u_\pi(b) = 1$ for all

$\pi \in [0, 1]$. It is also the case that u_π is strictly increasing for all $\pi \in [0, 1]$ so that Condition 1 is satisfied along with the condition that $u_\pi(x) \in [u(w), u(b)]$ for all x and π such that $u_\pi(x)$ is defined. Lastly, for any $0 \leq \pi < \mu \leq 1$, and $x \in (w, b)$ we know that $u_\pi(x) > u_\mu(x)$ so that Condition 2 is satisfied, ensuring that, for any $x \in (w, b)$, there exists $\pi, \mu \in [0, 1]$ such that $u_\pi(x) \neq u_\mu(x)$.

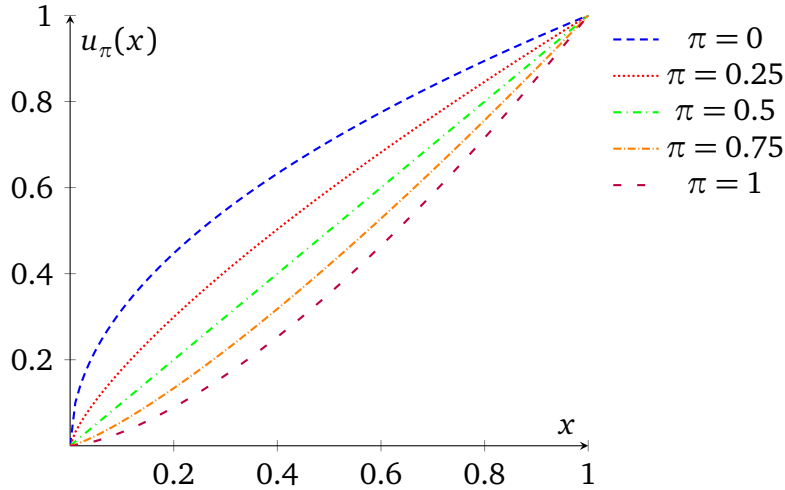


Figure 1: $u_\pi(x) = \left(\frac{x-w}{b-w}\right)^{0.5+\pi}$ with $w = 0, b = 1$

Note that we constructed the functional form so that the DM's risk attitude changes from risk-averse to risk-loving after a certain threshold as the probability placed on disappointing prizes increases. In Figure 1, for example, the utility function is concave when $\pi < 0.5$, linear when $\pi = 0.5$, and convex when $\pi > 0.5$.

5 Resulting Behavior

5.1 Preferences that are Inconsistent with Cumulative Prospect Theory

Our model allows us to accommodate many types of realistic preferences that seemingly stem from changing risk attitudes but may not be consistent with some popular models. The preference relation

provided in the Introduction is one such example. Recall that, in the example, the DM prefers the riskier lottery when both lotteries seem bound for doom/disappointment but prefers the lottery with less risk when a disappointing prize is replaced by a moderately valuable prize. We showed in the example that the preferences could not be generated using the cumulative prospect theory. To show how the ECU can engender such behavior, consider the following example of a binary ECU representation.

Example 1. Suppose we have a binary ECU where $X = [0, 300]$, $d = 20$, $\tau = 0.75$, and we have strictly increasing u such that $u(0) = 0, u(90) = 15, u(100) = 20, u(150) = 50, u(200) = 60$. Suppose further we have utility function v such that $v(0) = 0, v(90) = 8, v(100) = 10, v(150) = 25, v(200) = 50$. \square

It is easy to verify that the specifications in Example 1 are consistent with strictly increasing u and v and that we have $u(x) > v(x)$ for the values of x that we specified above other than 0. Consequently, we know that a binary ECU with the above specification exists. For the discussion that follows in this section we write an arbitrary lottery p as $p = (x_1, p_1; \dots; x_n, p_n)$ where, for any $1 \leq i \leq n$, $x_i \in X$ and p_i is the probability that x_i is obtained. In some cases, we may have $x_i = x_j$ for $i \neq j$. In this case, denoting this common prize as x , the probability that x in p is obtained is $\sum_{\{i: x_i=x\}} p_i$. Under this ECU, when the DM evaluates lotteries $p = (0, 0.9; 100, 0.05; 200, 0.05)$ and $q = (0, 0.9; 150, 0.1)$, the utility function v is used because these lotteries place too high a probability on the awarding of disappointing prizes. This gives us

$$V(p) = \int v dp = 0.9v(0) + 0.05v(100) + 0.05v(200) = 3$$

$$V(q) = \int v dq = 0.9v(0) + 0.1v(150) = 0.25.$$

This means that we have $(0, 0.9; 100, 0.05; 200, 0.05) \succ (0, 0.9; 150, 0.1)$. When we take lotteries $p' = (90, 0.9; 100, 0.05; 200, 0.05)$ and $q' = (90, 0.9; 150, 0.1)$, however, the DM uses utility function u to evaluate the lotteries because replacing 0 with 90 has pushed the probability that disappointments

are awarded below threshold τ . Therefore,

$$V(p') = \int u dp' = 0.9u(90) + 0.05u(100) + 0.05u(200) = 17.5$$

$$V(q') = \int u dq' = 0.9u(90) + 0.1u(150) = 22.5,$$

This gives us $(0, 0.9; 100, 0.05; 200, 0.05) \prec (0, 0.9; 150, 0.1)$, thus generating a change in risk-taking behavior from our initial example.

More generally, to generate preferences of the same type as in the introduction, suppose there is a common worst prize, say x , for two lotteries that has been assigned the same probability weight in both lotteries and is then replaced by a higher prize y . We can generate these types of preferences by choosing d to be in (x, y) and choosing τ to be lower than the probability weight assigned to x in both lotteries.

5.2 Allais Paradox

We next turn our attention to preferences associated with the common ratio and common consequence effects of the Allais (1953) Paradox. We use the specific preferences considered in the experiment conducted by Kahneman and Tversky (1979) to consider these two effects. Although the idea seems to carry over from the previous example inasmuch as DMs tend to be risk-loving when disappointment is highly likely but risk-averse when it becomes less likely, the way we modify the lotteries is slightly different from how Kahneman and Tversky (1979) did.

For the common consequence effect, we again use a binary ECU to generate the observed preferences.

Example 2. Suppose we have a binary ECU with $X = [0, 3000]$, $d = 10$, $\tau = 0.5$, and a strictly increasing u such that $u(0) = -1000$, $u(10) = 0$, $u(2400) = 2600$, $u(2500) = 2615$. Suppose further that we have a utility function v such that $v(0) = -1000$, $v(10) = -1$, $v(2400) = 2400$, $v(2500) =$

2500. \square

Take $p = (2500, 0.33; 0, 0.67)$ over $q = (2400, 0.34; 0, 0.66)$. With the specified model, we have $(2500, 0.33; 0, 0.67) \succ (2400, 0.34; 0, 0.66)$, and therefore the DM prefers the riskier lottery of the two when the probability that disappointment occurs exceeds τ . Suppose we replace the 0.66 probability that 0 prizes are awarded in both lotteries with 2400 prizes. The two lotteries, $q' = (2400, 1)$ and $p' = (2500, 0.33; 2400, 0.66; 0, 0.01)$, are evaluated using u instead of v because the probability that a disappointment occurs vanishes. This gives us preference $q' \succ p'$, and in this case the DM prefers the less risky lottery.

We use a binary ECU also for the common ratio effect.

Example 3. Suppose we have a binary ECU with $X = [0, 7000]$, $d = 1000$, $\tau = 0.9$, and a strictly increasing u such that $u(0) = 0$, $u(1000) = 2$, $u(3000) = 25$, $u(6000) = 40$. Suppose further that we have a strictly increasing utility function v such that $v(0) = 0$, $v(1000) = 1$, $v(3000) = 10$, $v(6000) = 30$. \square

With these parameter values, a simple calculation shows that we have $(6000, 0.001; 0, 0.999) \succ (3000, 0.002; 0, 0.998)$. Again, the DM prefers the riskier lottery when the probability that disappointment occurs is high. When we consider lotteries $(3000, 0.9; 0, 0.1)$ and $(6000, 0.45; 0, 0.55)$, where the disappointing prizes are less than or equal to the threshold value, though, we obtain $(3000, 0.9; 0, 0.1) \succ (6000, 0.45; 0, 0.55)$, so the DM is risk-averse when considering these lotteries.

5.3 Violation of the Betweenness Axiom

In addition to the class of rank-dependent models that distort probability weights, another important branch of the literature that studies axiomatic models of non-expected utility employs the class of models that satisfy the axiom of betweenness. The betweenness axiom is weaker than the independence axiom and we define it here.

Definition 3. A preference relation \succsim satisfies **betweenness** if, for any $p, q \in \mathcal{L}$, $p \succ q$ implies $p \succ \alpha p + (1 - \alpha)q$ for all $\alpha \in (0, 1)$, and $p \sim q$ implies $p \sim \alpha p + (1 - \alpha)q$ for all $\alpha \in [0, 1]$.

In reference to next set of preferences we discuss, we show how a DM whose risk-taking behavior varies can cause a violation of Betweenness. By showing that such preferences can be consistent with the ECU, we demonstrate that our model does not fall into the class of betweenness models. Unlike the examples presented thus far, this example involves a preference for a riskier lottery when the probability that a prize is disappointing is high while a less risky lottery is preferred when the probability is low. This case is similar to the idea presented by Honda (2022).

Consider lottery $p = (\$50, 1)$ and lottery $q = (\$100, 0.5; \$20, 0.5)$. Suppose that the DM considers the 0.5 probability that the lower prize of \$20 is awarded to be disturbingly high and so prefers the less-risky lottery p . Now, consider lottery $0.6p + (1 - 0.6)q = (\$100, 0.2; \$50, 0.6; \$20, 0.2)$. The lower probability that the disappointing \$20 is awarded causes the DM to consider it negligible. This encourages the DM to become risk-loving and take a chance on achieving the highest prize of \$100. This results in $0.6p + (1 - 0.6)q \succ p$, which is a violation of betweenness.

Example 4. Suppose we have a binary ECU. Let $X = [0, 200]$, $d = 30$, $\tau = 0.3$, with a strictly increasing u such that $u(20) = 8$, $u(50) = 10$, $u(100) = 200$. Suppose further that we have a utility function v such that $v(20) = 4$, $v(100) = 12$. \square

Lottery p places probability 0 on disappointing prizes so its value based on the ECU representation is $V(p) = 1u(50) = 10$. Lottery q , on the other hand, assigns a probability higher than $\tau = 0.3$ to prizes that are less than or equal to $d = 30$, so its value is given by $V(q) = 0.5v(100) + 0.5v(20) = 8$. This gives us $p \succ q$. In contrast, lottery $0.6p + (1 - 0.6)q$ assigns only a 0.2 probability to the \$20 prize, and this means that the lottery will be evaluated according to u . We thus have $V(0.6p + (1 - 0.6)q) = 0.2u(100) + 0.6u(50) + 0.2u(20) = 11.6$, which implies that $0.6p + 0.4q \succ p$. The intuition here is that the DM uses function v when evaluating lottery q because the high probability of disappointment makes the DM pessimistic. The lower probability that disappointment occurs when we take

the mixture $0.6p + 0.4q$, however, makes the DM feel optimistic and leads the DM to use function u when evaluating the lottery.

5.4 Illustrating Changes in Risk Attitudes

To provide a clearer intuition for how the model gives rise to all the mentioned behaviors, we illustrate the familiar Marschak-Machina triangles for the binary ECU. As usual, we let the top left corner represent the largest prize (H) among the three vertices, while the bottom left corner represents the medium value (M) and the bottom right corner represents the lowest value (L). Whenever all three prizes are greater than d ($H > M > L > d$), the MM-Triangle is identical to that of the standard expected utility. This is also the case if $d > H > M > L$. Thus, the only interesting cases are $H > M > d > L$ and $H > d > M > L$. We present these two cases in that order.

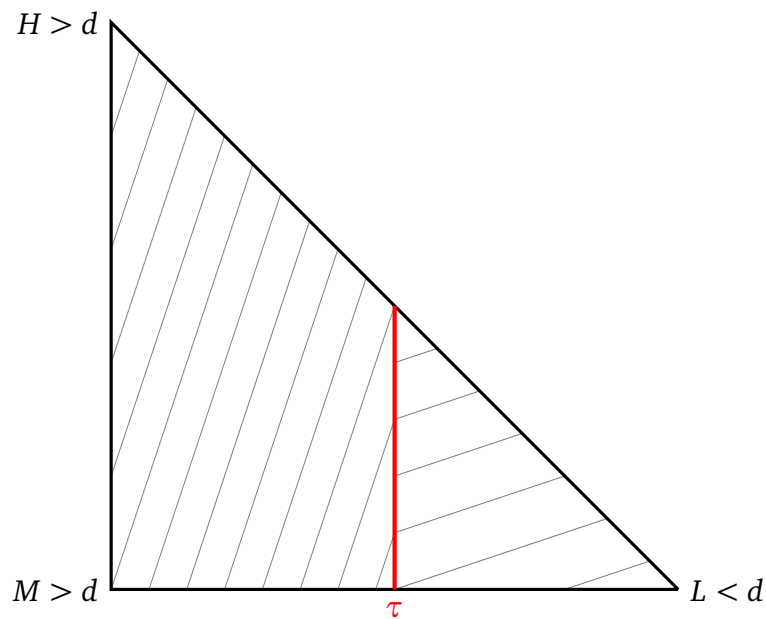


Figure 2: $H > M > d > L$

For this first case, because L is the only prize below d , we have the usual expected utility maximizer using function u as long as the probability placed on L is less than or equal to τ . Once the probability rises above τ at the red line, the DM switches to function v and this can generate the change in slopes of the indifference curves. So, instead of a gradual “fanning out”, we see a sudden fanning out when we leave the bottom right region of the triangle.

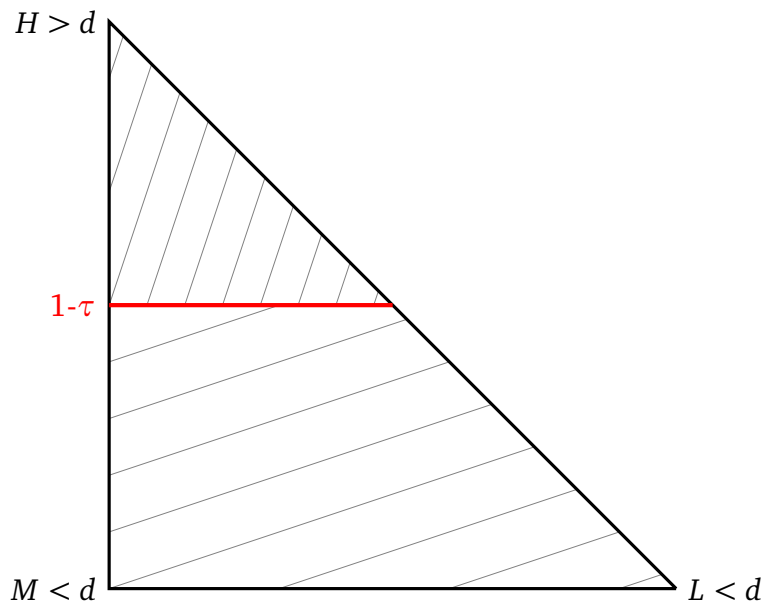


Figure 3: $H > d > M > L$

The idea informing the second case is similar. When H is the only prize above the threshold, we have a standard expected utility maximizer using function v whenever the probabilities placed on M and L exceed τ . This means the probability placed on H comes in below $1 - \tau$. When the value of H exceeds $1 - \tau$ at the red line, we obtain the change in functions from v to u .

Additionally, we provide the types of indifference curves given in Figure 4 of Gul (1991) using another binary ECU example. For this graph, we let the horizontal axis represent the value of a prize X that occurs in a lottery at probability p and the vertical axis represent the value of a prize Y that occurs in the same lottery at probability $1 - p$. So point (x, y) in our diagram corresponds

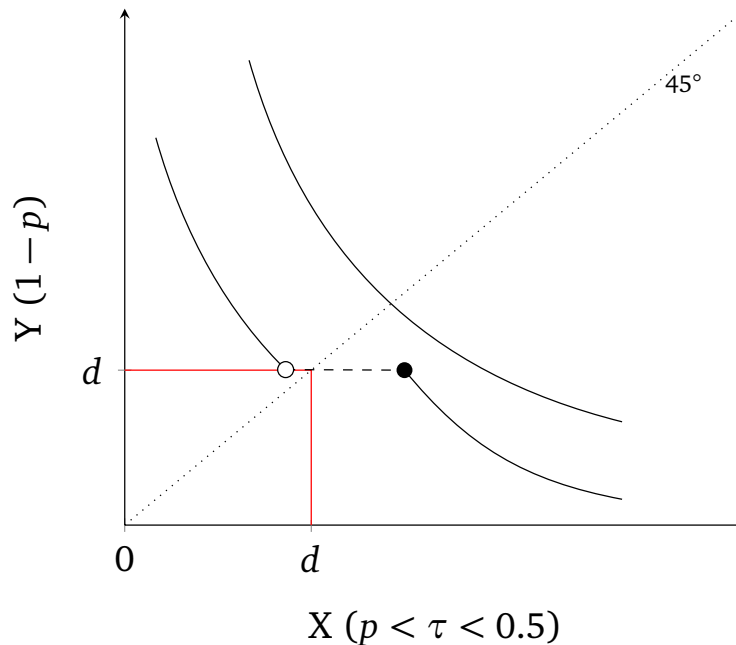


Figure 4: Gul (1991) Indifference Curve

to a lottery that awards prize x at probability p and prize y at probability $1 - p$. We focus on the case in which $p < \tau < 0.5$. Each indifference curve corresponds to lotteries that are indifferent to each other. For ease of comparison, we draw these curves for the case in which we obtain smooth indifference curves when the DM is a standard expected utility maximizer.

For Figure 4, in the bottom indifference curve we initially see the usual smooth curve until Y drops to value d because we have a standard expected utility maximizer using function u as a result of $p < \tau$. Once the value of Y reaches d , the probabilities of all prizes are less than or equal to d , and so the DM switches to function v . When this happens, the utility that the DM derives from prize Y suddenly drops from $u(Y)$ to $v(Y)$. To keep the DM indifferent, the value of X needs to rise much higher to compensate for this change, and hence there is a discontinuity at that point. As the higher indifference curve shows, we have a smooth indifference curve without any discontinuity whenever the probability placed on prizes that are less than or equal to d is below τ . We obtain this result because, of course, the DM will always use u to evaluate such lotteries.

6 Experiment

6.1 Experimental Design and Hypotheses

To explore a direct implication of our model, we are interested in creating reversals in participants' risk attitudes that are not compatible with rank-dependent utility models. We can achieve such reversals by creating rows of binary comparisons between two lotteries in a choice list or multiple price list table format, similar to Holt and Laury (2002), Chapman et al. (2017), and, more recently, Bernheim and Sprenger (2020). We include screenshots of the software interface used in the experiment in the appendix.

Each row of the table represents a task, which is our basic unit of observation. Each task represents a choice between 'Option A' and 'Option B,' where Option A is a two-outcome lottery and Option B is a three-outcome lottery, much as in the original Allais example. We group tasks into three stages for distinct purposes. While the actual experiment included a third stage as a robustness check, we first focus on the two main stages. For the third stage, please see the appendix for details.

In Stage 1, we present a table listing ten tasks. Holding all else equal, each task differs only by the size of its worst prize. For example, as shown in the screenshot of the experiment's software interface in Figure 7, we increase the size of the worst prize for both Option A and Option B by increments of 44 points within a range running from 0 to 400 points. Each point is converted to \$0.01 USD. We constructed the prizes using the same ratio as we used in the motivating example.

If a DM exhibits context-dependent risk attitudes, and if some of these values are disappointing, we expect the DM to switch from one option to another. Within the ECU framework, we can interpret such a switch as a demonstration of a scenario in which a participant uses multiple utility functions that vary with the context. For instance, comparing the first and the last rows of the table on their screen, a participant may opt to use one utility function for a disappointing outcome 0 that seems too

likely and another utility function for a less disappointing outcome of 400 points. Our first hypothesis tests whether participants indeed exhibit such context-dependent risk-preference reversals.

Hypothesis 1. *In Stage 1, participants will switch choices from Option A to Option B.*

We have fixed the ranks of the prizes as well as their corresponding probabilities, so any switch between the two columns violates expected utility/rank-dependent utility/cumulative prospect theory. Note that, if we focus only on a participant’s choices in Task 1 and Task 10, the format is analogous to that of the original Allais experiment and a choice reversal (choosing Option A in Task 1 and Option B in Task 10, or vice versa) between the two tasks is sufficient to demonstrate the violation of expected utility/rank-dependent utility/cumulative prospect theory, as argued previously. Adding the intermediary rows allows for better resolution for understanding their choices with respect to the context.

The interpretation of such choice reversals may, however, be more complicated than the Allais reversal with two tasks. For example, without restrictions on our general model, a switch from Option A to B in Task #2 implies that threshold d falls below the worst outcome in Task #3. In other words, in this experimental setup $d \in (-\infty, 88 \text{ points})$. Furthermore, while our general model imposes no restrictions on the number of switches that can be made across the ten rows, the interpretation of multiple switches is even less clear with potentially confounding factors, including irregular switching (Chew et al. (2022)) and a preference for randomization (Agranov and Ortoleva (2017) and Feldman and Rehbeck (2022)).

In this spirit, to facilitate the interpretation of the disappointment threshold d and the tolerance threshold τ , we restricted participants to making only one switch in the experiment.⁵ That is, we asked participants to draw a line above which they would prefer Option A and below which they would prefer Option B and vice versa. They can do so by clicking one of the arrows displayed in the center column of the table. This restriction allows us to interpret the switching point with respect

5. We allowed participants to switch freely between the two columns in the first two of our pilot sessions. We discuss the pilot sessions in the appendix.

to the context more easily. For example, assuming a binary ECU, a switch between Task k and Task $k + 1$ implies that the threshold d falls between the worst outcome in Task k and the worst outcome in Task $k + 1$: $d \in (44 \times k, 44 \times (k + 1))$.

We present another table of ten tasks in Stage 2. In particular, we construct the lotteries in the following format using the estimated d from Stage 1,

$$\left(d + \frac{b-d}{2}, 1-y; 0, y\right) \text{ vs. } \left(d + \frac{3b-3d}{4}, \frac{1-y}{2}; d + \frac{b-d}{4}, \frac{1-y}{2}; 0, y\right),$$

where d is recorded as $44 \times (k + 1)$ when a participant switches between Task k and Task $k + 1$. The left-side lottery places probability y on the supposedly disappointing outcome 0 and $1 - y$ to the non-disappointing outcome $d + (b - d)/2$. The right-side lottery splits the probability $1 - y$ assigned to two non-disappointing outcomes equally. This mean-preserving split makes the right-side lottery riskier.

With this construction, we fix the worst outcome at 0 and change the respective probability y in increments. As in Stage 1, here we can interpret such a switch as a demonstration of a scenario in which a participant uses one utility function for disappointing outcome 0 as too likely and another utility function for the same disappointing outcome 0 as less likely. Note that a switch between the two columns in Stage 2 violates expected utility but not necessarily rank-dependent utility/cumulative prospect theory, as we vary probabilities across tasks. Again, restricting participants to making only one switch and assuming binary ECU, we interpret the switching point from one option to another as the tolerance threshold τ . The interpretation may not be as straightforward, however, if a participant did not switch from Option A to Option B in Stage 1. Therefore, we present our second hypothesis, which we limit to observations conditional on Stage 1 switching behavior.

Hypothesis 2. *Participants who switch in Stage 1 will switch their choice from Option A to Option B in Stage 2.*

6.2 Results

Using Prolific, we recruited 150 participants in March 2024. All recruited participants were adults with a median age of 25.5 and were fluent in English. Of the 150 participants, 58 were female and 91 were male. One participant declined to report their gender. Each participant took approximately 19.5 minutes to complete the experiment, including reading the instructions, taking a screening quiz to ensure they understood the experimental procedures, and completing all stages of the main tasks in the experiment. All participants received a \$6 show-up fee. On average, each participant earned \$9.16 including the show-up fee.

We begin by demonstrating that a majority of participants indeed switched from one column to the other, providing support for the presence of context-dependent risk attitudes.

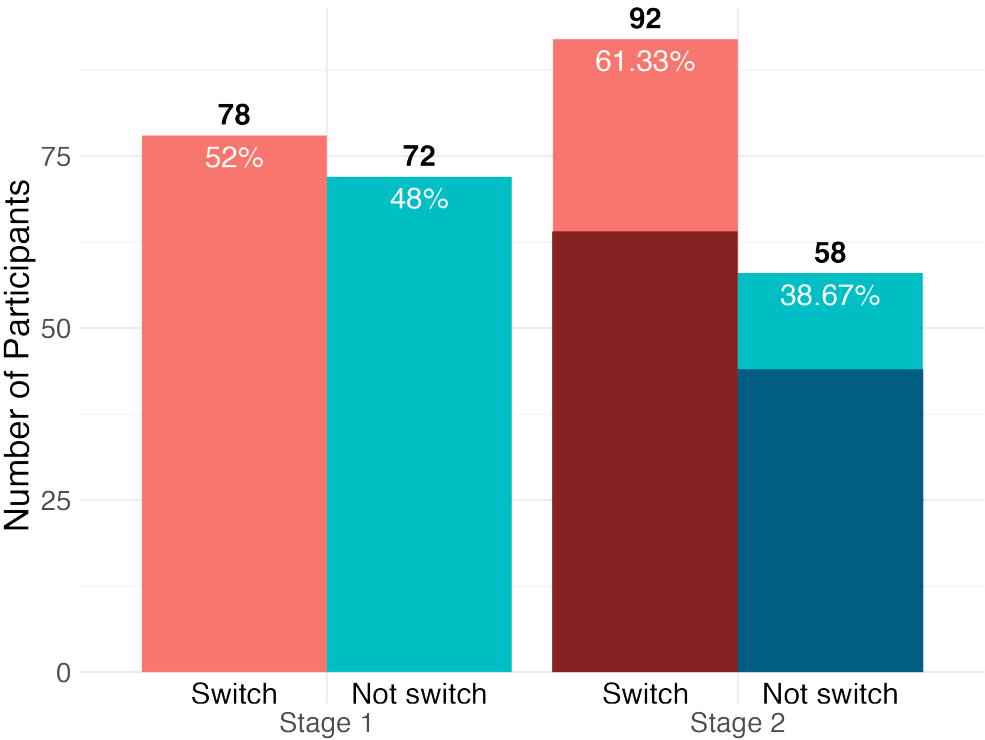


Figure 5: In our data, context-dependent risk-attitude reversals were observed in a majority of participants.

Result 1. *In stage 1, 78 out of 150 (52.0%) participants switched their choice between Option A and Option B.*

This result supports Hypothesis 1. The majority of participants switched between the options, consistent with the motivating example we began with in the introduction. That is, while we fixed the ranks of all prizes and the corresponding probabilities across the ten rows, participants responded to changes in the relative size of the worst prize. It is clear that the number of switchers is greater than zero, but one may find it less straightforward to exclude the possibility that random switching behavior is observed only in Stage 1 data. The next result in the Stage 2 data, in conjunction with the findings from the pilot sessions, suggests, however, that it is unlikely that the switching behavior resulted from a purely random motivation.

Result 2-1. *In Stage 2, 92 of 150 (61.3%) participants switched their choice from Option A to Option B.*

Result 2-2. *64 of 78 (82.1%) of participants who switched in Stage 1 did so in Stage 2. Of the 72 participants who did not switch in Stage 1, only 28 (38.9%) switched in Stage 2.*

Result 2-3. *44 of 150 (29.3%) of participants did not switch in both stages.*

These results support Hypothesis 2. The change in the probability placed on the worst outcome appears to have affected participants' risk attitudes. Furthermore, the correlation between participants' behavior in Stage 1 and Stage 2 suggests as well that the reversal in risk attitudes between the two stages are individual-specific and that the intuition that informs our understanding via both the prize and probability dimension thresholds is plausible. It is therefore possible to interpret the result that 64 of 150 participants switched as consistent with a model in which a DM exhibits context-dependent risk attitudes and the result that 44 of 150 participants did not switch (those who did not switch in both stages) is consistent with an expected utility maximizer. We also note a slight difference in participants' responses according to reported genders, but the main results are qualitatively unaffected. Please refer to the appendix C for details.

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Appendices

A Proofs

Proof of Lemma 1

Proof. Take any $x \in X$ and $\alpha \in [0, 1]$. First, consider the case where we have $y = w$. The lemma is immediately true if $\alpha = 0$ because we can take $\gamma = 1$. So suppose $\alpha \neq 0$. Consider $\alpha\delta_x + (1 - \alpha)\delta_y = \alpha\delta_x + (1 - \alpha)\delta_w$. By Axiom 4, there exists $\eta \in [0, 1]$ such that $\alpha\delta_x + (1 - \alpha)\delta_w \sim \eta\delta_b + (1 - \eta)\delta_w$. We know from Axiom 2 that η is a unique value.

We claim that $\eta \leq \alpha$. If this inequality is not true, we have $\eta > \alpha$. This implies that $\eta\delta_b + (1 - \eta)\delta_w \succ \alpha\delta_b + (1 - \alpha)\delta_w \succ \alpha\delta_x + (1 - \alpha)\delta_w$. The strict preference is implied by Axiom 2 while the weak preference is implied by Axiom 3. This contradicts $\alpha\delta_x + (1 - \alpha)\delta_w \sim \eta\delta_b + (1 - \eta)\delta_w$ and therefore establishes the claim.

Because $\eta \leq \alpha$ and $\alpha \neq 0$, we can define $\gamma = \frac{\eta}{\alpha} \in [0, 1]$. This gives us

$$\alpha\gamma\delta_b + \alpha(1 - \gamma)\delta_w + (1 - \alpha)\delta_b = \eta\delta_b + (1 - \eta)\delta_w \sim \alpha\delta_x + (1 - \alpha)\delta_w,$$

which is what we needed to show.

Next, take the case where $y = b$. The lemma is immediately true if $\alpha = 0$ because we can take $\gamma = 1$. Suppose then that $\alpha \neq 0$. Consider $\alpha\delta_x + (1 - \alpha)\delta_y = \alpha\delta_x + (1 - \alpha)\delta_b$. Axiom 4 implies that there exists $\eta \in [0, 1]$ such that $\alpha\delta_x + (1 - \alpha)\delta_b \sim (1 - \eta)\delta_b + \eta\delta_w$. We know from Axiom 2 that η is a unique value.

We claim that $\eta \leq \alpha$. If this inequality is not true, we have $\eta > \alpha$. This implies that $(1 - \eta)\delta_b + \eta\delta_w \prec (1 - \alpha)\delta_b + \alpha\delta_w \prec (1 - \alpha)\delta_b + \alpha\delta_x$. The strict preference is implied by Axiom 2 while the weak preference relation is implied by Axiom 3. This contradicts $\alpha\delta_x + (1 - \alpha)\delta_b \sim (1 - \eta)\delta_b + \eta\delta_w$

and therefore establishes the claim.

Because $\eta \leq \alpha$ and $\alpha \neq 0$, we can define $\gamma = 1 - \frac{\eta}{\alpha} \in [0, 1]$. This gives us

$$\alpha\gamma\delta_b + \alpha(1-\gamma)\delta_w + (1-\alpha)\delta_b = (1-\eta)\delta_b + \eta\delta_w \sim \alpha\delta_x + (1-\alpha)\delta_b,$$

which is what we needed to show. □

More Lemmas

We next provide additional lemmas that are useful for proving Theorem 1. The first lemma shows that the parameter we recover using set \mathcal{D} comes from the ECU representation.

Lemma 2. *If \succsim has an ECU representation, then $\tilde{d} = d$.*

Proof. Take the case where $d \neq b$. We show that $x \in \mathcal{D} \iff x \in [w, d]$. First, to show that $x \in [w, d] \implies x \in \mathcal{D}$, take $x \in [w, d]$. Because $\delta_x([w, d]) = 1$, $V(\delta_x) = u_1(x)$. Therefore, ϕ_x is the value such that $u_1(x) = \phi_x u(b) + (1 - \phi_x)u(w)$. To see this, if we let $p = \phi_x \delta_b + (1 - \phi_x)\delta_w$, then

$$\begin{aligned} V(p) &= \phi_x u(b) + (1 - \phi_x)u(w) \\ &= u_1(x) \\ &= V(\delta_x), \end{aligned}$$

so that $\delta_x \sim \phi_x \delta_b + (1 - \phi_x)\delta_w$. Now, take any $\alpha \in [0, 1]$. and define lotteries $q = \alpha\delta_x + (1 - \alpha)\delta_w$

and $r = \alpha\phi_x\delta_b + \alpha(1 - \phi_x)\delta_w + (1 - \alpha)\delta_w$. Because q places probability 1 on disappointing prizes,

$$\begin{aligned} V(q) &= \alpha u_1(x) + (1 - \alpha)u_1(w) \\ &= \alpha\phi_x u(b) + \alpha(1 - \phi_x)u(w) + (1 - \alpha)u(w) \\ &= V(\alpha\phi_x\delta_b + \alpha(1 - \phi_x)\delta_w + (1 - \alpha)\delta_w). \end{aligned}$$

This gives us $q \sim r$, and so $x \in \mathcal{D}$.

For the other direction, take any $x \in (d, b)$. Therefore, ϕ_x is the value such that $u_0(x) = \phi_x u(b) + (1 - \phi_x)u(w)$. To see this, if we let $p' = \phi_x\delta_b + (1 - \phi_x)\delta_w$, then

$$\begin{aligned} V(p') &= \phi_x u(b) + (1 - \phi_x)u(w) \\ &= u_0(x) \\ &= V(\delta_x). \end{aligned}$$

By assumption of an ECU representation, there exists a $\beta \in (0, 1)$ such that $u_\beta(x) \neq u_0(x)$. If we let $q' = (1 - \beta)\delta_x + \beta\delta_w$, we have

$$\begin{aligned} V(q') &= (1 - \beta)u_\beta(x) + \beta u_\beta(w) \\ &\neq (1 - \beta)u_0(x) + \beta u_0(w) \\ &= (1 - \beta)\phi_x u(b) + (1 - \beta)(1 - \phi_x)u(w) + \beta u(w) \\ &= V((1 - \beta)\phi_x\delta_b + (1 - \beta)(1 - \phi_x)\delta_w + \beta\delta_w). \end{aligned}$$

This means that we do not have $(1 - \beta)\delta_x + \beta\delta_w \sim (1 - \beta)\phi_x\delta_b + (1 - \beta)(1 - \phi_x)\delta_w + \beta\delta_w$, and so $x \notin \mathcal{D}$. This establishes that $\mathcal{D} = [w, d]$, which implies that $\sup \mathcal{D} = \tilde{d} = d$.

Next, consider the case where $d = b$. Take any $x \in [w, b)$. The same argument for the case with $d \neq b$ and $x \in [w, d]$ shows that $x \in \mathcal{D}$. This gives us $\mathcal{D} = [w, b)$, and so $\sup \mathcal{D} = \tilde{d} = b = d$. \square

Lemma 3. *Suppose \succsim has an ECU representation. If $\tilde{d} \neq b$ then, for any $x \in [w, d]$ and $\alpha \in (0, 1]$, we have $u_\alpha(x) = \phi_x^\alpha u(b) + (1 - \phi_x^\alpha)u(w)$.*

Proof. Take any $x \in [w, d]$ and $\alpha \in (0, 1]$. By Lemma 2, $\tilde{d} = d$, and so ϕ_x^α is the value such that $\alpha\delta_x + (1 - \alpha)\delta_b \sim \alpha\phi_x^\alpha\delta_b + \alpha(1 - \phi_x^\alpha)\delta_w + (1 - \alpha)\delta_b$. This means that

$$\begin{aligned} \alpha u_\alpha(x) + (1 - \alpha)u(b) &= V(\alpha\delta_x + (1 - \alpha)\delta_b) \\ &= V(\alpha\phi_x^\alpha\delta_b + \alpha(1 - \phi_x^\alpha)\delta_w + (1 - \alpha)\delta_b) \\ &= \alpha\phi_x^\alpha u(b) + \alpha(1 - \phi_x^\alpha)u(w) + (1 - \alpha)u(b). \end{aligned}$$

Applying algebra to the equation gives us

$$u_\alpha(x) = \phi_x^\alpha u(b) + (1 - \phi_x^\alpha)u(w).$$

\square

Lemma 4. *Suppose that \succsim has an ECU representation. If $\tilde{d} \neq b$ then, for any $x \notin [w, d]$ and $\alpha \in [0, 1)$, we have $u_\alpha(x) = \phi_x^\alpha u(b) + (1 - \phi_x^\alpha)u(w)$.*

Proof. Take any $x \notin [w, d]$ and $\alpha \in [0, 1)$. By Lemma 2, $\tilde{d} = d$, and so ϕ_x^α is the value such that

$\alpha\delta_w + (1-\alpha)\delta_x \sim \alpha\delta_w + (1-\alpha)\phi_x^\alpha\delta_b + (1-\alpha)(1-\phi_x^\alpha)\delta_w$. This means that

$$\begin{aligned}\alpha u(w) + (1-\alpha)u_\alpha(x) &= V(\alpha\delta_w + (1-\alpha)\delta_x) \\ &= V(\alpha\delta_w + (1-\alpha)\phi_x^\alpha\delta_b + (1-\alpha)(1-\phi_x^\alpha)\delta_w) \\ &= \alpha u(w) + (1-\alpha)\phi_x^\alpha u(b) + (1-\alpha)(1-\phi_x^\alpha)u(w).\end{aligned}$$

Applying algebra to the equation gives us

$$u_\alpha(x) = \phi_x^\alpha u(b) + (1-\phi_x^\alpha)u(w).$$

□

Lemma 5. *Suppose \succsim has an ECU representation. If $\tilde{d} = b$ then, for any $x \in [w, b]$, we have $u_1(x) = \phi_x^1 u_1(b) + (1-\phi_x^1)u_1(w)$.*

Proof. Take any $x \in [w, b]$. Because $\tilde{d} = b$, ϕ_x^1 is the value such that $\delta_x \sim \phi_x^1\delta_b + (1-\phi_x^1)\delta_w + (1-\alpha)\delta_b$. We also know by Lemma 2 that $b = \tilde{d} = d$. Ergo,

$$\begin{aligned}u_1(x) &= V(\delta_x) \\ &= V(\phi_x^1\delta_b + (1-\phi_x^1)\delta_w) \\ &= \phi_x^1 u_1(b) + (1-\phi_x^1)u_1(w).\end{aligned}$$

□

Proof of Theorem 1

Proof. Sufficiency:

We first prove the sufficiency of the axioms for an ECU representation. Suppose that Axioms 1-5 are satisfied.

Case 1: $\tilde{d} \neq b$

Let $\tilde{d} = d$ for the threshold of the ECU representation. By definition of \tilde{d} , it must be in $[w, b)$. We let $u_\pi(w) = 0$ for all $\pi \in (0, 1]$. Also let $u_\pi(b) = 1$ for all $\pi \in [0, 1)$. For any $x \in (w, d]$ and $\pi \in (0, 1]$, let $u_\pi(x) = \phi_x^\pi$. For any $x \in (d, b)$ and $\pi \in [0, 1)$, let $u_\pi(x) = \phi_x^\pi$.

The requirement that $u_\pi(w) < u_\pi(b)$ for all $\pi \in [0, 1]$ such that $u_\pi(w), u_\pi(b)$ are defined is clearly met. It is also easy to see that, for any $x \in \{w, b\}$ and $\pi, \mu \in [0, 1]$ such that $u_\pi(x), u_\mu(x)$ are defined, $u_\pi(x) = u_\mu(x)$. Because $\phi_x^\pi \in [0, 1]$ for all x and π such that ϕ_x^π are defined, $u_\pi(x) \in [u(w), u(b)]$ for all x and π such that ϕ_x^π are defined.

For any $x \in (w, b)$, because we assumed that there exists $\pi, \mu \in (0, 1)$ such that $\phi_x^\pi \neq \phi_x^\mu$, there exist $\pi, \mu \in (0, 1)$ such that $u_\pi(x) \neq u_\mu(x)$. Therefore, we have a set of contextual utilities.

We want to show that, for any $p, q \in \mathcal{L}$, we have $p \succsim q \iff V(p) \geq V(q)$. Suppose that $p([w, d]) = \alpha$ and $q([w, d]) = \gamma$. We have

$$\begin{aligned}
 p \succsim q &\iff \sum_{x \in \text{supp}(p)} p(x) [\phi_x^\alpha \delta_b + (1 - \phi_x^\alpha) \delta_w] \succsim \sum_{x \in \text{supp}(q)} q(x) [\phi_x^\gamma \delta_b + (1 - \phi_x^\gamma) \delta_w] \text{ by Axiom 5} \\
 &\iff \sum_{x \in \text{supp}(p)} p(x) \phi_x^\alpha \geq \sum_{x \in \text{supp}(q)} q(x) \phi_x^\gamma \text{ by Axiom 2} \\
 &\iff \sum_{x \in \text{supp}(p)} p(x) u_\alpha(x) \geq \sum_{x \in \text{supp}(q)} q(x) u_\gamma(x) \\
 &\iff \int u_\alpha dp \geq \int u_\gamma dq.
 \end{aligned}$$

Case 2: $\tilde{d} = b$

Let $\tilde{d} = b$ for the threshold of the ECU representation. We let $u_\pi(w) = 0$ and $u_\pi(b) = 1$ for all $\pi \in (0, 1]$. For any $x \in (w, b)$ and $\alpha \in (0, 1]$, we let $u_1(x) = \phi_x^1$.

As in Case 1, the requirement that $u_\pi(w) < u_\pi(b)$ for all $\pi \in [0, 1]$ such that $u_\pi(w), u_\pi(b)$ are defined is clearly met. It is also easy to see that for any $x \in \{w, b\}$ and $\pi, \mu \in [0, 1]$ such that $u_\pi(x), u_\mu(x)$ are defined, $u_\pi(x) = u_\mu(x)$. Because $\phi_x^\pi \in [0, 1]$ for all x and π such that ϕ_x^π are defined, $u_\pi(x) \in [u(w), u(b)]$ for all x and π such that $u_\pi(x)$ are defined. This gives us a set of utility functions that is contextual.

The argument for $p \succsim q \iff V(p) \geq V(q)$ is identical to that of the first case. This completes proof of sufficiency for Case 2.

Necessity:

We move on to prove necessity. Suppose that preference relation \succsim has an ECU representation. The necessity of Axiom 1 is trivial so the proof is omitted.

For Axiom 2, take any $0 \leq t_2 \leq t_1 \leq 1$. We have

$$\begin{aligned} t_1\delta_b + (1-t_1)\delta_w \succsim t_2\delta_b + (1-t_2)\delta_w &\iff t_1u(b) + (1-t_1)u(w) \geq t_2u(b) + (1-t_2)u(w) \\ &\iff t_1 \geq t_2 \end{aligned}$$

The second line is implied by $u_\alpha(b) \geq u_\alpha(w)$.

We next prove the necessity of Axiom 3. Take any $x \in X$ and $\alpha \in [0, 1]$. We have $V(\alpha\delta_b + (1-\alpha)\delta_w) = \alpha u(b) + (1-\alpha)u(w)$. Let $p = \alpha\delta_b + (1-\alpha)\delta_x$. Let $\pi = p([w, d])$. Then $V(p) = \alpha u_\pi(b) + (1-\alpha)u_\pi(x) = \alpha u(b) + (1-\alpha)u_\pi(x)$. Because $u_\pi(x) \in [u(w), u(b)]$, we have $V(p) = \alpha u(b) + (1-\alpha)u_\pi(x) \geq \alpha u(b) + (1-\alpha)u(w)$. This gives us $\alpha\delta_b + (1-\alpha)\delta_x \succsim \alpha\delta_b + (1-\alpha)\delta_w$.

Next, let $q = \alpha\delta_x + (1-\alpha)\delta_w$. Let $\nu = q([w, d])$. Then $V(q) = \alpha u_\nu(x) + (1-\alpha)u_\nu(w) = \alpha u_\nu(x) + (1-\alpha)u(w)$. Because $u_\nu(x) \in [u(w), u(b)]$, we have $V(q) = \alpha u_\nu(x) + (1-\alpha)u(w) \leq \alpha u(b) + (1-\alpha)u(w)$. This gives us $\alpha\delta_b + (1-\alpha)\delta_w \succsim \alpha\delta_x + (1-\alpha)\delta_w$. We conclude that Axiom 3 is satisfied.

For the necessity of Axiom 4, take any $p \in \mathcal{L}$. Let $p = (x_1, p_1; \dots; x_n, p_n)$ and $p([w, d]) = \rho$. Then

$$V(p) = \sum_{i=1}^n p_i u_\rho(x_i).$$

Because $u_\rho(x) \in [u(w), u(b)]$ for all $x \in X$, $V(p) \in [u(w), u(b)]$. An application of the Intermediate Value Theorem thus tells us that there exists γ such that $V(p) = \gamma u(b) + (1 - \gamma)u(w)$. Let $q = \gamma \delta_b + (1 - \gamma)\delta_w$. Then

$$\begin{aligned} V(q) &= \gamma u(b) + (1 - \gamma)u(w) \\ &= V(p), \end{aligned}$$

so that $p \sim q$.

The necessity of Axiom 5 follows. We prove this by cases. Take the case where $\alpha = 0$ and let p be such that $p([w, \tilde{d}]) = \alpha = 0$. This must mean that $\tilde{d} \neq b$. By Lemma 2, this also means that $p([w, d]) = 0$. If $d = b$ for the ECU representation, then $q([w, d]) = 1$ for all $q \in \mathcal{L}$. Because $p([w, d]) = 0 \neq 1$, we know $d \neq b$. It must also be the case that $x \notin [w, d]$ for all $x \in \text{supp}(p)$. We thus have

$$\begin{aligned} V(p) &= \sum_{x \in \text{supp}(p)} p(x) u_0(x) \\ &= \sum_{x \in \text{supp}(p)} p(x) [\phi_x^0 u(b) + (1 - \phi_x^0)u(w)] \\ &= V\left(\sum_{x \in \text{supp}(p)} p(x) [\phi_x^0 \delta_b + (1 - \phi_x^0)\delta_w]\right) \end{aligned}$$

$\implies p \sim \sum_{x \in \text{supp}(p)} p(x) [\phi_x^0 \delta_b + (1 - \phi_x^0)\delta_w]$. The second line is implied by Lemma 4.

Next, take the case where $\alpha = 1$ and let p be such that $p([w, \tilde{d}]) = \alpha = 1$. By Lemma 2, this means that $p([w, d]) = 1$. This must in turn mean that $x \in [w, d]$ for all $x \in \text{supp}(p)$. We thus have

$$\begin{aligned} V(p) &= \sum_{x \in \text{supp}(p)} p(x)u_1(x) \\ &= \sum_{x \in \text{supp}(p)} p(x)[\phi_x^1 u(b) + (1 - \phi_x^1)u(w)] \\ &= V\left(\sum_{x \in \text{supp}(p)} p(x)[\phi_x^1 \delta_b + (1 - \phi_x^1)\delta_w]\right) \end{aligned}$$

$\implies p \sim \sum_{x \in \text{supp}(p)} p(x)[\phi_x^0 \delta_b + (1 - \phi_x^0)\delta_w]$. The second line is implied by Lemma 3 if $\tilde{d} \neq b$ or by Lemma 5 if $\tilde{d} = b$.

For the last case, take $\alpha \in (0, 1)$ and let p be such that $p([w, \tilde{d}]) = \alpha$. Because $\alpha \neq 1$, $\tilde{d} \neq b$. By Lemma 2, we also have $p([w, d]) = \alpha$. If $d = b$ for the ECU representation, then $q([w, d]) = 1$ for all $q \in \mathcal{L}$. Because $p([w, d]) \neq 1$, we know that $d \neq b$. We then have

$$\begin{aligned} V(p) &= \sum_{x \in \text{supp}(p)} p(x)u_\alpha(x) \\ &= \sum_{x \in \text{supp}(p)} p(x)[\phi_x^\alpha u(b) + (1 - \phi_x^\alpha)u(w)] \\ &= V\left(\sum_{x \in \text{supp}(p)} p(x)[\phi_x^\alpha \delta_b + (1 - \phi_x^\alpha)\delta_w]\right) \end{aligned}$$

$\implies p \sim \sum_{x \in \text{supp}(p)} p(x)[\phi_x^0 \delta_b + (1 - \phi_x^0)\delta_w]$. The second line is implied by a combination of Lemma 3 and Lemma 4.

Finally, we prove the necessity of the assumption that we did not list as an axiom. This is the condition that, if $\tilde{d} \neq b$, then, for any $x \in (w, b)$, there exists $\mu, \pi \in [0, 1)$ such that $\phi_x^\pi \neq \phi_x^\mu$.

Suppose that $\tilde{d} \neq b$. Because $\tilde{d} \neq b$, Lemma 2 tells us that $d \neq b$. Consider the case where $x \in (w, d]$. Because the set of utility functions is contextual, there exist $\pi, \mu \in (0, 1)$ such that $u_\pi(x) \neq u_\mu(x)$. In combination with Lemma 3, this means that

$$\begin{aligned} & \phi_x^\pi u_\pi(b) + (1 - \phi_x^\pi) u_\pi(w) \neq \phi_x^\mu u_\mu(b) + (1 - \phi_x^\mu) u_\mu(w) \\ \implies & \phi_x^\pi u_\pi(b) + (1 - \phi_x^\pi) u_\pi(w) \neq \phi_x^\mu u_\pi(b) + (1 - \phi_x^\mu) u_\pi(w) \\ \implies & \phi_x^\pi \neq \phi_x^\mu \end{aligned}$$

For the case where $x \in (d, b)$, we can use an identical argument using Lemma 4 instead of Lemma 3. □

Proof of Theorem 2

Proof. The "if" direction is immediate by the linearity of integrals. So, we need only to prove the "only if" direction. Suppose that a preference relation \succsim has an ECU with $\{d_1, (u_\alpha)_{\alpha \in [0,1]}\}$ and $\{d_2, (v_\alpha)_{\alpha \in [0,1]}\}$. For value $V(p)$ of any lottery p given in the definition of an ECU, we use the notation $V_1(p)$ when using the ECU with parameters $\{d_i, (u_\alpha)_{\alpha \in [0,1]}\}$, while we use $V_2(p)$ when we use the ECU with parameters $\{d_2, (v_\alpha)_{\alpha \in [0,1]}\}$. As was done with functions u_α , we often simplify and write $v(w)$ or $v(b)$ for the value of $v_\alpha(w)$ and $v_\alpha(b)$, which are common across all $\alpha \in [0, 1]$.

Suppose that $d_1 \neq d_2$. WLOG assume $d_1 < d_2$. Take any $x \in (d_1, d_2)$. Let $\phi_x \in [0, 1]$ be the value such that $v_1(x) = \phi_x v(b) + (1 - \phi_x) v(w)$, meaning that $\delta_x \sim \phi_x \delta_b + (1 - \phi_x) \delta_w$. Such a value exists

by an application of the Intermediate Value Theorem. For any $\alpha \in [0, 1]$ we have

$$\begin{aligned}
V_2(\alpha\delta_x + (1-\alpha)\delta_w) &= \alpha v_1(x) + (1-\alpha)v_1(w) \\
&= \alpha\phi_x v(b) + \alpha(1-\phi_x)v(w) + (1-\alpha)v(w) \\
&= V_2(\alpha\phi_x\delta_b + \alpha(1-\phi_x)\delta_w + (1-\alpha)\delta_w),
\end{aligned}$$

so that $\alpha\delta_x + (1-\alpha)\delta_w \sim \alpha\phi_x\delta_b + \alpha(1-\phi_x)\delta_w + (1-\alpha)\delta_w$.

Inasmuch as we have $\delta_x \sim \phi_x\delta_b + (1-\phi_x)\delta_w$, it must also be the case that $u_0(x) = \phi_x u(b) + (1-\phi_x)u(w)$. Because we have an ECU with $d_1 \neq b$, there exists $\alpha \in (0, 1)$ such that $u_\alpha(x) \neq u_0(x)$. This tells us that

$$\begin{aligned}
V_1((1-\alpha)\delta_x + \alpha\delta_w) &= (1-\alpha)u_\alpha(x) + \alpha u(w) \\
&\neq (1-\alpha)u_0(x) + \alpha u(w) \\
&= (1-\alpha)\phi_x u(b) + (1-\alpha)(1-\phi_x)u(w) + \alpha u(w) \\
&= V_1((1-\alpha)\phi_x\delta_b + (1-\alpha)(1-\phi_x)\delta_w + \alpha\delta_w),
\end{aligned}$$

so that we do not have $(1-\alpha)\delta_x + \alpha\delta_w \sim (1-\alpha)\phi_x\delta_b + (1-\alpha)(1-\phi_x)\delta_w + \alpha\delta_w$. This contradicts $\{d_1, (u_\alpha)_{\alpha \in [0,1]}\}$ and $\{d_2, (v_\alpha)_{\alpha \in [0,1]}\}$ representing the same preferences. We conclude that $d_1 = d_2$. For the remainder of the proof, we refer to the common value as d .

We next show that $v_\alpha = ku_\alpha + c$ for all $\alpha \in [0, 1]$ with $k \in \mathbb{R}_{++}$ and $c \in \mathbb{R}$. We prove this in two steps. First, we show that, for any $\alpha \in (0, 1]$ and $x \in [w, d]$, we have $v_\alpha(x) = ku_\alpha(x) + c$. In the second step, we show that for any $\alpha \in [0, 1)$ and $x \in [w, d]$, we have $v_\alpha(x) = ku_\alpha(x) + c$. In the first step, we do not need $\alpha = 0$ because u_0 and v_0 are not defined on $[w, d]$. Similarly, for the second step, we do not consider $\alpha = 1$ because u_1, v_1 are not defined on $X \setminus [w, d]$.

Take any $\alpha \in (0, 1]$ and $x \in [w, d]$. Let $p = (x, \alpha; b, 1 - \alpha)$. We have

$$V_1(p) = \alpha u_\alpha(x) + (1 - \alpha)u(b).$$

Letting $\gamma_x = \frac{\alpha[u_\alpha(x) - u(b)]}{u(w) - u(b)}$, we have $\gamma_x u(w) + (1 - \gamma_x)u(b) = V_1(p)$. It is easy to see that $\gamma_x \in [0, 1]$.

Therefore, letting $q = (w, \gamma_x; b, 1 - \gamma_x)$, we have $p \sim q$. Because the two ECU representations represent the same preferences, we must also have

$$\begin{aligned} \alpha v_\alpha(x) + (1 - \alpha)v(b) &= \gamma_x v(w) + (1 - \gamma_x)v(b) \\ \implies v_\alpha(x) &= u_\alpha(x) \left[\frac{v(w) - v(b)}{u(w) - u(b)} \right] + \frac{v(b)u(w) - u(b)v(w)}{u(w) - u(b)}. \end{aligned}$$

We let $k = \left[\frac{v(w) - v(b)}{u(w) - u(b)} \right]$ and $c = \frac{v(b)u(w) - u(b)v(w)}{u(w) - u(b)}$. It is easy to see that $k \in \mathbb{R}_{++}$.

Now, take any $\alpha \in [0, 1)$ and $x \notin [w, d]$. Let $p = (w, \alpha; x, 1 - \alpha)$. We have

$$V_1(p) = \alpha u(w) + (1 - \alpha)u_\alpha(x).$$

Letting $\gamma_x = \frac{\alpha u(w) + (1 - \alpha)u_\alpha(x) - u(b)}{u(w) - u(b)}$, we have $\gamma_x u(w) + (1 - \gamma_x)u(b) = V_1(p)$. It is easy to see that $\gamma_x \in [0, 1]$.

Therefore, letting $q = (w, \gamma_x; b, 1 - \gamma_x)$, we have $p \sim q$. Because the two ECU representations represent the same preferences, we must also have

$$\begin{aligned} \alpha v(w) + (1 - \alpha)v_\alpha(x) &= \gamma_x v(w) + (1 - \gamma_x)v(b) \\ \implies v_\alpha(x) &= u_\alpha(x) \left[\frac{v(w) - v(b)}{u(w) - u(b)} \right] + \frac{v(b)u(w) - u(b)v(w)}{u(w) - u(b)} \\ \implies v_\alpha(x) &= k u_\alpha(x) + c. \end{aligned}$$

□

Proof of Proposition 1

Proof. Take $p, q \in \mathcal{L}$ such that p first-order stochastically dominates q . Let $\pi = p([w, d])$ and $\mu = q([w, d])$. One equivalent definition of first-order stochastic dominance says that, for any non-decreasing function $u : X \rightarrow \mathbb{R}$, we have $\int u dp \geq \int u dq$. Because we have an ECU representation with Condition 1, u_μ is non-decreasing. Therefore, $\int u_\mu dq \leq \int u_\mu dp$. Also by definition of first-order stochastic dominance, $\pi \leq \mu$. Ergo, $u_\pi(x) \geq u_\mu(x)$ for all $x \in X$. This means that $\int u_\pi dp \geq \int u_\mu dp$. This gives us $\int u_\pi dp \geq \int u_\mu dq$, and so $p \succeq q$. □

B Statistical testing

In addition to testing Hypothesis 1, we conducted an exact binomial test to determine whether the probability that participants switch choices was significantly greater than 0.5. Observing 78 successes out of 150 trials does not reject the null hypothesis, with a p-value of 0.3416. The 95% confidence interval for the probability that success is achieved ranged from 0.4497 to 1.0000, with a sample estimate of 0.52. The pilot session showed more pronounced switching behavior, however, with 42 successes out of 67 trials and a p-value of 0.0249, suggesting that a larger sample may provide a more definitive conclusion.

	Value
Number of successes	78
Number of trials	150
p-value	0.3416
Alternative hypothesis	True probability of success is greater than 0.5
95 percent confidence interval	[0.4497293, 1.0000000]
Probability of success	0.52

Table 1: Stage 1 Exact Binomial Test Results for Main Data

	Value
Number of successes	64
Number of trials	78
p-value	4.291e-09
Alternative hypothesis	True probability of success is greater than 0.5
95 percent confidence interval	[0.7337, 1.0000]
Probability of success	0.8205

Table 2: Stage 2 Exact Binomial Test Results for Main Data

	Value
Number of successes	42
Number of trials	67
p-value	0.0249
Alternative hypothesis	True probability of success is greater than 0.5
95 percent confidence interval	[0.5194, 1.0000]
Probability of success	0.6269

Table 3: Stage 1 Exact Binomial Test Results for Pilot Session

C Gender Differences

We note a slight difference in participants' responses by reported gender. Table 6 shows that female participants were less likely to reverse preferences in Stage 1. Of 149 participants with reported gender data, 58 were female and 91 were male. We also see that, of the 58 female participants, 34

	Value
Number of successes	37
Number of trials	42
p-value	2.217e-07
Alternative hypothesis	True probability of success is greater than 0.5
95 percent confidence interval	[0.7658, 1.0000]
Probability of success	0.8810

Table 4: Stage 2 Exact Binomial Test Results for Pilot Session

	Female	Male	Total
No switch	27	31	58
Switch	31	60	91
Total	58	91	149
Fisher's Exact (2-sided): 0.168			
Fisher's Exact (1-sided): 0.088			

Table 5: Cross-Tabulation of Gender Comparison in Stage 2 Data

(58.6%) chose not to switch, while, of the 91 males, 53 (58.2%) chose to switch. The results derived from Fisher's exact test shows the presence of a clear gender effect in the Stage 1 data. A Pearson's Chi-squared test also shows a similar result. Stage 2 data show a similar result. The difference in their responses, however, disappears when we consider the subgroup of participants who switched in Stage 1. Within our ECU framework, these results might indicate that the parameters of our model, which influence participants' switching behavior, may be correlated with gender. By controlling for these parameters during Stage 1, we account for gender differences, as a result of which no significant gender differences were observed in Stage 2 among the subgroup of participants who switched in Stage 1 or whose threshold d was between zero and 400 points in our setup. This finding suggests the presence of an unexplored angle regarding gender differences in risk attitudes. We hope future research using our ECU framework can provide a deeper understanding of the conventional belief that females are typically more risk-averse than males. For example, future research could

explore contexts in which gender differences in risk preferences are more pronounced.

	Female	Male	Total
No switch	34	38	72
Switch	24	53	77
Total	58	91	149
Fisher's Exact (2-sided): 0.064			
Fisher's Exact (1-sided): 0.033			

Table 6: Cross-Tabulation of Gender Comparison in Stage 1 Data

	Female	Male	Total
No switch	6	8	14
Switch	18	45	63
Total	24	53	77
Fisher's Exact (2-sided): 0.345			
Fisher's Exact (1-sided): 0.231			

Table 7: Cross-Tabulation of Gender Comparison in Stage 2 for Subgroup that Switched in Stage 1

D Robustness check: Stage 3

Hypotheses 1 and 2 and the corresponding results obtained in Stages 1 and 2 are clear-cut tests of the existing models and provide supporting evidence for the occurrence of context-dependent risk attitudes. These tests may not however be sufficient for identifying the underlying model given the degree of freedom in our proposed ECU model. For example, the failure of a participant to switch between Option A and Option B may reflect the participant's threshold level falling outside the proposed experimental parameters; or, alternatively, it is also possible that the participant is a perfect expected utility maximizer. We therefore conduct additional tests in a robustness check.

As shown in Figures 2 and 3, the DM in our binary ECU model will change their risk attitudes when crossing the threshold line on their Marschak-Machina triangles. That is, we should expect to see an Allais-like reversal if we give them choice tasks for lotteries on opposite sides of the threshold line. On the other hand, we should not expect to observe such a reversal if we provide lotteries only on one side of the threshold line. With this intuition, we use the estimated d and τ from the first two stages to construct the following lotteries for Stage 3.

1. Pairs of lotteries for which we expect more Allais-like risk attitude reversal based on our model prediction.

- common consequence

$$\left(d + \frac{b-d}{2}, 1\right) \text{ vs. } \left(d + \frac{3b-3d}{4}, \frac{1-\tau-\varepsilon}{2}; d + \frac{b-d}{2}, \tau + \varepsilon; 0, \frac{1-\tau-\varepsilon}{2}\right)$$

$$\left(d + \frac{b-d}{2}, 1-\tau-\varepsilon; 0, \tau + \varepsilon\right) \text{ vs. } \left(d + \frac{3b-3d}{4}, \frac{1-\tau-\varepsilon}{2}; 0, 1 - \frac{1-\tau-\varepsilon}{2}\right)$$

The lottery on the left side in the first pair of lotteries places probability 1 on a prize that is above the recovered threshold d for what is considered a disappointing prize. Therefore, the DM would use the optimistic function u to evaluate the lottery. The two lotteries in the first pair have a “common consequence” of $d + \frac{b-d}{2}$, so for $\varepsilon > 0$, we transfer $\tau + \varepsilon$ of the probability on this prize to the prize 0 for each lottery. We chose ε to be 0.01 since it must be sufficiently small to have well-defined lotteries. These transfers give us the second pair of lotteries. This results in the lottery on the left side having probability greater than τ on the prize 0. This would cause the DM to evaluate the lottery using v instead of u , making it possible to have a reversal in risk-taking behavior and generate the common consequence effect.

- common ratio

$$(b, 0.8; 0, 0.2) \text{ vs. } \left(d + \frac{b-d}{2}, 1\right)$$

$$(b, 0.8 \times (1 - \tau - \varepsilon); 0, 1 - 0.8 \times (1 - \tau - \varepsilon)) \text{ vs. } \left(d + \frac{b-d}{2}, 1 - \tau - \varepsilon; 0, \tau + \varepsilon\right)$$

There is probability 1 of getting a prize greater than d for the lottery on the right side in the first pair of lotteries. This means that the DM would use the function u . However, the lottery on the right side for the second pair places probability greater than τ on 0. This suggests that the DM would use v . This makes it possible that, in the first pair, the less risky lottery on the right side is more preferred since the optimistic function is used to evaluate the lottery, but the lottery on the left side is more preferred for the second pair since the one on the right is now evaluated with the pessimistic function v . Note that this would be an observation of the common ratio effect since the ratio of probabilities of getting a “good” prize (better than d) for the lottery on the left and the right is the same value of 0.8 for the two pairs of lotteries.

2. Pairs of lotteries for which we do not expect Allais-like risk attitude reversal based on our model prediction.

- common consequence

$$\left(\frac{d}{10} + \frac{b+d}{2}, 1\right) \text{ vs. } \left(\frac{d}{5} + \frac{b+d}{2}, 0.5; \frac{d}{10} + \frac{b+d}{2}, 0.1; 0, 0.4\right)$$

$$\left(\frac{d}{10} + \frac{b+d}{2}, 0.9; 0, 0.1\right) \text{ vs. } \left(\frac{d}{5} + \frac{b+d}{2}, 0.5; 0, 0.5\right)$$

- common ratio

$$\left(\frac{d+\varepsilon}{4} + \frac{b+d}{2}, 1\right) \text{ vs. } \left(\frac{d+\varepsilon}{3} + \frac{b+d}{2}, 0.5; 0, 0.5\right)$$

$$\left(\frac{d+\varepsilon}{4} + \frac{b+d}{2}, 0.9; 0, 0.1\right) \text{ vs. } \left(\frac{d+\varepsilon}{3} + \frac{b+d}{2}, 0.45; 0, 0.55\right)$$

The procedure for obtaining the second pair of lotteries from the first pair is nearly identical to the previous examples. Hence, a preference for the less risky lottery in the first pair and a switch to a preference for the riskier lottery in the second pair would be the type of behavior we see in the common consequence effect and the common ratio effect. However, notice that every one of these lotteries consist of only prizes greater than d . This would lead to the DM using the function u to evaluate all of these lotteries and preclude the observation of such behavior.

In Stage 3, we present one choice task at a time to avoid inducing preferences for randomization (Agranov and Ortoleva (2017) and Feldman and Rehbeck (2022)). The construction of lotteries leads to our third hypothesis.

Hypothesis 3. *Participants who switch in Stage 1 and Stage 2 will make Allais-like reversals in the first set of choice tasks. Participants will not make Allais-like reversals in the latter set of choice tasks.*

Result 3. *Participants were more likely to make risk-preference reversals in the set of choices where our model predicted Allais behavior.*

1. *In the set of choice tasks where our model predicted Allais behavior:*

- *26 of 150 participants (17.3%) switched in the common-consequence tasks. Of the 64 participants who made risk preference reversals in both Stages 1 and 2, 12 participants (18.8%) switched in the common-consequence tasks.*
- *60 of 150 participants (40%) switched in the common-ratio tasks. Among the 64 participants who made risk-preference reversals in both Stages 1 and 2, 25 participants (39.1%) switched in the common-ratio tasks.*

2. *In the set of choice tasks where our model predicted that no Allais behavior would occur:*

- *14 of 150 participants (9.3%) switched in the common-consequence tasks. Of the 64 participants who made risk-preference reversals in both Stages 1 and 2, 8 participants (12.5%) switched in the common-consequence tasks.*

- 8 of 150 participants (5.3%) switched in the common-ratio tasks. Of the 64 participants who made risk-preference reversals in both Stages 1 and 2, 4 participants (6.3%) switched in the common-ratio tasks.

This summary statistic weakly supports Hypothesis 3. We do observe more participants making risk-preference reversals in the set of choice tasks where our model predicted Allais behavior. To conduct a more detailed analysis, we ran a logit regression in which we explain participants' preference reversals by individual characteristics and by a treatment dummy,

$$y_{t,i} = \begin{cases} 1 & \text{if } \beta_0 + \beta_1\theta_t + \beta_2d_i + \beta_3\tau_i + \beta_4\text{gender}_i + \varepsilon_{t,i} > 0, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

with robust standard errors clustered at the participant level, where $y_{t,i}$ is the binary variable representing whether participants i reverse their risk preferences in the given set of tasks t , θ_t is the treatment dummy where $\theta_t = 1$ if the set t contains choice tasks where our model predicted Allais behavior and 0 otherwise, and $d_i, \tau_i, \text{gender}_i$ are individual characteristics of participant i .

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-1.681	0.473	-3.555	0.000 ***
is_Allais	1.405	0.363	3.876	0.000 ***
d	-0.002	0.002	-0.856	0.392
tau	0.604	0.827	0.731	0.465
genderMale	-0.692	0.418	-1.654	0.098 .

*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

Table 8: Logistic Regression Results

E Pilot sessions

We recruited 95 student participants from a participant pool at the Economic Science Institute (ESI) Laboratory at Chapman University in Orange, California. We conducted five experimental sessions, each lasting approximately 30 minutes. These sessions included instructions that participants read before the main experiment and a non-incentivized questionnaire upon completion of the main experiment. In the first two sessions, we allowed participants to switch freely between the two columns in Stages 1 and 2. In the three later sessions the participants were allowed to switch only once in each stage.

For all five sessions, participants received a \$7 show-up fee. The dollar amounts they could earn for each option varied from \$0 to a maximum of \$24 in all except the second session. Hence, the total amount they could earn ranged from \$7 to \$31. In the second session, the amount they could earn ranged from \$7 to \$103 for the 30-minute session. No participant participated in more than one session.

The overall results obtained from the pilot sessions are qualitatively consistent with our main findings. We list them separately because of two main differences in the sessions. First, the pilot sessions were all conducted in a physical lab environment whereas the main data were collected in an online environment. Second, the pilot sessions were all conducted using dollar amounts, whereas the online experiments used points as the main denomination.

E.1 Pilot sessions with multiple switches allowed

We conducted two sessions where we allowed participants to switch freely between the two columns. While this represents a natural design when testing the validity of our model, the data show that the majority of participants switched multiple times. Most importantly, these multiple switches obstructed effective estimation of the model parameters. Although it is possible to generalize our model to include more than two utility functions for a better fit, and such generalization would

include the model of Becker and Sarin (1987) as a special case, we chose to focus on the dual utility representation given its simplicity and practicality for applications. The raw data, which demonstrate the extent of multiple switches in Stage 1 and Stage 2 in these sessions, are presented at the end of this section. We begin by summarizing the findings from the first session.

Result 1-1. *10 of 14 (71.4%) participants switched their choices between Option A and Option B in Stage 1. Only one participant switched once. Conditional on switching at least once, participants switched 4.1 times on average.*

Result 2-1. *12 of 14 (85.7%) participants switched their choices between Option A and Option B in Stage 2. Three participants switched once. Conditional on switching at least once, participants switched 3.25 times on average.*

9 of 10 (90.0%) participants who switched in Stage 1 did so in Stage 2. Of the 4 participants who did not switch in Stage 1, 3 (75.0%) switched in Stage 2.

These results support Hypotheses 1 and 2, but the excessive frequency of switches between the options suggested either that many of them were irrational as classified in Chew et al. (2022) or preferred randomization (Agranov and Ortoleva (2017)). Interestingly, a larger proportion of participants exhibited risk-attitude reversal than in the main experiment, where we restricted the number of switches to one. Combining the observations, it is possible that participants might have perceived the compensation as unworthy of their cognitive effort and chose to randomize over their choices irrationally. To test this hypothesis, we conducted another pilot session with much higher stakes. We offered prizes worth four times the original amounts so that the amounts they could earn would range from \$7 to \$103 for the 30-minute session.

Result 1-2. *11 of 14 (78.6%) participants switched their choices between Option A and Option B in Stage 1. 5 participants switched once. Conditional on switching at least once, participants switched 2.55 times on average.*

Result 2-2. *13 of 14 (92.9%) participants switched their choice between Option A and Option B in Stage 2. 4 participants switched once. Conditional on switching at least once, participants switched 3.46 times on average.*

10 of 11 (90.9%) participants who switched in Stage 1 did so in Stage 2. of the 3 participants who did not switch in Stage 1, all 3 (100%) switched in Stage 2.

While the small sample size precludes offering conclusive explanations of these results, the higher stakes did increase the number of participants who switched only once. It also reduced the average number of switches in Stage 1. Interestingly, it did not increase the number of non-switchers.

E.2 Pilot sessions with one switch

In the later three sessions, to facilitate the estimation of the disappointment threshold d and the tolerance threshold τ in the experiment, we restricted participants to only one switch per session. The results are qualitatively consistent with the main findings.

Result 1-3. *42 of 67 (62.7%) participants switched their choices between Option A and Option B in Stage 1.*

This result supports Hypothesis 1. The majority of participants switched between the options, consistent with the motivating example we started with in the introduction. That is, while we fixed the ranks of all the prizes and their corresponding probabilities across the ten rows, the participants responded to the change in the relative size of the worst prize.

Result 2-3. *49 of 67 (73.1%) participants switched their choices between Option A and Option B in Stage 2. In particular, 37 of 42 (88.1%) participants who switched in Stage 1 did so in Stage 2. Of the 25 participants who did not switch in Stage 1, only 12 (48.0%) switched in Stage 2.*

This result supports Hypothesis 2. The correlation between participants' behavior in Stages 1 and 2 suggests as well that reversals in risk attitudes over the two stages are individual-specific, indicating

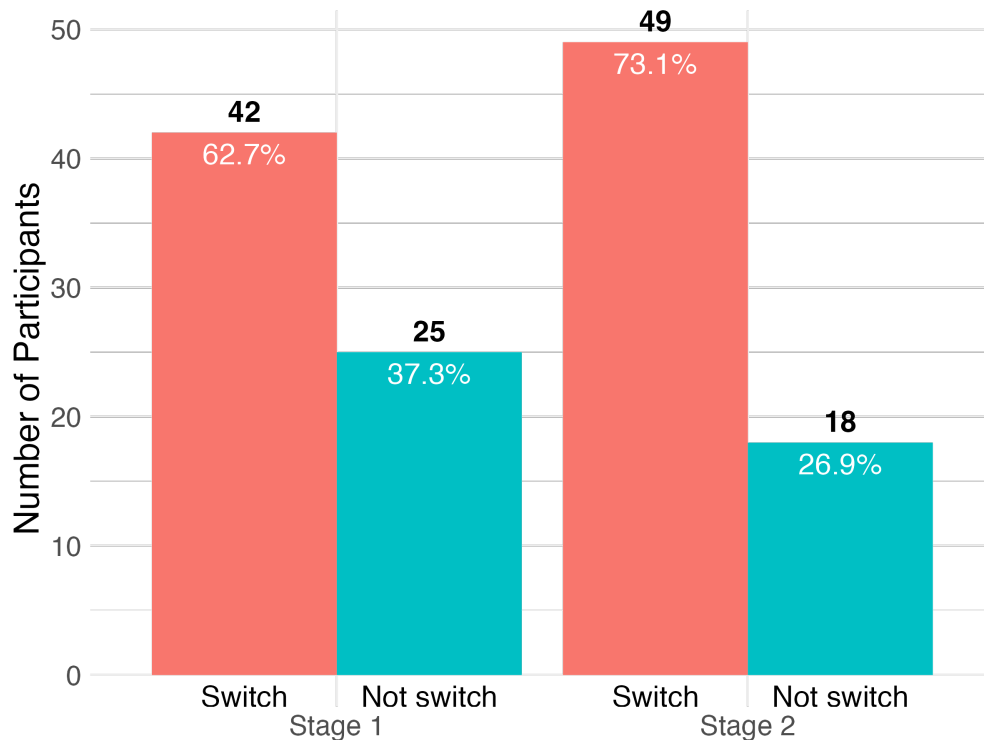


Figure 6: The violation of EU/RDU/CPT appears is more pronounced in the pilot sessions.

the plausibility of our intuition that thresholds on both the prize and probability dimensions explain the results.

Result 3-3.

1. In the set of choice tasks where our model predicted Allais behavior:

- 13 of 67 (19.4%) participants switched in the common-consequence tasks; and
- 21 of 67 (31.3%) participants switched in the common-ratio tasks.

2. In the set of choice tasks where our model predicted a lack of Allais behavior:

- 3 of 67 (4.5%) participants switched in the common-consequence tasks; and
- 0 of 67 (0%) participants switched in the common-ratio tasks.

These results support Hypothesis 3. The empirically observed switching frequency in the Allais-predicted sets of choice tasks is consistent with findings reported by Blavatskyy, Ortmann, and Panchenko (2022). That meta-study reports that approximately 15% of experiments using real incentives reveal fanning-out, while 20% reveal fanning-in, in contrast to the much higher frequencies reported in experiments employing hypothetical incentives with large award amounts, as in Kahneman and Tversky (1979).

E.3 Raw data from Sessions 1 and 2

Table 9: Raw data from Session 1

Subject	(a) Stage 1										(b) Stage 2									
	Task 1	2	3	4	5	6	7	8	9	10	Task 1	2	3	4	5	6	7	8	9	10
1	1	1	1	0	0	0	1	1	1	1	0	1	1	1	1	0	0	0	0	1
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1
3	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
4	1	0	1	0	1	1	0	1	0	1	0	1	0	0	1	0	1	0	0	1
5	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1	0	1	1	0	0
6	1	0	1	1	0	0	0	0	0	0	1	0	1	1	1	1	0	0	0	0
7	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0	1	1	1	1	1
8	1	1	1	1	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
11	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	0	0	1	0	1	0	1	0	0	0	1	1	0	1	0	0	1	1
14	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0

Table 10: Raw data from Session 2 (high stake)

	(a) Stage 1										(b) Stage 2									
Subject	Task 1	2	3	4	5	6	7	8	9	10	Task 1	2	3	4	5	6	7	8	9	10
15	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
16	0	1	0	1	1	1	1	1	0	1	1	1	1	1	0	1	0	1	0	1
17	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
18	0	0	1	0	1	0	1	0	1	1	0	0	1	0	1	0	0	1	1	1
19	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1
20	0	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	0	0	0
21	0	1	1	1	1	0	0	1	1	0	0	1	0	0	0	1	1	0	0	0
22	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1
23	1	1	1	0	0	0	0	1	1	0	0	1	0	1	0	1	0	1	0	1
24	0	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	1	1	0	1
25	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	1	1
26	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	1	1	1	1	1
28	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1

F Screenshots of the experiment's software interface

Stage 1: Task #1-10

Task#	Option A		or	Option B		
	Prize (Chance)	Prize (Chance)		Prize (Chance)	Prize (Chance)	Prize (Chance)
1	600 (10%)	0 (90%)	< >	800 (5%)	400 (5%)	0 (90%)
2	600 (10%)	44 (90%)	< >	800 (5%)	400 (5%)	44 (90%)
3	600 (10%)	88 (90%)	< >	800 (5%)	400 (5%)	88 (90%)
4	600 (10%)	133 (90%)	< >	800 (5%)	400 (5%)	133 (90%)
5	600 (10%)	177 (90%)	< >	800 (5%)	400 (5%)	177 (90%)
6	600 (10%)	222 (90%)	< >	800 (5%)	400 (5%)	222 (90%)
7	600 (10%)	266 (90%)	< >	800 (5%)	400 (5%)	266 (90%)
8	600 (10%)	311 (90%)	< >	800 (5%)	400 (5%)	311 (90%)
9	600 (10%)	355 (90%)	< >	800 (5%)	400 (5%)	355 (90%)
10	600 (10%)	400 (90%)	< >	800 (5%)	400 (5%)	400 (90%)

Your choice:

[Open Instructions](#) [Next](#)

Figure 7: A screenshot of the experiment's software interface displaying Stage 1.

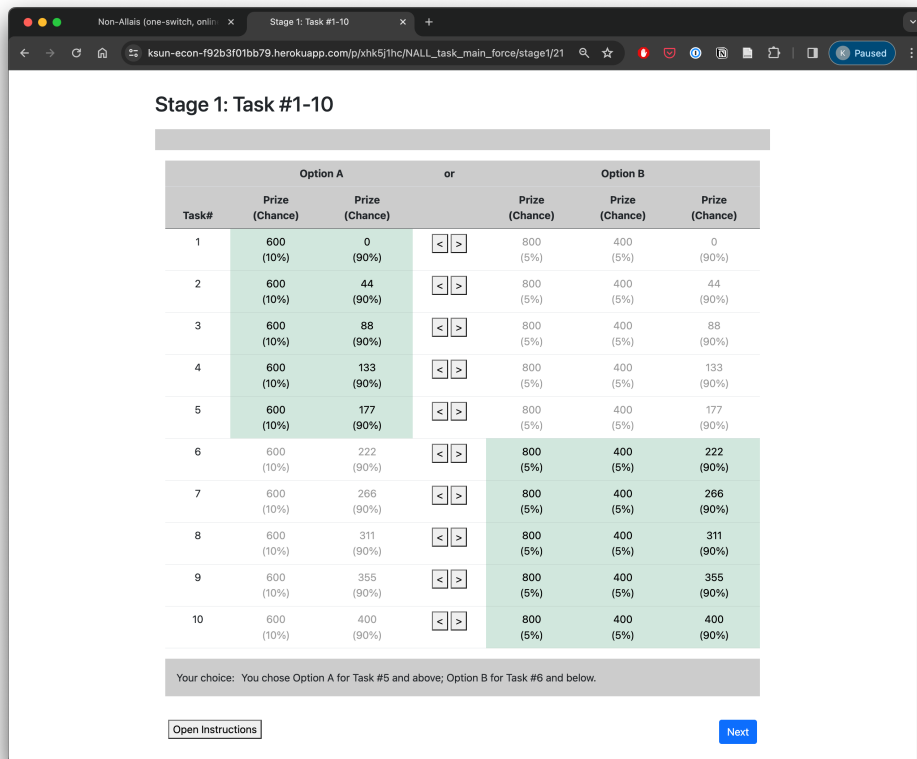


Figure 8: A screenshot of the experiment's software interface displaying Stage 1 where the participant chose Option A for Task #5 and above.

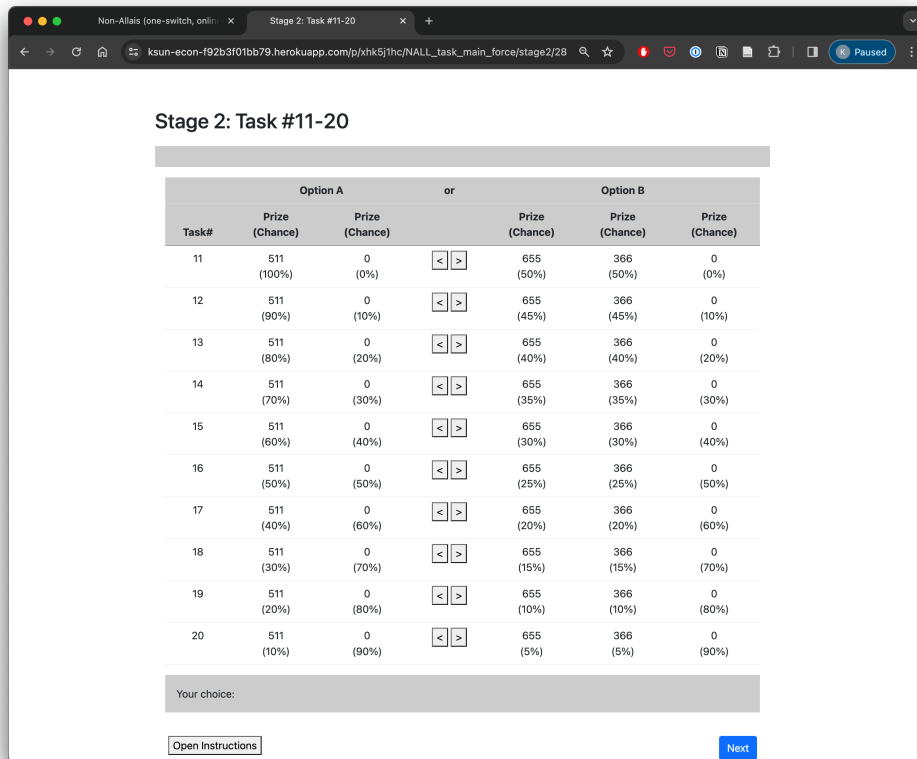


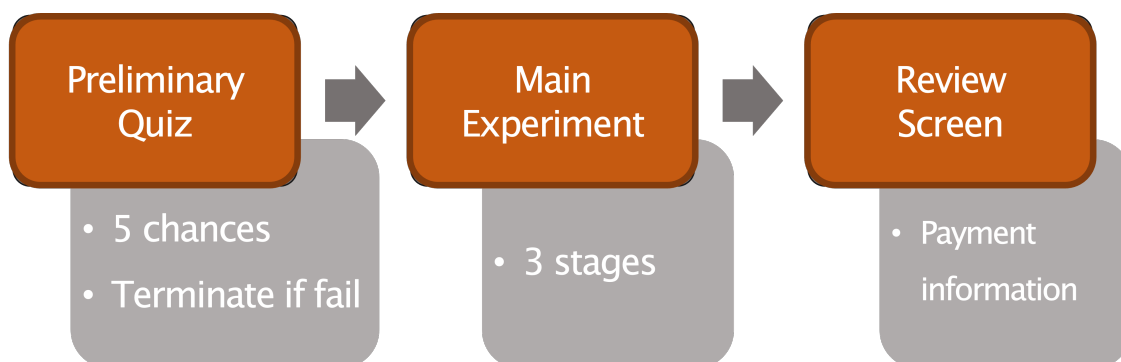
Figure 9: A screenshot of the experiment's software interface displaying Stage 2.

G Experimental Instructions

Welcome to the experiment! Thank you very much for participating today. This research is conducted by researchers of the University of Manitoba, Canada and Fairfield University, USA.

The experiment you will be participating in is an experiment in individual decision making. You will receive your compensation via Prolific. You will receive the show-up fee of \$6.00 for completing the experiment, with the additional bonus amount that depends on your decisions and on chance. The details of the compensation will be described later. All instructions and descriptions that you will be given in the experiment are accurate and true. At no point will we attempt to deceive you in any way. When you are ready, please click the button below to read the instructions.

There are three parts in this experiment.



First, there is a short quiz at the end of the instructions to ensure your understanding of the procedures. You will be able to repeat the quiz if you make mistakes. You will have **five** chances to attempt the quiz. If you fail to get all questions correct after five attempts, you may **not** participate in the main experiment. If you are unable to pass the quiz, please understand that we can only compensate those who participate in the main part of the experiment and we will not be able to provide payment. In such an event, we kindly request that you return your submission on Prolific.

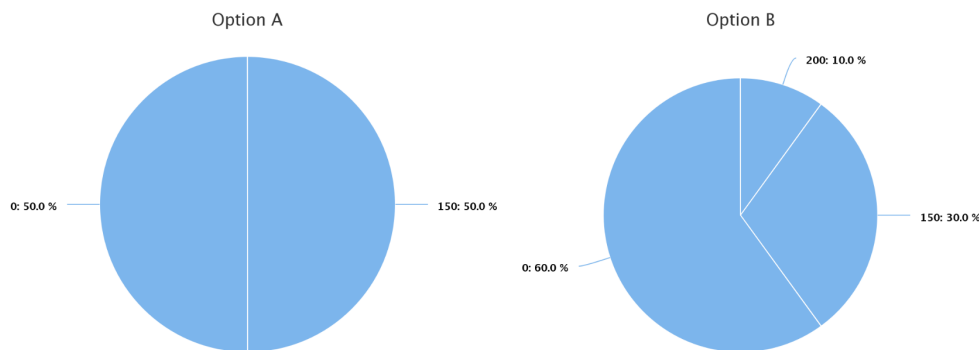
In the main part of the experiment, we will ask you to perform tasks of choosing between two options. We will shortly explain what they are.

At the end of the experiment, we will display a review screen to verify your payment information.

We will now give you more details about the main experiment. The basic unit of the experiment is a stage of tasks. Each stage consists of one or more tasks.

In each stage, we ask you to perform a series of tasks of choosing between two available options. Each option in a task represents a lottery by which you can earn certain amounts of points depending on chances. Below is an example of such options.

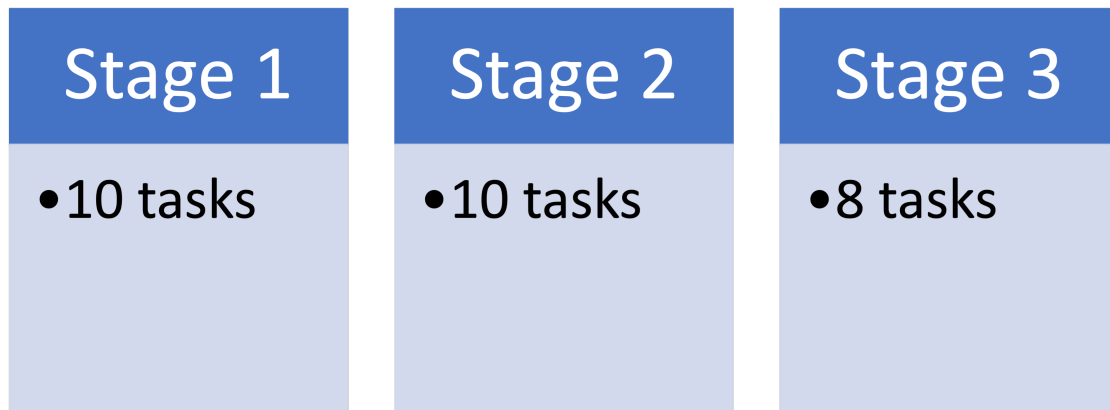
Task#	Option A		or	Option B		
	Prize (Chance)	Prize (Chance)		Prize (Chance)	Prize (Chance)	Prize (Chance)
0	150 (50%)	0 (50%)	< >	200 (10%)	150 (30%)	0 (60%)



The columns and the image on the left represent a hypothetical Option A that gives you 150 points with 50% chance and 0 points with 50% chance. The columns and the image on the right represent a hypothetical Option B that gives you 200 points with 10% chance, 150 points with 30% chance, and 0 points with 60% chance. Your task is to indicate which of the two options you prefer by clicking the arrow buttons located between the two lotteries.

The combination of options and the chances associated with them vary across tasks. The amount

of points you may earn in each Option varies from 0 to the maximum of 800. The chances associated with them vary from 0% to 100%.



There are three stages in today's experiment. In Stage 1 and Stage 2, we present a table that contains a series of ten (10) tasks. Each row represents a task of choosing between Option A and Option B.

In Stage 1, we will fix the chances and vary the amounts of points you can receive in each Option across the ten rows. In Stage 2, we will fix the amounts of points you can receive and vary the chances associated with them across the ten rows.

In Stages 1 and 2, we will ask you to simply indicate a threshold line above which you would rather have Option A and below which you would rather have Option B, and vice versa. You can do so by clicking one of the arrows displayed in the center column of the table. Once you make your choice, a confirmation prompt will appear at the bottom of the screen to remind you of your selection. This will simplify the experiment and reduce the amount of repetitive procedures you need to complete. By doing so, you will only need to perform this procedure once instead of ten times for each row.

In Stage 3, we will present one task at a time. There will be eight (8) tasks in Stage 3, and we will vary both the amounts of points you can receive and the chances associated with them.

Combining all three Stages, there are total of 28 tasks in today's experiment. After completing all 28 tasks, the computer will randomly select one task for the payment. Each task has an equal chance of being selected.

The final amount you receive will depend on your choice in the randomly chosen task. The computer will simulate a lottery draw based on the option of your choice.

For example, if the option of your choice were Option A in the hypothetical example, you will receive 150 points with 50% chance and 0 points with 50% chance.

For your convenience, imagine this drawing process as the following. Suppose there is a box that contains 50 red balls, and 50 yellow balls. The computer shuffles the box and take one ball out at random. If the ball is red, you receive 150 points, hence 50% chance. If the ball is yellow, you receive 0 points, hence 50% chance.

If the option of your choice were Option B, on the other hand, you will receive 200 points with 10% chance, 150 points with 30% chance, and 0 points with 60% chance.

For your convenience, imagine this drawing process as the following. Suppose there is a box that contains 10 red balls, 30 yellow balls, and 60 green balls. The computer shuffles the box and take one ball out at random. If the ball is red, you receive 200 points, hence 10% chance. If the ball is yellow, you receive 150 points, hence 30% chance. If the ball is green, you receive 0 points, hence 60% chance.

We will convert 100 points to \$1.00. That is, each point you receive is worth \$0.01.

This is the end of the instructions.

You may use the 'Previous' button to read the previous pages. Please make sure you understand the instructions well before starting the quiz.

When you are ready, please click the next button to start the quiz.