

Electron-electron interactions and two-dimensional- two-dimensional tunneling

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Abstract

We derive and evaluate expressions for the dc tunneling conductance between interacting two-dimensional electron systems at non-zero temperature. The possibility of using the dependence of the tunneling conductance on voltage and temperature to determine the temperature-dependent electron-electron scattering rate at the Fermi energy is discussed. The finite electronic lifetime produced by electron-electron interactions is calculated as a function of temperature for quasiparticles near the Fermi circle. Vertex corrections to the random phase approximation substantially increase the electronic scattering rate. Our results are in an excellent quantitative agreement with experiment.

I. INTRODUCTION

The development of high mobility double-quantum-well structures and techniques to independently contact the two wells even when separated by only tens of nanometers, have together opened a new fruitful area of research in low-dimensional physics. For example, inter-layer electron-electron interactions have been investigated through the frictional drag voltage occurring when charge in one layer is moved relative to charge in the nearby layer^{1,2}. Separately contacted double-well structures have turned out to be extremely useful not only in studies of properties unique to bilayer systems but also for examining some properties of a single-layer 2D electron gas. In the case of a gated heterostructure with remotely spaced quantum wells, where the inter-layer interactions could be safely neglected, Eisenstein *et al.*³ were able to relate the electric field leaking between layers in response to the voltage change on a remote gate to the compressibility of the electron layer closer to the gate. 2D-2D tunneling^{4,5} provides another example where double-well systems can be used to probe the electronic properties of individual electron layers in new ways. An important feature of ideal 2D-2D tunneling is the conservation of electron momentum; for 3D-3D tunneling the component of momentum in the direction of the tunneling barrier is not conserved. Momentum conservation is not perfect because of disorder in the tunneling barrier and in the electron layers, and also because of inelastic electron-phonon and electron-electron scattering. An electron conserving its energy *and* momentum can tunnel only when the subband edges of the two layers are precisely aligned, resulting in large peak-to valley ratios in the observed resonant tunneling peaks. Experimental studies in which the Fermi surfaces of the 2D electron systems were mapped by measuring the tunneling conductance in magnetic fields applied parallel to the 2D planes⁵ have provided, arguably, the most striking demonstration that electron momentum is conserved to a remarkable degree in GaAs/AlGaAs double-well structures. The heights of resonant peaks in 2D-2D tunneling conductances are limited by scattering processes which do not conserve the momenta of individual electrons. At zero temperature, peak heights and widths may be used to measure

the elastic scattering rate due to disorder.⁶ It has been suggested^{7,8} that the temperature dependence of 2D-2D tunneling conductance peak heights and widths can be used to probe inelastic electron-electron or electron-phonon scattering processes in the individual 2D layers. By comparing with the measured mobility of their system, Murphy *et al.*⁷ concluded that electron-phonon scattering could not account for their observations and suggested that the experiment provides information predominantly about electron-electron scattering rates.

In this article we discuss the connection between electron-electron scattering rates in individual layers and 2D-2D tunneling.⁹ We find in Section II that the energy dependence of the electron-electron scattering rate makes the relationship to tunneling conductance more complicated than in the case of elastic disorder scattering. The formal expressions derived in Section II nevertheless allow the tunneling conductance to be calculated from the energy-dependent quasiparticle lifetime. The problem of calculating the quasiparticle lifetime due to Coulomb interactions is a standard one in the many-body theory of the electron gas; for most purposes the random-phase-approximation (RPA) is reasonably accurate. For the two-dimensional electron gas, however, confusing disagreements exist among various analytic evaluations of the RPA expressions.^{10–13} To clarify the situation, we present in Section III a detailed derivation of the approximate analytic formulas for the temperature and energy dependence of the electron-electron scattering rate. We follow a line similar to that described, in detail, in Ref. 11 and emphasize the points where present work departs from previous studies. In Section III, we also discuss calculations of the quasiparticle lifetime which go beyond the RPA by including local-field corrections to the effective electron-electron interaction. The corrections approximately account for density and spin-density correlations present in the ground state of the interacting electron system. When they are included in lifetime calculations along with the energy-dependence of the electron-electron scattering rate, excellent agreement is obtained, as we discuss in Section IV, with the 2D-2D tunneling experiments of Murphy *et al.*⁷ In Section V, we briefly summarize our results. An account of a preliminary version of this work has been presented previously.¹⁴

II. 2D-2D TUNNELING CONDUCTANCE FOR $T \neq 0$

We consider a GaAs/AlGaAs heterostructure with two identical quantum wells and equal layer densities. For typical separations between the 2D-layers ($\sim 300\text{\AA}$) inter-layer interaction effects, including the screening of the intra-layer Coulomb potential by electrons from the opposite layer, are weak. All such effects will be ignored in following calculations. The Hamiltonian can be written as the sum of three terms:

$$H = H_R + H_L + H_T$$

$$H_T = - \sum_{\vec{k}, \vec{k}'} \left(t_{\vec{k}, \vec{k}'} c_{\vec{k}, R}^{\dagger} c_{\vec{k}', L} + \text{h.c.} \right) . \quad (1)$$

H_R and H_L are the Hamiltonian for isolated electrons in right and left wells including, in general, contributions from intra-layer interactions and from disorder in each layer. The tunneling Hamiltonian, H_T , couples the two systems; we assume that this term can be treated at leading order in perturbation theory. For tunneling barriers which are invariant under translations perpendicular to the barrier $t_{\vec{k}, \vec{k}'}$ is zero for $\vec{k} \neq \vec{k}'$ and is independent of \vec{k} ; $t_{\vec{k}, \vec{k}'} = t \delta_{\vec{k}, \vec{k}'}$. For noninteracting electrons t determines the difference in energy between symmetric and antisymmetric combinations of subband states for the two-layers.

The Kubo formula, which we will use to calculate the tunneling current I , treats the H_T to leading order in perturbation theory.¹⁵ For 2D-2D tunneling our theory is valid to leading order in $2t\tau/\pi\hbar$ where τ is the lifetime of an electron in the individual layers. For noninteracting 2D-electrons in disorder free double-quantum-well systems it is never valid to treat H_T as a perturbation. The condition for the validity of the weak perturbation assumption is that the mean time for an electron to move between layers ($\tau_t = \pi\hbar/2t$), is much longer than the lifetime of electrons due to scattering within a layer, τ . An electron hopping from one well to the other will then scatter, *i.e.* change its in-plane momentum, many times before it jumps back to the first well.

Following a familiar line^{15,16} of reasoning we obtain

$$I(V) = \frac{2e}{\hbar} t^2 S \int \frac{d^2k}{(2\pi)^2} \int_{-E_F}^{\infty} \frac{dE}{2\pi} A(E, \vec{k}) A(E + eV, \vec{k}) [n_F(E) - n_F(E + eV)] , \quad (2)$$

where S is the area of the two-dimensional electron systems, eV is the difference between chemical potentials in the right and left quantum wells, $n_F(E)$ is the Fermi distribution function and $A(E, \vec{k})$ is the spectral function related to the retarded Green's function by

$$A(E, \vec{k}) = -2\text{Im}G_{ret}(E, \vec{k}) = \frac{-2\text{Im}\Sigma_{ret}(E, \vec{k})}{\left[E - \xi_k - \text{Re}\Sigma_{ret}(E, \vec{k})\right]^2 + \left[\text{Im}\Sigma_{ret}(E, \vec{k})\right]^2}. \quad (3)$$

We choose to measure energies from the Fermi energy so that $\xi_k = \hbar^2 k^2 / 2m - E_F$. Near the quasiparticle peak the spectral function can be approximated by a Lorentzian:

$$A(E, \vec{k}) = \frac{\Gamma(\xi_k, \vec{k})}{(E - \xi_k)^2 + (\Gamma(\xi_k, \vec{k})/2)^2}. \quad (4)$$

Here we have neglected the real part of the self-energy which leads to a physically unimportant rigid shift in the quasiparticle energies, causes the quasiparticle effective mass to differ slightly from its free electron value, and slightly reduces the weight of the quasiparticle peak in the spectral function. These effects play a minor role in 2D-2D tunneling experiments¹⁷ and we neglect them here in favor of the main effect which comes from the broadening of the quasiparticle pole. The width of the Lorentzian peak is related to the self-energy by $\Gamma(\xi_k, \vec{k}) \equiv -2\text{Im}\Sigma_{ret}(\xi_k, \vec{k})$.

In the limit of $\Gamma(\xi_k, \vec{k}) \rightarrow 0$, corresponding to the noninteracting disorder-free 2D-electron gas, Eq.(2) can be written as

$$I(V) = \frac{2e}{\hbar} t^2 S \frac{g_0}{2} \int_{-\infty}^{\infty} \frac{dx}{2\pi} \delta(x) \delta(x + eV) \int_{-E_F}^{\infty} d\xi_k [n_F(x + \xi_k) - n_F(x + \xi_k + eV)], \quad (5)$$

where $g_0 = m/\pi\hbar^2$ is the free-particle density of states for a 2D electron system and $x \equiv E - \xi_k$. Because of the δ -functions in Eq.(5) the integral over ξ_k gives eV and we obtain for $|eV| \ll E_F$ that the tunneling conductance $G(V) \equiv I(V)/V$ is proportional to $\delta(V)$. This sharp voltage-dependence of the tunneling conductance is a direct consequence of electron energy *and* momentum conservation during the tunneling. An electron with kinetic energy ξ_k can tunnel only when its potential energy is conserved, *i.e.*, when the energy levels in the quantum wells are aligned. For identical wells this condition is satisfied only at zero voltage.

Scattering processes lead to an uncertainty in the energy of an electron with a given momentum \vec{k} resulting in a finite width of the spectral function and broadened peaks in the $G(V)$ curve. At zero temperature the broadening is dominated by elastic scattering. Replacing the spectral width $\Gamma(\xi_k, \vec{k})$ in Eq.(4) by \hbar/τ_{el} , where τ_{el} is the constant elastic scattering lifetime, Eqs. (2) and (4) give for $\hbar/\tau_{el} \ll E_F$ and $|eV| \ll E_F$:

$$G(V) = \frac{2e}{\hbar} t^2 S \frac{g_0}{2} \frac{2\hbar/\tau_{el}}{(eV)^2 + (\hbar/\tau_{el})^2} . \quad (6)$$

We define Γ_G to be the half width of the $G(V)$ curve. The dependence of the tunneling parameter t on the bias voltage can, and will, be neglected. To see this note that $t \sim \exp(-\kappa d_B)$ where d_B is the width of the barrier between the quantum wells, $\kappa = (2mV_B/\hbar^2)^{1/2}$ gives the decay rate of the wavefunction in the barrier, and V_B is the barrier height. When a bias potential is applied the average value of V_B is changed by $\sim eV$. Using $d\kappa/dV_B \sim \kappa/V_B$ the t should change by a factor of $\sim \exp(-\kappa d_B(eV)/V_B)$. In the experiments to which we refer $V_B \sim 300$ meV and the maximum value of $|eV|$ is $\sim 0.1E_F \sim 0.5$ meV. We see that the magnitude of the argument of the exponential above is much smaller than one. It follows from Eq. (6) that $\Gamma_G = \hbar/\tau_{el}$ for a system with only elastic scattering. It has been established experimentally^{8,7} that Γ_G is temperature dependent, indicating that some inelastic scattering process is contributing to the quasiparticle scattering rate. Samples with different levels of disorder have $G(V)$ peaks whose half-widths appear to differ by temperature-independent constants. This property enables elastic and inelastic contributions to the scattering rate to be separated experimentally; all the calculations in this article are for a disorder-free system. Our calculations will help confirm the experimental analysis of Murphy *et al.* who attributed the temperature dependent broadening of the peak in the $G-V$ characteristic to electron-electron interactions. If we approximate the spectral width due to electron-electron interactions, $\Gamma_{e-e}(\xi_k, T)$, by $\Gamma_{e-e}(0, T)$ we arrive again at Eq.(6) with \hbar/τ_{el} replaced by $\Gamma_{e-e}(0, T)$. We show later that this neglect of the energy dependence of $\Gamma_{e-e}(\xi_k, T)$ underestimates Γ_G .

III. ELECTRON-ELECTRON SCATTERING RATE

As explained above, both the temperature dependence and the energy dependence of Γ_{e-e} need to be calculated in order to compare theory to measured $G - V$ characteristics. In this section we present calculations for a pure 2D-electron gas in the random phase approximation and in the local-field-corrected RPA. The contribution of electron-electron scattering to the spectral width of the one-particle Greens function can be expressed^{20,9} in terms of the scattering rates of electrons (\hbar/τ_e) and holes (\hbar/τ_h):

$$\Gamma_{e-e}(\xi_k, T) = \frac{\hbar}{\tau_e(\xi_k, T)} + \frac{\hbar}{\tau_h(\xi_k, T)}$$

$$\frac{\hbar}{\tau_e(\xi_k, T)} = \sum_{\sigma'} \int \frac{d^2k'}{(2\pi)^2} \int \frac{d^2p}{(2\pi)^2} W^{\sigma\sigma'} n_F(\xi_p) [1 - n_F(\xi_{k'})] [1 - n_F(\xi_{p'})] \delta(\xi_k + \xi_p - \xi_{k'} - \xi_{p'})$$

$$\frac{\hbar}{\tau_h(\xi_k, T)} = \frac{n_F(\xi_k)}{[1 - n_F(\xi_k)]} \frac{\hbar}{\tau_e(\xi_k, T)}. \quad (7)$$

In Eq.(7), \vec{k}, \vec{p} are the initial electron momenta and \vec{k}', \vec{p}' are the final electron momenta. Because of the conservation of the total momentum in electron-electron scattering processes, $\vec{p}' = \vec{k} + \vec{p} - \vec{k}'$. $W^{\sigma\sigma'}$ is the scattering function which we now discuss.

A. Scattering function in the random phase approximation

In the RPA, the scattering function $W^{\sigma\sigma'}$ is spin-independent ($W_{RPA}^{\uparrow\uparrow} = W_{RPA}^{\downarrow\downarrow} \equiv W_{RPA}$) and depends only on the momentum transfer $q \equiv |\vec{k}' - \vec{k}| = |\vec{p}' - \vec{p}|$ and energy transfer $\hbar\omega \equiv \xi_{k'} - \xi_k = \xi_p - \xi_{p'}$:

$$W_{RPA} = 2\pi \left| \frac{v(q)}{\varepsilon_{RPA}(q, \hbar\omega)} \right|^2. \quad (8)$$

In Eq.(8) $v(q) = g_0^{-1} q_{TF}/q$ is the unscreened Coulomb interaction and $\varepsilon_{RPA}(q, \hbar\omega) = 1 - v(q)\chi_0(q, \hbar\omega)$ is the RPA dielectric function^{18,11,12} for the 2D-electron gas. (Here $q_{TF} = g_0 e^2/2\epsilon$ is Thomas-Fermi screening wavevector and $\chi_0(q, \hbar\omega)$ is the susceptibility of a noninteracting electron gas^{18,19,11,12}.)

At low temperatures the energy transferred in electron-electron scattering ($\sim k_B T$) is small compared to the Fermi energy and the magnitude of the momentum transfer is restricted to the interval $(0, 2k_F)$. In this limit the real part of the susceptibility $\text{Re}\chi_0(q, \hbar\omega) \approx -g_0$, the imaginary part $\text{Im}\chi_0(q, \hbar\omega) \approx 0$, and we can write

$$W_{RPA} \approx \frac{2\pi}{g_0^2} \frac{(q_{TF}/q)^2}{(1 + q_{TF}/q)^2}. \quad (9)$$

Previous analytic evaluations of the RPA quasiparticle lifetimes have employed the approximation $W_{RPA} \approx 2\pi/g_0^2$, which would be reasonable if $q_{TF} \gg 2k_F$ or if the slowly varying function $W_{RPA}(q)$ is multiplied, in the integrand in Eq.(7), by a function sharply peaked near $q = 0$. Neither of these assumptions is valid however since: (i) The condition $q_{TF} \gg 2k_F$ corresponds to the dimensionless parameter $r_s = (q_{TF}/2k_F)/\sqrt{2}$, conventionally used to render the density of an electron gas, being much larger than 1. At such low electron densities it is known that the RPA fails. In fact, in the next section it is shown that even for $r_s \sim 1$ vertex corrections to the RPA substantially increase the electron electron scattering rate. Moreover, in GaAs, for electron concentrations typical of tunneling experiments the Thomas-Fermi screening wavevector is comparable to the Fermi wavevector; (ii) As we now discuss in detail the integrand in Eq.(7) has sharp peaks near both forward ($q = 0$) and backward ($q = 2k_F$) scattering momentum transfers.

In polar coordinates $d^2k' = k' dk' d\theta_{k'}$ and $d^2p = p dp d\theta_p$, where the angles $\theta_{k'}$ and θ_p are measured with respect to momentum \vec{k} . The integral over θ_p may be performed using the energy-conservation δ -function. Since $\xi_{p'}$ depends on θ_p we may write

$$\delta(\xi_k + \xi_p - \xi_{k'} - \xi_{p'}) = \sum_i \left| \frac{\partial \xi_{p'}}{\partial \theta_p} \right|_{\theta_p = \theta_{p,i}}^{-1} \delta(\theta_p - \theta_{p,i}), \quad (10)$$

where $\theta_{p,i}$, is an angle at which both the energy conservation

$$p'^2 = k^2 + p^2 - k'^2 \quad (11)$$

and momentum conservation

$$p'^2 = (\vec{k} + \vec{p} - \vec{k}')^2 = k^2 + p^2 + k'^2 + 2kp \cos \theta_p - 2kk' \cos \theta_{k'} - 2pk' \cos(\theta_{k'} - \theta_p) \quad (12)$$

conditions are satisfied. From Eq.(12) it follows that

$$\frac{\partial p'^2}{\partial \theta_p} = -2kp \sin \theta_p + 2pk' \sin(\theta_p - \theta_{k'}) = -2pk' \sin \theta_{k'} \frac{\cos(\theta_p + z)}{\cos z}, \quad (13)$$

where

$$\tan z = -\frac{k - k' \cos \theta_{k'}}{k' \sin \theta_{k'}} \quad (14)$$

The combination of Eqs.(11), (12) and (14) gives

$$k'^2 = \frac{pk' \sin \theta_{k'}}{\cos z} \sin(\theta_p + z) + kk' \cos \theta_{k'} \quad (15)$$

and together with Eq.(13) we finally obtain

$$\left. \frac{\partial \xi_{p'}}{\partial \theta_p} \right|_{\theta_p = \theta_{p,i}}^{-1} = \frac{1}{2} \left[A + (E_F + \xi_k)(E_F + \xi_{k'}) \sin^2 \theta_{k'} \right]^{-1/2} \quad (16)$$

$$A = (\xi_p - \xi_{k'}) \left[\xi_k + \xi_{k'} + 2E_F - (\xi_k + E_F)^{1/2} (\xi_{k'} + E_F)^{1/2} \cos \theta_{k'} \right]$$

In the limit of small T and ξ_k the Fermi functions restrict energies (measured from the Fermi energy) of particles involved in the scattering process to a small region near zero energy. As seen from Eq.(16), A is then small and the integrand in (7) has equivalent sharp peaks in the available phase space for scattering near $\theta_{k'} = 0$ and $\theta_{k'} = \pi$. The main contributions to the electron-electron scattering rate come from processes with small wavevector transfer (forward scattering) *and* wavevector transfer $\sim 2k_F$ (backward scattering). This suggests that $W_{RPA}(q)$ may be approximated by the average of its value at $q = 0$ and its value at $q = 2k_F$:

$$W_{RPA} \approx \frac{2\pi}{g_0^2} w_{RPA}^{f,b}, \quad (17)$$

where

$$w_{RPA}^{f,b} = \frac{1 + \left(1 + \frac{1}{r_s \sqrt{2}}\right)^{-2}}{2}. \quad (18)$$

B. Scattering function in the local-field-corrected RPA

In estimating quasiparticle scattering rates the RPA does not account for electronic correlations in the interacting electron gas. Technically, the RPA for the electronic self-energy neglects vertex corrections to the dynamically screened exchange energy. On physical grounds correlations are expected to suppress scattering between like-spin electrons since these electrons are required to avoid each other by the Pauli exclusion principle and to enhance scattering between opposite-spin electrons. In general the four-point scattering amplitude for electrons depends on the energies and momenta of all electrons and not just on the momentum (q) and energy (ω) transferred in the scattering event. Nevertheless, a number of workers^{21,22} have suggested similar approximations in which the four-point scattering wavefunction is replaced by an effective electron-electron interaction dependent only on q and ω . In these approximations correlations are accounted for by modifying the bare electron-electron interaction to take account of the correlation clouds carried around by each quasiparticle.

Here we estimate corrections to the RPA by adopting the effective electron-electron interaction suggested by MacDonald and Geldart.²² Their effective interaction has density-density (t_{nn}) and spin-spin (t_{mm}) components:

$$\begin{aligned} W_{LFRPA}^{\uparrow\uparrow} &= 2\pi(t_{nn} + t_{mm})^2 \\ W_{LFRPA}^{\uparrow\downarrow} &= 2\pi(t_{nn} - t_{mm})^2 . \end{aligned} \quad (19)$$

Here the density-density interaction,

$$t_{nn} = \frac{v(q) + F_{nn}(q)}{1 - \chi_0(q, \hbar\omega)(v(q) + F_{nn}(q))} , \quad (20)$$

is a screened Coulomb interaction and the spin-spin interaction,

$$t_{mm} = \frac{F_{mm}(q)}{1 - \chi_0(q, \hbar\omega)F_{mm}(q)} , \quad (21)$$

can be thought of as exchange interaction which favors parallel spin alignment. The local field factors $F_{nn}(q)$ and $F_{mm}(q)$ which appear in Eq.(20) and Eq.(21) are related to the static density ($\chi_{nn}(q)$) and spin ($\chi_{mm}(q)$) response functions of the electron gas:

$$\begin{aligned}
F_{nn}(q) &= \chi_0(q)^{-1} - \chi_{nn}(q)^{-1} \\
F_{mm}(q) &= \chi_0(q)^{-1} - \chi_{mm}(q)^{-1}.
\end{aligned}
\tag{22}$$

From Eq.(22) we see explicitly that the local fields vanish in the RPA. Quantum Monte-Carlo calculations of the response functions indicate that the wavevector dependence of the local fields is weak²³ for $q \leq 2k_F$. Here we approximate the local fields by their $q \rightarrow 0$ limits which are²⁴ accurately known. Using the same arguments as in the previous section, the four point scattering function $W_{LFRPA}^{\sigma\sigma'}$ can then be approximated by the average between its forward and backward scattering limits, *i.e.*,

$$\frac{W_{LFRPA}^{\uparrow\uparrow} + W_{LFRPA}^{\uparrow\downarrow}}{2} = 2\pi(t_{nn}^2 + t_{mm}^2) \approx \frac{2\pi}{g_0^2} w_{LFRPA}^{f,b}, \tag{23}$$

where

$$w_{LFRPA}^{f,b} = \frac{1 + \left(1 + \frac{1}{r_s\sqrt{2+g_0F_{nn}}}\right)^{-2}}{2} + \left(\frac{g_0F_{mm}}{1 + g_0F_{mm}}\right)^2. \tag{24}$$

In Figure 1 we compare scattering functions in the random phase approximation and in the local-field-corrected RPA as a function of density. The RPA result is recovered in the high-density limit where the local fields go to zero. For $r_s = 1$, corresponding to the electron density of the sample in the experiment⁷ of Murphy *et al.*, the local-field-corrections increase the scattering rate by approximately 30%. As also shown in Figure 1, there is a $\approx 50\%$ difference at $r_s = 1$ between the forward scattering approximation for the scattering function and the approximation obtained by averaging forward and backward limits. (In the forward scattering limit $w_{RPA}^f = 1$ and $w_{LFRPA}^f = 1 + [g_0F_{mm}/(1 + g_0F_{mm})]^2$.)

C. Approximate analytic results for low temperatures and energies

Analytic expressions for the RPA electron lifetime \hbar/τ_e at low energies and/or temperatures have been derived previously in four independent studies^{10–13} with slightly different results obtained in each case. None of these expressions agree with the analytic results we present below, whose accuracy has been verified by comparing with independent numerical

calculations. While some of the discrepancies appear to be due to inadvertent algebraic errors, some are associated with significant aspects of the physics of electron-electron scattering in two-dimensional electron systems which we draw attention to below. Because of the existing confusion we give a detailed description of our calculation. We have found the detailed analysis presented in Ref. 11 to be very helpful and have followed this calculation closely.

We have shown above that the phase space for electron-electron scattering at low-temperatures is dominated by contributions with equal weight near the forward and backward scattering limits. This property invalidates the approximation, made in previous analytic evaluations, in which the RPA screened interaction is approximated by its forward scattering limit. For our analytic calculation we have adopted an approximation which is in the same spirit by replacing the scattering function by Eqs.(17),(18) or Eqs.(23),(24). Making this replacement we can take the interaction strength outside all integrals. We integrate first over the angle $\theta_{k'}$ between the incoming and scattered momenta in Eq. (7). Using equations similar to (10-16) we find that

$$\left| \frac{\partial \xi_{p'}}{\partial \theta_{k'}} \right|_{\theta_{k'}=\theta_{k',i}}^{-1} = \frac{1}{2} \left[(\xi_{k'} - \xi_p)(\xi_k - \xi_{k'}) + (E_F + \xi_k)(E_F + \xi_p) \sin^2 \theta_{k'} \right]^{-1/2}. \quad (25)$$

As illustrated in Figure 2 there are two different angles, $\theta_{k',1}$ and $\theta_{k',2}$, with identical integration weights (25) which satisfy the energy and momentum conservation conditions. The corresponding momenta are related by a mirror symmetry with respect to the vector $\vec{k} + \vec{p}$. (The associated factor of two in the scattering rate appears to have been missed in some previous work.) At low T we can take

$$(E_F + \xi_k)(E_F + \xi_p) \approx E_F^2. \quad (26)$$

Defining $\tilde{\xi}_k \equiv \xi_k/k_B T$, $\tilde{\xi}_{k'} \equiv \xi_{k'}/k_B T$, and $\tilde{\xi}_p \equiv \xi_p/k_B T$ leads to

$$\begin{aligned} \frac{\hbar}{\tau_e(\tilde{\xi}_k, T)} &\approx 4 \frac{m^2}{(2\pi\hbar)^4} \frac{(k_B T)^2}{E_F} \frac{2\pi}{g_0^2} w^{f,b} \int_{-\infty}^{\infty} d\tilde{\xi}_{k'} \int_{-\infty}^{\infty} d\tilde{\xi}_p n_F(\tilde{\xi}_p) \\ &\times \left[1 - n_F(\tilde{\xi}_{k'}) \right] \left[1 - n_F(\tilde{\xi}_k + \tilde{\xi}_p - \tilde{\xi}_{k'}) \right] I(B), \end{aligned} \quad (27)$$

where $w^{f,b}$ stands for either $w_{RPA}^{f,b}$ or $w_{LFRPA}^{f,b}$ and

$$B = \left(\frac{k_B T}{E_F} \right)^2 (\tilde{\xi}_{k'} - \tilde{\xi}_p)(\tilde{\xi}_k - \tilde{\xi}_{k'}) \quad (28)$$

The angular integral $I(B)$ reads

$$\begin{aligned} I(B) &= 2 \int_{\theta_0}^{\pi - \theta_0} \frac{d\theta_p}{2 (B + \sin^2 \theta_p)^{1/2}} \\ &= 2 \int_{u_0}^1 \frac{du}{(1 - u^2)^{1/2} (B + u^2)^{1/2}} . \end{aligned} \quad (29)$$

Here $u_0 = 0$ for $B \geq 0$ and $u_0 = \sqrt{|B|}$ for $B < 0$. To find the leading low-temperature behavior of the electron-electron scattering rate we replace the expression $(B + u^2)^{1/2}$ by first two terms of its Taylor expansion and approximate the elliptic integral Eq.(29) by

$$I(B) \approx \ln(8) - \ln |B| . \quad (30)$$

Inserting Eq.(30) into Eq.(27) and using the identity

$$\int_{-\infty}^{\infty} d\tilde{\xi}_{k'} \int_{-\infty}^{\infty} d\tilde{\xi}_p n_F(\tilde{\xi}_p) [1 - n_F(\tilde{\xi}_{k'})] [1 - n_F(\tilde{\xi}_k + \tilde{\xi}_p - \tilde{\xi}_{k'})] = \frac{1}{2} (\pi^2 + \tilde{\xi}_k^2) [1 - f(\tilde{\xi}_k)] \quad (31)$$

we arrive at

$$\begin{aligned} \frac{\hbar/\tau_e(\tilde{\xi}_k, T)}{E_F} &\approx \frac{w^{f,b}}{\pi} \left(\frac{k_B T}{E_F} \right)^2 [1 - n_F(\tilde{\xi}_k)] \\ &\times \left[\frac{1}{2} (\pi^2 + \tilde{\xi}_k^2) \left(\frac{\ln 8}{2} + \ln \left(\frac{E_F}{k_B T} \right) \right) - F(\tilde{\xi}_k) \right] , \end{aligned} \quad (32)$$

where

$$\begin{aligned} F(\tilde{\xi}_k) &= \frac{1}{2} [1 - n_F(\tilde{\xi}_k)]^{-1} \int_{-\infty}^{\infty} d\tilde{\xi}_{k'} \int_{-\infty}^{\infty} d\tilde{\xi}_p \ln |(\tilde{\xi}_{k'} - \tilde{\xi}_p)(\tilde{\xi}_k - \tilde{\xi}_{k'})| \\ &\times n_F(\tilde{\xi}_p) [1 - n_F(\tilde{\xi}_{k'})] [1 - n_F(\tilde{\xi}_k + \tilde{\xi}_p - \tilde{\xi}_{k'})] . \end{aligned} \quad (33)$$

In the limit of $\tilde{\xi}_k \rightarrow 0$, *i.e.*, for an electron on the Fermi surface, $F(0) = 0.41388$ and we obtain the leading contribution to the electron-electron scattering rate at low temperatures

$$\frac{\hbar/\tau_e(0, T)}{E_F} \approx w^{f,b} \frac{\pi}{4} \left(\frac{k_B T}{E_F} \right)^2 \left[\ln \left(\frac{E_F}{k_B T} \right) + \frac{\ln 8}{2} - .083 \right] . \quad (34)$$

As follows from Eq.(7) the scattering rates of electrons and holes are equivalent at $\xi_k = 0$, *i.e.*,

$$\Gamma_{e-e}(0, T) = 2 \frac{\hbar}{\tau_e(0, T)}. \quad (35)$$

At zero temperature the calculation is simplified by the fact that the Fermi distribution functions are reduced to step functions. We find that for $\xi_k > 0$

$$\frac{\Gamma_{e-e}(\xi_k, 0)}{E_F} = \frac{\hbar/\tau_e(\xi_k, 0)}{E_F} \approx w^{f,b} \frac{1}{2\pi} \left(\frac{\xi_k}{E_F} \right)^2 \left[\ln \left(\frac{E_F}{\xi_k} \right) + \frac{\ln 8}{2} + .5 \right] \quad (36)$$

and for $\xi_k < 0$

$$\frac{\Gamma_{e-e}(\xi_k, 0)}{E_F} = \frac{\hbar/\tau_h(\xi_k, 0)}{E_F} \approx w^{f,b} \frac{1}{2\pi} \left(\frac{\xi_k}{E_F} \right)^2 \left[\ln \left(-\frac{E_F}{\xi_k} \right) + \frac{\ln 8}{2} + .5 \right]. \quad (37)$$

IV. NUMERICAL RESULTS

All the calculations we discuss in this section have been performed for $r_s = 1$. In Figure 3 the low-temperature and low-energy approximate analytic results (Eqs.(34),(35) and (36),(37)) are compared with numerical results obtained within the constant interaction approximation (23),(24). For temperatures or excitation energies up to 10% of the Fermi temperature or Fermi energy, resp., the terms with higher powers of T or ξ_k can be safely neglected. The discrepancies here are due only to the low-temperature and low-energy approximations. We see that including the T^2 and ξ_k^2 corrections to the leading $-T^2 \ln(T)$ and $-\xi_k^2 \ln |\xi_k|$ terms greatly extends the range of validity of these expressions. Note that the spectral width due to electron-electron interactions is *not* precisely an even function of energy. In the constant interaction approximation at zero temperature the difference between scattering rates of an electron with energy $\xi_k > 0$ and of a hole with energy $-\xi_k$ is due to the term given by Eq.(25). Since $(E_F + |\xi_k|) > (E_F - |\xi_k|)$ the integrand in Eq.(7) is smaller for electrons than for holes with the same absolute value of energy. Making the approximation of Eq.(26) which only affects terms of higher order in ξ_k than $\xi_k^2 \ln |\xi_k|$ and ξ_k^2 gives the leading low-energy behavior which is even in ξ_k .

The validity of the constant scattering amplitude approximation (Eq.(23) and Eq.(24)) was examined by performing three different numerical calculations of $\Gamma_{e-e}(0, T)$ and $\Gamma_{e-e}(\xi_k, 0)$. In Figure 4 we show results obtained using (i) the constant scattering function $W_{LFRPA}^{f,b} = 2\pi/g_0^2 w_{LFRPA}^{f,b}$ (These results are identical to the curves labeled "Numerical" in Figure 3.), (ii) the q -dependent approximation $W^a(q)$ to the scattering function which results when the wavevector and frequency-dependence of the susceptibility $\chi_0(q, \omega)$ is neglected ($\chi_0(q, \omega) \approx -g_0$), and (iii) the full q and ω dependent scattering function ($W_{LFRPA}(q, \omega)$) in the local-field-corrected RPA where both the real and imaginary parts of $\chi_0(q, \omega)$ are taken into account. We see that averaging between forward and backward scattering limits provides an excellent approximation for the q -dependence of $W^a(q)$. The discrepancy between Γ_{e-e} calculated with the constant ($W_{LFRPA}^{f,b}$) and full scattering function ($W_{LFRPA}(q, \omega)$) is primarily due to neglecting the imaginary part of χ_0 which is non-zero at finite frequencies.

At $T \neq 0$ quasielectrons and quasiholes are both present both above and below the Fermi energy. The energy-dependent spectral width $\Gamma_{e-e}(\xi_k)$ is then a sum of electron and hole scattering rates. Numerical results showing electron and hole scattering rate contributions are shown for $T = 0.3 T_F$, in Figure 5. Spectral widths calculated at several different temperatures are plotted in Figure 6. Dotted curves show the results of calculations where the temperature dependence of the susceptibility χ_0 is neglected. This approximation greatly simplifies non-zero temperature numerical calculations, however, we see here that it can be safely used only for temperatures $T < 0.1 T_F$.

Inserting $\Gamma_{e-e}(\xi_k)$ into Eq.(4) we can evaluate Eq.(2) for the tunneling current and calculate the tunneling conductance as a function of applied voltage at a fixed temperature. At low temperatures and energies the spectral width $\Gamma_{e-e}(\xi_k, T)$ is, roughly, the sum of $\Gamma_{e-e}(0, T)$ and $\Gamma_{e-e}(\xi_k, 0)$ (see Figure 6). Looking at the approximate analytical formulas (34)-(37) we see that if $T/T_F \ll 1$ and $|\xi_k|/E_F \ll 1$ then also $\Gamma_{e-e}(\xi_k, T)/E_F \ll 1$. For $|eV|/E_F \ll 1$ we can therefore replace the difference between Fermi functions in Eq.(2) by $\partial n_F(E)/\partial E \times eV$ resulting in the following formula for the tunneling conductance

$$\begin{aligned}
G(V) = & \frac{2e}{\hbar} t^2 S \frac{g_0}{2} \int_{-\infty}^{\infty} \frac{dE}{2\pi} \int_{-E_F}^{\infty} d\xi_k \frac{\Gamma_{e-e}(\xi_k)}{(E - \xi_k)^2 + (\Gamma_{e-e}(\xi_k)/2)^2} \\
& \times \frac{\Gamma_{e-e}(\xi_k - eV)}{(E + eV - \xi_k)^2 + (\Gamma_{e-e}(\xi_k - eV)/2)^2} \frac{\partial n_F(E)}{\partial E}. \tag{38}
\end{aligned}$$

Using Eq.(38) we determined numerically the effect of the energy dependence of the spectral width due to electron-electron interactions on $\Gamma_G(T)$. As shown in Figure 7, approximating $\Gamma_{e-e}(\xi_k, T)$ by $\Gamma_{e-e}(0, T)$ underestimates $\Gamma_G(T)$ by a factor ~ 1.1 - 1.16 in the range of temperatures used in the tunneling experiment⁷. In Figure 8, the calculated $\Gamma_G(T)$ is compared to the measured half-width of the tunneling conductance peak, showing excellent agreement between the theoretical result obtained in the local-field-corrected RPA and experimental data. Note that the agreement between theory and experiment has been achieved without introducing any adjustable parameters. As for the scattering rate calculation, local-field-corrections increase the RPA $\Gamma_G(T)$. At $T/T_F = 0.15$ the correction factor is ~ 1.3 and improves agreement between theory and experiment. While it is certainly possible that the excellent agreement with experiment is partly fortuitous, our results leave little doubt that the temperature dependence observed in the 2D-2D tunneling experiments is due to the finite quasiparticle lifetimes which result from electron-electron scattering.

V. SUMMARY

In 2D-2D tunneling experiments, resonant tunneling features have no width in the absence of quasiparticle scattering processes. This property of 2D phase space allows 2D-2D tunneling experiments to measure quasiparticle scattering rates. In this article we have examined the role of electron-electron scattering in recent 2D-2D tunneling experiments. For elastic scattering the width of the resonant tunneling peak is proportional to the scattering rate for electrons at the Fermi energy. For electron-electron scattering we find that the relationship is complicated by the non-negligible dependence of the scattering rate on quasiparticle energy relative to the Fermi energy. We have shown that, although the energy

dependence of the scattering rate results in a non-Lorentzian lineshape for the resonance, it has only a modest influence on the temperature dependence of the half-width of resonant tunneling peak. Analytic expressions have been derived which approximate RPA and local-field-corrected RPA results for the scattering rates at low temperatures and energies. The accuracy of these analytic expressions has been confirmed by comparison with independent exact numerical evaluations of the scattering rates in these approximations. For the electron densities of existing experiments local-field-corrections increase the RPA scattering rate by a factor ~ 1.3 and are important in improving agreement between theory and experiment.

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FIGURES

FIG. 1. Forward (f) and the average between forward and backward (f, b) limits of the scattering function calculated in the random phase approximation (RPA) and in the local-field-corrected RPA (LFRPA).

FIG. 2. Schematic illustration of angles between the incoming and scattered momenta for two-dimensional electron-electron scattering.

FIG. 3. Spectral width as a function of temperature at zero energy (upper panel) and as a function of energy at zero temperature (lower panel) calculated using the constant scattering amplitude approximations discussed in the text. These results are for the local-field-corrected RPA. Numerical calculations (full lines) are compared to these approximate analytic results.

FIG. 4. Spectral width as a function of temperature at zero energy (upper panel) and as a function of energy at zero temperature (lower panel) calculated numerically in the local-field-corrected RPA. Exact LFRPA calculations (full lines) are compared to results obtained with q -dependent approximate scattering function (dashed lines) and with the constant scattering amplitude (dotted lines).

FIG. 5. Spectral width (full line) as a sum of electron and hole scattering rates (dotted lines) at $T = 0.3T_F$.

FIG. 6. Spectral widths (full lines) calculated at different temperatures. Dotted lines show results obtained using the zero temperature analytic form of free electron susceptibility.

FIG. 7. Half-width at half-maximum of the tunneling conductance peak relative to the spectral width at $\xi_k = 0$. The inset shows the tunneling conductance at $T = 0.05T_F$ (dotted line) and at $T = 0.3T_F$ (dashed line) and the tunneling conductance calculated with $\Gamma_{e-e}(\xi_k, T)$ replaced by $\Gamma_{e-e}(0, T)$. The energy dependence results in a line-shape for the 2D-2D tunneling resonance which is not precisely Lorentzian.

FIG. 8. Half-width at half-maximum for 2D-2D resonant tunneling peaks: theoretical results including local-field corrections to the RPA (full line), theoretical results in the RPA (dashed line), Murphy *et al.* experiment (dotted line).















