# A quantum holographic principle from decoherence

Sougato Bose<sup>†</sup> and Anupam Mazumdar \*

† Optics Section, Blackett Laboratory, Imperial College, London, SW7 2BZ, U. K.

\* Astrophysics Group, Blackett Laboratory, Imperial College, London, SW7 2BZ, U. K.

(November 18, 2018)

We present a fully quantum version of the holographic principle in terms of quantum systems, subsystems, and their interactions. We use the concept of environment induced decoherence to prove this principle. We discuss the conditions under which the standard (semi-classical) holographic principle is obtained from this quantum mechanical version.

PACS numbers: 98.80.Cq

Cq IMPERIAL preprint IMPERIAL-AST 99/9-5, gr-qc/yymmnn

## I. INTRODUCTION

It was first propounded by 't Hooft [1] that the observable degrees of freedom of a 3 + 1 diemsnional world can as well be realised from the boundary of the system. This idea fits in perfectly with Bekenstein's result about the total entropy of a black-hole not exceeding a quarter of the area of its event horizon [2]. Following 't Hooft's, this suggests that all the information of a black hole can be collected from the surface of its horizon. There also exists a conjecture in supergravity Yang-Mills theories that the bulk information can be successfully stored at the boundary of an anti-de-Sitter space [3]. All these results lead to the conjecture : all information about a system can be obtained from its boundary, which is known as the holographic principle.

However, the application of holographic principle, as it stands, becomes restricted to situations where one can uniquely define the boundary of a system. What could be the boundary of a purely quantum mechanical system? One could imagine it to be the boundary of the classical potential which traps the system. From such considerations, for a particle in a box, the box would be its boundary. However, a quantum system can always tunnel out from any finite classical potential. Moreover, in reality, classical potential wells are the results of the interaction of a quantum system with other quantum fields and the notion of a classical potential well does not even exist in an entirely quantum universe. This motivates us to formulate a version of the holographic principle entirely in terms of quantum systems and their interactions. To prove this version of the principle, we rely on the notion of environment induced decoherence (EID) [4].

EID is a process used to explain the emergence of classicality from the quantum world. When an isolated quantum system interacts with it's environment, its state evolves to a diagonal mixed states in a specific pointer basis [5]. This is also a mechanism for entropy production in an open quantum system. We show that if this is the sole entropy generation mechanism in a system, then a quantum version of the holographic principle can be proved. In other words, we assume that all the entropy of a quantum system arises from its interactions with its environment. This assumption is justified if all systems in our universe are quantum. Their entropy cannot increase due to their own unitary evolutions. They gain entropy only when they interact with other systems.

We will first give an operational definition of the boundary of a quantum system in terms of the notions of systems, sub-systems and their interactions. Next we will show how the decoherence of the system leads to all information about the system being stored in its boundary ( as defined by us ). After this, we will use the stability of the states of the pointer basis in decoherence to justify why the information *remains stored* in this way. To conclude the paper, we will point out the conditions required for our quantum holographic principle to go over to the standard ( semi-classical) holographic principle.

#### **II. THE QUANTUM HOLOGRAPHIC PRINCIPLE**

Let there be a quantum system S. All quantum systems interacting with it together constitute another system, called the environment E. However, in general, all constituents of the system S will not interact with E directly. Therefore, we divide the system S into two subsystems A and B with B being the subsystem which *directly interacts* with E. We define B to be the boundary of the system S. Using this definition of the boundary, we can formulate the following holographic principle : *all information about* S *can be obtained from* B.

As we have assumed that all the entropy of the system is due to its interaction with its environment, we can start off the system S, before any interaction with E, in the pure state  $|\psi\rangle_{AB}$ . Let us assume that  $\hat{X}_{B}$  is the operator whose

eigenstates  $|X_i\rangle_B$  form the pointer basis for the subsystem B. This means, that B interacts with E via an interaction Hamiltonian of the type

$$H_{\rm BE} = g_1 \hat{X}_{\rm B} \hat{Y}_{\rm E} \,, \tag{1}$$

where  $\hat{Y}_{\rm E}$  is some operator in the Hilbert space of the system E and  $g_1$  is the coupling strength. If E is a sufficiently large system, and if B has many degrees of freedom interacting with E, then this interaction leads to a diagonalization of the state B in the basis  $|X_i\rangle$ . The initial pure state of the system S can be written in this basis as

$$|\psi\rangle_{\rm AB} = \sum_{i} c_i |\phi_i\rangle_{\rm A} |X_i\rangle_{\rm B} , \qquad (2)$$

where the state  $|\phi_i\rangle_A$  need not be orthogonal, but are assumed to be normalized. After interaction of B with E for a span of time more than the decoherence time scale [4] this state evolves to

$$\rho_{\rm AB} = \sum_{i} |c_i|^2 |\phi_i\rangle_{\rm A} \langle \phi_i|_{\rm A} \otimes |X_i\rangle_{\rm B} \langle X_i|_{\rm B} \,. \tag{3}$$

At this stage the von-Neumann entropy  $S^{\nu}(\rho_{AB})$  of the system S is given by

$$S^{\nu}(\rho_{AB}) = -\text{Tr}(\rho_{AB} \ln \rho_{AB}) = -\sum_{i} |c_{i}|^{2} \ln |c_{i}|^{2}, \qquad (4)$$

The Von-Newmann entropy of the boundary B is obtained from  $\rho_{\rm B} = \text{Tr}_{\rm A}(\rho_{\rm AB})$  to be

$$S^{v}(\rho_{\rm B}) = -\text{Tr}(\rho_{\rm B}\ln\rho_{\rm B}) = -\sum_{i} |c_{i}|^{2}\ln|c_{i}|^{2} = S^{v}(\rho_{\rm AB}).$$
(5)

It is known that the von-Neumann entropy of a system is equal to the amount of information one can acquire on observing the system's state [7]. Eq.(5) shows that the entire information  $S^v(\rho_{AB})$  about the system S can be gained from just acquiring the information  $S^v(\rho_B)$  stored on it's boundary B.

However, how stable is this information stored in the boundary? In fact, what about the possibility of Eq.(5) loosing it's validity, due to the interaction between subsystems A and B of the system S? This possibility can be prevented if B is a subsystem with a large number of degrees of freedom (i.e a macroscopic system). Then by definition of the pointer basis, the states of the basis  $|X_i\rangle$  are stable states. In fact, it is the *stability of states in the pointer basis* [6], which makes them ideal candidates for classical states. This demands that the subsystem B interacts with the subsystem A via an interaction Hamiltonian of the type

$$H_{\rm AB} = g_2 X_{\rm B} Z_{\rm A} \,, \tag{6}$$

where  $\hat{Z}_{A}$  is an operator in the Hilbert space of A and  $g_{2}$  is the coupling strength. In Eq.(6),  $\hat{X}_{B}$  can be replaced by any operator that commutes with  $\hat{X}_{B}$ . As long as  $H_{AB}$  is of the form described by Eq.(6), the state of the system S will evolve only to states of the form

$$\rho_{\rm AB}' = \sum_{i} |c_i|^2 |\phi_i'\rangle_{\rm A} \langle \phi_i'|_{\rm A} \otimes |X_i\rangle_{\rm B} \langle X_i|_{\rm B} , \qquad (7)$$

where  $|\phi'_i\rangle_{\rm A} = e^{-ig_2 \hat{X}_i \hat{Z}_{\rm A} t} |\phi_i\rangle_{\rm A}$  ( in which t denotes the time ). Thus Eq.(5) continues to be satisfied, and the entire information about the system S can be learnt from its boundary B.

In the above proof of the quantum holographic principle, the facts that B was macroscopic enough to decohere, and that it was coupled to both A and E by the same operator  $\hat{X}_{\rm B}$ , were important. We can give a counter example of the quatum holographic principle when the above conditions do not hold. Let S be a very simple system comprised of two spin  $\frac{1}{2}$  particles A and B. Only B directly interacts with another spin  $\frac{1}{2}$  particle E, and as such, is the boundary of S. We take an initial state of A, B and E to be the following form

$$(|\uparrow\rangle_{A}|\uparrow\rangle_{B} + |\downarrow\rangle_{A}|\downarrow\rangle_{B}) \otimes |\downarrow\rangle_{E}.$$
(8)

At this stage, the system S has a zero entropy and so does the environment E. Let the unitary interaction  $U_{\text{BE}}$  between B and E be the following

$$\begin{aligned} |\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{E}} \to |\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{E}} , \\ |\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{E}} \to |\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{E}} . \end{aligned} \tag{9}$$

Then the final state of A, B and E is as follows

$$(|\uparrow\rangle_{\mathcal{A}}|\uparrow\rangle_{\mathcal{E}} + |\downarrow\rangle_{\mathcal{A}}|\downarrow\rangle_{\mathcal{E}}) \otimes |\downarrow\rangle_{\mathcal{B}}.$$
(10)

All entropy of the system S is stored in A. The boundary B has no entropy. This counter example illustrates the importance of the Hamiltonians  $H_{\text{BE}}$  and  $H_{\text{AB}}$  being of the form given by Eqs.(1) and (6) for the quantum holographic principle to hold true.

### **III. CONCLUSION**

We have shown that under a specific set of conditions, the entire information about a quantum system S can be obtained from it's subsystem B which directly interacts with the environment. In the language of quantum measurement theory, the boundary B acts as an apparatus for the state of the whole system S. While the standard (semi-classical) holographic principle helps in making the 3 dimensional world effectively 2 dimensional, the quantum version could lead to a reduction of the Hilbert space dimensions of a problem.

For the quantum version of the principle to reduce to the semi-classical version, we require:

(1) The physical boundary of the system in the semi-classical version to coincide with the subsystem B. In other words, we require the boundary to be interacting strongly with the environment and the bulk to have insignificant interaction with the outside world.

(2) The boundary itself to have quite a large number of degrees of freedom, so that it is macroscopic and strongly decohering (i.e nearly a classical system). It must also be coupled both to it's environment and to it's bulk by the same operator  $\hat{X}_{\rm B}$ .

(3) The system to be an open system. So it would not apply to closed systems such as the entire universe.

To conclude, we emphasize that there may be other methods to derive the holographic principle which apply to less restricted situations. But if we assume that all systems are essentially quantum, and gain entropy only from interaction with their environment, then the requirements (1) - (3) are probably essential for the validity of the standard holographic principle.

#### ACKNOWLEDGMENTS

Authors are supported by the INLAKS foundation and the ORS award.

[1] G. 't Hooft, Dimensional reduction in quantum gravity, gr-qc/9310026; L. Susskind, J. Math. Phys. 36, 6377 (1995).

- [2] J. D. Bekenstein, Phys. Rev. **D** 23, 287 (1981).
- [3] J. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998), hep-th/9711200.
- [4] W. H. Zurek, Phys. Today 44, 36 (1991).
- [5] W. H. Zurek, Phys. Rev. **D** 24, 1519 (1981).
- [6] W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett 70, 1187 (1993).
- [7] B. Schumacher, Phys. Rev. A 51, 2738 (1995).