

## Two-time autocorrelation function in phase-ordering kinetics from local scale-invariance

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**Abstract.** – The time-dependent scaling of the two-time autocorrelation function of spin systems without disorder undergoing phase-ordering kinetics is considered. Its form is shown to be determined by an extension of dynamical scaling to a local scale-invariance which turns out to be a new version of conformal invariance. The predicted autocorrelator is in agreement with Monte-Carlo data on the autocorrelation function of the 2D kinetic Ising model with Glauber dynamics quenched to a temperature below criticality.

Understanding the kinetics of phase-ordering after a rapid quench from an initial disordered state into the ordered phase has since a long time posed a continuing challenge (see [1–4] for reviews). A key insight has been the observation that many of the apparently erratic and history-dependent properties of such systems can be organized in terms of a simple scaling picture [5]. This means that there is a single time-dependent length-scale  $L(t)$  which is identified with the typical linear size of ordered clusters. It turns out that the ageing behaviour is more fully revealed in observables such as the two-time autocorrelation function  $C(t, s)$  or the two-time linear autoresponse function  $R(t, s)$  defined as

$$C(t, s) := \langle \phi(t)\phi(s) \rangle, \quad R(t, s) := \left. \frac{\delta \langle \phi(t) \rangle}{\delta h(s)} \right|_{h=0} \quad (1)$$

where  $\phi(t)$  denotes the time-dependent order-parameter,  $h(s)$  is the time-dependent conjugate magnetic field,  $t$  is referred to as *observation time* and  $s$  as *waiting time*. One says that the system undergoes *ageing* if  $C$  or  $R$  depend on both  $t$  and  $s$  and not merely on the difference  $\tau = t - s$ . These two-time functions are expected to show dynamical scaling in the ageing regime  $t, s \gg t_{\text{micro}}$  and  $t - s \gg t_{\text{micro}}$ , where  $t_{\text{micro}}$  is some microscopic time scale. Then

$$C(t, s) = M_{\text{eq}}^2 f_C(t/s), \quad R(t, s) = s^{-1-a} f_R(t/s) \quad (2)$$

such that the scaling functions  $f_{C,R}(y)$  satisfy the following asymptotic behaviour

$$f_C(y) \sim y^{-\lambda_C/z}, \quad f_R(y) \sim y^{-\lambda_R/z} \quad (3)$$

as  $y \rightarrow \infty$  and where  $\lambda_C$  and  $\lambda_R$ , respectively, are known as the autocorrelation [6,7] and autoresponse exponents [8] and  $z$  is the dynamical exponent, defined through  $L(t) \sim t^{1/z}$ . Throughout, we consider simple ferromagnets without disorder and with a non-conserved order-parameter. Then  $z = 2$  is known [9]. For spin systems with short-ranged equilibrium correlators (e.g. the  $d > 1$  Glauber-Ising model) it has been checked in detail that  $a = 1/z = 1/2$  [10,11], but the closed-form and *ad hoc* OJK approximation gives  $a = (d-1)/2$  [12,13]. The exponents  $\lambda_{C,R}$  are independent of the equilibrium exponents and of  $z$  [2,3,14]. Although the equality  $\lambda_C = \lambda_R$  had been taken for granted (reconfirmed in a recent second-order perturbative analysis of the time-dependent Ginzburg-Landau equation [15]), counterexamples exist for long-ranged initial correlations in ageing ferromagnets [8] and in the random-phase sine-Gordon model [16]. For *short-ranged* initial correlations, dynamical scaling together with Galilei-invariance at temperature  $T = 0$  are sufficient for  $\lambda_C = \lambda_R$  [17].

We are interested in the form of the scaling functions  $f_{C,R}(y)$ . Indeed, it is known that for any given value of  $z$  there exist infinitesimal local scale-transformations  $t \mapsto (1 + \varepsilon)^z t$ ,  $\mathbf{r} \mapsto (1 + \varepsilon)\mathbf{r}$  with an infinitesimal  $\varepsilon = \varepsilon(t, \mathbf{r})$  which may depend on both time and space [18]. Furthermore, the *local* scale-transformations so constructed act as dynamical symmetries of certain linear field equations which might be viewed as some effective renormalized equation of motion. From the assumption that the response functions of the theory transform covariantly under local scale-transformations, the exact form of the scaling function  $f_R(y)$  is found [17–19]

$$f_R(y) = r_0 y^{1+a'-\lambda_R/z} (y-1)^{-1-a'} \quad (4)$$

where  $a'$  is a new exponent [17] and  $r_0$  is a normalization constant. Eq. (4) with  $a = a'$  is recovered in many spin systems quenched to a temperature  $T \leq T_c$  and whose dynamics is described by a master equation [4,17–21], but in models such as the 1D Glauber-Ising model at  $T = 0$  [17] or the OJK approximation of phase-ordering [12,13,15] eq. (4) holds but with  $a \neq a'$ . If a phase-ordering system is also Galilei-invariant at  $T = 0$ , then  $f_R(y)$  is independent of both the thermal and the initial noises [17].

While the scaling form of the autoresponse function thus seems to be understood, the problem of finding the scaling function  $f_C(y)$  of the autocorrelation function appears to be considerably more difficult. A by now classical attempt recognizes that for  $T < T_c$ , temperature should be irrelevant [1] and hence sets  $T = 0$ . Building on the Ohta-Jasnow-Kawasaki approximation (see [1]) in the kinetic  $O(n)$ -model one introduces an auxiliary field for which a gaussian closure procedure is assumed. This leads to [22–24]

$$f_{C,\text{BFT}}(y) = \frac{n}{2\pi} \left[ B \left( \frac{1}{2}, \frac{n+1}{2} \right) \right]^2 \left( \frac{4y}{(y+1)^2} \right)^{d/4} {}_2F_1 \left( \frac{1}{2}, \frac{1}{2}; \frac{n+2}{2}; \left( \frac{4y}{(y+1)^2} \right)^{d/2} \right) \quad (5)$$

where  $B$  is Euler's beta function and  ${}_2F_1$  a hypergeometric function. However, this closed form implies  $\lambda_C = d/2$  which only holds in certain limiting cases (for example,  $\lambda_C = d/2 + \alpha n^{-1}$ , to leading order in  $n$ , in the  $O(n)$ -model and with a known value of  $\alpha > 0$  [25]. See also [26]).

Here we investigate to what extent  $f_C(y)$  may be determined from a local scale-invariance (LSI). We concentrate on phase-ordering where  $T < T_c$  and thus  $z = 2$  [9]. The group of local scale-transformations is then the Schrödinger group [27,28] which for example arises as the maximal kinematic group of the free Schrödinger (or diffusion) equation. In particular, the Schrödinger group contains dilatations with  $z = 2$  and Galilei-transformations. For local theories, there is a Ward identity such that these two symmetries imply full Schrödinger-invariance [29]. However, Galilei-invariance is incompatible with thermal or initial noises.

We consider a coarse-grained order-parameter  $\phi(t, \mathbf{r})$  satisfying a Langevin equation [1,30]

$$\frac{\partial \phi(t, \mathbf{r})}{\partial t} = -D \frac{\delta \mathcal{H}}{\delta \phi} - Dv(t)\phi(t, \mathbf{r}) + \eta(t, \mathbf{r}) \quad (6)$$

where  $\mathcal{H}$  is the classical Hamiltonian, and  $D$  stands for the diffusion constant. Zero-mean thermal noise is characterized by its variance  $\langle \eta(t, \mathbf{r})\eta(s, \mathbf{r}') \rangle = 2DT \delta(t-s) \delta(\mathbf{r}-\mathbf{r}')$  where  $T$  is the bath temperature. The initial conditions are specified in terms of  $a(\mathbf{r}-\mathbf{r}') := \langle \phi(0, \mathbf{r})\phi(0, \mathbf{r}') \rangle$  and where we already anticipated spatial translation invariance, hence  $a(\mathbf{r}) = a(-\mathbf{r})$ . The potential  $v = v(t)$  acts as a Lagrange multiplier. For  $z = 2$  it is easy to see that if

$$k(t) := \exp\left(-D \int_0^t du v(u)\right) \sim t^F, \quad F = 1 + a - \frac{\lambda_R}{2} \quad (7)$$

and if  $a = a'$  then eq. (4) is reproduced from Schrödinger-invariance [17]. The Langevin equation (6) may be turned into a field-theory using the Martin-Siggia-Rose formalism. Provided that field-theory is *Galilei-invariant in the absence of thermal and of initial noise* (i.e.  $T = 0$  and  $a(\mathbf{r}) = 0$ ) then the two-time autocorrelation function can be expressed in terms of noiseless response functions. Precise data on the form of the space-time response function in the Glauber-Ising model in  $2D$  and  $3D$  provide strong direct evidence in favour of its Galilei-invariance [20]. We concentrate on the case of a fully disordered initial state with  $a(\mathbf{R}) = 2a_0\delta(\mathbf{R})$  where  $a_0$  is a normalization constant. Then it is shown in [17] that

$$C(t, s) = a_0 \int d\mathbf{R} R_0^{(3)}(t, s, 0; \mathbf{R}) + DT \int du d\mathbf{R} R_0^{(3)}(t, s, u; \mathbf{R}) \quad (8)$$

$$R_0^{(3)}(t, s, u; \mathbf{r}) := \left\langle \phi(t; \mathbf{y})\phi(s; \mathbf{y})\tilde{\phi}(u; \mathbf{r} + \mathbf{y})^2 \right\rangle_0 = \frac{k(t)k(s)}{k^2(u)} \mathcal{R}_0^{(3)}(t, s, u; \mathbf{r}) \quad (9)$$

where the index 0 refers to the noiseless part of the field-theory. Here, the field  $\phi$  has the scaling dimension  $x = 1 + a$  and  $\tilde{\phi}^2$  is a composite field with scaling dimension  $2\tilde{x}_2$  (only for free fields  $\tilde{x}_2 = x$ ). The well-known three-point response function  $\mathcal{R}_0^{(3)}$  for  $v(t) = 0$  is given by Schrödinger-invariance [31]

$$\begin{aligned} \mathcal{R}_0^{(3)}(t, s, u; \mathbf{r}) &= \mathcal{R}_0^{(3)}(t, s, u) \exp\left[-\frac{\mathcal{M}}{2} \frac{t+s-2u}{(s-u)(t-u)} \mathbf{r}^2\right] \Psi\left(\frac{t-s}{2(t-u)(s-u)} \mathbf{r}^2\right) \\ \mathcal{R}_0^{(3)}(t, s, u) &= \Theta(t-u)\Theta(s-u) (t-u)^{-\tilde{x}_2} (s-u)^{-\tilde{x}_2} (t-s)^{-x+\tilde{x}_2} \end{aligned} \quad (10)$$

where  $\Psi = \Psi(\rho)$  is an arbitrary scaling function and  $\mathcal{M} = 1/(2D)$  is a non-universal constant. Eqs. (8,9,10) are the foundation of our analysis of the autocorrelation function.

Comparing (8,9,10) with the scaling form (2,3), we have  $\tilde{x}_2 - x = d/2 - \lambda_C$  and [17]

$$C(t, s) = a_0 y^{\lambda_C/2} (y-1)^{-\lambda_C} \Phi\left(\frac{y+1}{y-1}\right), \quad \Phi(w) := \int_{\mathbb{R}^d} d\mathbf{R} \exp\left[-\frac{\mathcal{M}w}{2} \mathbf{R}^2\right] \Psi(\mathbf{R}^2) \quad (11)$$

where  $y = t/s$ . The second term in (8) merely gives a finite-time correction and may be dropped, in agreement with  $T$  being irrelevant for  $T < T_c$  [1]. The form of  $f_C(y)$  still depends on the unknown function  $\Phi(w)$ . A simple heuristic way to fix its form is to argue that the noiseless response function  $R_0^{(3)}(t, s, 0; \mathbf{r})$  which describes a response of the autocorrelation  $C(t, s) = \langle \phi(t)\phi(s) \rangle$  should be non-singular at  $t = s$ . This leads to  $\Phi(w) \simeq \Phi_0 w^{-\lambda_C}$  as  $w \rightarrow \infty$ . If this were valid for all  $w$ , we would obtain the following simple form [17]

$$C(t, s) \approx a_0 \Phi_0 \left((y+1)^2/y\right)^{-\lambda_C/2} \quad (12)$$

TABLE I – Parameters of the autocorrelation function of the 2D Glauber-Ising model.

$T$	$C_\infty$	$C_{\text{ref}}$	$y_{\text{ref}}$	$A$	$B$	$E$
0.0	1.65(3)	0.605	3.5	-0.601	3.94	0.517
1.5	1.69(3)	0.48	5.5	-5.41	18.4	1.24

which at least gives the correct asymptotic behaviour as  $y \rightarrow \infty$ . It has been checked that this form is exact for systems described by an underlying free-field theory [17].

We now outline how to find the scaling function  $\Phi(w)$  in (11) more systematically. To achieve this by a dynamical symmetry argument, an extension of the Schrödinger group used so far as dynamical group has to be found. Indeed, when considering the dynamical symmetries of the free Schrödinger equation  $(2\mathcal{M}\partial_t - \partial_r^2)\phi = 0$ , it is possible to consider also the ‘mass’  $\mathcal{M}$  as a dynamical variable [32]. Then the dynamical symmetry group extends to the conformal group in  $d + 2$  dimensions [29, 33]. We postulate that, at zero temperature, this *conformal symmetry is a dynamical symmetry of phase-ordering*. Now a quasiprimary field depends on three variables  $\phi = \phi(\mathcal{M}, t, \mathbf{r})$ . For conformal invariance, it is sufficient that  $\phi$  transforms covariantly under the extra generators [29] (for simplicity we also set  $d = 1$ )

$$V_- = -i\frac{\partial^2}{\partial\mathcal{M}\partial r} + ir\frac{\partial}{\partial t}, \quad N_0 = -t\frac{\partial}{\partial t} - 1 - \mathcal{M}\frac{\partial}{\partial\mathcal{M}} \quad (13)$$

We look for the general form of a three-point function  $\mathcal{R}_0^{(3)} = \langle \phi_a \phi_b \tilde{\phi}_c \rangle$  of a theory without noise and which is conformally invariant. Recall that in the MSR formalism, response functions are written as correlators of  $\phi$  and  $\tilde{\phi}$ . The response field  $\tilde{\phi}$  conjugate to the field  $\phi$  has the ‘mass’  $\tilde{\mathcal{M}} = -\mathcal{M} \leq 0$  [29] and for  $\mathcal{R}_0^{(3)}$  the ‘mass’ conservation  $\mathcal{M}_a + \mathcal{M}_b + \tilde{\mathcal{M}}_c = 0$  holds [31]. The response function is given, because of Schrödinger-invariance, by eq. (10). Since now the ‘masses’ are also considered as variables, we expect  $\Psi = \Psi(\rho, \mathcal{M}_a, \mathcal{M}_b)$ . Covariance under the conformal group means  $V_- \mathcal{R}_0^{(3)} = N_0 \mathcal{R}_0^{(3)} = 0$  and leads to

$$\left( \lambda_C - a - \frac{d}{2} + \mathcal{M}_a \frac{\partial}{\partial \mathcal{M}_a} + \frac{\partial^2}{\partial \rho \partial \mathcal{M}_a} \right) \Psi = 0, \quad \left( \lambda_C - a - \frac{d}{2} + \mathcal{M}_b \frac{\partial}{\partial \mathcal{M}_b} + \frac{\partial^2}{\partial \rho \partial \mathcal{M}_b} \right) \Psi = 0$$

$$\left( \lambda_C - 2a - \frac{d}{2} - \rho \frac{\partial}{\partial \rho} + \mathcal{M}_a \frac{\partial}{\partial \mathcal{M}_a} + \mathcal{M}_b \frac{\partial}{\partial \mathcal{M}_b} \right) \Psi = 0 \quad (14)$$

hence  $\Psi = \rho^{\lambda_C - 2a - d/2} K(\eta, \zeta)$  with  $\eta = (\mathcal{M}_a + \mathcal{M}_b)\rho/2$  and  $\zeta = (\mathcal{M}_a - \mathcal{M}_b)\rho/2$ . We need the response of the autocorrelator  $\langle \phi_a \phi_a \rangle$ , thus  $\mathcal{M}_a = \mathcal{M}_b = \mathcal{M}$ , hence  $\zeta = 0$ . Then

$$\left[ (2\lambda_C - d - 2a) + (\lambda_C + 1 - 2a - d/2) \partial_\eta + \eta \partial_\eta + \eta \partial_\eta^2 \right] K(\eta, 0) = 0 \quad (15)$$

and we finally obtain the required scaling function ( $\psi_{0,1}$  are arbitrary constants)

$$\Psi(\rho, \mathcal{M}, \mathcal{M}) = \psi_0 \rho^{\lambda_C - 2a - d/2} {}_1F_1(2\lambda_C - d - 2a, \lambda_C + 1 - 2a - d/2; -\mathcal{M}\rho)$$

$$+ \psi_1 \mathcal{M}^{2a + d/2 - \lambda_C} {}_1F_1(\lambda_C - d/2, 1 + 2a + d/2 - \lambda_C; -\mathcal{M}\rho) \quad (16)$$

Before we can insert this into (11), we should consider the conditions required such that the derivation of (16) is valid. In particular, it is based on dynamical scaling and we recall the condition  $t - s \gg t_{\text{micro}}$  for its validity (similar difficulties have been encountered before for integrated response functions, see [10, 11, 34]). From (10), this means that for small arguments  $\rho \rightarrow 0$  the form of the function  $\Psi(\rho)$  is not given by local scale-invariance. Rather, for  $\rho \ll 1$

we expect that the response of the two-time autocorrelation function  $C(t, s) = C(s, t)$  should be symmetric and especially non-singular in the limit  $t - s \rightarrow 0$  [17]. This suggests that  $\Psi(\rho) \simeq \Psi_0 \rho^{\lambda_C - d/2}$  if  $\rho \leq \varepsilon$  and  $\Psi(\rho)$  is given by (16) only if  $\rho \geq \varepsilon$  where  $\varepsilon$  sets the scale which separates the two regimes. The constant  $\Psi_0$  is determined from the condition that  $\Psi(\rho)$  is continuous at  $\rho = \varepsilon$ . A straightforward but slightly lengthy calculation gives

$$\begin{aligned} \Phi(w) = & B \left[ \Gamma(d/2) {}_2F_1(\lambda_C - d/2, d/2; 1 + 2a + d/2 - \lambda_C; -1/w) - \gamma(d/2, Ew) \right] w^{-d/2} \\ & + A \left[ \Gamma(\lambda_C - 2a) {}_2F_1(2\lambda_C - d - 2a, \lambda_C - 2a; \lambda_C + 1 - 2a - d/2; -1/w) w^{2a - \lambda_C} \right. \\ & \quad \left. - \gamma(\lambda_C - 2a, Ew) w^{2a - \lambda_C} \right] + AE^{-2a} w^{-\lambda_C} \gamma(\lambda_C, Ew) + BE^{d/2 - \lambda_C} w^{-\lambda_C} \gamma(\lambda_C, Ew) \\ & + AE^{1-2a} \frac{2\lambda_C - d - 2a}{\lambda_C + 1 - 2a - d/2} w^{-\lambda_C} \left[ (Ew)^{2a-1} \gamma(\lambda_C + 1 - 2a, Ew) - \gamma(\lambda_C, Ew) \right] \quad (17) \end{aligned}$$

where  $\gamma(a, z)$  is an incomplete gamma-function,  $E = \mathcal{M}\varepsilon$  and  $A, B$  are constants related to  $\psi_{0,1}$ . Eqs. (11,17) together give the autocorrelation function  $C(t, s)$ . This is our main result.

Some simple consistency checks are easy to perform. First, for free fields  $\lambda_C = d/2$  and eq. (12) is recovered for  $A = 0$ . Second, we find  $\Phi(w) \sim w^{-\lambda_C}$  for  $w \rightarrow \infty$  as expected and if  $A \neq 0$ , we obtain the additional constraint  $2a \leq 1$ .

For a non-trivial test, we consider the phase-ordering kinetics of the 2D Glauber-Ising model, which we realize through a standard heat-bath rule, and lattices up to  $800^2$ . We consider quenches to temperatures  $T = 0$  and  $T = 1.5$ , both in the ordered phase. For  $T = 0$  ( $T = 1.5$ ) we went up to  $y = t/s = 100$  ( $y = 60$ ), with  $s = 1600$  being the longest waiting time studied, and averaged over typically 500 independent runs for the largest lattices. While the exponent  $a = 1/2$  is known [3, 10], we repeated the determination of  $\lambda_C$  and find  $\lambda_C = 1.25(1)$ , in agreement with earlier results [6, 26]. Next, we determined the amplitude  $C_\infty$  from  $C(t, s) \simeq C_\infty (t/s)^{-\lambda_C/2}$  as  $t/s \rightarrow \infty$  which produces a first constraint for the fit of the constants  $A, B, E$ . A second constraint follows from the observation that at a special value  $y_{\text{ref}}$  the curves for different values of  $s$  cross. We write  $C(y_{\text{ref}}) = C_{\text{ref}}$ . The results are listed in table I, together with the values of the parameters  $A, B, E$  into which we absorbed the normalization constant  $a_0$ . In figure 1, we finally compare our Monte Carlo data obtained for  $T = 0$  with several theoretical predictions. Clearly, the waiting times considered are large enough to be inside the dynamical scaling-regime.

Considering first a large range of values of  $t/s$  (see figure 1a) we observe that although the prediction (5) [22–24] is quite close to the data for  $t/s$  small (even so it lies systematically above the numerical data), there is a strong deviation for  $t/s \gtrsim 3$ , see [26]. On the other hand, the simple approximation (12) works well for  $t/s$  large but not surprisingly fails for  $t \gtrsim s$  since the model at hand is not described by a free field. Finally, local scale-invariance (LSI) as given by eqs. (11,17) and the parameters of table I produces a nice overall agreement with the data, up to the smallish region  $t/s \lesssim 2$ . That region is examined closer in figure 1b. We remark that for  $t \simeq s$  dynamical scaling no longer holds true [34] (see the inset in figure 1b) and we cannot hope to be able to find  $C(t, s)$  from a dynamical symmetry argument. The approximate analytical theories proposed in [13, 35] are only qualitatively correct which indicates that OJK-style approximations might not capture fully the quantitative aspects of phase-ordering.

In figure 2, we present  $C(t, s)$  in a more traditional way usually preferred by experimentalists, for both  $T = 0$  and  $T = 1.5$ , which makes the simultaneous dependence of  $C(t, s)$  on  $t - s$  and on  $s$  explicit. Again, we find a nice agreement between LSI and the numerical data, but with larger finite-time corrections to scaling for  $T = 1.5$  than for  $T = 0$ . This finding is strong evidence that the extension of dynamical scaling to Schrödinger-invariance and further to conformal invariance involving also the ‘masses’ as variables is indeed a true dynamical sym-

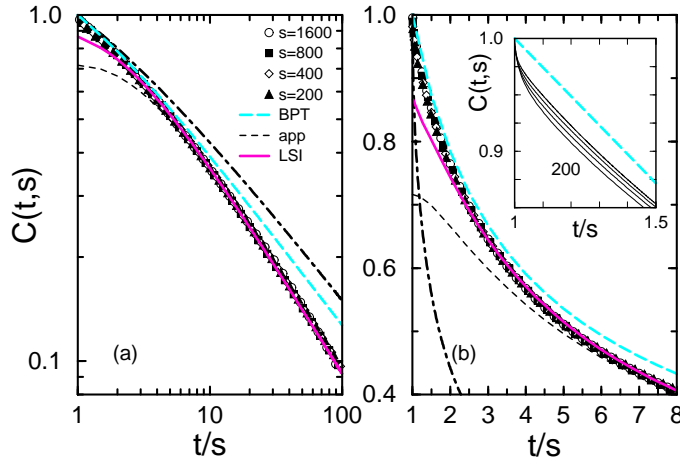


Fig. 1 – Scaling of the autocorrelation function  $C(t, s)$  of the 2D Glauber-Ising model at  $T = 0$ . The curves are as follows: BPT corresponds to (5), app to (12) and LSI to (11,17). Error bars are much smaller than the symbol sizes. The dash-dotted line in a) gives a 2<sup>nd</sup>-order perturbative correction of (5) [13] and the dash-dotted line in b) is the closed-form approximation of [35]. The inset shows  $C(t, s)$  for  $s = 200, 400, 800$  and  $1600$  (from bottom to top), as well as the BPT line eq. (5).

metry of phase-ordering kinetics for all temperatures  $T < T_c$ . We recall that the LSI-prediction (11,17) depends on  $\phi$  and  $\hat{\phi}^2$  being quasiprimary under Schrödinger/conformal transformations [18]. It has turned out that the magnetic order-parameter of the Glauber-Ising model is indeed quasiprimary. For the XY-model, however, the spin magnetization  $\mathcal{S}(t, \mathbf{r})$  cannot be identified with a quasiprimary field but the spin-wave approximation suggests that the phase variable  $\phi(t, \mathbf{r})$  should take that rôle [17].

Summarizing, we have proposed to extend the usual dynamical scaling found in phase-

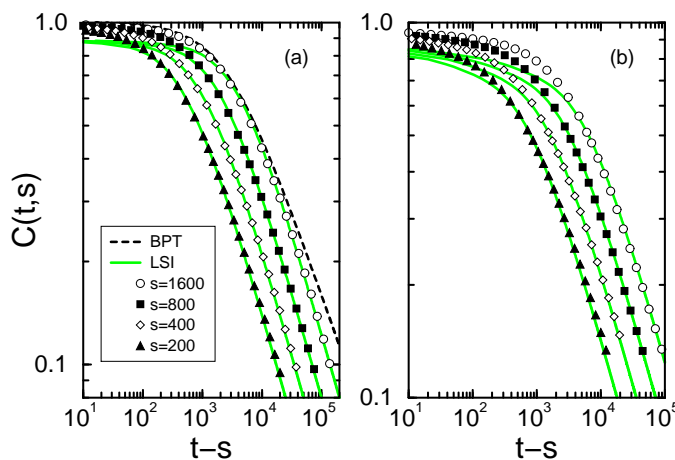


Fig. 2 – Autocorrelator of the 2D Glauber-Ising model at temperatures (a)  $T = 0$  and (b)  $T = 1.5$ . LSI is the prediction eqs. (11,17) and for  $s = 1600$  the curve BPT eq. (5) is also shown.

ordering kinetics to a *local* scale-invariance by (i) requiring Galilei-invariance at  $T = 0$  and (ii) considering the dimensionful ‘masses’ of the order-parameter and response fields as further variables. This has led us to postulate a new kind of time-dependent conformal invariance in phase-ordering kinetics. We have derived the explicit prediction eqs. (11,17) for the two-time autocorrelation function. This expression is in agreement with numerical results of the  $2D$  Glauber-Ising model and also agrees with several exactly solvable systems [17].

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