In the pursuit for the nature of the $P_c(4457)$ and related pentaquarks

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The magnetic moment of a hadron is an important spectroscopic parameter as its mass and encodes valuable information about its internal structure. In this present study, we systematically study magnetic moments of the $P_c(4457)$ and its related hidden-charm pentaquark states with and without strangeness employing a comprehensive analysis that encompasses diquark-diquark-antiquark scheme with $J^P = \frac{1}{2}^-$, $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^-$ quantum numbers. Some of the obtained magnetic moments agree well with the available theoretical results. The predicted magnetic moment values together with alongside results existing in the literature, may offer insights into their underlying structures, and consequently their spin-parity quantum numbers.

I. INTRODUCTION

Quark model which was introduced by Gell-Mann and Zweig independently [1, 2] describes mesons as bound states of a quark and an antiquark, and baryons as bound states of three quarks. Quark model have successfully explained physical features of experimentally observed meson and baryon states till to the first years of 2000s. The paradigm has changed when the observation of $\chi_{c1}(3872)$ (also known as X(3872)) was announced, a state that has unusual properties which cannot be fitted to the quark model [3]. Since then many states have been reported which cannot be collected as ordinary mesons ($q\bar{q}$) or baryons (qqq). These are called exotic states, neither quark model nor quantum chromodynamics (QCD) prohibit the existence of exotic states.

QCD is the quantum field theory of strong interaction, not only governs the interactions between quarks and gluons but also the interaction between color-neutral hadrons. Short and long distance behaviours of QCD is of primary interest since perturbative and nonperturbative interactions are related to these distances. Investigating hadronhadron interactions based on QCD presents opportunities for multiquark states. For example, four-quark states can be studied either compact tetraquark or molecular states. In both cases, physical interpretations can lead to different physical results. For this reason, unravelling internal structure of the multiquark states is one of the major topic in hadron physics.

To date back, the existence of multiquark states were conjectured in the original works of Gell-Mann and Zweig. They showed that it is possible to obtain color-neutral objects by combining four, five or even six quarks. Among these multiquark states, five quark states (namely pentaquarks) are important members of multiquark states. In 2015, LHCb Collaboration reported the observation of pentaquark states. $P_c(4380)$ and $P_c(4450)$ states were confirmed in the $J/\psi + p$ decay channel. Four years later, LCHb Collaboration accumulated tied more data on these states which yielded important results. Previously reported $P_c(4450)$ state had split into $P_c(4440)$ and $P_c(4457)$ states, and another resonance, named as $P_c(4312)^+$ had been discovered. The status of the other previously reported $P_c(4380)$ state remains unresolved: neither confirmed nor refuted. Following the year 2019, in 2020 LHCb Collaboration observed a pentaquark state, $P_{cs}(4459)$, in the invariant mass spectrum of $J/\psi\Lambda$ in the $\Xi_b^0 \rightarrow J/\psi\Lambda K^-$ decay. In 2022, the LHCb collaboration announced observation of a new structure $P_{cs}(4338)$ in the $J/\psi\Lambda$ mass distribution of the $B^- \rightarrow J/\psi\Lambda^- p$ decays. The spectroscopic parameters, minimal valence quark contents, and observed channels for these states are presented in Table I.

State	Mass (MeV)	Width (MeV)	Content	Observed channels
$P_c(4380)^+$ [4]	$4380\pm8\pm29$	$215 \pm 18 \pm 86$	$uudc\bar{c}$	$ \Lambda_b^0 \to J/\psi p K^-$
$P_c(4312)^+$ [5]	$4311.9 \pm 0.7 \begin{array}{c} +6.8 \\ -0.6 \end{array}$	$9.8 \pm 2.7 \ ^{+3.7}_{-4.5}$	$uudc\bar{c}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_c(4440)^+$ [5]	$4440.3 \pm 1.3 \ ^{+4.1}_{-4.7}$	$20.6 \pm 4.9 \ ^{+8.7}_{-10.1}$	$uudc\bar{c}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_c(4457)^+$ [5]	$4457.3 \pm 0.6 \ ^{+4.1}_{-1.7}$	$6.4 \pm 2.0 \ ^{+5.7}_{-1.9}$	$uudc\bar{c}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_{cs}(4459)^0$ [6]	$4458.8 \pm 2.9 \ ^{+4.7}_{-1.1}$	$17.3 \pm 6.5 \ ^{+8.0}_{-5.7}$	$udsc\bar{c}$	$\Xi_b^- \to J/\psi \Lambda K^-$
$P_{cs}(4338)^0$ [7]	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$udsc\bar{c}$	$B^- \rightarrow J/\psi \Lambda \bar{p}$

TABLE I. Hidden-charm pentaquark states observed by the LHCb Collaboration.

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The observation of the aforementioned pentaquark states triggered many theoretical studies with various methods to elucidate inner structure, exact nature, and determine quantum numbers of these nonconventional states. The pentaquark states consist of five quarks and clustering these quarks into substructures seems reasonable. The popular schemes are (i) molecular scheme, (ii) diquark-diquark-antiquark scheme, and (iii) diquark-triquark scheme. These pictures have different quark rearrangements. The primary assumptions of these pictures are as follows:

- (i) Molecular state scheme: In this scheme, pentaquark state consists of a meson and a baryon. Color neutral constituents (meson and baryon) form color singlet pentaquark state in molecular configuration.
- (ii) Diquark-diquark-antiquark scheme: In this case, diquark prefers to form color antitriplet $\bar{3}_c$. Two color antiplet diquarks together with color antitriplet quark $\bar{3}_c(\text{diquark}) \otimes \bar{3}_c(\text{diquark}) \otimes \bar{3}_c(\text{antiquark})$ form color singlet pentaquark.
- (iii) Diquark-triquark scheme: This scheme has a similar configuration with respect to molecular scheme. In this type of configuration, triquark consists two quarks and antiquark. Color triplet triquark 3_c (triquark) and color antitriplet diquark $\bar{3}_c$ (diquark) form diquark-triquark configuration 3_c (triquark) $\otimes \bar{3}_c$ (diquark) as color singlet a pentaquark state.

The above models are frequently used to study spectroscopic parameters of the hidden-charm pentaquark states. Although the literature consist many studies about hidden-charm pentaquark states, mostly about mass spectra, their internal nature and quantum numbers are still incomplete. However spectroscopic parameters alone may be inadequate to enlighten the controversial nature of these states. Consequently, to unveil the internal structure and configurations of these states, further studies including magnetic moments, radiative decays and weak decays are required. Among these, the information that magnetic moment prevails is essential to gain insight into the nature and internal structure of exotic states, which almost all of them are controversial in terms of internal structure. The magnetic moment is a measure of the distribution of quark-antiquark pairs within the hadron. It is also related to the shape of the hadron, electric radii and magnetic radii of the hadrons. In addition to this, pentaquark magnetic moments have a significant effect on differential and total cross-sections in electroproduction and photoproduction of pentaquarks. In this respect, magnetic moment is a crucial ingredient in calculation of cross sections of J/ψ photoproduction which can provide observation of further pentaquark states [8, 9].

In this present work, we adopt diquark-diquark-antiquark picture for the $P_c(4457)$ and related hidden-charm pentaquark states with and without strangeness and study magnetic moments. This picture can be visualized in Figure 1.



FIG. 1. Schematic diagram of pentaquark in diquark-diquark-antiquark scheme.

The diquark-antiquark model provides a framework for understanding the structure and properties of these pentaquark states. This model has been instrumental in describing the masses and decay properties of various observed

states. For instance, the LHCb data on pentaquarks with open charm emphasize the importance of hadron components in their structure, suggesting that these states can be described both in terms of quarks and hadrons [10]. It is pointed out in Ref. [11] that the development of lattice-regularized QCD and new theoretical approaches to the continuum bound-state problem have strongly suggested the importance of soft quark+quark (diquark) interactions in hadron physics. A recent LHCb analysis on the nature of $\chi_{c1}(3872)$ reveals that the radiative decays of $\chi_{c1}(3872) \rightarrow \psi(2S)\gamma$ and $\chi_{c1}(3872) \rightarrow J/\psi\gamma$ indicate that $D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}$ picture for the $\chi_{c1}(3872)$ is questionable and favors a sizeable compact component in the $\chi_{c1}(3872)$ state [12]. The other point is that diquark-triquark model has a similar quark configuration as the molecular model has, the exception is just the color representation.

Despite the fact that magnetic moment gives valuable information on physical parameters that are directly correlated with the inner structure of the state, the number of studies related to magnetic moment are not at the desired level [9, 13–31]. Although the magnetic moments of the hadrons give valuable information about the internal structure, magnetic moment measurement is a real challenge. Magnetic moment is the response of a state to an external magnetic field and if the particle lifetime is relatively short, it is hard to measure. However, magnetic moment measurement is possible indirectly, provided sufficient data is accumulated. For example, $\Delta^+(1232)$ baryon has also a very short lifetime, ($\sim 5 \times 10^{-24}$ s), however its magnetic moment was obtained through $\gamma N \rightarrow \Delta \rightarrow \Delta \gamma \rightarrow \pi N \gamma$ process [32–34]. A comparable process may be employed to achieve the magnetic moment of the P_c pentaquarks. QCD based models can also be employed to obtain magnetic moments. In Refs. [35, 36], magnetic moments of baryons containing two charm quarks have been extracted by lattice QCD formalism. These analyses can be extended to the exotic hadrons.

We organize this paper in the following manner: In Sec. II we present the method which is used to calculate the magnetic moment of the considered states, named as P_c . Sec. III is dedicated to numerical results of the magnetic moments of the P_c states. Finally, Sec. IV is reserved for summary of this work.

II. THEORETICAL FORMALISM

A. Wave function

The total wave function of a hadronic state depend on both spatial degrees of freedom and the internal degrees of freedom of color, flavor and spin

$$\Psi_{\text{total}} = \phi_{flavor} \chi_{spin} \epsilon_{color} \eta_{spatial}.$$
 (1)

The wave function of a pentaquark state can be constructed by imposing symmetry considerations: (i) the total wave function of pentaquark should be antisymmetric, and (ii) all physical states should be colorless, i.e., color neutral.

According to the color configurations, the pentaquark state may consist of either two or three clusters. In our model, pentaquark is composed of three clusters: two diquarks plus an antiquark. In color space, a diquark is in the $\bar{3}_c$ representation of the SU(3) color group. Introducing the notation

$$(S_t, R_f, R_c), \tag{2}$$

where, S_t denotes the spin of two quarks, R_f and R_c are the irreducible representations of two-quark states in flavor $SU_f(3)$ and color $SU_c(3)$ spaces, respectively; two quarks in S-wave can have the following configurations

$$(0, 6_f, 6_c), (1, \bar{3}_f, 6_c), (0, \bar{3}_f, \bar{3}_c), (1, 6_f, \bar{3}_c).$$
 (3)

In spin-flavor space, two quarks inside a diquark should be symmetric in order to satisfy Fermi statistics, due to the fact that they are antisymmetric in color space. The possible diquark configurations are then $(0, \bar{3}_f, \bar{3}_c)$ and $(1, 6_f, \bar{3}_c)$. As is clear from these representations, a diquark prefers to form $\bar{3}_c$ in color space and 6_f or $\bar{3}_f$ in flavor space. The flavor wave function of the hidden-charm pentaquark states can be constructed as follows in the notation of ϕ (Flavor representation, Isospin, Strangeness)

$$\phi(8_{1f}, 1/2, 0) = -\{qq\} (qc)\bar{c},\tag{4}$$

$$\phi(8_{1f}, 0, -1) = -\{qs\}(qc)\bar{c},\tag{5}$$

$$\phi(8_{1f}, 1, -1) = \sqrt{\frac{2}{3}} \{qq\} (sc)\bar{c} - \sqrt{\frac{1}{3}} \{qs\} (qc)\bar{c}, \tag{6}$$

$$\phi(8_{1f}, 1/2, -2) = \sqrt{\frac{1}{3}} \{qs\}(sc)\bar{c} - \sqrt{\frac{2}{3}} \{ss\}(qc)\bar{c},\tag{7}$$

$$\phi(8_{2f}, 1/2, 0) = [qq](qc)\bar{c}, \tag{8}$$

$$\phi(8_{2f}, 1/2, -2) = [qs](sc)c, \tag{9}$$

$$\phi(8_{2f}, 0, -1) = -\sqrt{\frac{2}{3}}[qq](sc)\bar{c} - \sqrt{\frac{1}{3}}[qs](qc)\bar{c}, \qquad (10)$$

$$\phi(8_{2f}, 1, -1) = [qs](qc)\bar{c},\tag{11}$$

$$\phi(10_f, 3/2, 0) = \{qq\}(qc)\bar{c},\tag{12}$$

$$\phi(10_f, 1, -1) = \sqrt{\frac{2}{3}} \{qs\}(qc)\bar{c} + \sqrt{\frac{1}{3}} \{qq\}(sc)\bar{c}, \tag{13}$$

$$\phi(10_f, 1/2, -2) = \sqrt{\frac{2}{3}} \{qs\}(sc)\bar{c} + \sqrt{\frac{1}{3}} \{ss\}(qc)\bar{c}, \tag{14}$$

$$\phi(10_f, 0, -3) = \{ss\}(sc)\bar{c}.$$
(15)

Here q represents the light quarks, u and d. In the above flavor wave functions, the bracket " $\{ \}$ " for the first diquark denotes that two quarks are symmetric (spin 1) inside it and the bracket "[]" for the first diquark denotes two quarks are antisymmetric (spin 0) inside it in the SU(3) flavor space. For the second diquark, the bracket "()" means that flavor symmetry of the two quarks inside this diquark is not definite. It is possible to use SU(2) Clebcsh-Gordon coefficients to expand the flavor wave functions in Eqs. (4-15) to obtain flavor wave functions in 8_f representation (Table III) and 10_f representation (Table III):

TABLE II. Flavor wave functions of P_c and related states in 8_f representation

(I, I_3)	Wave function- 8_{1f}	Wave function- 8_{2f}
$(\frac{1}{2}, \frac{1}{2})$	$-\sqrt{\frac{1}{3}} \{ud\}(cu)\bar{c} + \sqrt{\frac{2}{3}} \{uu\}(cd)\bar{c}$	$[ud](cu)\overline{c}$
$\left(\frac{1}{2},-\frac{1}{2}\right)$	$\sqrt{rac{1}{3}} \{ud\}(cd)ar{c} - \sqrt{rac{2}{3}} \{dd\}(cu)ar{c}$	$[ud](cd)ar{c}$
(1, 1)	$\sqrt{\frac{1}{3}} \{us\}(cu)\overline{c} - \sqrt{\frac{2}{3}} \{uu\}(cs)\overline{c}$	$[us](cu)ar{c}$
(1, 0)	$\sqrt{\frac{1}{6}}[\{us\}(cd)\bar{c} + \{ds\}(cu)\bar{c}] - \sqrt{\frac{2}{3}}\{ud\}(cs)\bar{c}$	$\frac{1}{\sqrt{2}}\{[us](cd)\bar{c}+[ds](cu)\bar{c}\}$
(1, -1)	$\sqrt{rac{1}{3}} \{ds\}(cd)ar{c} - \sqrt{rac{2}{3}} \{dd\}(cs)ar{c}$	$[ds](cd)ar{c}$
(0,0)	$\sqrt{\frac{1}{2}}[\{ds\}(cu)ar{c}-\{us\}(cd)ar{c}]$	$\left \frac{1}{\sqrt{6}}\left\{\left[us\right](cd)\bar{c}-\left[ds\right](cu)\bar{c}-2\left[ud\right](cs)\bar{c}\right\}\right.$
$\left(\frac{1}{2},\frac{1}{2}\right)$	$\sqrt{\frac{1}{3}} \{us\}(cs)\bar{c} - \sqrt{\frac{2}{3}} \{ss\}(cu)\bar{c}$	$[us](cs)\bar{c}$
$\left(\frac{1}{2},-\frac{1}{2}\right)$	$\sqrt{\frac{1}{3}} \{ ds \} (cs) \bar{c} - \sqrt{\frac{2}{3}} \{ ss \} (cd) \bar{c}$	$[ds](cs)ar{c}$

In spin space, the wave function of the pentaquark can be written as

$$|S_{12}, S_{34}, S_{1234}, S\rangle = |\left\{ \left[(s_1 s_2)_{S_{12}} \otimes (s_3 s_4)_{S_{34}} \right]_{S_{1234}} \otimes s_{\bar{5}} \right\}_S \otimes \ell \rangle, \tag{16}$$

where $S_{12} = 1$ for symmetric flavor wave function and $S_{12} = 0$ for antisymmetric wave function for the first diquark. This pattern follows for the second diquark, where $S_{34} = 1$ or 0 for symmetric or antisymmetric flavor wave functions, respectively. ℓ represents the orbital angular momentum.

B. Magnetic moment

Magnetic moments of the hadrons give valuable information about the internal structure, charge distribution and geometric shape of the hadron. As mentioned before, the Fermi statistics requires that the overall wave function of

TABLE III. Flavor wave functions of P_c and related states in 10_f representation

(I, I_3)	Wave function- 10_f
$(\frac{3}{2}, \frac{3}{2})$	$\{uu\}(cu)ar{c}$
$(\frac{3}{2}, \frac{1}{2})$	$\sqrt{\frac{2}{3}} \{ud\}(cu)\bar{c} + \sqrt{\frac{1}{3}} \{uu\}(cd)\bar{c}$
$(\frac{3}{2}, -\frac{1}{2})$	$\sqrt{\frac{2}{3}} \{ud\}(cd)\overline{c} + \sqrt{\frac{1}{3}} \{dd\}(cu)\overline{c}$
$(\frac{3}{2}, -\frac{3}{2})$	$\{dd\}(cd)\overline{c}$
(1, 1)	$\sqrt{\frac{2}{3}} \{us\}(cu)\bar{c} + \sqrt{\frac{1}{3}} \{uu\}(cs)\bar{c}$
(1, 0)	$\left \sqrt{\frac{1}{3}}[\{us\}(\underline{c}d)\bar{c} + \{ds\}(cu)\bar{c}] + \sqrt{\frac{1}{3}}\{ud\}(cs)\bar{c}\right $
(1, -1)	$\sqrt{rac{2}{3}} \{ds\}(cd)ar{c} + \sqrt{rac{1}{3}} \{dd\}(cs)ar{c}$
$\left(\frac{1}{2},\frac{1}{2}\right)$	$\sqrt{\frac{1}{3}} \{ss\}(cu)\bar{c} + \sqrt{\frac{2}{3}} \{us\}(cs)\bar{c}$
$(\frac{1}{2}, -\frac{1}{2})$	$\sqrt{\frac{1}{3}}\{ss\}(cd)\bar{c} + \sqrt{\frac{2}{3}}\{ds\}(cs)\bar{c}$
$(\bar{0},0)$	$\{ss\}(cs)ar{c}$

a pentaquark should be antisymmetric. In the ground state, color wave function ϵ_{color} is antisymmetric and space wave function $\eta_{spatial}$ is symmetric which yields $\epsilon_{color} \otimes \eta_{spatial}$ be antisymmetric. Therefore, one needs to consider spin-flavor wave function $\chi_{spin} \otimes \phi_{flavor}$ for the calculation of magnetic moment.

The quark configuration for the diquark-diquark-antiquark scheme is $(Qq_1)(q_2q_3)Q$. Specifically, the diquark-diquark-antiquark configuration is $(cq_1)(q_2q_3)(\bar{c})$ and the wave function for this state becomes

$$|\text{Pentaquark}\rangle = \left| \left\{ \left[(cq_1)_{s_H} \otimes (q_2q_3)_{s_L} \right]_{s_{HL}} \otimes \bar{c} \right\}_S \otimes \ell \right\rangle.$$
(17)

Here s_H is the spin of the (cq_1) diquark, s_L is the spin of the (q_2q_3) diquark which couples $s_H \otimes s_L$ to the spin s_{HL} of the four-quark structure. This four-quark structure then couples to the spin of anticharm \bar{c} to form the total spin of the pentaquark state. ℓ denotes the orbital excitation. At the quark level, magnetic moment operators can be written as

$$\hat{\mu}_{\rm spin} = \sum_{i} \frac{q_i}{2m_i} \hat{\sigma}_i,\tag{18}$$

where q_i is the quark charge, m_i is the quark mass, and σ_i is the Pauli's spin matrix. Each constitute may contribute to the magnetic moment of the related state. For the up, down, strange and charm quarks, we use the constituent quark masses as input [37]:

$$m_u = m_d = 0.336 \text{ GeV}, \ m_s = 0.540 \text{ GeV}, \ m_c = 1.660 \text{ GeV}.$$
 (19)

III. NUMERICAL RESULTS AND DISCUSSION

We present S-wave magnetic moment results of 8_{1f} and 8_{2f} representations. The reason of choosing these configurations is that $P_c(4380)$ and $P_c(4450)$ states with isospin $(I, I_3) = (\frac{1}{2}, \frac{1}{2})$ are in the 8_{1f} or 8_{2f} representation in the diquark-diquark-antiquark scheme [13]. In addition to this, $P_{cs}(4459)$ is supposed to be in 8_{2f} representation [18].

Before presenting magnetic moments of the $P_c(4457)$ and its related states, we want to denote the labelling convention of the states in this work. We use a labelling convention (P_{cri} , where r denotes for the related $P_c(4457)$ states and i counts the sates) in Table IV in order to distinguish the states¹. The results are listed in Tables V, VI, VII, VIII, IX, X, for $P_c(4457)$, P_{cr1} , P_{cr2} , P_{cr3} , P_{cr4} and P_{cr5} states, respectively.

Based on the predictions presented, we highlight our results as follows:

• As can be seen in the tables, all the states in the $0^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration of $J^P = \frac{1}{2}^-$ quantum number in 8_{2f} flavor representation and $(1^+ \otimes 1^+)_0 \otimes \frac{1}{2}^- \otimes 0^+$ configuration of $J^P = \frac{1}{2}^-$ quantum number in 8_{1f} flavor representation have same magnetic moments, $\mu = -0.377 \ \mu_N$. This is the magnetic moment of the anticharm quark. Therefore, only anticharm quark contributes to the pentaquark magnetic moment in this case.

¹ For the related $P_c(4457)$ states, we adopt the convention in Ref. [38]

Type of states	Wave function- 8_{1f}	Wave function- 8_{2f}
$P_{c}(4457)$	$\left -\sqrt{\frac{1}{3}} \{ud\}(cu)\bar{c} + \sqrt{\frac{2}{3}} \{uu\}(cd)\bar{c}\right $	$[ud](cu)\bar{c}$
P_{cr1}	$\sqrt{\frac{1}{3}} \{ ud \} (cd) \bar{c} - \sqrt{\frac{2}{3}} \{ dd \} (cu) \bar{c}$	$[ud](cd)ar{c}$
P_{cr2}	$\sqrt{\frac{1}{3}} \{us\}(cu)\bar{c} - \sqrt{\frac{2}{3}} \{uu\}(cs)\bar{c}$	$[us](cu)\bar{c}$
P_{cr3}	$\sqrt{\frac{1}{3}} \{ ds \} (cd) \bar{c} - \sqrt{\frac{2}{3}} \{ dd \} (cs) \bar{c}$	$[ds](cd)ar{c}$
P_{cr4}	$\sqrt{\frac{1}{3}} \{us\}(cs)\bar{c} - \sqrt{\frac{2}{3}} \{ss\}(cu)\bar{c}$	$[us](cs)\bar{c}$
P_{cr5}	$\sqrt{\frac{1}{3}} \{ ds \} (cs) \bar{c} - \sqrt{\frac{2}{3}} \{ ss \} (cd) \bar{c}$	$[ds](cs)ar{c}$

TABLE IV. Flavor wave functions of P_c and related states in 8_f representation

TABLE V. The magnetic moment results of the $P_c(4457)$ pentaquark. $J_H^{P_H}$ corresponds to the angular momentum and parity of (cq_1) , $J_L^{P_L}$ is for (q_2q_3) , $J_{\bar{c}}^{P_{\bar{c}}}$ is for \bar{c} , and $J_{\ell}^{P_{\ell}}$ is for orbital. The results are presented in unit of nuclear magneton μ_N .

	8_{2f} representation					
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results			
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377			
		$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.244			
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.930			
		8_{1f} representation				
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results			
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes rac{1}{2}^- \otimes 0^+$	2.607			
	-	$(1^+\otimes 1^+)_0\otimes {1\over 2}^-\otimes 0^+$	-0.377			
		$(1^+\otimes 1^+)_1\otimes {1\over 2}^-\otimes 0^+$	1.182			
$\frac{3}{2}$ -	${}^{4}S_{\frac{3}{2}}$	$(0^+\otimes 1^+)\otimes \frac{1}{2}^-\otimes 0^+$	3.345			
		$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	1.207			
		$(1^+\otimes 1^+)_2\otimes {1\over 2}^-\otimes 0^+$	3.077			
$\frac{5}{2}^{-}$	${}^6S_{rac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	2.792			

• In $P_c(4457)$ state, the magnetic moments of the 8_{1f} and 8_{2f} flavor representations are different, except the two cases mentioned above. In 8_{2f} representation, all the magnetic moments are negative whereas except $(1^+ \otimes 1^+)_0 \otimes \frac{1}{2}^- \otimes 0^+$ with $J^P = \frac{1}{2}^-$ in 8_{1f} representation, all the magnetic moments are positive. In Ref. [16], magnetic moment of $P_c(4457)$ with $\frac{1}{2}^-$ quantum number was studied by using light-cone QCD sum rule formalism in diquark-diquark-antiquark and molecular pictures where the results for magnetic moments are $\mu_{P_c(4457)} = 0.88^{+0.32}_{-0.29} \mu_N$. Our result of $J^P = \frac{1}{2}^ (1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$ in 8_{1f} representation is $\mu_N = 1.182 \ \mu_N$ and compatible with the prediction of reference work. Using quark model in a framework with and without coupled channel and D-wave effects [17], magnetic moment of $P_c(4457)$ with $J^P = \frac{3}{2}^-$ quantum number was obtained as $\mu_{P_c(4457)} = (1.145 - 1.365) \ \mu_N$. Our result of $J^P = \frac{3}{2}^- (1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$ in 8_{1f} representation is $\mu = 1.207 \ \mu_N$ and agree well with the given reference. In Ref. [38], magnetic moment of $P_c(4457)$ with $J^P = \frac{3}{2}^-$ in diquark-diquark-antiquark scheme is obtained as $\mu_{P_c(4457)} = -1.96^{+0.50}_{-0.37} \ \mu_N$ by using light-cone QCD sum rule formalism. The results of 8_{2f} representation are negative but are not compatible with this result in terms of magnitude. The result of $1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$ of $J^P = \frac{3}{2}^-$ in 8_{2f} representation is $\mu = -0.930 \ \mu_N$ and close to the value of given reference within the up and down limits.

	8_{2f} representation				
J^P	$ ^{2s+1}L_J$	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results		
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377		
		$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	1.617		
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	1.861		
		8_{1f} representation			
J^P	$ ^{2s+1}L_J$	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results		
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes rac{1}{2}^- \otimes 0^+$	-1.115		
		$(1^+\otimes 1^+)_0\otimes {1\over 2}^-\otimes 0^+$	-0.377		
		$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	0.251		
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$(0^+\otimes 1^+)\otimes rac{1}{2}^-\otimes 0^+$	-2.238		
		$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	-0.188		
		$(1^+ \otimes 1^+)_2 \otimes \frac{1}{2}^- \otimes 0^+$	0.565		
$\frac{5}{2}^{-}$	${}^{6}S_{\frac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	0		

TABLE VI. Same as in Table V but for the the P_{cr1} pentaquark.

TABLE VII. Same as in Table V but for the the P_{cr2} pentaquark.

8_{2f} representation				
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results	
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377	
		$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.009	
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.579	
		8_{1f} representation		
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results	
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	2.607	
		$(1^+\otimes 1^+)_0\otimes rac{1}{2}^-\otimes 0^+$	-0.377	
		$(1^+\otimes 1^+)_1\otimes {1\over 2}^-\otimes 0^+$	1.299	
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$(0^+\otimes 1^+)\otimes rac{1}{2}^-\otimes 0^+$	3.345	
		$(1^+\otimes 1^+)_1\otimes {1\over 2}^-\otimes 0^+$	1.383	
		$(1^+ \otimes 1^+)_2 \otimes \frac{1}{2}^- \otimes 0^+$	3.394	
$\frac{5}{2}^{-}$	${}^6S_{rac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	3.143	

• In the P_{cr1} type pentaquark, magnetic moment of the $1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{5}{2}^-$ quantum

	8_{2f} representation				
J^P	$ ^{2s+1}L_J$	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results		
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377		
		$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.009		
$\frac{3}{2}^{-}$	$^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.579		
		8_{1f} representation			
J^P	$ ^{2s+1}L_J$	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results		
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	-1.115		
		$(1^+ \otimes 1^+)_0 \otimes \frac{1}{2}^- \otimes 0^+$	-0.377		
		$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	-0.562		
$\frac{3}{2}^{-}$	$^{4}S_{\frac{3}{2}}$	$(0^+ \otimes 1^+) \otimes \frac{1}{2}^- \otimes 0^+$	-2.238		
	_	$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	-1.408		
		$(1^+ \otimes 1^+)_2 \otimes \frac{1}{2}^- \otimes 0^+$	-1.631		
$\frac{5}{2}^{-}$	${}^{6}S_{rac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	-2.440		

TABLE VIII. Same as in Table V but for the the P_{cr3} pentaquark.

TABLE IX. Same as in Table V but for the the P_{cr4} pentaquark.

8_{2f} representation				
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results	
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377	
		$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	1.617	
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	1.861	
		8_{1f} representation		
J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results	
$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.646	
		$(1^+\otimes 1^+)_0\otimes {1\over 2}^-\otimes 0^+$	-0.377	
		$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	0.485	
$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$(0^+\otimes 1^+)\otimes rac{1}{2}^-\otimes 0^+$	-1.535	
		$(1^+\otimes 1^+)_1\otimes {1\over 2}^-\otimes 0^+$	0.163	
		$(1^+ \otimes 1^+)_2 \otimes \frac{1}{2}^- \otimes 0^+$	1.198	
$\frac{5}{2}^{-}$	${}^6S_{rac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	0.703	

number turns out to be zero. Both $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ quantum numbers in 8_{1f} and 8_{2f} representations

	8_{2f} representation				
	J^P	$\left {}^{2s+1}L_J \right $	$\left J_{H}^{P_{H}}\otimes J_{L}^{P_{L}}\otimes J_{\bar{c}}^{P_{\bar{c}}}\otimes J_{\ell}^{P_{\ell}}\right $	Results	
	$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+\otimes 0^+\otimes rac{1}{2}^-\otimes 0^+$	-0.377	
			$1^+ \otimes 0^+ \otimes rac{1}{2}^- \otimes 0^+$	-0.244	
_	$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.930	
_			8_{1f} representation		
_	J^P	$\Big ^{2s+1}L_J$	$J_{H}^{P_{H}} \otimes J_{L}^{P_{L}} \otimes J_{\bar{c}}^{P_{\bar{c}}} \otimes J_{\ell}^{P_{\ell}}$	Results	
_	$\frac{1}{2}^{-}$	$^{2}S_{\frac{1}{2}}$	$0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	-0.646	
			$(1^+ \otimes 1^+)_0 \otimes \frac{1}{2}^- \otimes 0^+$	-0.377	
			$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	-0.445	
	$\frac{3}{2}^{-}$	${}^{4}S_{\frac{3}{2}}$	$(0^+ \otimes 1^+) \otimes \frac{1}{2}^- \otimes 0^+$	-1.535	
			$(1^+ \otimes 1^+)_1 \otimes \frac{1}{2}^- \otimes 0^+$	-1.233	
			$(1^+ \otimes 1^+)_2 \otimes \frac{1}{2}^- \otimes 0^+$	-1.315	
_	$\frac{5}{2}^{-}$	${}^6S_{rac{5}{2}}$	$1^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$	-2.088	

TABLE X. Same as in Table V but for the the P_{cr5} pentaquark.

are quite different and a possible observation of magnetic moment may help to identify quantum number of this state. Ref. [38] obtained magnetic moment of this state with $J^P = \frac{3}{2}^-$ quantum number as $\mu_N = -2.04^{+0.46}_{-0.41} \mu_N$ using light-cone QCD sum rule formalism. Our result of $J^P = \frac{3}{2}^-$ (0⁺ \otimes 1⁺) $\otimes \frac{1}{2}^- \otimes 0^+$ in 8_{1f} representation is $\mu = -2.238 \ \mu_N$ and agree with this result. Compared to $P_c(4457)$ state, it can be seen that quark rearrangements in diquarks drastically change the magnetic moment results.

- In the P_{cr2} type pentaquark, all the results in 8_{2f} representation are negative. In the 8_{1f} representation, except $(1^+ \otimes 1^+)_0 \otimes \frac{1}{2}^- \otimes 0^+$ with $J^P = \frac{1}{2}^-$, all the results are positive. None of the our results agree with the prediction in Ref. [38] which is $\mu = -2.08^{+0.53}_{-0.39} \mu_N$ for $J^P = \frac{3}{2}^-$ quantum number.
- In the P_{cr3} type pentaquark, all the results of magnetic moments are negative in both 8_{1f} and 8_{2f} representations. Ref. [38] obtained magnetic moment as $\mu_N = -2.13^{+0.53}_{-0.40} \mu_N$ for $J^P = \frac{3}{2}^-$ for quantum number. Our results for $J^P = \frac{3}{2}^- (0^+ \otimes 1^+) \otimes \frac{1}{2}^- \otimes 0^+$ configuration is $\mu = -2.238 \ \mu_N$ agree well the result of reference value.
- In the P_{cr4} type pentaquark, the magnetic moment results scatter between negative and positive values. Ref. [38] obtained magnetic moment as $\mu = -2.29^{+0.53}_{-0.39} \mu_N$ for $J^P = \frac{3}{2}^-$ quantum number. Our result for $J^P = \frac{3}{2}^ (0^+ \otimes 1^+) \otimes \frac{1}{2}^- \otimes 0^+$ configuration is $\mu = -1.535 \mu_N$ which is compatible.
- In the P_{cr5} type pentaquark, all the results of magnetic moments are negative in both 8_{1f} and 8_{2f} representations. Our result of $0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{3}{2}^-$ is compatible the result of Ref. [38], which is $\mu = -2.33^{+0.53}_{-0.41} \mu_N$.
- In 8_{2f} representations, $0^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{1}{2}^-$ and $1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{3}{2}^-$ of $P_c(4457)$ and P_{cr5} , P_{cr1} and P_{cr4} , P_{cr2} and P_{cr3} states have the same magnetic moments. The reason for this could be that the contribution of light quarks to the total magnetic moment is dominant.
- In 8_{1f} representations, $0^+ \otimes 1^+ \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{1}{2}^-$ and $(0^+ \otimes 1^+) \otimes \frac{1}{2}^- \otimes 0^+$ configuration with $J^P = \frac{3}{2}^-$ of $P_c(4457)$ and P_{cr2} , P_{cr1} and P_{cr3} , P_{cr4} and P_{cr5} states have the same magnetic moments. The reason for this could be that the contribution of light quarks to the total magnetic moment is dominant.

• The results of present study together with the available results in the literature magnetic moments of the hiddencharm pentaquark states may give information about internal structure and their spin-parity quantum numbers. These predictions can help distinguish between various theoretical models based on experimental measurements.

IV. SUMMARY

In this present study, we systematically study magnetic moments of the $P_c(4457)$ and its related hidden-charm pentaquark states with and without strangeness employing a comprehensive analysis that encompasses diquarkdiquark-antiquark scheme with $J^P = \frac{1}{2}^-$, $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^-$ quantum numbers. We also compare our results with the available results reported in the literature.

The magnetic moments of hidden-charm pentaquark states vary with phenomenological models. These variations can help distinguish between different structural models such as molecular, diquark-diquark-antiquark, and diquark-triquark models. Different phenomenological models predict distinct magnetic moments for hidden-charm pentaquark states. These predictions can help distinguish between various theoretical models based on experimental measurements. It is clear that the experimental measurement of the magnetic moment of the pentaquarks can help distinguish their inner structure. We hope that our results will help the endeavour for the probing the internal structure of $P_c(4457)$ and its related pentaquarks.

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