

A new field–theoretical formulation for the motion of an electron in a quenched disorder potential ^{*}

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Abstract

Following a proposal by Aronov and Ioselevich, we express the Green functions (GF) of a noninteracting disordered Fermi system as a functional integral on a real time/frequency lattice. The normalizing denominator of this functional integral is equal to unity, because of identities satisfied by the GF. The GF can then be simply averaged with respect to the random disorder potential. We describe the fermionic fields not belonging to the external frequency by means of a bosonic auxiliary field g . The Hubbard–Stratonovich field Q is introduced only with respect to the fermionic fields for the external frequency.

1 Introduction

We consider noninteracting Fermions (electrons) under the influence of a random quenched disorder potential. The Green functions (GF) or products of GF are usually expressed as functional integrals with a normalizing denominator. It is this normalizing denominator which prevents simple averaging with respect to a model–like Gaussian distributed random potential. The normalizing denominator is equal to unity in the replica method [1], the supersymmetry method [2], the Keldysh method [3], and a method proposed by Aronov and Ioselevich [4], using one real time axis.

In this work we are concerned with further development of the method of Aronov and Ioselevich. In Section 2, we start like these authors with a definition of the GF on a time lattice, what allows definition of the functional integral as a multiple integral. In frequency representation the definition of the functional integral needs

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a frequency lattice; we call the method ‘‘Many–frequency–technique’’. We define here the GF as periodic in the time difference by adding a boundary term in the definition. From the definition of the GF follow well defined identities for the GF (Eqs. (6), (8) below), necessary for the vanishing of all closed loops. In order to avoid complications due to so many frequencies, we treat in Section 2.1 the fermionic fields not belonging to the external frequency by means of one bosonic auxiliary field g . A differentiation ‘‘mechanism’’ with respect to $g \rightarrow 0$ takes care of eliminating the contribution of the closed loops. Finally, in Section 2.2, we introduce the Hubbard–Stratonovich field Q only for the fermionic fields referring to the external frequency.

2 Many–frequency–technique

On the real time interval $[-T, T)$ a time lattice is introduced by

$$\Delta = \frac{T}{\mathcal{N}}, \quad t_n = n\Delta, \quad n = -\mathcal{N}, -\mathcal{N} + 1, \dots, \mathcal{N} - 1. \quad (1)$$

We define the real time retarded and advanced GF corresponding to an eigenvalue E_λ for the one–particle system by

$$\begin{aligned} (G_{R\lambda})^{-1}(t_{n_1} - t_{n_2}) &= i\delta_{n_1, n_2} - ib_{R\lambda}[\delta_{n_1-1, n_2}(1 - \delta_{n_1, -\mathcal{N}}) + \delta_{n_1, -\mathcal{N}}\delta_{-\mathcal{N}-1, n_2}], \\ (G_{A\lambda})^{-1}(t_{n_1} - t_{n_2}) &= -i\delta_{n_1, n_2} + ib_{A\lambda}[\delta_{n_1+1, n_2}(1 - \delta_{n_1, \mathcal{N}-1}) + \delta_{n_1, \mathcal{N}-1}\delta_{-\mathcal{N}, n_2}], \\ b_{R, A\lambda} &= \exp[\mp i(E_\lambda - E_F \mp i\eta)\Delta] \approx 1 \mp i(E_\lambda - E_F \mp i\eta)\Delta; \end{aligned} \quad (2)$$

E_F is the Fermi energy. The definition (2) coincides with that of Aronov and Joselevich [4] besides the added boundary terms. The following procedure for the limits is used:

$$\text{first } \mathcal{N} \rightarrow \infty \text{ with } \mathcal{N}\Delta = T = \text{const}, \text{ then } T \rightarrow \infty, \text{ finally } \eta \rightarrow +0. \quad (3)$$

The Fourier transform of the GF is

$$\begin{aligned} G_{\alpha\lambda}(\omega_l) &= \frac{1}{\Delta} G_{\alpha\lambda}^{\text{cont}}(\omega_l) = \frac{-i\alpha}{1 - \exp\{i\alpha(\omega_l - E_\lambda + E_F + i\alpha\eta)\Delta\}} \\ &\approx \frac{1}{\Delta} \frac{1}{\omega_l - E_\lambda + E_F + i\alpha\eta} \quad \text{for } |\omega_l\Delta| \ll 1. \end{aligned} \quad (4)$$

Here $\alpha = R, A$ or $+1, -1$; $\omega_l = (2\pi/2T)l$ with $l = -\mathcal{N}, -\mathcal{N} + 1, \dots, \mathcal{N} - 1$. Expression (4) shows the coincidence of the lattice GF with the continuum GF in the limit $\Delta \rightarrow 0$.

The GF are expressed (in unperturbed basis) by the Graßmann functional integral

$$\begin{aligned} G_{\alpha; \lambda_1 \lambda_2}(\omega_l) &= \int \mathcal{D}\psi^+ \mathcal{D}\psi \psi_{\alpha l \lambda_1} \psi_{\alpha l \lambda_2}^+ \exp(-S), \\ S &= \sum_{i l \lambda_1 \lambda_2} \psi_{\alpha_i l \lambda_1}^+ (G_{\alpha_i}(\omega_l))_{\lambda_1 \lambda_2}^{-1} \psi_{\alpha_i l \lambda_2}, \\ (G_\alpha(\omega_l))_{\lambda_1 \lambda_2}^{-1} &= (\lambda_1 | i\alpha [1 - \exp\{i\alpha(\omega_l - h^{(0)} + E_F + i\alpha\eta - v(\mathbf{r}))\Delta\}] | \lambda_2) \\ &\approx (\lambda_1 | i\alpha [1 - \exp\{i\alpha\omega_l\Delta\}] \{1 + i\alpha(h^{(0)} - E_F - i\alpha\eta + v(\mathbf{r}))\Delta\} | \lambda_2). \end{aligned} \quad (5)$$

Here, instead of the eigenvalue E_λ , the one-particle Hamiltonian $h^{(0)} - E_F + v(\mathbf{r})$ is used with $h^{(0)}$ as an unperturbed Hamiltonian with eigenstates $|\lambda\rangle$ and eigenvalues $E_\lambda^{(0)}$; $v(\mathbf{r})$ is the random potential. Because averages of the product of two GF are needed, two Grassmann fields are introduced; the α_i ($i = 1, 2$) are R, R or A, A or R, A , respectively.

Eq. (5) contains *no* normalizing denominator; it is an identity because of the identity (written in diagonal representation) for the GF (see [4]):

$$Z := \int \mathcal{D}\psi^+ \mathcal{D}\psi \exp(-S) = \prod_{\alpha\lambda} \det \left[(G_{\alpha\lambda})^{-1} (t_{n_1} - t_{n_2}) \right] = \prod_{\alpha l \lambda} (G_{\alpha\lambda})^{-1}(\omega_l) = 1. \quad (6)$$

The matrix $(G_{R\lambda})^{-1}(n_1 - n_2)$ in n_1, n_2 has only elements for $n_2 = n_1$, for $n_2 = n_1 - 1$ and a single element for $n_1 = -\mathcal{N}, n_2 = \mathcal{N} - 1$. Expansion of the determinant with respect to the elements of the first row gives in accordance with the procedure (3) for the limits:

$$\det \left[(G_{R\lambda})^{-1} \right] = 1 + ib_{R\lambda} (-ib_{R\lambda})^{2\mathcal{N}-1} = 1 - \exp[-i(E_\lambda - i\eta)(2\mathcal{N} - 1)\Delta] \rightarrow 1. \quad (7)$$

From (6) follows the identity

$$\sum_l \Delta \exp\{i\alpha\omega_l\Delta\} G_{\alpha\lambda}(\omega_l) = 0 \quad (8)$$

by differentiation with respect to E_λ ; similar identities hold for l -sums over products of *only* retarded or *only* advanced GF in any basis. In a perturbation expansion these identities guaranty the vanishing of all closed loops, because a closed loop leads to a frequency sum.

Now, the absence of a normalizing denominator in (5) allows simple averaging (denoted by $\langle \dots \rangle_v$) over the Gaussian distributed random potential:

$$\begin{aligned} \langle G_{\alpha_i\lambda_1\lambda_2}(\omega_l) \rangle_v &= \langle \psi_{\alpha_l\lambda_1} \psi_{\alpha_l\lambda_2}^+ \rangle_v = \int \mathcal{D}\psi^+ \mathcal{D}\psi \psi_{\alpha_l\lambda_1} \psi_{\alpha_l\lambda_2}^+ \exp(-S_{eff}), \\ S_{eff} &= \sum_{i l \lambda} \psi_{\alpha_i l \lambda}^+ (G_{\alpha_i \lambda}^{(0)}(\omega_l))^{-1} \psi_{\alpha_i l \lambda} \end{aligned} \quad (9)$$

$$\begin{aligned} & - \frac{\gamma \Delta^2}{2} \sum_{i l \lambda_1 \lambda_2; j \bar{l} \bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} \exp\{i\alpha_i \omega_l \Delta\} \psi_{\alpha_i l \lambda_1}^+ \psi_{\alpha_i l \lambda_2} \exp\{i\alpha_j \omega_{\bar{l}} \Delta\} \psi_{\alpha_j \bar{l} \bar{\lambda}_1}^+ \psi_{\alpha_j \bar{l} \bar{\lambda}_2}, \\ \gamma \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} &= (\lambda_1 |\bar{\lambda}_1| \langle v(\mathbf{r}) v(\bar{\mathbf{r}}) \rangle_v |\bar{\lambda}_2| \lambda_2); \end{aligned} \quad (10)$$

γ is the strength of the potential correlator.

2.1 Introduction of the g -field

Besides the retarded and the advanced fields for a fixed external frequency ω_{l_0} , all fields belonging to other frequencies shall be treated “globally” by a bosonic auxiliary field g . For this, we introduce into the functional integral the identities

$$1 = \int_{-\infty}^{\infty} dg_{\lambda_1 \lambda_2} \delta \left\{ g_{\lambda_1 \lambda_2} - \sum_{i l; (l \neq l_0)} \exp\{i\alpha_i \omega_l \Delta\} \psi_{\alpha_i l \lambda_1}^+ \psi_{\alpha_i l \lambda_2} \right\}. \quad (11)$$

This changes the measure in the functional integral to $\exp\{-S_\psi\} :=$

$$\begin{aligned}
&= \exp \left[- \sum_{i\lambda_1\lambda_2l; (l \neq l_0)} \psi_{\alpha_i l \lambda_1}^+ \left\{ (G_{\alpha_i \lambda_1}^{(0)}(\omega_l))^{-1} \delta_{\lambda_1 \lambda_2} - \sqrt{\gamma} \Delta \exp\{i\alpha_i \omega_l \Delta\} \frac{\partial}{\partial g_{\lambda_1 \lambda_2}} \right\} \psi_{\alpha_i l \lambda_2} \right] * \\
&\exp \left[\frac{1}{2} \sum_{\lambda_1 \lambda_2; \bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} g_{\lambda_1 \lambda_2} g_{\bar{\lambda}_1 \bar{\lambda}_2} - \sum_{i\lambda_1 \lambda_2} \psi_{\alpha_i \lambda_1}^+ \left\{ (G_{\alpha_i \lambda_1}^{(0)})^{-1} \delta_{\lambda_1 \lambda_2} - \sqrt{\gamma} \Delta * \right. \right. \\
&\left. \left. \sum_{\bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} g_{\bar{\lambda}_1 \bar{\lambda}_2} \right\} \psi_{\alpha_i \lambda_2} + \frac{\gamma \Delta^2}{2} \sum_{\lambda_1 \lambda_2; \bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} \left\{ \sum_i \psi_{\alpha_i \lambda_1}^+ \psi_{\alpha_i \lambda_2} \right\} \left\{ \sum_j \psi_{\alpha_j \bar{\lambda}_1}^+ \psi_{\alpha_j \bar{\lambda}_2} \right\} \right] \Bigg|_{g=0}. \tag{12}
\end{aligned}$$

In obtaining (12) we used the integral representation of the δ -function (with a field $\vartheta_{\lambda_1 \lambda_2}$ in the exponent), and rescaled the integration variables, $\sqrt{\gamma} \Delta g = \tilde{g}$, $\vartheta / \sqrt{\gamma} \Delta = \tilde{\vartheta}$ (the tilde is omitted). Further, for $i\vartheta_{\lambda_1 \lambda_2}$ we substituted $\partial / \partial g_{\lambda_1 \lambda_2}$ acting on $\exp\{i \sum_{\lambda_1 \lambda_2} \vartheta_{\lambda_1 \lambda_2} g_{\lambda_1 \lambda_2}\}$; after partial integration, $g_{\lambda_1 \lambda_2} = 0$ results because of δ -functions. The $\psi_{\alpha \lambda}$ refer to the frequency ω_{l_0} . Now, the ψ with frequencies with $l \neq l_0$ appear only bilinearly in the first line of (12) and can be integrated out.

2.2 Introduction of the Q-Field

We introduce the Q -field by first introducing the identity for the Hermitean field \bar{Q} ,

$$1 = \int \mathcal{D}\bar{Q} \exp \left\{ - \frac{1}{2\gamma} \text{Tr} \{ \bar{Q}^2 \} \right\}, \tag{13}$$

into the functional integral, and by applying a shift,

$$\bar{Q}_{\alpha_i \lambda_1; \alpha_j \lambda_2} = Q_{\alpha_i \lambda_1; \alpha_j \lambda_2} + i\gamma \Delta \sum_{\bar{\lambda}_1 \bar{\lambda}_2} (\Gamma^{1/2})_{\bar{\lambda}_2 \lambda_2}^{\lambda_1 \bar{\lambda}_1} \psi_{\alpha_2 \bar{\lambda}_2}^+ \psi_{\alpha_1 \bar{\lambda}_1}, \tag{14}$$

cancelling the fourth order term in the ψ in (12); the composition rule for the Γ ,

$$\sum_{\bar{\lambda}_1 \bar{\lambda}_2} (\Gamma^s)_{\bar{\lambda}_2 \lambda_2}^{\lambda_1 \bar{\lambda}_1} (\Gamma^t)_{\bar{\lambda}_2 \lambda_2}^{\bar{\lambda}_1 \lambda_1'} = (\Gamma^{s+t})_{\bar{\lambda}_2 \lambda_2}^{\lambda_1 \lambda_1'}, \tag{15}$$

was used. By integrating out the remaining $\psi_{\alpha \lambda}$ we obtain finally for the measure of the functional integral $\exp\{-S_Q\} :=$

$$\begin{aligned}
&= \exp \left[\text{Tr} \sum_{l; (l \neq l_0)} \ln \left\{ (G_{\alpha_i \lambda_1}^{(0)}(\omega_l))^{-1} \delta_{\lambda_1 \lambda_2} - \sqrt{\gamma} \Delta \exp\{i\alpha_i \omega_l \Delta\} \frac{\partial}{\partial g_{\lambda_1 \lambda_2}} \right\} \right] * \\
&\exp \left[\frac{1}{2} \sum_{\lambda_1 \lambda_2; \bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} g_{\lambda_1 \lambda_2} g_{\bar{\lambda}_1 \bar{\lambda}_2} - \frac{1}{2\gamma} \sum_{i\lambda_1; j\lambda_2} |Q_{\alpha_i \lambda_1; \alpha_j \lambda_2}|^2 + \text{Tr} \ln \left\{ (G_{\alpha_i \lambda_1}^{(0)})^{-1} \delta_{ij} \delta_{\lambda_1 \lambda_2} \right. \right. \\
&\left. \left. - \sqrt{\gamma} \Delta \sum_{\bar{\lambda}_1 \bar{\lambda}_2} \Gamma_{\bar{\lambda}_1 \bar{\lambda}_2}^{\lambda_1 \lambda_2} g_{\bar{\lambda}_1 \bar{\lambda}_2} \delta_{ij} + i\Delta \sum_{\bar{\lambda}_1 \bar{\lambda}_2} (\Gamma^{1/2})_{\bar{\lambda}_2 \lambda_2}^{\lambda_1 \bar{\lambda}_1} Q_{\alpha_i \bar{\lambda}_1; \alpha_j \bar{\lambda}_2} + i\Delta \hat{Q}_{\alpha_i \lambda_1; \alpha_j \lambda_2} \right\} \right] \Bigg|_{g=0};
\end{aligned} \tag{16}$$

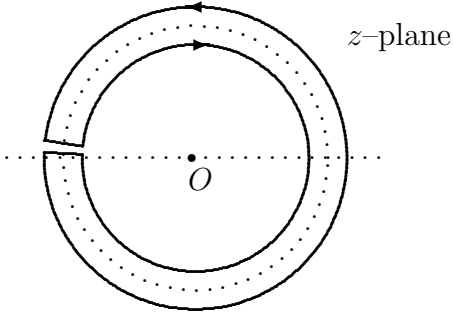
Tr is defined with respect to i, λ , and we added a source term with \hat{Q} .

Appendix

We remark that the identity (8) can be also obtained by evaluating directly the sum over l by means of the residuum theorem:

$$\begin{aligned} \sum_{l=-\mathcal{N}}^{\mathcal{N}-1} f\left(\exp\left\{i\frac{2\pi}{2\mathcal{N}}l\right\}\right) &= \oint_{C_1} \frac{dl}{\exp\{i2\pi l\} - 1} f\left(\exp\left\{i\frac{2\pi}{2\mathcal{N}}l\right\}\right) \\ &= \frac{2\mathcal{N}}{2\pi i} \oint_{C_2} \frac{dz}{z} \frac{1}{z^{2\mathcal{N}} - 1} f(z) = -2\mathcal{N} \operatorname{Res}_{(\text{outer domain})} \left[\frac{1}{z} \frac{1}{z^{2\mathcal{N}} - 1} f(z) \right]. \end{aligned} \quad (17)$$

The path C_1 in the complex l -plane surrounds the poles of the denominator in the first line of (17). Introducing $z = \exp\{i2\pi l/2\mathcal{N}\}$, it is substituted by the path C_2 (see Fig.). In analogy to the situation in the continuum, the lattice GF G_R is analytic outside the unit circle, and G_A is analytic inside the unit circle.



The path C_2 in the complex z -plane. The unit circle and the real axis are dotted. The poles of $1/(z^{2\mathcal{N}} - 1)$ on the unit circle, lying in the interval $-\pi \leq \arg z < \pi$, are surrounded (inner domain) by C_2 ; the poles of $f(z)$ lie in the outer domain (containing the origin O and the infinite point).

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