

Decuplet Baryon Masses in Partially Quenched Chiral Perturbation Theory

Brian C. Tiburzi* and André Walker-Loud†

*Department of Physics
University of Washington
P.O. Box 351560
Seattle, WA 98195-1560
(Dated: September 9, 2018)*

Abstract

The masses of the spin- $\frac{3}{2}$ baryons are calculated to next-to-next-to-leading order in heavy baryon chiral perturbation theory and partially quenched heavy baryon chiral perturbation theory. The calculations are performed for three light flavors in the isospin limit. These results are necessary for extrapolating QCD and partially quenched QCD lattice calculations of the decuplet baryon masses.

PACS numbers: 12.38.Gc

arXiv:hep-lat/0407030v2 13 Sep 2004

*bctiburz@u.washington.edu

†walkloud@u.washington.edu

I. INTRODUCTION

The study of low-energy QCD provides insight into the non-perturbative dynamics of quarks and gluons. One of the most fundamental observables is the spectrum of the theory. Quarks and gluons are confined into color neutral hadronic states that are seen in nature. Understanding the mass spectrum of these lowest-lying states directly from QCD remains an on-going challenge.

A model independent tool to study low-energy QCD is through chiral perturbation theory (χ PT). This is an effective theory written in terms of low-energy degrees of freedom, e.g. the octet mesons in $SU(3)$ flavor are assumed to be the pseudo-Goldstone bosons that appear from chiral symmetry breaking. In this effective theory observable quantities receive both long-range and short-range contributions. The long-range contributions arise from non-analytic meson loops, while the short-range physics is encoded in a number of low-energy constants (LECs). Symmetry constrains the number of such constants but their values must be determined from experiment or from theoretical calculations.

Lattice QCD can provide first principles numerical determination of QCD observables, in particular the mass spectrum of baryons. There have been numerous calculations of baryon masses in quenched QCD (QQCD) and a few in QCD [1, 2, 3, 4, 5] Partially quenched QCD (PQQCD) calculations in the baryonic sector are so far very limited. A problem that haunts lattice calculations is that computing power currently restricts the simulations to quark masses that are larger than the physical light quark masses. Therefore, to make physical predictions it is necessary to extrapolate from the heavier lattice quark masses to those found in nature. For QQCD, where the fermionic determinant that arises from the path integral is set to a constant, quenched chiral perturbation theory (Q χ PT) has been developed to aid in the extrapolation [6, 7, 8, 9, 10, 11, 12]. There is no general connection of QQCD observables to QCD because QQCD does not have an axial anomaly. This feature of the quenched approximation leads to new operators in Q χ PT that have no analogue in χ PT. Moreover the LECs of Q χ PT are numerically different than in χ PT.

The problems of the quenched approximation can be remedied by using partially quenched QCD (PQQCD). In PQQCD contributions from sea quarks are retained and the fermionic determinant is hence no longer a constant. Additionally the masses of sea quarks are varied independently of the valence quarks. By efficaciously giving the sea quarks larger masses it is much less costly to calculate observables than in ordinary QCD. The low-energy effective theory of PQQCD is partially quenched χ PT (PQ χ PT) [13, 14, 15, 16, 17, 18, 19, 20, 21]. Since PQQCD retains an axial anomaly, the singlet field can be integrated out. Therefore the LECs of χ PT appear in PQ χ PT. By fitting PQ χ PT to PQQCD lattice data one can determine the LECs and thereby make physical predictions for QCD. There has been much activity recently in calculating baryon properties in PQ χ PT [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. There has also been interest in using chiral effective theories to extrapolate quenched and unquenched lattice data on the baryon masses, see for example [34, 35, 36].

In this work we calculate the masses of the decuplet baryons to next-to-next-to leading order (NNLO) in χ PT. We also provide the NNLO calculation of the decuplet baryon masses in PQ χ PT. These calculations are performed in the isospin limit of $SU(3)$ and in the partially quenched calculation we use three valence, three ghost and three sea quarks, with two of the sea quarks degenerate. The expressions we derive can be used to extrapolate lattice QCD and partially quenched lattice QCD data to the physical quark masses. Additionally the LECs can be determined up to $\mathcal{O}(m_q^2)$, where m_q represents a quark mass.

This paper has the following organization. First in Section II, we review the basics of χ PT including the inclusion of the octet and decuplet baryons to leading and next-to-leading order in the heavy baryon expansion. In this work we use rank-three flavor tensors for both the octet and decuplet baryons. Next in Section III, we calculate the decuplet masses to next-to-next-to-leading order in χ PT. We work in the $SU(3)$ flavor group and in the isospin limit. In Section IV, we review PQ χ PT in a way that parallels our discussion of χ PT, then in Section V we calculate the decuplet masses at next-to-next-to-leading order in PQ χ PT. Finally a summary (Section VI) ends the paper.

II. HEAVY BARYON CHIRAL PERTURBATION THEORY

In this section, we briefly review chiral perturbation theory. We start first in the meson sector and then review heavy baryon chiral perturbation theory to leading and next-to-leading order in the inverse baryon mass. For the baryons, we differ from the standard formulation by embedding the octet states into rank-three flavor tensors.

A. Pseudo-Goldstone bosons

For massless quarks, the QCD Lagrangian exhibits a chiral symmetry, $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$, which is spontaneously broken down to $SU(3)_V \otimes U(1)_V$. Chiral perturbation theory, the low-energy effective theory of QCD, emerges by expanding about the physical vacuum state. Without the explicit breaking of chiral symmetry provided by the quark mass term of the Lagrangian, eight particles (the pions, kaons and eta) would emerge as the Goldstone bosons of the broken $SU(3)_A$ symmetry. Given that the quark masses are small compared to the scale of chiral symmetry breaking, however, the lowest lying mesons emerge as an octet of pseudo-Goldstone bosons.

The pseudo-Goldstone bosons are collected in an exponential matrix

$$\Sigma = \exp\left(\frac{2i\Phi}{f}\right) = \xi^2, \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (1)$$

With the above convention, the pion decay constant f is 132 MeV. The effective Lagrangian describing the dynamics of these mesons at leading order in χ PT is [37]

$$\mathcal{L} = \frac{f^2}{8} \text{tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + \lambda \text{tr}(m_Q^\dagger \Sigma + m_Q \Sigma^\dagger). \quad (2)$$

Above, leading order in the power counting is $\mathcal{O}(m_Q)$ and hence $\mathcal{O}(m_Q) \sim \mathcal{O}(q^2)$, where q is an external pion momentum. In the isospin limit, the quark mass matrix m_Q is given by

$$m_Q = \text{diag}(m_u, m_u, m_s). \quad (3)$$

Expanding the Lagrangian Eq. (2) to leading order, one finds that mesons with quark content QQ' are canonically normalized when the meson masses are given by

$$m_{QQ'}^2 = \frac{4\lambda}{f^2}(m_Q + m_{Q'}). \quad (4)$$

B. Baryons

Besides the mesons, there are also three quark states in the spectrum of QCD. To include these octet and decuplet baryons systematically into chiral perturbation theory, we use heavy baryon χ PT (HB χ PT) [38, 39, 40, 41]. For the octet, baryon fields $B(x)$ are redefined in terms of velocity dependent fields $B_v(x)$,

$$B_v(x) = \frac{1 + \not{v}}{2} e^{iM_B v \cdot x} B(x), \quad (5)$$

where v_μ is the four-velocity of the baryon, B . This field redefinition corresponds to parameterizing the baryon momentum as

$$p_\mu = M_B v_\mu + k_\mu, \quad (6)$$

where k_μ is the residual momentum. This efficacious redefinition eliminates the Dirac mass term for baryons

$$\bar{B} (i\not{\partial} - M_B) B = \bar{B}_v i\not{\partial} B_v + \mathcal{O}\left(\frac{1}{M_B}\right). \quad (7)$$

From Eq. (5), it is easy to verify that derivatives acting on B_v bring down powers of the residual momentum k . Thus, higher dimension operators of the heavy baryon field B_v are suppressed by powers of M_B and a consistent derivative expansion emerges. Henceforth we shall omit the subscript v from all baryon fields treating them implicitly as heavy baryons.

In this work we embed all baryons in rank-3 flavor tensors. The convenience of this choice will become readily apparent when we generalize to PQ χ PT. The $SU(3)$ matrix of the lowest-lying spin- $\frac{1}{2}$ baryon fields is

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (8)$$

These baryon states are embedded in the tensor B_{ijk} in the following way [12]

$$B_{ijk} = \frac{1}{\sqrt{6}} (\varepsilon_{ijl} \mathbf{B}_k^l + \varepsilon_{ikl} \mathbf{B}_j^l). \quad (9)$$

The flavor tensor has the symmetry properties

$$B_{ijk} = B_{ikj} \quad \text{and} \quad B_{ijk} + B_{jik} + B_{kji} = 0. \quad (10)$$

When the spin- $\frac{3}{2}$ decuplet baryons T are included in the theory, an additional mass parameter Δ appears. This parameter is the leading-order mass splitting between the octet and the decuplet in the chiral limit and must be included in the power counting. We treat Δ as $\mathcal{O}(q)$, where q is a typical small pion momentum. The spin- $\frac{3}{2}$ decuplet baryons can be described by a Rarita-Schwinger field, $(T^\mu)_{ijk}$, which is totally symmetric under the interchange of flavor indices. We employ the normalization convention in which $T^{111} = \Delta^{++}$. The heavy baryon Rarita-Schwinger field satisfies the constraints, $v \cdot T = S \cdot T = 0$, where S^μ is the covariant spin-vector.

The Lagrangian to leading order in the $1/M_B$ expansion can be written as

$$\begin{aligned}
\mathcal{L} = & (\overline{B} i v \cdot D B) + 2\alpha_M (\overline{B} B \mathcal{M}_+) + 2\beta_M (\overline{B} \mathcal{M}_+ B) + 2\sigma_M (\overline{B} B) \text{tr}(\mathcal{M}_+) \\
& - (\overline{T}^\mu [i v \cdot D - \Delta] T_\mu) + 2\gamma_M (\overline{T}^\mu \mathcal{M}_+ T_\mu) - 2\overline{\sigma}_M (\overline{T}^\mu T_\mu) \text{tr}(\mathcal{M}_+) \\
& + 2\alpha (\overline{B} S^\mu B A_\mu) + 2\beta (\overline{B} S^\mu A_\mu B) + 2\mathcal{H} (\overline{T}^\nu S^\mu A_\mu T_\nu) \\
& + \sqrt{\frac{3}{2}} \mathcal{C} [(\overline{T}^\nu A_\nu B) + (\overline{B} A_\nu T^\nu)].
\end{aligned} \tag{11}$$

Above, D_μ is the chiral-covariant derivative which acts on the B and T fields as

$$(D^\mu B)_{ijk} = \partial^\mu B_{ijk} + (V^\mu)_i^l B_{ljk} + (V^\mu)_j^l B_{ilk} + (V^\mu)_k^l B_{ijl}. \tag{12}$$

The vector and axial-vector meson fields appearing in the Lagrangian are given by

$$V_\mu = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad A_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi), \tag{13}$$

and

$$\mathcal{M}_+ = \frac{1}{2} (\xi^\dagger m_Q \xi^\dagger + \xi m_Q \xi). \tag{14}$$

In Eq. (11) the brackets () denote a contraction of the flavor indices and are defined in [12]. Such contractions ensure the proper transformations of the field bilinears under chiral transformations. To compare with the coefficients used in the standard two-index baryon formulation [38, 39, 40, 41], it is straightforward to show

$$\alpha = \frac{2}{3}D + 2F, \quad \beta = -\frac{5}{3}D + F, \tag{15}$$

and

$$\alpha_M = \frac{2}{3}b_D + 2b_F, \quad \beta_M = -\frac{5}{3}b_D + b_F, \quad \sigma_M = b_D - b_F + \sigma. \tag{16}$$

C. Higher dimensional operators

The Lagrangian in Eq. (11) contains some, but not all, terms of $\mathcal{O}(q^2)$. To calculate the decuplet masses to $\mathcal{O}(m_Q^2)$ we must include all $\mathcal{O}(q^2)$ relevant operators, which contribute to the mass via loops, and all $\mathcal{O}(q^4)$ relevant operators which contribute at tree level. The Lagrangian also includes operators of $\mathcal{O}(q^3)$, but they do not contribute to the self-energy.

The baryon mass is treated as $M_B \sim \Lambda_\chi$. As the LECs are *a priori* unknown, we can combine the $1/M_B$ and $1/\Lambda_\chi$ expansions into one expansion in powers of $1/\Lambda_\chi$. There is one exception: constraints from reparameterization invariance (RI) determine the coefficients of some of the higher dimension operators arising in the $1/M_B$ expansion [42, 43]. Thus these $1/M_B$ corrections must be kept distinct to insure the Lorentz invariance of the heavy baryon theory to a given order.

The heavy baryon momentum parameterization in terms of v^μ and k^μ in Eq. (6) is unique only up to $1/M_B$ corrections. When the velocity and residual momentum are simultaneously transformed in the following way

$$v \rightarrow v + \frac{\epsilon}{M_B}, \quad k \rightarrow k - \epsilon, \tag{17}$$

the momentum p_μ in Eq. (6) is unchanged. Reparameterization invariance requires the effective Lagrangian to be invariant under such transformations and thus ensures the theory is Lorentz invariant to a given order in $1/M_B$. Furthermore, utilizing RI has non-trivial consequences as it connects operators of different orders in the $1/M_B$ expansion and thereby exactly fixes the coefficients of some of the higher dimensional operators with respect to the lower ones. We find that the fixed coefficient Lagrangian is

$$\mathfrak{L} = - \left(\bar{B} \frac{D_\perp^2}{2M_B} B \right) + \left(\bar{T}^\mu \frac{D_\perp^2}{2M_B} T_\mu \right) + \mathcal{H} \left[\left(\bar{T}^\mu \frac{i\overleftarrow{D} \cdot S}{M_B} v \cdot A T_\mu \right) - \left(\bar{T}^\mu v \cdot A \frac{S \cdot i\overrightarrow{D}}{M_B} T_\mu \right) \right] \quad (18)$$

where $D_\perp^2 = D^2 - (v \cdot D)^2$ and we have kept only terms relevant to the calculation of decuplet masses.

The Lagrangian contains additional $\mathcal{O}(q^2)$ operators as well as $\mathcal{O}(q^4)$ operators which are invariant under the $SU(3)$ chiral transformations. In part, these operators absorb some effects of the unphysical, off-shell degrees of freedom [44]. The operators relevant to the self energy of the decuplet baryons are

$$\begin{aligned} \mathfrak{L} = \frac{1}{4\pi f} & \left[t_1^A \bar{T}_\mu^{kji} (A_\nu A^\nu)_i{}^{i'} T_{i'jk}^\mu + t_2^A \bar{T}_\mu^{kji} (A_\nu)_i{}^{i'} (A^\nu)_j{}^{j'} T_{i'j'k}^\mu + t_3^A (\bar{T}_\mu T^\mu) \text{tr}(A_\nu A^\nu) \right. \\ & + t_1^{\tilde{A}} \bar{T}_\mu^{kji} (A^\mu A_\nu)_i{}^{i'} T_{i'jk}^\nu + t_2^{\tilde{A}} \bar{T}_\mu^{kji} (A^\mu)_i{}^{i'} (A_\nu)_j{}^{j'} T_{i'j'k}^\nu + t_3^{\tilde{A}} (\bar{T}_\mu T^\nu) \text{tr}(A^\mu A_\nu) \\ & + t_1^{vA} \bar{T}_\mu^{kji} (v \cdot A v \cdot A)_i{}^{i'} T_{i'jk}^\mu + t_2^{vA} \bar{T}_\mu^{kji} (v \cdot A)_i{}^{i'} (v \cdot A)_j{}^{j'} T_{i'j'k}^\mu + t_3^{vA} (\bar{T}_\mu T^\mu) \text{tr}(v \cdot A v \cdot A) \\ & + t_1^M \bar{T}_\mu^{kji} (\mathcal{M}_+ \mathcal{M}_+)_i{}^{i'} T_{i'jk}^\mu + t_2^M \bar{T}_\mu^{kji} (\mathcal{M}_+)_i{}^{i'} (\mathcal{M}_+)_j{}^{j'} T_{i'j'k}^\mu + t_3^M (\bar{T}_\mu T^\mu) \text{tr}(\mathcal{M}_+ \mathcal{M}_+) \\ & \left. + t_4^M (\bar{T}_\mu \mathcal{M}_+ T^\mu) \text{tr}(\mathcal{M}_+) + t_5^M (\bar{T}_\mu T^\mu) \text{tr}(\mathcal{M}_+) \text{tr}(\mathcal{M}_+) \right]. \quad (19) \end{aligned}$$

All the LECs t_i^A , $t_i^{\tilde{A}}$, t_i^{vA} , and t_i^M are dimensionless. In principle, additional $1/M_B$ operators with the same chiral symmetry properties as those contained in Eq. (19) can be generated. However, these $1/M_B$ operators do not have their coefficients constrained by RI, therefore they shall be absorbed into the definition of the various t_i^A , $t_i^{\tilde{A}}$, t_i^{vA} , and t_i^M . Since the flavor, spin and Lorentz structure of these omitted $1/M_B$ operators is identical to those above, we are guaranteed that the values absorbed in the LECs of Eq. (19) remain the same for all processes and thus the determination of the above LECs is in that sense universal.

Including the decuplet fields in χ PT requires additional operators involving Δ/Λ_χ because Δ is a chiral singlet. Thus arbitrary functions $f(\Delta/\Lambda_\chi)$ can multiply any term in the Lagrangian without changing the properties under chiral transformations. To the order we are working, all constants in the calculation must be arbitrary polynomial functions of Δ/Λ_χ , and expanded out to the appropriate order. For example

$$\gamma_M \rightarrow \gamma_M \left(\frac{\Delta}{\Lambda_\chi} \right) = \gamma_M \left[1 + \gamma_1 \frac{\Delta}{\Lambda_\chi} + \gamma_2 \frac{\Delta^2}{\Lambda_\chi^2} + \mathcal{O} \left(\frac{\Delta^3}{\Lambda_\chi^3} \right) \right]. \quad (20)$$

Furthermore at this order the decuplet-octet mass splitting in the chiral limit is a polynomial function of Δ/Λ_χ . We shall not explicitly write down the operators that contribute to the Δ dependence because determination of their LECs requires the ability to vary Δ . Additionally, the LECs are also implicit functions of μ , to absorb the divergences arising from the loop integrals.

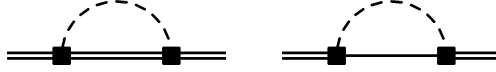


FIG. 1: One loop graphs which give contributions to $M_{T_i}^{(3/2)}$. The single, double and dashed lines correspond to octet baryons, decuplet baryons and mesons, respectively. The filled squares denote the axial coupling given in Eq. (11).

TABLE I: The tree level coefficients m_T , $(m^2)_T$, and $(mm')_T$ in χ PT and PQ χ PT for decuplet states T .

	m_T	$(m^2)_T$	$(mm')_T$
Δ	$3m_u$	$3m_u^2$	$3m_u^2$
Σ^*	$2m_u + m_s$	$2m_u^2 + m_s^2$	$m_u^2 + 2m_u m_s$
Ξ^*	$m_u + 2m_s$	$m_u^2 + 2m_s^2$	$2m_u m_s + m_s^2$
Ω^-	$3m_s$	$3m_s^2$	$3m_s^2$

III. DECUPLET MASSES IN χ PT

The masses of the octet and decuplet baryons have been investigated considerably in χ PT [33, 41, 45, 46, 47, 48, 49]. Here we calculate the masses of the decuplet baryons to NNLO in χ PT. The mass of the i^{th} decuplet baryon in the chiral expansion can be written as

$$M_{T_i} = M_0(\mu) + M_{T_i}^{(1)}(\mu) + M_{T_i}^{(3/2)}(\mu) + M_{T_i}^{(2)}(\mu) + \dots \quad (21)$$

Here, $M_0(\mu)$ is the renormalized mass of the decuplet baryons in the chiral limit which is independent of m_Q and also of the T_i . $M_{T_i}^{(n)}$ is the contribution to the i^{th} decuplet baryon of the order $m_Q^{(n)}$, and μ is the renormalization scale. For this calculation we use dimensional regularization with a modified minimal subtraction ($\overline{\text{MS}}$) scheme.

In calculating the masses, the leading dependence upon m_Q arises from the terms the Lagrangian Eq. (11) with coefficients γ_M and $\bar{\sigma}_M$. The $\mathcal{O}(m_Q^{3/2})$ contributions arise from the one-loop diagrams shown in Fig. 1, which are formed from the operators in the Lagrangian with coefficients \mathcal{H} and \mathcal{C} . The $\mathcal{O}(m_Q^2)$ contributions arise from the one-loop diagrams shown in Fig. 2, from the tree-level contributions of the operators with coefficients, t_i^M , and from the NLO wave-function corrections.

We find that the leading-order contributions to the decuplet masses are

$$M_T^{(1)} = \frac{2}{3} \gamma_M m_T - 2\bar{\sigma}_M \text{tr}(m_Q), \quad (22)$$

where the coefficients m_T appear in Table I. The next-to-leading order contributions are

$$M_T^{(3/2)} = -\frac{5\mathcal{H}^2}{72\pi f^2} \sum_{\phi} A_{\phi}^T m_{\phi}^3 - \frac{\mathcal{C}^2}{(4\pi f)^2} \sum_{\phi} B_{\phi}^T \mathcal{F}(m_{\phi}, -\Delta, \mu), \quad (23)$$

TABLE II: The coefficients A_ϕ^T and B_ϕ^T in χ PT for decuplet states T .

ϕ	A_ϕ^T			B_ϕ^T		
	π	K	η	π	K	η
Δ	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	0
Σ^*	$\frac{4}{9}$	$\frac{8}{9}$	0	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{3}$
Ξ^*	$\frac{1}{6}$	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
Ω^-	0	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{4}{3}$	0

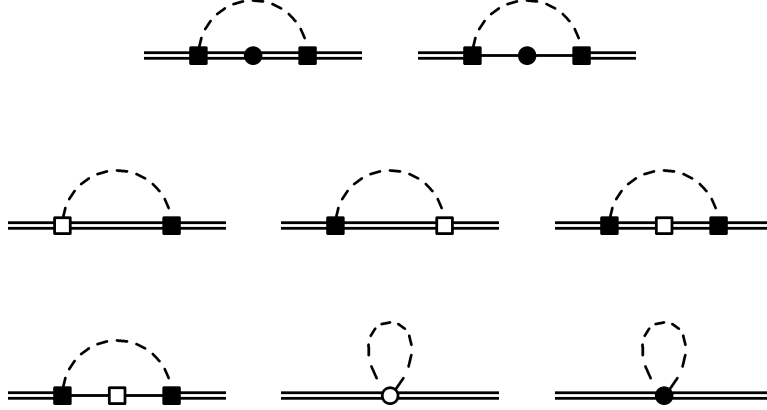


FIG. 2: One-loop graphs which give contributions to $M_{T_i}^{(2)}$. Single, double and dashed lines correspond to octet baryons, decuplet baryons and mesons, respectively. The filled squares and filled circles denote axial couplings and insertions of \mathcal{M}_+ given in Eq. (11). The empty squares denote insertions of fixed $1/M_B$ operators from Eq. (18), and the empty circles denote two-baryon-axial couplings defined in Eq. (19).

where the function \mathcal{F} is defined by

$$\mathcal{F}(m, \delta, \mu) = (m^2 - \delta^2) \left[\sqrt{\delta^2 - m^2} \log \left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\varepsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\varepsilon}} \right) - \delta \log \left(\frac{m^2}{\mu^2} \right) \right] - \frac{1}{2} \delta m^2 \log \left(\frac{m^2}{\mu^2} \right). \quad (24)$$

In Eq. (23) the sum on ϕ runs over loop mesons with the mass m_ϕ . The coefficients A_ϕ^T and B_ϕ^T above are the sums of squares of Clebsch-Gordon coefficients. The numerical values of these coefficients depend on the decuplet state T and are listed in Table II for loop mesons with mass m_ϕ .

Finally the next-to-next-to-leading contributions to the decuplet mass are

TABLE III: The coefficients C_ϕ^T and \bar{C}_ϕ in χ PT. Coefficients are listed for the decuplet states T and grouped into contributions from loop mesons with mass m_ϕ . \bar{C}_ϕ is identical for each decuplet state.

C_ϕ^T	π	K	η
Δ	$3m_u$	$m_u + m_s$	$\frac{1}{3}m_u$
Σ^*	$2m_u$	$\frac{4}{3}(m_u + m_s)$	$\frac{2}{9}(m_u + 2m_s)$
Ξ^*	m_u	$\frac{5}{3}(m_u + m_s)$	$\frac{1}{9}(m_u + 8m_s)$
Ω^-	0	$2(m_u + m_s)$	$\frac{4}{3}m_s$
\bar{C}_ϕ	$6m_u$	$4(m_u + m_s)$	$\frac{2}{3}(m_u + 2m_s)$

TABLE IV: The coefficients D_ϕ^T and \bar{D}_ϕ in χ PT. Coefficients are listed for the decuplet states T and grouped into contributions from loop mesons with mass m_ϕ . \bar{D}_ϕ is identical for each decuplet state.

D_ϕ^T	π	K	η
Δ	$\frac{3}{2}$	1	$\frac{1}{6}$
Σ^*	1	$\frac{4}{3}$	$\frac{1}{3}$
Ξ^*	$\frac{1}{2}$	$\frac{5}{3}$	$\frac{1}{2}$
Ω^-	0	2	$\frac{2}{3}$
\bar{D}_ϕ	3	4	1

$$\begin{aligned}
M_T^{(2)} &= (Z - 1)M_T^{(1)} \\
&+ \frac{1}{4\pi f} \left\{ \frac{1}{3}t_1^M (m^2)_T + \frac{1}{3}t_2^M (mm')_T + t_3^M \text{tr}(m_Q^2) + \frac{1}{3}t_4^M m_T \text{tr}(m_Q) + t_5^M [\text{tr}(m_Q)]^2 \right\} \\
&- \frac{2\gamma_M}{(4\pi f)^2} \sum_\phi C_\phi^T \mathcal{L}(m_\phi, \mu) + \frac{2\bar{\sigma}_M}{(4\pi f)^2} \sum_\phi \bar{C}_\phi \mathcal{L}(m_\phi, \mu) \\
&+ \frac{1}{(4\pi f)^3} \sum_\phi (t_1^A D_\phi^T + t_2^A E_\phi^T + t_3^A \bar{D}_\phi) \bar{\mathcal{L}}(m_\phi, \mu) \\
&+ \frac{1}{4(4\pi f)^3} \sum_\phi \left[(t_1^{\tilde{A}} + t_1^{vA})D_\phi^T + (t_2^{\tilde{A}} + t_2^{vA})E_\phi^T + (t_3^{\tilde{A}} + t_3^{vA})\bar{D}_\phi \right] \left[\bar{\mathcal{L}}(m_\phi, \mu) - \frac{1}{2}m_\phi^4 \right] \\
&- \frac{5}{8} \frac{\mathcal{H}^2}{(4\pi f)^2 M_B} \sum_\phi A_\phi^T \left[\bar{\mathcal{L}}(m_\phi, \mu) + \frac{19}{10}m_\phi^4 \right] - \frac{15}{16} \frac{\mathcal{C}^2}{(4\pi f)^2 M_B} \sum_\phi B_\phi^T \left[\bar{\mathcal{L}}(m_\phi, \mu) - \frac{1}{10}m_\phi^4 \right] \\
&- \frac{10\mathcal{H}^2\bar{\sigma}_M \text{tr}(m_Q)}{3(4\pi f)^2} \sum_\phi A_\phi^T \left[\mathcal{L}(m_\phi, \mu) + \frac{26}{15}m_\phi^2 \right] - \frac{3\mathcal{C}^2\sigma_M \text{tr}(m_Q)}{(4\pi f)^2} \sum_\phi B_\phi^T \mathcal{J}(m_\phi, -\Delta, \mu) \\
&+ \frac{10\mathcal{H}^2\gamma_M}{9(4\pi f)^2} \sum_\phi F_\phi^T \left[\mathcal{L}(m_\phi, \mu) + \frac{26}{15}m_\phi^2 \right] - \frac{3\mathcal{C}^2}{2(4\pi f)^2} \sum_\phi G_\phi^T \mathcal{J}(m_\phi, -\Delta, \mu). \tag{25}
\end{aligned}$$

Here the tree-level contributions are expressed in terms of coefficients m_T , $(m^2)_T$, and $(mm')_T$ which are listed in Table I. Above the wavefunction renormalization Z is given

TABLE V: The coefficients E_ϕ^T in χ PT. Coefficients are listed for the decuplet states T and grouped into contributions from loop mesons with mass m_ϕ .

E_ϕ^T	π	K	η
Δ	$\frac{1}{2}$	0	$\frac{1}{6}$
Σ^*	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$
Ξ^*	0	$\frac{2}{3}$	0
Ω^-	0	0	$\frac{2}{3}$

TABLE VI: The coefficients F_ϕ^T in χ PT. Coefficients are listed for the decuplet states T and grouped into contributions from loop mesons with mass m_ϕ .

F_ϕ^T	π	K	η
Δ	$\frac{5}{2}m_u$	$\frac{1}{3}(2m_u + m_s)$	$\frac{1}{2}m_u$
Σ^*	$\frac{4}{9}(2m_u + m_s)$	$\frac{8}{9}(2m_u + m_s)$	0
Ξ^*	$\frac{1}{6}(m_u + 2m_s)$	$\frac{1}{3}(4m_u + 5m_s)$	$\frac{1}{6}(m_u + 2m_s)$
Ω^-	0	$\frac{2}{3}(m_u + 2m_s)$	$2m_s$

by

$$Z - 1 = -\frac{5\mathcal{H}^2}{3(4\pi f)^2} \sum_\phi A_\phi^T \left[\mathcal{L}(m_\phi, \mu) + \frac{26}{15}m_\phi^2 \right] - \frac{3\mathcal{C}^2}{2(4\pi f)^2} \sum_\phi B_\phi^T \mathcal{J}(m_\phi, -\Delta, \mu). \quad (26)$$

The above equations (25) and (26) also employ abbreviations for non-analytic functions arising from loop contributions. These functions are

$$\mathcal{L}(m, \mu) = m^2 \log \frac{m^2}{\mu^2} \quad (27)$$

$$\bar{\mathcal{L}}(m, \mu) = m^4 \log \frac{m^2}{\mu^2} \quad (28)$$

$$\mathcal{J}(m, \delta, \mu) = (m^2 - 2\delta^2) \log \frac{m^2}{\mu^2} + 2\delta\sqrt{\delta^2 - m^2} \log \left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}} \right). \quad (29)$$

As in Eq. (23), the sums over ϕ in Eqs. (25) and (26) run through loop mesons with mass m_ϕ . The coefficients in these sums are themselves the sums of squares of Clebsch-Gordon coefficients and/or quark mass insertions. These are listed in Tables III–VII. To be clear, the above expressions Eqs. (22), (23), and (25) are quark mass expressions, i.e., the meson masses in these expressions are merely replacements for the quark masses via Eq. (4). To utilize these expressions for chiral extrapolations of lattice QCD data, one must write the quark masses in terms of physical meson masses (as determined on the lattice). To work consistently to $\mathcal{O}(m_q^2)$, one needs to use the NLO expressions for the meson masses in the LO results in Eq. (22)

TABLE VII: The coefficients G_ϕ^T in χ PT. Coefficients are listed for the decuplet states T and grouped into contributions from loop mesons with mass m_ϕ .

G_ϕ^T	π	K	η
Δ	$\frac{4}{3}m_u(\alpha_M + \beta_M)$	$\frac{2}{9}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$	0
Σ^*	$\frac{1}{27}[m_u(19\alpha_M + 22\beta_M) + m_s(11\alpha_M + 8\beta_M)]$	$\frac{2}{27}[m_u(7\alpha_M + 10\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	$\frac{1}{9}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$
Ξ^*	$\frac{1}{9}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	$\frac{4}{9}[2m_u(\alpha_M + \beta_M) + m_s(\alpha_M + \beta_M)]$	$\frac{1}{9}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$
Ω^-	0	$\frac{4}{9}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	0

IV. PQ χ PT

In PQQCD, the quark part of the Lagrangian is

$$\mathfrak{L} = \sum_{j,k=1}^9 \bar{Q}^j (i\not{D} - m_Q)_j^k Q_k. \quad (30)$$

This differs from the $SU(3)$ Lagrangian of QCD by the inclusion of six extra quarks; three bosonic ghost quarks, $(\tilde{u}, \tilde{d}, \tilde{s})$, and three fermionic sea quarks, (j, l, r) , in addition to the light physical quarks (u, d, s) . The nine quark fields transform in the fundamental representation of the graded $SU(6|3)$ group. They have been accommodated in the nine-component vector

$$Q^\dagger = (u, d, s, j, l, r, \tilde{u}, \tilde{d}, \tilde{s}). \quad (31)$$

The quark fields obey the graded equal-time commutation relation

$$Q_i^\alpha(\mathbf{x})Q_j^{\beta\dagger}(\mathbf{y}) - (-1)^{\eta_i\eta_j}Q_j^{\beta\dagger}(\mathbf{y})Q_i^\alpha(\mathbf{x}) = \delta^{\alpha\beta}\delta_{ij}\delta^3(\mathbf{x} - \mathbf{y}), \quad (32)$$

where α, β are spin and i, j are flavor indices. Analogous graded equal-time commutation relations can be written for two Q 's and two Q^\dagger 's. The grading factors

$$\eta_k = \begin{cases} 1 & \text{for } k = 1, 2, 3, 4, 5, 6 \\ 0 & \text{for } k = 7, 8, 9 \end{cases} \quad (33)$$

take into account the different fermionic and bosonic statistics of the quark fields. In the isospin limit the quark mass matrix of $SU(6|3)$ is given by

$$m_Q = \text{diag}(m_u, m_u, m_s, m_j, m_j, m_r, m_u, m_u, m_s). \quad (34)$$

Because the ghost quark masses are identically equal to the valence quark masses there is an exact cancellation in the path integral between the valence quark determinant and the ghost quark determinant. The sea quark determinant is unaffected. Thus in PQQCD, one has the ability to vary the valence and sea quark masses independently. QCD is recovered in the limit $m_j \rightarrow m_u$ and $m_r \rightarrow m_s$.

A. Pseudo-Goldstone Mesons

For massless quarks, the theory corresponding to the Lagrangian in Eq. (30) has a graded $SU(6|3)_L \otimes SU(6|3)_R \otimes U(1)_V$ symmetry which is assumed to be spontaneously broken down to $SU(6|3)_V \otimes U(1)_V$ in analogy with QCD. The effective low-energy theory obtained by perturbing about the physical vacuum state of PQQCD is PQ χ PT. The result is 80 pseudo-Goldstone mesons with dynamics described at leading order in the chiral expansion by the Lagrangian

$$\mathcal{L} = \frac{f^2}{8} \text{str} (\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + \lambda \text{str} (m_q \Sigma^\dagger + m_q^\dagger \Sigma) + \alpha_\Phi \partial^\mu \Phi_0 \partial_\mu \Phi_0 - m_0^2 \Phi_0^2, \quad (35)$$

where

$$\Sigma = \exp \left(\frac{2i\Phi}{f} \right) = \xi^2, \quad (36)$$

and the meson fields appear in

$$\Phi = \begin{pmatrix} M & \chi^\dagger \\ \chi & \tilde{M} \end{pmatrix}. \quad (37)$$

The operation $\text{str}()$ in Eq. (35) is the supertrace over flavor indices. The quantities α_Φ and m_0 are non-vanishing in the chiral limit. M and \tilde{M} are matrices containing bosonic mesons (with quantum numbers of $q\bar{q}$ pairs and $\tilde{q}\tilde{q}$ pairs, respectively), while χ and χ^\dagger are matrices containing fermionic mesons (with quantum numbers of $\tilde{q}\bar{q}$ pairs and $q\tilde{q}$ pairs, respectively). The upper 3×3 block of M is the usual octet of pseudo-scalar mesons and the remaining components are mesons formed with one or two sea quarks, see e.g. [22].

The flavor singlet field is defined to be $\Phi_0 = \text{str}(\Phi)/\sqrt{6}$. PQQCD has a strong axial anomaly $U(1)_A$ and therefore the mass of the singlet field m_0 can be taken to be the order of the chiral symmetry breaking scale, $m_0 \rightarrow \Lambda_\chi$ [17]. In this limit, the η two-point correlation functions deviate from their form in χ PT. For $a, b = u, d, s, j, l, r, \tilde{u}, \tilde{d}, \tilde{s}$, the $\eta_a \eta_b$ propagator with 2 + 1 sea-quarks at leading order is

$$\mathcal{G}_{\eta_a \eta_b} = \frac{i\epsilon_a \delta_{ab}}{q^2 - m_{\eta_a}^2 + i\epsilon} - \frac{i}{3} \frac{\epsilon_a \epsilon_b (q^2 - m_{jj}^2) (q^2 - m_{rr}^2)}{(q^2 - m_{\eta_a}^2 + i\epsilon) (q^2 - m_{\eta_b}^2 + i\epsilon) (q^2 - m_X^2 + i\epsilon)}, \quad (38)$$

where

$$\epsilon_a = (-1)^{1+\eta_a}. \quad (39)$$

The mass m_{xy} is the mass of a meson composed of (anti)-quarks of flavor x and y , while the mass m_X is defined as $m_X^2 = \frac{1}{3} (m_{jj}^2 + 2m_{rr}^2)$. The singlet propagator can be conveniently rewritten as

$$\mathcal{G}_{\eta_a \eta_b} = \epsilon_a \delta_{ab} P_a + \epsilon_a \epsilon_b \mathcal{H}_{ab} (P_a, P_b, P_X), \quad (40)$$

where

$$P_a = \frac{i}{q^2 - m_{\eta_a}^2 + i\epsilon}, \quad P_b = \frac{i}{q^2 - m_{\eta_b}^2 + i\epsilon}, \quad P_X = \frac{i}{q^2 - m_X^2 + i\epsilon}$$

$$\mathcal{H}_{ab}(A, B, C) = -\frac{1}{3} \left[\frac{(m_{jj}^2 - m_{\eta_a}^2)(m_{rr}^2 - m_{\eta_a}^2)}{(m_{\eta_a}^2 - m_{\eta_b}^2)(m_{\eta_a}^2 - m_X^2)} A - \frac{(m_{jj}^2 - m_{\eta_b}^2)(m_{rr}^2 - m_{\eta_b}^2)}{(m_{\eta_a}^2 - m_{\eta_b}^2)(m_{\eta_b}^2 - m_X^2)} B \right. \\ \left. + \frac{(m_X^2 - m_{jj}^2)(m_X^2 - m_{rr}^2)}{(m_X^2 - m_{\eta_a}^2)(m_X^2 - m_{\eta_b}^2)} C \right]. \quad (41)$$

B. Baryons

In PQ χ PT the baryons are composed of three quarks $Q_i Q_j Q_k$ where $i - k$ can be valence, sea or ghost quarks. One decomposes the irreducible representations of $SU(6|3)_V$ into irreducible representations of $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost} \otimes U(1)$. The method for including the octet and decuplet baryons into PQ χ PT is to use the interpolating field [12, 22]

$$\mathcal{B}_{ijk}^\gamma \sim \left(Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} - Q_i^{\alpha,a} Q_j^{\gamma,c} Q_k^{\beta,b} \right) \epsilon_{abc} (C\gamma_5)_{\alpha\beta}. \quad (42)$$

Under the interchange of flavor indices, one finds [12]

$$\mathcal{B}_{ijk} = (-)^{1+\eta_j\eta_k} \mathcal{B}_{ikj} \quad \text{and} \quad \mathcal{B}_{ijk} + (-)^{1+\eta_i\eta_j} \mathcal{B}_{jik} + (-)^{1+\eta_i\eta_j+\eta_j\eta_k+\eta_i\eta_k} \mathcal{B}_{kji} = 0. \quad (43)$$

We require that $\mathcal{B}_{ijk} = B_{ijk}$, defined in Eq. (9), when the indices, i, j, k are restricted to 1–3. Thus the octet baryons are contained as an $(\mathbf{8}, \mathbf{1}, \mathbf{1})$ of $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost} \otimes U(1)$ in the **240** representation. In addition to the conventional octet baryons composed of valence quarks, \mathcal{B}_{ijk} also contains baryon fields composed of sea and ghost quarks. In this work we only need states of the **240** which contain at most one sea or ghost quark, and these states have been explicitly constructed in [22].

Similarly, one can construct the spin- $\frac{3}{2}$ baryons which make up the **138**, and have an interpolating field

$$\mathcal{T}_{ijk}^{\alpha,\mu} \sim \left(Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} + Q_i^{\beta,b} Q_j^{\gamma,c} Q_k^{\alpha,a} + Q_i^{\gamma,c} Q_j^{\alpha,a} Q_k^{\beta,b} \right) \epsilon_{abc} (C\gamma^\mu)_{\beta\gamma}. \quad (44)$$

Under the interchange of flavor indices, one finds that

$$\mathcal{T}_{ijk} = (-)^{1+\eta_i\eta_j} \mathcal{T}_{jik} = (-)^{1+\eta_j\eta_k} \mathcal{T}_{ikj}. \quad (45)$$

We require that $\mathcal{T}_{ijk} = T_{ijk}$, when the indices i, j, k are restricted to 1–3. Under $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost} \otimes U(1)$ they transform as a $(\mathbf{10}, \mathbf{1}, \mathbf{1})$. In addition to the conventional decuplet resonances composed of valence quarks, \mathcal{T}_{ijk} contains fields with sea and ghost quarks. As with the **240**, for our calculation the required states of the **138** have been constructed in [22].

To write down the PQ χ PT Lagrangian, we must also include the appropriate grading factors in the contraction of flavor indices. These are included in the $()$ notation as was

originally defined in [12, 22] The leading order PQ χ PT Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = & (\bar{\mathcal{B}} iv \cdot D \mathcal{B}) + 2\alpha_M (\bar{\mathcal{B}}\mathcal{B}\mathcal{M}_+) + 2\beta_M (\bar{\mathcal{B}}\mathcal{M}_+\mathcal{B}) + 2\sigma_M (\bar{\mathcal{B}}\mathcal{B}) \text{str}(\mathcal{M}_+) \\
& - (\bar{\mathcal{T}}^\mu [iv \cdot D - \Delta] \mathcal{T}_\mu) + 2\gamma_M (\bar{\mathcal{T}}^\mu \mathcal{M}_+ \mathcal{T}_\mu) - 2\bar{\sigma}_M (\bar{\mathcal{T}}^\mu \mathcal{T}_\mu) \text{str}(\mathcal{M}_+) \\
& + 2\alpha (\bar{\mathcal{B}}S^\mu \mathcal{B}A_\mu) + 2\beta (\bar{\mathcal{B}}S^\mu A_\mu \mathcal{B}) + 2\mathcal{H} (\bar{\mathcal{T}}^\nu S^\mu A_\mu \mathcal{T}_\nu) \\
& + \sqrt{\frac{3}{2}} \mathcal{C} [(\bar{\mathcal{T}}^\nu A_\nu \mathcal{B}) + (\bar{\mathcal{B}}A_\nu \mathcal{T}^\nu)]. \tag{46}
\end{aligned}$$

The low-energy constants appearing above have the same numerical values as those in χ PT.

At higher orders, the situation is quite similar to the χ PT case considered in Section II. Recall that at higher orders, the Lagrangian can contain arbitrary functions of Δ/Λ_χ . We take this into account by implicitly treating the leading order coefficients as functions of Δ/Λ_χ expanded out to the required order. To enforce Lorentz invariance on the theory, we use RI to generate the higher dimensional operators with fixed coefficients. In PQ χ PT the fixed coefficient Lagrangian is given by

$$\mathcal{L} = - \left(\bar{\mathcal{B}} \frac{D_\perp^2}{2M_B} \mathcal{B} \right) + \left(\bar{\mathcal{T}}^\mu \frac{D_\perp^2}{2M_B} \mathcal{T}_\mu \right) + \mathcal{H} \left[\left(\bar{\mathcal{T}}^\mu \frac{i\vec{D} \cdot S}{M_B} v \cdot A \mathcal{T}_\mu \right) - \left(\bar{\mathcal{T}}^\mu v \cdot A \frac{S \cdot i\vec{D}}{M_B} \mathcal{T}_\mu \right) \right]. \tag{47}$$

As in χ PT there are additional operators with unfixed coefficients. These higher dimensional operators relevant for our calculation are collected in the PQ χ PT Lagrangian¹

$$\begin{aligned}
\mathcal{L} = & \frac{1}{4\pi f} \left[t_1^A \bar{\mathcal{T}}_\mu^{kji} (A_\nu A^\nu)_i{}^{i'} \mathcal{T}_{i'jk}^\mu + t_2^A (-)^{\eta_{i'}(\eta_j + \eta_{j'})} \bar{\mathcal{T}}_\mu^{kji} (A_\nu)_i{}^{i'} (A^\nu)_{j'}{}^{j'} \mathcal{T}_{i'j'k}^\mu + t_3^A (\bar{\mathcal{T}}_\mu \mathcal{T}^\mu) \text{str}(A_\nu A^\nu) \right. \\
& + t_1^{\bar{A}} \bar{\mathcal{T}}_\mu^{kji} (A^\mu A_\nu)_i{}^{i'} \mathcal{T}_{i'jk}^\nu + t_2^{\bar{A}} \bar{\mathcal{T}}_\mu^{kji} (A^\mu)_i{}^{i'} (A_\nu)_{j'}{}^{j'} \mathcal{T}_{i'j'k}^\nu + t_3^{\bar{A}} (\bar{\mathcal{T}}_\mu \mathcal{T}^\nu) \text{tr}(A^\mu A_\nu) \\
& + t_1^{vA} \bar{\mathcal{T}}_\mu^{kji} (v \cdot A v \cdot A)_i{}^{i'} \mathcal{T}_{i'jk}^\mu + t_2^{vA} \bar{\mathcal{T}}_\mu^{kji} (v \cdot A)_i{}^{i'} (v \cdot A)_{j'}{}^{j'} \mathcal{T}_{i'j'k}^\mu + t_3^{vA} (\bar{\mathcal{T}}_\mu \mathcal{T}^\mu) \text{tr}(v \cdot A v \cdot A) \\
& + t_1^M \bar{\mathcal{T}}_\mu^{kji} (\mathcal{M}_+ \mathcal{M}_+)_{i'}{}^{i'} \mathcal{T}_{i'jk}^\mu + t_2^M (-)^{\eta_{i'}(\eta_j + \eta_{j'})} \bar{\mathcal{T}}_\mu^{kji} (\mathcal{M}_+)_{i'}{}^{i'} (\mathcal{M}_+)_{j'}{}^{j'} \mathcal{T}_{i'j'k}^\mu \\
& \left. + t_3^M (\bar{\mathcal{T}}_\mu \mathcal{T}^\mu) \text{tr}(\mathcal{M}_+ \mathcal{M}_+) + t_4^M (\bar{\mathcal{T}}_\mu \mathcal{M}_+ \mathcal{T}^\mu) \text{tr}(\mathcal{M}_+) + t_5^M (\bar{\mathcal{T}}_\mu \mathcal{T}^\mu) \text{tr}(\mathcal{M}_+) \text{tr}(\mathcal{M}_+) \right]. \tag{48}
\end{aligned}$$

The coefficients of these operators t_i^A , $t_i^{\bar{A}}$, t_i^{vA} , and t_i^M have the same numerical values as in χ PT.

V. DECUPLET MASSES IN PQ χ PT.

Only the masses of the octet baryons have been investigated in PQ χ PT [22, 33]. Here we calculate the masses of the decuplet baryons to NNLO in PQ χ PT. The chiral expansion

¹ We have omitted three PQ χ PT operators of the form $\bar{\mathcal{T}}^\mu [A_\mu, A_\nu] \mathcal{T}^\nu$ and three of the form $\bar{\mathcal{T}}^\mu [A_\nu, A_\rho] S^\nu S^\rho \mathcal{T}_\mu$ since their contributions to the decuplet masses vanish to the order we are working. Such operators identically vanish in χ PT.

TABLE VIII: The coefficients A_ϕ^T and $A_{\phi\phi'}^T$ in PQ χ PT. Coefficients are listed for the decuplet states T , and for A_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $A_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons.

	A_ϕ^T							$A_{\phi\phi'}^T$		
	π	K	η_s	ju	ru	js	rs	$\eta_u\eta_u$	$\eta_u\eta_s$	$\eta_s\eta_s$
Δ	$\frac{2}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0	1	0	0
Σ^*	$\frac{2}{9}$	$\frac{4}{9}$	0	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$
Ξ^*	0	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$
Ω^-	0	0	$\frac{2}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0	1

TABLE IX: The coefficients B_ϕ^T and $B_{\phi\phi'}^T$ in PQ χ PT. Coefficients are listed for the decuplet states T , and for B_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $B_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons.

	B_ϕ^T							$B_{\phi\phi'}^T$		
	π	K	η_s	ju	ru	js	rs	$\eta_u\eta_u$	$\eta_u\eta_s$	$\eta_s\eta_s$
Δ	$-\frac{2}{3}$	0	0	$\frac{4}{3}$	$\frac{2}{3}$	0	0	0	0	0
Σ^*	$-\frac{2}{9}$	$-\frac{4}{9}$	0	$\frac{8}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$-\frac{4}{9}$	$\frac{2}{9}$
Ξ^*	0	$-\frac{4}{9}$	$-\frac{2}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{8}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$-\frac{4}{9}$	$\frac{2}{9}$
Ω^-	0	0	$-\frac{2}{3}$	0	0	$\frac{4}{3}$	$\frac{2}{3}$	0	0	0

of the decuplet baryon masses in PQ χ PT has the same form as in χ PT.

$$M_{T_i} = M_0(\mu) + M_{T_i}^{(1)}(\mu) + M_{T_i}^{(3/2)}(\mu) + M_{T_i}^{(2)}(\mu) + \dots \quad (49)$$

The contributions at each order are similar to those in Section III. However, we must also include hairpin contributions from the flavor diagonal propagator, see Fig. 3 and Fig. 4.

To leading order in PQ χ PT the decuplet masses are

$$M_T^{(1)} = \frac{2}{3}\gamma_M m_T - 2\bar{\sigma}_M \text{str}(m_Q), \quad (50)$$

where the coefficients m_T appear in Table I. At next-to-leading order, we have

$$M_T^{(3/2)} = -\frac{5\mathcal{H}^2}{72\pi f^2} \left[\sum_\phi A_\phi^T m_\phi^3 + \sum_{\phi\phi'} A_{\phi\phi'}^T \mathcal{M}^3(m_\phi, m_{\phi'}) \right] - \frac{\mathcal{C}^2}{(4\pi f)^2} \left[\sum_\phi B_\phi^T \mathcal{F}(m_\phi, -\Delta, \mu) + \sum_{\phi\phi'} B_{\phi\phi'}^T \mathcal{F}(m_\phi, m_{\phi'}, -\Delta, \mu) \right], \quad (51)$$

where $\mathcal{F}(m, \Delta, \mu)$ is given by Eq. (24). Additionally we have employed the abbreviations

$$\mathcal{M}^n(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}(m_\phi^n, m_{\phi'}^n, m_X^n), \quad (52)$$

$$\mathcal{F}(m_\phi, m_{\phi'}, \delta, \mu) = \mathcal{H}_{\phi\phi'}[\mathcal{F}(m_\phi, \delta, \mu), \mathcal{F}(m_{\phi'}, \delta, \mu), \mathcal{F}(m_X, \delta, \mu)], \quad (53)$$

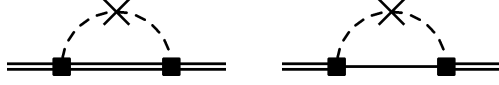


FIG. 3: In addition to the one-loop diagrams in Fig. 1, $M_{T_i}^{(3/2)}$ also receives contributions from the singlet (hairpins) in PQ χ PT. Single and double line correspond to **240**-baryons and **138**-baryons respectively. The crossed dashed line denotes a hairpin propagator. The filled squares denote the axial coupling given in Eq. (46).

for contributions arising from the hairpin diagrams depicted in Figure 3. The sums involving these two functions are over pairs of flavor neutral states in the quark basis, e.g., above $\phi\phi'$ runs over $\eta_u\eta_u$, $\eta_u\eta_s$, and $\eta_s\eta_s$. In this way there is no double counting. The coefficients A_ϕ^T and $A_{\phi\phi'}^T$ appear in Table VIII and B_ϕ^T and $B_{\phi\phi'}^T$ appear in Table IX.

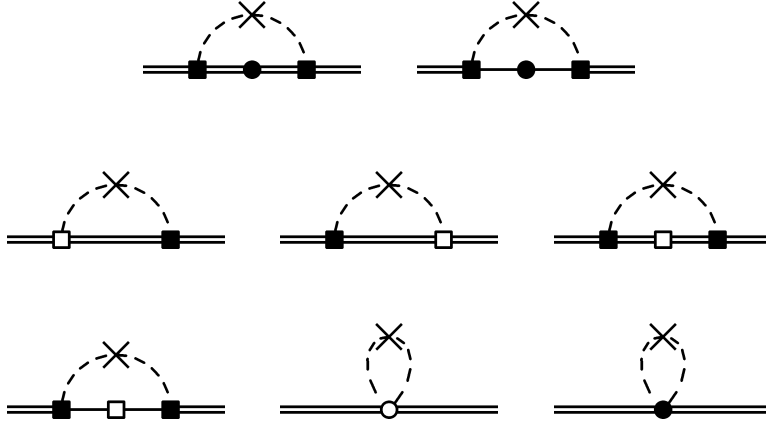


FIG. 4: In addition to the one-loop diagrams in Fig. 2, $M_{T_i}^2$ also receives contributions from the singlet (hairpins) in PQ χ PT. Single and double lines correspond to **240**-baryons and **138**-baryons respectively. The crossed dashed line denotes a hairpin insertion. Filled squares denote the axial coupling given in Eq. (46). Filled circles denote a coupling to \mathcal{M}_+ given in Eq. (46). Empty squares and empty circles denote insertions of fixed $1/M_B$ operators given in Eq. (47), and insertions of a two-baryon-two-axial coupling given in Eq. (48) respectively.

At next-to-next-to leading order, we have the contribution to the decuplet mass

$$\begin{aligned}
M_T^{(2)} = & (Z - 1)M_T^{(1)} \\
& + \frac{1}{4\pi f} \left\{ \frac{1}{3}t_1^M (m^2)_T + \frac{1}{3}t_2^M (mm')_T + t_3^M \text{str}(m_Q^2) + \frac{1}{3}t_4^M m_T \text{str}(m_Q) + t_5^M [\text{str}(m_Q)]^2 \right\} \\
& - \frac{2\gamma_M}{(4\pi f)^2} \left[\sum_{\phi} C_{\phi}^T \mathcal{L}(m_{\phi}, \mu) + \sum_{\phi\phi'} C_{\phi\phi'}^T \mathcal{L}(m_{\phi}, m_{\phi'}, \mu) \right] \\
& + \frac{2\bar{\sigma}_M}{(4\pi f)^2} \left[\sum_{\phi} \bar{C}_{\phi} \mathcal{L}(m_{\phi}, \mu) + \sum_{\phi\phi'} \bar{C}_{\phi\phi'} \mathcal{L}(m_{\phi}, m_{\phi'}, \mu) \right] \\
& + \frac{1}{(4\pi f)^3} \sum_{\phi} (t_1^A D_{\phi}^T + t_2^A E_{\phi}^T + t_3^A \bar{D}_{\phi}) \bar{\mathcal{L}}(m_{\phi}, \mu) \\
& + \frac{1}{(4\pi f)^3} \sum_{\phi\phi'} (t_1^A D_{\phi\phi'}^T + t_2^A E_{\phi\phi'}^T + t_3^A \bar{D}_{\phi\phi'}) \bar{\mathcal{L}}(m_{\phi}, m_{\phi'}, \mu) \\
& + \frac{1}{4(4\pi f)^3} \sum_{\phi} \left[(t_1^{\bar{A}} + t_1^{vA}) D_{\phi}^T + (t_2^{\bar{A}} + t_2^{vA}) E_{\phi}^T + (t_3^{\bar{A}} + t_3^{vA}) \bar{D}_{\phi} \right] \left[\bar{\mathcal{L}}(m_{\phi}, \mu) - \frac{1}{2}m_{\phi}^4 \right] \\
& + \frac{1}{4(4\pi f)^3} \sum_{\phi\phi'} \left[(t_1^{\bar{A}} + t_1^{vA}) D_{\phi\phi'}^T + (t_2^{\bar{A}} + t_2^{vA}) E_{\phi\phi'}^T + (t_3^{\bar{A}} + t_3^{vA}) \bar{D}_{\phi\phi'} \right] \\
& \quad \times \left[\bar{\mathcal{L}}(m_{\phi}, m_{\phi'}, \mu) - \frac{1}{2}\mathcal{M}^4(m_{\phi}, m_{\phi'}) \right] \\
& - \frac{5}{8} \frac{\mathcal{H}^2}{(4\pi f)^2 M_B} \left\{ \sum_{\phi} A_{\phi}^T \left[\bar{\mathcal{L}}(m_{\phi}, \mu) + \frac{19}{10}m_{\phi}^4 \right] + \sum_{\phi\phi'} A_{\phi\phi'}^T \left[\bar{\mathcal{L}}(m_{\phi}, m_{\phi'}, \mu) + \frac{19}{10}\mathcal{M}^4(m_{\phi}, m_{\phi'}) \right] \right\} \\
& - \frac{15}{16} \frac{\mathcal{C}^2}{(4\pi f)^2 M_B} \left\{ \sum_{\phi} B_{\phi}^T \left[\bar{\mathcal{L}}(m_{\phi}, \mu) - \frac{1}{10}m_{\phi}^4 \right] + \sum_{\phi\phi'} B_{\phi\phi'}^T \left[\bar{\mathcal{L}}(m_{\phi}, m_{\phi'}, \mu) - \frac{1}{10}\mathcal{M}^4(m_{\phi}, m_{\phi'}) \right] \right\} \\
& - \frac{10\mathcal{H}^2\bar{\sigma}_M \text{str}(m_Q)}{3(4\pi f)^2} \left\{ \sum_{\phi} A_{\phi}^T \left[\mathcal{L}(m_{\phi}, \mu) + \frac{26}{15}m_{\phi}^2 \right] + \sum_{\phi\phi'} A_{\phi\phi'}^T \left[\mathcal{L}(m_{\phi}, m_{\phi'}, \mu) + \frac{26}{15}\mathcal{M}^2(m_{\phi}, m_{\phi'}) \right] \right\} \\
& - \frac{3\mathcal{C}^2\sigma_M \text{str}(m_Q)}{(4\pi f)^2} \left[\sum_{\phi} B_{\phi}^T \mathcal{J}(m_{\phi}, -\Delta, \mu) + \sum_{\phi\phi'} B_{\phi\phi'}^T \mathcal{J}(m_{\phi}, m_{\phi'}, -\Delta, \mu) \right] \\
& + \frac{10\mathcal{H}^2\gamma_M}{9(4\pi f)^2} \left\{ \sum_{\phi} F_{\phi}^T \left[\mathcal{L}(m_{\phi}, \mu) + \frac{26}{15}m_{\phi}^2 \right] + \sum_{\phi\phi'} F_{\phi\phi'}^T \left[\mathcal{L}(m_{\phi}, m_{\phi'}, \mu) + \frac{26}{15}\mathcal{M}^2(m_{\phi}, m_{\phi'}) \right] \right\} \\
& - \frac{3\mathcal{C}^2}{2(4\pi f)^2} \left[\sum_{\phi} G_{\phi}^T \mathcal{J}(m_{\phi}, -\Delta, \mu) + \sum_{\phi\phi'} G_{\phi\phi'}^T \mathcal{J}(m_{\phi}, m_{\phi'}, -\Delta, \mu) \right]. \tag{54}
\end{aligned}$$

The above expression is written quite compactly. It involves the tree-level coefficients m_T , $(m^2)_T$, and $(mm')_T$, which are listed in Table I, and the wavefunction renormalization Z ,

TABLE X: The coefficients C_ϕ^T , $C_{\phi\phi'}^T$, \overline{C}_ϕ , and $\overline{C}_{\phi\phi'}$ in PQ χ PT. The coefficients \overline{C}_ϕ and $\overline{C}_{\phi\phi'}$ are identical for all decuplet states. The remaining coefficients are listed for the decuplet states T , and for C_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $C_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons. If a particular meson or pair of flavor neutral mesons is not listed, then the value of the coefficient is zero for all decuplet states.

	C_ϕ^T				$C_{\phi\phi'}^T$	
	ju	ru	js	rs	$\eta_u\eta_u$	$\eta_s\eta_s$
Δ	$2(m_u + m_j)$	$m_u + m_r$	0	0	$2m_u$	0
Σ^*	$\frac{4}{3}(m_u + m_j)$	$\frac{2}{3}(m_u + m_r)$	$\frac{2}{3}(m_s + m_j)$	$\frac{1}{3}(m_s + m_r)$	$\frac{4}{3}m_u$	$\frac{2}{3}m_s$
Ξ^*	$\frac{2}{3}(m_u + m_j)$	$\frac{1}{3}(m_u + m_r)$	$\frac{4}{3}(m_s + m_j)$	$\frac{2}{3}(m_s + m_r)$	$\frac{2}{3}m_u$	$\frac{4}{3}m_s$
Ω^-	0	0	$2(m_s + m_j)$	$m_s + m_r$	0	$2m_s$
	jj	jr	rr		$\eta_j\eta_j$	$\eta_r\eta_r$
\overline{C}_ϕ	$8m_j$	$4(m_j + m_r)$	$2m_r$	$\overline{C}_{\phi\phi'}$	$4m_j$	$2m_r$

which we find to be

$$\begin{aligned}
Z - 1 = & -\frac{5\mathcal{H}^2}{3(4\pi f)^2} \left\{ \sum_\phi A_\phi^T \left[\mathcal{L}(m_\phi, \mu) + \frac{26}{15}m_\phi^2 \right] + \sum_{\phi\phi'} A_{\phi\phi'}^T \left[\mathcal{L}(m_\phi, m_{\phi'}, \mu) + \frac{26}{15}\mathcal{M}^2(m_\phi, m_{\phi'}) \right] \right\} \\
& -\frac{3\mathcal{C}^2}{2(4\pi f)^2} \left[\sum_\phi B_\phi^T \mathcal{J}(m_\phi, -\Delta, \mu) + \sum_{\phi\phi'} B_{\phi\phi'}^T \mathcal{J}(m_\phi, m_{\phi'}, -\Delta, \mu) \right]. \tag{55}
\end{aligned}$$

The new non-analytic functions arising from loop integrals in Eqs. (54) and (55) are defined to be

$$\mathcal{L}(m_\phi, m_{\phi'}, \mu) = \mathcal{H}_{\phi\phi'}[\mathcal{L}(m_\phi, \mu), \mathcal{L}(m_{\phi'}, \mu), \mathcal{L}(m_X, \mu)], \tag{56}$$

$$\overline{\mathcal{L}}(m_\phi, m_{\phi'}, \mu) = \mathcal{H}_{\phi\phi'}[\overline{\mathcal{L}}(m_\phi, \mu), \overline{\mathcal{L}}(m_{\phi'}, \mu), \overline{\mathcal{L}}(m_X, \mu)], \tag{57}$$

$$\mathcal{J}(m_\phi, m_{\phi'}, \delta, \mu) = \mathcal{H}_{\phi\phi'}[\mathcal{J}(m_\phi, \delta, \mu), \mathcal{J}(m_{\phi'}, \delta, \mu), \mathcal{J}(m_X, \delta, \mu)] \tag{58}$$

and arise from the hairpin contributions shown in Figure 4. The various coefficients in the above sums are listed in Tables VIII–XIV. One may check that in the limit $m_j \rightarrow m_u$, $m_r \rightarrow m_s$ the χ PT results are obtained from the PQ χ PT expressions.

As in the χ PT case, the above expressions Eqs. (50), (53), and (54) are quark mass expressions, i.e., the meson masses in these expressions are merely replacements for the quark masses via Eq. (4). To utilize these expressions for chiral extrapolations of lattice QCD data, one must write the quark masses in terms of the lattice meson masses. To work to $\mathcal{O}(m_q^2)$ means the LO contributions in Eq. (50) must be written in meson masses out to NLO.

VI. SUMMARY

We have calculated the masses of the decuplet baryons in the isospin limit of three-flavor χ PT and have also derived the decuplet masses in the analogous partially quenched theory.

TABLE XI: The coefficients D_ϕ^T , $D_{\phi\phi'}^T$, \bar{D}_ϕ , and $\bar{D}_{\phi\phi'}$ in PQ χ PT. The coefficients \bar{D}_ϕ and $\bar{D}_{\phi\phi'}$ are identical for all decuplet states. The remaining coefficients are listed for the decuplet states T , and for D_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $D_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons. If a particular meson or pair of flavor neutral mesons is not listed, then the value of the coefficient is zero for all decuplet states.

	D_ϕ^T				$D_{\phi\phi'}^T$	
	ju	ru	js	rs	$\eta_u\eta_u$	$\eta_s\eta_s$
Δ	2	1	0	0	1	0
Σ^*	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
Ξ^*	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
Ω^-	0	0	2	1	0	1

\bar{D}_ϕ	jj	jr	rr	$\bar{D}_{\phi\phi'}$	$\eta_j\eta_j$	$\eta_r\eta_r$
	4	4	1			2

TABLE XII: The coefficients E_ϕ^T and $E_{\phi\phi'}^T$ in PQ χ PT. Coefficients are listed for the decuplet states T , and for E_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $E_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons.

	E_ϕ^T			$E_{\phi\phi'}^T$		
	π	K	η_s	$\eta_u\eta_u$	$\eta_u\eta_s$	$\eta_s\eta_s$
Δ	1	0	0	1	0	0
Σ^*	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	0
Ξ^*	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$
Ω^-	0	0	1	0	0	1

We have kept all relevant terms to $\mathcal{O}(m_q^2)$, including terms fixed by reparameterization invariance that ensure the Lorentz invariance of the heavy baryon effective theory. Knowledge of the low-energy behavior of QCD and PQQCD is crucial to properly extrapolate lattice calculations from the light quark masses used on the lattice to those in nature.

Working to $\mathcal{O}(m_q^2)$ in the chiral expansion forces the introduction of a large number of low-energy constants. For any predictive power, the low-energy constants must be fit from experiment or lattice results. Lattice calculations will eventually allow first principles determination of these constants and thus predictions from QCD. In the foreseeable future, partially quenched simulations will enable these rigorous predictions. Our PQ χ PT results for decuplet baryon masses are required for the proper extrapolation of PQQCD lattice data and hence for physical predictions from QCD. As the decuplet baryons are resonances, a short procedural comment is in order. For large enough pion masses, the decuplet states are stable to strong decays and can be studied on the lattice, *cf.* our expressions are all real valued for these pion masses. Thus for $m_\pi \sim 300 \text{ MeV}$,² where one still trusts the chiral expansion, the decuplet properties can be calculated on the lattice. One then uses the expressions we have derived to fit the low-energy constants. The expressions at next-to-next-

² This is a conservative estimate. Volume effects will modify the decay threshold of the decuplet.

TABLE XIII: The coefficients F_ϕ^T and $F_{\phi\phi'}^T$ in PQ χ PT. Coefficients are listed for the decuplet states T , and for F_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $F_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons.

	F_ϕ^T			
	π	K	η_s	
Δ	$2m_u$	0	0	
Σ^*	$\frac{2}{9}(2m_u + m_s)$	$\frac{4}{9}(2m_u + m_s)$	0	
Ξ^*	0	$\frac{4}{9}(m_u + 2m_s)$	$\frac{2}{9}(m_u + 2m_s)$	
Ω^-	0	0	$2m_s$	

	ju	ru	js	rs
	Δ	$\frac{2}{3}(2m_u + m_j)$	$\frac{1}{3}(2m_u + m_r)$	0
Σ^*	$\frac{4}{9}(m_u + m_s + m_j)$	$\frac{2}{9}(m_u + m_s + m_r)$	$\frac{2}{9}(2m_u + m_j)$	$\frac{1}{9}(2m_u + m_r)$
Ξ^*	$\frac{2}{9}(2m_s + m_j)$	$\frac{1}{9}(2m_s + m_r)$	$\frac{4}{9}(m_u + m_s + m_j)$	$\frac{2}{9}(m_u + m_s + m_r)$
Ω^-	0	0	$\frac{2}{3}(2m_s + m_j)$	$\frac{1}{3}(2m_s + m_r)$

	$F_{\phi\phi'}^T$		
	$\eta_u\eta_u$	$\eta_u\eta_s$	$\eta_s\eta_s$
Δ	$3m_u$	0	0
Σ^*	$\frac{4}{9}(2m_u + m_s)$	$\frac{4}{9}(2m_u + m_s)$	$\frac{1}{9}(2m_u + m_s)$
Ξ^*	$\frac{1}{9}(m_u + 2m_s)$	$\frac{4}{9}(m_u + 2m_s)$	$\frac{4}{9}(m_u + 2m_s)$
Ω^-	0	0	$3m_s$

to-leading order are necessary for reducing uncertainty in the chiral extrapolation. Armed with the low-energy constants, one can then make predictions for the decuplet resonances, e.g. their decay widths can be found via

$$\Im m(M_T) = -\frac{\mathcal{C}^2}{8\pi f^2}(\Delta^2 - m_\pi^2)^{3/2} B_\pi^T \quad (59)$$

$$- \frac{\mathcal{C}^2}{4\pi f^2} \Delta \sqrt{\Delta^2 - m_\pi^2} \left\{ [\gamma_M m_T + 3(\sigma_M - \bar{\sigma}_M) \text{tr}(m_Q)] B_\pi^T + \frac{3}{2} G_\pi^T \right\}. \quad (60)$$

To reiterate: the low-energy constants appearing in χ PT appear in PQ χ PT and by fitting them using PQQCD lattice calculations one can make QCD predictions. The decuplet baryon masses are no exception; our PQ χ PT results exhibit a smooth limit to χ PT as the sea quark masses vary. Furthermore, lattice simulations with unphysically stable decuplet states can be used in conjunction with χ PT and PQ χ PT to predict properties of the physical resonances.

Acknowledgments

We thank Martin Savage for many useful discussions and David Lin for comments on the manuscript. This work was supported in part by the U.S. Department of Energy under

TABLE XIV: The coefficients G_ϕ^T and $G_{\phi\phi'}^T$ in PQ χ PT. Coefficients are listed for the decuplet states T , and for G_ϕ^T are grouped into contributions from loop mesons with mass m_ϕ , while for $G_{\phi\phi'}^T$ are grouped into contributions from pairs of quark-basis η_q mesons.

	G_ϕ^T		
	π	K	η_s
Δ	$-\frac{4}{3}m_u(\alpha_M + \beta_M)$	0	0
Σ^*	$-\frac{4}{27}[m_u(\alpha_M + 4\beta_M) + m_s(2\alpha_M - \beta_M)]$	$-\frac{4}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$	0
Ξ^*	0	$-\frac{4}{27}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	$-\frac{4}{27}[m_u(2\alpha_M - \beta_M) + m_s(\alpha_M + 4\beta_M)]$
Ω^-	0	0	$-\frac{4}{3}m_s(\alpha_M + \beta_M)$

	ju	ru	js	rs
	Δ	$\frac{4}{9}[m_u(5\alpha_M + 2\beta_M) + m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{9}[m_u(5\alpha_M + 2\beta_M) + m_r(\alpha_M + 4\beta_M)]$	0
Σ^*	$\frac{4}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(5\alpha_M + 2\beta_M) + 2m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(5\alpha_M + 2\beta_M) + 2m_r(\alpha_M + 4\beta_M)]$	$\frac{4}{27}[m_u(5\alpha_M + 2\beta_M) + m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{27}[m_u(5\alpha_M + 2\beta_M) + m_r(\alpha_M + 4\beta_M)]$
Ξ^*	$\frac{4}{27}[m_s(5\alpha_M + 2\beta_M) + m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{27}[m_s(5\alpha_M + 2\beta_M) + m_r(\alpha_M + 4\beta_M)]$	$\frac{4}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(5\alpha_M + 2\beta_M) + 2m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(5\alpha_M + 2\beta_M) + 2m_r(\alpha_M + 4\beta_M)]$
Ω^-	0	0	$\frac{4}{9}[m_s(5\alpha_M + 2\beta_M) + m_j(\alpha_M + 4\beta_M)]$	$\frac{2}{9}[m_s(5\alpha_M + 2\beta_M) + m_r(\alpha_M + 4\beta_M)]$

	$G_{\phi\phi'}^T$		
	$\eta_u\eta_u$	$\eta_u\eta_s$	$\eta_s\eta_s$
Δ	0	0	0
Σ^*	$\frac{2}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$	$-\frac{4}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$	$\frac{2}{27}[m_u(5\alpha_M + 2\beta_M) + m_s(\alpha_M + 4\beta_M)]$
Ξ^*	$\frac{2}{27}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	$-\frac{4}{27}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$	$\frac{2}{27}[m_u(\alpha_M + 4\beta_M) + m_s(5\alpha_M + 2\beta_M)]$
Ω^-	0	0	0

Grant No. DE-FG03-97ER4014.

-
- [1] S. Aoki et al. (CP-PACS), Phys. Rev. Lett. **84**, 238 (2000), hep-lat/9904012.
 - [2] A. Ali Khan et al. (CP-PACS), Phys. Rev. **D65**, 054505 (2002), hep-lat/0105015.
 - [3] C. R. Allton et al. (UKQCD), Phys. Rev. **D65**, 054502 (2002), hep-lat/0107021.
 - [4] S. Aoki et al. (JLQCD), Phys. Rev. **D68**, 054502 (2003), hep-lat/0212039.
 - [5] J. M. Zanotti et al. (CSSM Lattice), Phys. Rev. **D68**, 054506 (2003), hep-lat/0304001.
 - [6] A. Morel, J. Phys. (France) **48**, 1111 (1987).
 - [7] S. R. Sharpe, Phys. Rev. **D46**, 3146 (1992), hep-lat/9205020.
 - [8] C. W. Bernard and M. Golterman, Nucl. Phys. Proc. Suppl. **26**, 360 (1992).
 - [9] C. W. Bernard and M. F. L. Golterman, Phys. Rev. **D46**, 853 (1992), hep-lat/9204007.
 - [10] M. F. L. Golterman, Acta Phys. Polon. **B25**, 1731 (1994), hep-lat/9411005.
 - [11] S. R. Sharpe and Y. Zhang, Phys. Rev. **D53**, 5125 (1996), hep-lat/9510037.
 - [12] J. N. Labrenz and S. R. Sharpe, Phys. Rev. **D54**, 4595 (1996), hep-lat/9605034.

- [13] C. W. Bernard and M. F. L. Golterman, Phys. Rev. **D49**, 486 (1994), hep-lat/9306005.
- [14] S. R. Sharpe, Phys. Rev. **D56**, 7052 (1997), hep-lat/9707018.
- [15] M. F. L. Golterman and K.-C. Leung, Phys. Rev. **D57**, 5703 (1998), hep-lat/9711033.
- [16] S. R. Sharpe and N. Shoresh, Nucl. Phys. Proc. Suppl. **83**, 968 (2000), hep-lat/9909090.
- [17] S. R. Sharpe and N. Shoresh, Int. J. Mod. Phys. **A16S1C**, 1219 (2001), hep-lat/0011089.
- [18] S. R. Sharpe and N. Shoresh, Phys. Rev. **D62**, 094503 (2000), hep-lat/0006017.
- [19] S. R. Sharpe and N. Shoresh, Phys. Rev. **D64**, 114510 (2001), hep-lat/0108003.
- [20] N. Shoresh (2001), Ph.D. thesis, University of Washington, UMI-30-36529.
- [21] S. R. Sharpe and R. S. Van de Water, Phys. Rev. **D69**, 054027 (2004), hep-lat/0310012.
- [22] J.-W. Chen and M. J. Savage, Phys. Rev. **D65**, 094001 (2002), hep-lat/0111050.
- [23] S. R. Beane and M. J. Savage, Nucl. Phys. **A709**, 319 (2002), hep-lat/0203003.
- [24] J.-W. Chen and M. J. Savage, Phys. Rev. **D66**, 074509 (2002), hep-lat/0207022.
- [25] S. R. Beane and M. J. Savage, Phys. Rev. **D67**, 054502 (2003), hep-lat/0210046.
- [26] D. B. Leinweber, Phys. Rev. **D69**, 014005 (2004), hep-lat/0211017.
- [27] D. Arndt, S. R. Beane, and M. J. Savage, Nucl. Phys. **A726**, 339 (2003), nucl-th/0304004.
- [28] S. R. Beane and M. J. Savage, Phys. Rev. **D68**, 114502 (2003), hep-lat/0306036.
- [29] D. Arndt and B. C. Tiburzi, Phys. Rev. **D68**, 094501 (2003), hep-lat/0307003.
- [30] D. Arndt and B. C. Tiburzi, Phys. Rev. **D68**, 114503 (2003), hep-lat/0308001.
- [31] D. Arndt and B. C. Tiburzi, Phys. Rev. **D69**, 014501 (2004), hep-lat/0309013.
- [32] D. Arndt Ph.D. thesis, University of Washington (2004), hep-lat/0406011.
- [33] A. Walker-Loud (2004), hep-lat/0405007.
- [34] R. D. Young, D. B. Leinweber, A. W. Thomas, and S. V. Wright, Phys. Rev. **D66**, 094507 (2002), hep-lat/0205017.
- [35] A. Ali Khan et al. (QCDSF-UKQCD), Nucl. Phys. **B689**, 175 (2004), hep-lat/0312030.
- [36] S. R. Beane, Nucl. Phys. **B695**, 192 (2004), hep-lat/0403030.
- [37] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
- [38] E. Jenkins and A. V. Manohar (1991), talk presented at the Workshop on Effective Field Theories of the Standard Model, Dobogoko, Hungary, Aug 1991.
- [39] E. Jenkins and A. V. Manohar, Phys. Lett. **B255**, 558 (1991).
- [40] E. Jenkins and A. V. Manohar, Phys. Lett. **B259**, 353 (1991).
- [41] E. Jenkins, Nucl. Phys. **B368**, 190 (1992).
- [42] M. E. Luke and A. V. Manohar, Phys. Lett. **B286**, 348 (1992), hep-ph/9205228.
- [43] A. V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1 (2000).
- [44] T. R. Hemmert, B. R. Holstein, and J. Kambor, J. Phys. **G24**, 1831 (1998), hep-ph/9712496.
- [45] R. F. Lebed, Nucl. Phys. **B430**, 295 (1994), hep-ph/9311234.
- [46] R. F. Lebed and M. A. Luty, Phys. Lett. **B329**, 479 (1994), hep-ph/9401232.
- [47] M. K. Banerjee and J. Milana, Phys. Rev. **D52**, 6451 (1995), hep-ph/9410398.
- [48] B. Borasoy and U.-G. Meissner, Annals Phys. **254**, 192 (1997), hep-ph/9607432.
- [49] M. Frink and U.-G. Meissner, JHEP **07**, 028 (2004), hep-lat/0404018.