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Inconsistency between alternative approaches to Quantum Decoherence in special systems

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Abstract

We study the decoherence properties of a certain class of Markovian quantum open systems from both the Decohering Histories and Environment Induced Superselection paradigms. The class studied includes many familiar quantum optical cases. For this class, we show that there always exists a basis which leads to *exactly* consistent histories for any coarse graining *irrespective* of the initial conditions. The magnitude of the off–diagonal elements of the reduced density matrix ρ in this basis however, depends on the initial conditions. Necessary requirements for classicality as advanced by the two paradigms are thus in direct conflict in these systems.

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In this letter we describe a class of quantum open Markovian systems and examine the decoherence properties from the Decohering Histories approach (DH) [1] and from the Environment Induced Superselection approach (EIS) [2]. We curiously discover that for this class of systems, histories defined as projections onto a particular basis ($|n\rangle$ say) are *exactly* consistent for any coarse graining *irrespective* of the initial condition of the system. Thus one can ascribe separate histories with probabilities which obey the classical probability laws. In the framework of Decohering Histories the attainment of a consistent set of alternative histories is a necessary requirement for the existence of a "Quasiclassical Domain" [3]. In this same class of systems, however, the magnitude of the off–diagonal elements of the reduced density matrix for the system *does* depend on the initial state. Although the reduced density matrix becomes diagonal in the basis $|n\rangle$ for times greater than the dissipation timescale, the "decoherence" timescale in EIS sensitively depends on the initial state of the system [2]. In EIS the criteria for classicality is uncertain but generally a system is said to behave in a classical manner when the reduced density matrix is sufficiently diagonal in the "pointer basis" over a sufficiently long period of time (for the most recent report on EIS see [4]). Thus, in these systems it appears that the requirements for classicality in EIS depends on the initial system state whereas in DH the requirements are independent of the initial state. This result highlights the differences between these two approaches and brings into question the validity of either in describing the transition from quantum to classical in these models.

a. Models Consider the class of quantum systems which are linearly coupled to an infinite bath of harmonic oscillators in the regime where the system's dynamics are Markovian. For these models one can write the master equation for the reduced density matrix of the system in a standard form as shown by Linblad [5,6]

$$\dot{\rho} = L\rho = -\frac{i}{\hbar}[H,\rho] + \sum_J [2A_J\rho A_J^\dagger - \rho A_J^\dagger A_J - A_J^\dagger A_J\rho] \ , \tag{1}$$

where H is some Hermitian operator and the A_J are arbitrary. The rigorous proofs require the A_J to be bounded operators while a number of authors have successfully applied Linblad's theory to unbounded operators [7]. We will concentrate only on those models where the diagonal matrix elements of (1) in some basis $|n\rangle$ involve only diagonal matrix elements of ρ. One such model occurs when the A_J's are a representation of a Lie algebra $\mathcal{L}$ and where H is constructed solely from elements of the Cartan subalgebra h of $\mathcal{L}$. Here $|n\rangle$ is given by the eigenbasis of $A_J^\dagger A_J$ for some J ie. an eigenbasis of an element of the Cartan subgroup h. One can, however, construct other models where again $|n\rangle$ and H are as above while the A_J's can be complicated functions of single photon creation and annihilation operators eg. $A = a^m$, $A = f(a^\dagger a)$ or $A = a + a^\dagger$. In all the above examples bar the last, the basis $|n\rangle$ is discrete. For these types of models the diagonal matrix elements of (1) in $|n\rangle$ can be written as

$$\langle n|\dot{\rho}|n\rangle = \dot{\rho}_{nn} = \sum_{m=0}^{\infty} c_{nm}\rho_{mm} \quad . \tag{2}$$

The dynamics of the diagonal elements of ρ in this particular basis decouples completely from the off–diagonal elements. We effectively have a dynamical superselection rule. In fact, since the diagonal elements are the probabilities, (2) may be rewritten as

$$\partial_t\, P(n) = \sum_{m=0}^{\infty} c_{nm} P(m) \quad , \tag{3}$$

which is a discrete version of a classical Markov process – a birth death stochastic process [6].

We now examine the Decoherence Functional for such models where we ignore the bath of harmonic oscillators and concentrate on coarse grained histories of the system defined through projections onto coarse grainings of $|n\rangle\langle n|$ at particular instants of time. We can write the Decoherence Functional as

$$\mathbf{D}(\alpha', \alpha) = \tag{4}$$

$$Tr\left[P_i^{\alpha_i}\left(e^{\int_{\Delta t_i} Ldt}\left(P_{i-1}^{\alpha_{i-1}}\left(e^{\int_{\Delta t_{i-1}} Ldt}(\cdots)\right)P_{i-1}^{\alpha_{i-1}\prime}\right)\right)\right] \quad , \tag{5}$$

where we have taken the trace over the environment and used the master equation (1) to evolve forward the reduced density matrix of the system between subsequent projections P_{i-2} and P_{i-1} etc. We describe the histories through the P_i^α where

$$P_i^{\alpha} = \sum_{n \in n_{\alpha_i}} |n\rangle\langle n| \quad , \tag{6}$$

where the n_α represents a complete and exclusive binning of the basis $|n\rangle$ into alternatives labeled by α_i. Let us concentrate on the final t_{i-1} to t_i in the Decoherence Functional (4). We have

$$\mathbf{D}(\alpha, \alpha') = \sum_{\substack{n_i \in n_{\alpha_i} \\ n_{i-1} \in n_{\alpha_{i-1}} \\ n'_{i-1} \in n_{\alpha'_{i-1}}}} \langle n_i | e^{\int_{\Delta t_i} L dt} \left(\tilde{\rho}_{n_{i-1}\, n'_{i-1}} |n_{i-1}\rangle\langle n'_{i-1}| \right) |n_i\rangle \quad , \tag{7}$$

where

$$\tilde{\rho}_{n_{i-1}\, n'_{i-1}} = \langle n_{i-1} | e^{\int_{\Delta t_{i-1}} L dt} (\cdots) | n'_{i-1}\rangle \quad . \tag{8}$$

However, from equation (2) the final trace in the Decoherence Functional implies that the right hand side of (7) vanishes for $n_{i-1} \neq n'_{i-1}$. This argument can be repeated for times t_i and t_{i-2} and so on. We ultimately find that the Decoherence Functional exactly vanishes when the two histories α and α' differ. Thus, for the types of models discussed above the Decoherence Functional is *exactly* diagonal in the histories. Further, from (7) we see that the result does not depend on the specific values of $\tilde{\rho}_{n_{i-1} n'_{i-1}}$. The achievement of consistency is therefore completely insensitive to the initial state of the system. We also note that this result holds for all possible binnings occurring in the construction of the projectors (6).

The dynamics of the off–diagonal elements of the reduced density matrix of the system in the $|n\rangle$ basis is, of course, dependent on the initial state. One can begin with an initial state where the magnitude of the off–diagonal elements are very large. One example where the off-diagonal elements decouple exactly from diagonal elements is that of a spin with a magnetic moment M in a fluctuating magnetic field [8]. Other examples can be found in number of models commonly used in quantum optics. Specifically, we look at master equations derived in the Markov-Born limit with weak coupling to the bath and with the Rotating Wave Approximation (RWA). The master equations for such quantum optical models are only approximate but are generally regarded as good descriptions of the physical processes in the proper regimes. We will have more to say concerning the validity of these master equations later.

We take the case of a general system coupled to a thermal bath and a broadband squeezed vacuum. The quantum optical master equation is [9]

$$\begin{aligned}
\dot{\rho}_{\rm sys} = & -\frac{i}{\hbar}[H_{\rm sys},\rho] + \frac{1}{2}\gamma(N+1)(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c) \\
& +\frac{1}{2}\gamma N(2c^\dagger\rho c - cc^\dagger\rho - \rho cc^\dagger) \\
& -\frac{1}{2}\gamma M(2c^\dagger\rho c^\dagger - c^\dagger c^\dagger\rho - \rho c^\dagger c^\dagger) \\
& -\frac{1}{2}\gamma M^*(2c\rho c - cc\rho - \rho cc) \ ,
\end{aligned} \tag{9}$$

where γ is the coupling strength to the bath, c is a system operator which effectively couples to the creation operator of the bath (see [6]), N is the number of quantum per mode in the reservoir and M with $N(N+1) \geq |M|^2$ is a measure of the squeezing.

Looking first at the case where the system is coupled to a thermal vacuum, i.e. $N = 0$, $M = 0$, the types of system Hamiltonians which match the models described in this note depend on the type of system operator which couples to the bath. The simplest is where $c = a$, the system lowering operator. The master equation is already in the Linblad form where $|n\rangle$ is the eigenbasis of $a^\dagger a$. We can take $H_{\rm sys} = f(a^\dagger a)$ where f is arbitrary. In particular we can set $H_{\rm sys} = \hbar\omega(a^\dagger a + 1/2)$, the simple harmonic oscillator. We can also set $c = a^\dagger a$ with $|n\rangle$ and H as before to obtain the phase damped harmonic oscillator. The decay of off-diagonal coherences for these examples has been studied by Walls and Milburn [10].

One can include a thermal bath and still retain the diagonal property of the master equation. Note however, that the inclusion of a driving field destroys this property. Thus, histories in the photon number basis will be automatically consistent in these models while the off-diagonal elements in the number basis of ρ may be quite large over periods much greater than the Markov time.

In considering the additional coupling to a squeezed vacuum we follow [9] and rewrite (9) as

$$\dot{\rho} = -\frac{i}{\hbar}[H_{\rm sys}, \rho] + \Lambda\rho \ . \tag{10}$$

We may recast $\Lambda\rho$ in the Linblad form,

$$\Lambda\rho = \frac{1}{2}\sum_{\kappa=1}^{2} \lambda_\kappa(2N+1)[2a_\kappa\rho a_\kappa^\dagger - a_\kappa^\dagger a_\kappa\rho - \rho a_\kappa^\dagger a_\kappa] \ , \tag{11}$$

where

$$a_\kappa = \sum_{\kappa=1}^{2} c_i V_{i\kappa} \qquad\qquad (\kappa = 1, 2) \ , \tag{12}$$

$$\lambda_{1,2} = \frac{\gamma}{2}(2N + 1 \pm \sqrt{1 + 4|M|^2}) \ , \tag{13}$$

$$V = \begin{pmatrix} \cos\frac{\theta}{2}\ e^{-i\phi/2} & -\sin\frac{\theta}{2}\ e^{-i\phi/2} \\ \sin\frac{\theta}{2}\ e^{i\phi/2} & \cos\frac{\theta}{2} e^{i\phi/2} \end{pmatrix} \ , \tag{14}$$

and $c_1 \equiv c$, $c_2 \equiv c^\dagger$. We only consider the pure state squeezed vacuum case where $N(N+1) = |M|^2$. In this case $\lambda_1 = \gamma(2N+1)$, $\lambda_2 = 0$ with

$$a_1 = (\sqrt{N+1}e^{-i\phi/2}c + \sqrt{N}e^{i\phi/2}c^\dagger) \ , \tag{15}$$

and $a_2 = 0$. We can show $[a_1, a_1^\dagger] = 1$ and thus $\Lambda\rho$ will only couple diagonal elements of ρ in the $a_1^\dagger a_1$ eigenbasis. For the complete master equation to couple only diagonal matrix elements we must have $H_{\rm sys} = f\,[a_1^\dagger a_1]$ or

$$H_{\rm sys} = f\,[(2N+1)a^\dagger a + \sqrt{N(N+1)}e^{i\phi}a^{\dagger\,2} + Ne^{-i\phi}a^2] \ , \tag{16}$$

where f is again an arbitrary function. Although such a system Hamiltonian is quite artificial it is curious that such a model yields consistent histories which obey classical probability laws. The addition of a^2 and $a^{\dagger\,2}$ to the system Hamiltonian usually results in the phenomena of “squeezing” [6] and can produce very non–classical states.

b. Discussion We have described a class of models consisting of system+interaction+bath for which the definitions of "decoherence" in the Decohering Histories approach and Environment Induced Superselection approach disagree. We find that there is an effective dynamical superselection which decouples the diagonal matrix elements of the reduced density matrix from the off-diagonal elements in a particular basis.

Before discussing the possible implications of this result let us comment on the validity of the quantum optical master equation (9) and in particular the Rotating Wave Approximation (RWA). The original coupling of the system to the bath in these models is taken to be of the position–position type ie. $H_{\text{int}} \sim (a^\dagger + a)(b^\dagger + b)$ where the a and b are destruction operators for the system and bath respectively. One then neglects the high frequency components of this interaction, $a^\dagger b^\dagger$ and ab to finally obtain an interaction of the form $a^\dagger b + ab^\dagger$. This procedure is almost standard in all quantum optical calculations. However, as pointed out by Lindenberg and West [8], the fully–coupled model (ie. without the RWA) possesses couplings between the diagonal and off-diagonal elements of ρ in the number basis. These couplings are weak in the appropriate regimes and can usually be neglected but their existence, no matter how small, eliminates such position–position couplings from the class of models studied in this paper. Interactions between the system and the bath which do fall into the class of models described in this note are those which involve linear couplings in *both* position–position (x-x) and momentum–momentum (p-p). These types of system–bath interactions can be written in the $a^\dagger b + ab^\dagger$ form without any approximation for any mixture of x-x and p-p coupling using a canonical transformation. This dual interaction in both position and momentum has received little attention in the literature. It has been treated explicitly by Leggett [11] with a number of examples of pure x-x ("normal"), pure (p-p) ("anomalous") and "mixed" x-x and p-p coupling. He concludes that in any realistic physical system the dissipation is unlikely to be pure "anomalous" but can be of the "mixed" type. Mixed x-x and p-p couplings in Josephson junctions and quantum tunnelling have also been treated in [12,13]. Momentum dependent interactions also occur frequently in nuclear physics, for example in meson interactions [14,15] and in [16]. From the framework of Environment

Induced Superselection we might also expect a mixed x-x and p-p interaction. We normally picture classical mechanics as existing on phase space. If we wish the correlations between x and p for a quantum system coupled to a bath to exhibit little or no interference on phase space, then according to Zurek, one should couple the system to the bath in both position and momentum.

It is clear that for the class of systems treated here the two definitions of decoherence differ greatly. Further, in the DH approach, the $|n\rangle$ projected histories are exactly consistent irrespective of the initial state of the system for all grainings. If one chooses to accept that this class of models can represent physically realistic situations, (as we argue above) then one is forced to conclude that either one or both of the "decoherence" paradigms (EIS and DH) is incorrect as a sole indicator of "classical" behaviour. In DH the criteria for a quasi–classical world requires, at least, one exactly consistent set of histories. This is achieved almost automatically for all times greater than the Markov time in these models. However, in EIS we can only say that the "collapse" of the wavefunction to the "observed" basis ($|n\rangle$) occurs for times greater than the decoherence time t_d. This decoherence time depends on the initial state of the system and can be of the order of the relaxation timescale for sufficiently low temperatures and small wavepacket spread. For very weak coupling t_d may even exceed the age of the universe! It has been argued that if the system rapidly decoheres in the EIS approach for a particular basis then coarse grained histories over this basis should be approximately consistent [4]. An example where EIS decoherence is achieved and DH decoherence is not has been discovered by Laflamme and Matacz [17]. We have found models displaying the converse. Clearly, greater understanding of the physical implications of these two paradigms is needed.

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