$\Lambda(1405)$ and Negative-Parity Baryons in Lattice QCD

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We review briefly recent studies of the $\Lambda(1405)$ spectrum in Lattice QCD. Ordinary three-quark pictures of the $\Lambda(1405)$ in quenched Lattice QCD fail to reproduce the mass of the experimental value, which seems to support the penta-quark picture for the $\Lambda(1405)$ such as a $\bar{K}N$ molecule-like state. It is also noted that the present results suffer from relatively large systematic uncertainties coming from the finite volume effect, the chiral extrapolation and the quenching effect.

I. INTRODUCTION

The $\Lambda(1405)$ is still a mysterious particle due to its small mass. It is the lightest negative-parity baryon although it contains a strange valence quark. The conventional quark model, in which the $\Lambda(1405)$ is assigned as the flavor-singlet state in the 70 dimensional representation under the spin-flavor SU(6) symmetry, cannot explain this small mass. Since the mass is just below the $\bar{K}N$ threshold, there is another interpretation based on a five-quark picture, a $\bar{K}N$ bound state or a $\pi\Sigma$ resonance. It is likely that the actual $\Lambda(1405)$ will be a mixed state of these two pictures. However, knowing which picture is the dominant contribution is quite important for hyper-nuclear physics and astro-nuclear physics, because the proton in the K^-p bound state suffers from the Pauli blocking effect in nuclear matter. It may influence on the existence of the kaonic nuclei and kaon condensation in neutron stars.

Quenched lattice QCD is a useful tool to distinguish between the three- and the five-quark pictures of the $\Lambda(1405)$, because it is more valence-like than dynamical QCD due to the absence of sea quarks. Recently we have calculated the $\Lambda(1405)$ and other low-lying negative-parity baryon spectra in anisotropic quenched lattice QCD [1]. In this talk we mainly report our results and comment on other recent two lattice studies on the $\Lambda(1405)$ spectrum [2, 3].

II. LATTICE FORMULATION

We employ the standard Wilson gauge action and the $O(a)$ improved Wilson quark action with tadpole improvement. We adopt the anisotropic lattice since fine resolution in the temporal direction makes us easy to follow the change of heavy-baryon correlations and to determine the fitting ranges for their masses. We take the renormalized anisotropy as $\xi = a_\sigma/a_\tau = 4$, where a_σ and a_τ are the spatial and temporal lattice spacings, respectively. The simulation is carried out on the three lattices where parameters are well tuned and errors are rather well evaluated [4]. The sizes of the lattices are $12^3 \times 96(\beta = 5.75$, i.e., $a_\sigma^{-1} = 1.034(6)$GeV), $16^3 \times 128(\beta = 5.95$, i.e., $a_\sigma^{-1} = 1.499(9)$GeV), and $20^3 \times 160(\beta = 6.10$, i.e., $a_\sigma^{-1} = 1.871(14)$GeV). The scale a_σ^{-1} is determined from the K^* meson mass. Determination of the other parameters is described in Ref.[4].

As for the quark mass, we adopt four different values which roughly cover around strange quark mass and correspond to the pion mass being about $0.6 - 0.9$GeV. (See also Fig. 1). We use the standard baryon operators which survive in the nonrelativistic limit: the $(\bar{q}^T C\gamma_5)q$ form for the octet and the $(\bar{q}^T C\gamma_\mu)q$ form for the decuplet. Here C is the charge conjugation matrix. For the $\Lambda(1405)$ we use the flavor-singlet operator following the assignment of the quark model,

$$(u^T C\gamma_5 d)s + (d^T C\gamma_5 u)s + (s^T C\gamma_5 u)d, \quad (1)$$

where we have omitted the color indices for simplicity. Both the positive- and negative-parity baryon spectra can be obtained from these operators using the parity projection, because they couple to both the parity states. In the source operator, each quark is spatially smeared with the Gaussian function with the width ~ 0.4 fm for better overlap with the low-lying states.

III. NUMERICAL RESULTS

The numerical results and the fit results of the baryon spectrum for the finest $\beta = 6.10(a_\sigma^{-1} \simeq 1.9$GeV) lattice are shown in Fig.1. The results from the other lattices are similar [1]. The physical u, d and s quark masses are determined with the π and K meson masses. For each baryon, two of quark masses are taken to be the same value, m_1, and the other quark mass m_2 is taken to be an independent value. The baryon masses are then expressed by the function of m_1 and m_2, but the numerical results seem to be well described with the linear form, $m_B(m_1, m_2) = m_B(0, 0) + B_B \cdot (2m_1 + m_2)$. Therefore we fit the baryon spectrum to the linear form in the sum of corresponding pseudoscalar meson mass squared, $\langle m^2_{\rm PS}(m_i)\rangle = 1/N_q \sum_{i=1}^{N_q} m^2_{\rm PS}(m_i, m_i) = 2B \sum_{i=1}^{N_q} m_i$ with $N_q = 3$ for baryons, as shown in Fig. 1.

We have also calculated the vector meson spectrum with the same way mentioned above and have used the same fitting method. Our results for the vector meson

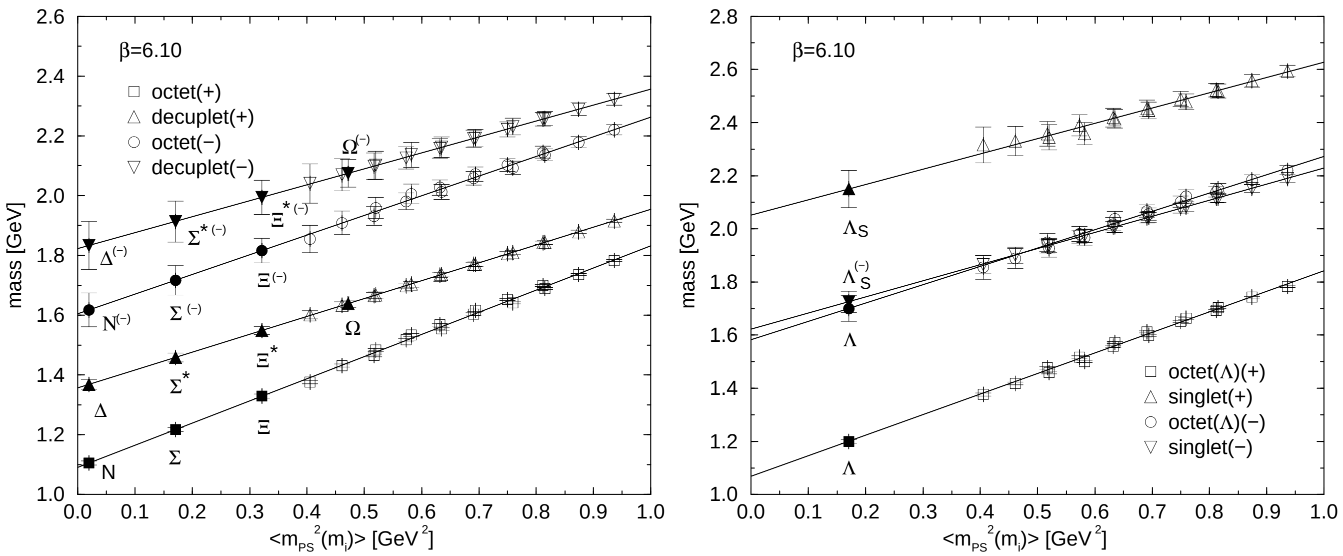

FIG. 1: The positive- and negative-parity baryon spectra for the $\beta = 6.10(a_\sigma^{-1} \simeq 1.9\text{GeV})$ lattice. The octet and the decuplet baryons are shown in the left figure and the octet(Λ) and the singlet baryons in the right. The open symbols denote the lattice data and the filled symbols the fitted results from the linear chiral extrapolation.

and the positive-parity baryons are consistent with those obtained in Ref. [4].

IV. DISCUSSIONS

From Fig.1, the chiral extrapolated results of the negative-parity baryon spectrum are heavier than those of the corresponding positive-parity sectors for octet and decuplet, as expected. The flavor-singlet negative-parity baryon is, however, lighter than the positive-parity one. This is consistent with the conventional quark model, in which the flavor-singlet positive-parity baryon is assigned as a state with the principal quantum number $N = 2$, while the negative-parity sector has $N = 1$.

Various baryon masses at the $\beta = 6.10$ lattice together with the experimental values are shown in Fig.2. As for the negative-parity baryons, most of the lattice results comparatively well reproduce the experiment. The flavor-singlet baryon is, however, exceptional: its calculated mass of about 1.7GeV is much heavier than the $\Lambda(1405)$ with a difference of more than 300 MeV. The difference between them is actually the largest in all the hadrons in consideration. We also note that the flavor-singlet baryon has one strange valence quark and therefore the systematic error from the chiral extrapolation should be less than that of the nucleon and the delta. Even if one takes the systematic error coming from the quenching effect of 10% level into account, this discrepancy of more than 300 MeV cannot be accepted. Therefore it is natural to consider the flavor-singlet baryon is physically different from the $\Lambda(1405)$. The flavor-octet negative-parity Λ baryon mass is almost the same as the flavor-singlet one and thus it also is not likely to be a candidate of the $\Lambda(1405)$.

We turn to other two lattice studies on the $\Lambda(1405)$. Melnitchouk *et al.* investigated excited state baryons using an improved quenched Wilson gauge and an improved Wilson quark actions [2]. They computed an operator composed of the terms common to the flavor-octet Λ operator and the flavor-singlet operator, i.e.,

$$(d^T C\gamma_5 s)u + (s^T C\gamma_5 u)d \qquad (2)$$

to study the $\Lambda(1405)$. This allows for mixing between octet and singlet states by the favor SU(3) symmetry breaking due to the strange quark mass. Although they did not take the chiral extrapolation of the results, the result for the common operator is still much heavier than the $\Lambda(1405)$, about 1.8GeV, if one assumes the simple linear form for the chiral extrapolation. Therefore, as their conclusion, the $\Lambda(1405)$ does not couple strongly to the operator (2) and further investigations such as with lighter quark masses or dynamical quarks are needed.

Lee *et al.* studied excited state baryons using the overlap fermions [3]. They did not employ any special operators for the $\Lambda(1405)$ but the usual flavor-octet operators to compare with experiment. The overlap fermion is able to access light quark masses than ever because of the good chiral property. In fact they took very light quark masses for which the lightest pion mass becomes about 0.18GeV. The quark mass dependence of the baryons is apparently different from the other two studies: non-linear forms of the chiral extrapolation are clearly seen in the light quark mass region ($m_\pi^2 < 0.3\text{GeV}^2$). The flavor-octet negative-parity spectrum rapidly decreases leading to about 0.14GeV as the quark mass decreases. (Our result [1] and the result of Ref.[2] do not show such behavior, because their pion masses are about 0.6GeV at most.)

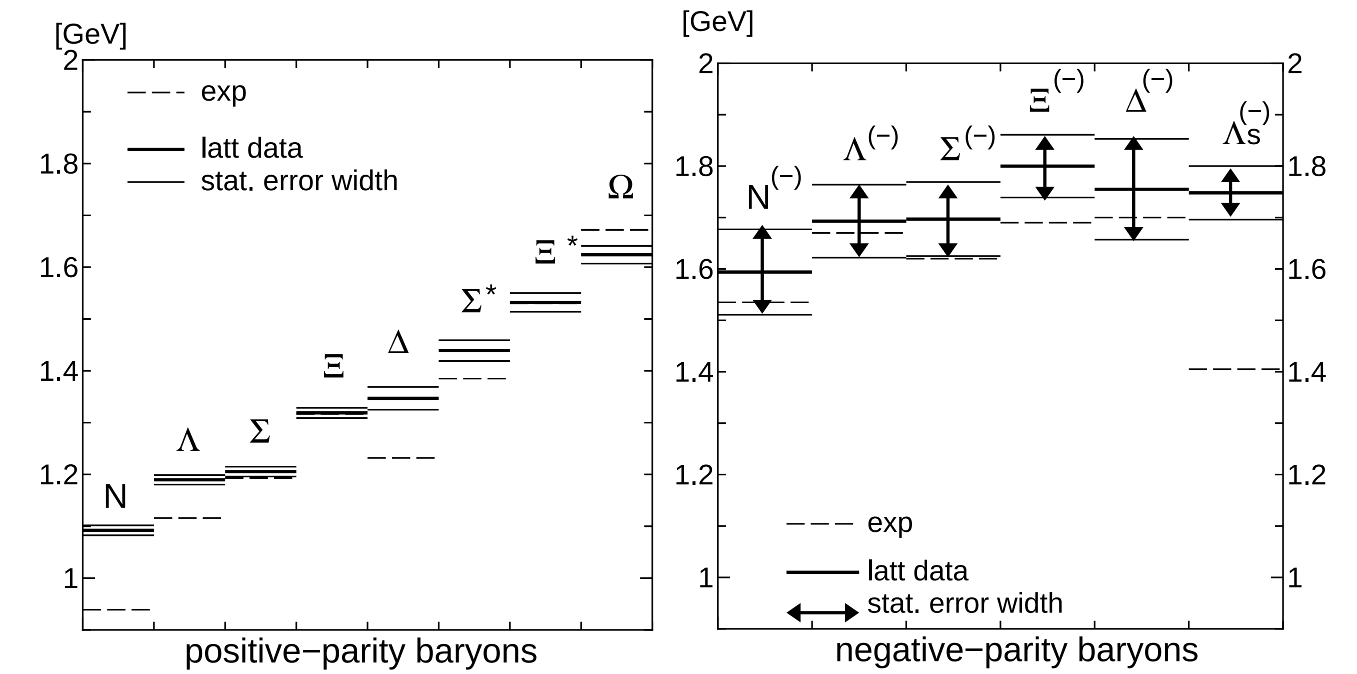

FIG. 2: Calculated and experimental baryon masses. The lattice results are taken from the $\beta = 6.10$ lattice. For the negative-parity baryons, the experimental values are $N(1535), \Lambda(1670), \Sigma(1620), \Xi(1690), \Delta(1700)$ and $\Lambda(1405)$.

Such a light quark mass region can, however, suffer from large systematic uncertainties even in the chiral sophisticated fermions. In particular some ghost effects coming from unitarity violation of the quenched approximation may be significant in this region. Also a rather coarse lattice ($a^{-1} \sim 1$GeV) which they used would give hadron spectrum an additional uncertainty. Hence further systematic error analyses are needed also in this case.

V. SUMMARY

We have focused on the $\Lambda(1405)$ spectrum in quenched lattice QCD. In our calculation with the anisotropic lattice, the flavor-singlet baryon mass measured with the three-quark operator is found to be about 1.7GeV, which is much heavier than the $\Lambda(1405)$. In Ref.[2], an operator with terms common to the flavor-octet Λ and the flavor-singlet states has been used to describe the $\Lambda(1405)$, and the result seems to be very heavy after the naive chiral extrapolation. Both the results present the difficulty in identifying the $\Lambda(1405)$ as the flavor-singlet three-quark baryon, which seems to indicate that the $\Lambda(1405)$ is dominated by an exotic state such as a $\bar{K}N$ molecule-like state.

On the other hand, it is reported in Ref.[3] that the linear chiral extrapolation of the excited baryon spectrum fails in the very light quark mass region using overlap fermions. Their results are closer to experiment than the others, while further systematic error analyses coming from the quenching effects are needed in that light quark region.

Thus for more definite understanding of the $\Lambda(1405)$ on the lattice, it is desired to carry out more extensive work such as a simulation with dynamical quarks or pentaquark states like the $\bar{K}N$ state.

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