Semileptonic B Decays with $N_f = 2 + 1$ Dynamical Quarks

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Semileptonic, $B \to \pi l \overline{\nu}$, decays are studied on the MILC dynamical configurations using NRQCD heavy and Asqtad light quarks. We work with light valence quark masses ranging between m_s and $m_s/8$. Preliminary simple linear chiral extrapolations have been carried out for form factors f_{\parallel} and f_{\perp} at fixed E_{π} . The chirally extrapolated results for the form factors $f_{+}(q^2)$ and $f_0(q^2)$ are then fit to the Becirevic-Kaidalov (BK) ansatz. Preliminary estimates of the CKM matrix element $|V_{ub}|$ are presented based on recently published branching fractions for $B^0 \to \pi^- l^+ \nu$ exclusive decays by the CLEO collaboration.

1. Introduction

First principles calculations of B meson semileptonic decay form factors are crucial for determining the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$. Recent progress on the lattice towards this goal comes from two major developments: the ability to go beyond the quenched approximation with close to realistic dynamical quark content [1,2] and the use of improved staggered light quarks in heavy-light simulations [3]. We report here on unquenched studies of $B \to \pi$, $l\overline{\nu}$ decays on the lattice using one of the coarse MILC $N_f = 2 + 1$ dynamical sets [1], NRQCD b quarks and improved staggered (Asqtad) light quarks. The light dynamical quark mass is fixed at $m_{dyn} = m_s/4$ and we vary the light valence quark mass between m_s and $m_s/8$.

2. Form Factors

Semileptonic form factors parameterise the hadronic matrix elements of electroweak currents between a B meson and a π or a ρ . In particular,

one has

$$\langle \pi | V^{\mu} | B \rangle = f_{+}(q^{2}) \left[p_{B}^{\mu} + p_{\pi}^{\mu} - \frac{M_{B}^{2} - m_{\pi}^{2}}{q^{2}} q^{\mu} \right]$$

$$+ f_{0}(q^{2}) \frac{M_{B}^{2} - m_{\pi}^{2}}{q^{2}} q^{\mu} \qquad (1)$$

$$= \sqrt{2M_{B}} \left[v^{\mu} f_{\parallel} + p_{\perp}^{\mu} f_{\perp} \right] \qquad (2)$$

with $v^{\mu} = \frac{p_B^{\mu}}{M_B}, \ p_{\perp}^{\mu} = p_{\pi}^{\mu} - (p_{\pi} \cdot v) \, v^{\mu}, \ q^{\mu} = p_B^{\mu} - p_{\pi}^{\mu}.$

A lattice calculation of the relevant matrix element starts with the three-point correlator

$$C^{(3)}(\vec{p}_{\pi}, \vec{p}_{B}, t, T_{B}) =$$

$$\sum_{\vec{z}} \sum_{\vec{y}} \langle \Phi_{\pi}(0) V_{lat}^{\mu}(\vec{z}, t) \Phi_{B}^{\dagger}(\vec{y}, T_{B}) \rangle$$

$$\times e^{i\vec{p}_{B} \cdot \vec{y}} e^{i(\vec{p}_{\pi} - \vec{p}_{B}) \cdot \vec{z}}$$

$$(3)$$

where Φ_{π} and Φ_{B} are interpolating operators for the π and B mesons respectively. The three-point correlator is fit to the form

$$C^{(3)}(\vec{p}_{\pi}, \vec{p}_{B}, t, T_{B}) \rightarrow$$

$$\sum_{k=0}^{N_{\pi}-1} \sum_{j=0}^{N_{B}-1} (-1)^{k*(t-1)} (-1)^{j*(T_{B}-t)}$$

$$\times A_{j,k} e^{-E_{\pi}^{(k)}(t-1)} e^{-E_{B}^{(j)}(T_{B}-t)}$$
(4)

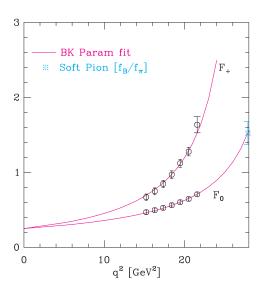


Figure 1. The form factors $f_+(q^2)$ and $f_0(q^2)$, extrapolated to the physical pion. Only statistical errors are shown. The full curves correspond to a Becirevic-Kaidalov (BK) parametrization fit to the data (see text).

and the goal is to extract the ground state contribution A_{00} . Working in the B meson rest frame, the ground state amplitudes $A_{00}(V^{\mu})$ are related to the form factors f_{\parallel} and f_{\perp} in a simple way.

$$f_{\parallel} = \frac{A_{00}(V^0)}{\sqrt{\zeta_{\pi}^{(0)}\zeta_B^{(0)}}} \sqrt{2E_{\pi}} Z_{V_0}$$
 (5)

$$f_{\perp} = \frac{A_{00}(V^k)}{\sqrt{\zeta_{\pi}^{(0)}\zeta_B^{(0)}} p_{\pi}^k} \sqrt{2E_{\pi}} Z_{V_k}$$
 (6)

where $\zeta_{\pi,B}^{(0)}$ are the groundstate amplitudes of the π and B two-point correlators respectively and $Z_{V_{\mu}}$ are the matching factors between the lattice heavy-light current V_{lat}^{μ} and the current in the continuum theory. We use one-loop estimates for these matching factors [4].

3. The Form Factors $f_+(q^2)$ and $f_0(q^2)$ at the Physical Pion

Although our simulations have been carried out with light quark masses as low as $m_s/8$ one still needs to extrapolate the form factors, determined above, to the physical pion. To date, we have only carried out simple linear chiral extrapolations. We first interpolate f_{\parallel} and f_{\perp} to common values of E_{π} , the pion energy in the B rest frame. These are then extrapolated linearly to the physical pion for several fixed values of E_{π} .

From the chirally extrapolated f_{\parallel} and f_{\perp} one obtains the form factors $f_{+}(q^2)$ and $f_0(q^2)$ at the physical pion. This is shown as the circles in Fig.1. Our results are currently limited to the $q^2 \geq 15 \, GeV^2$ region. Recently, a very promising approach to low q^2 form factors has been developed, namely "Moving NRQCD", which will allow us to overcome this limitation [5,6]. In the mean time, however, we will rely on a model ansatz to extend our form factor results into the low q^2 regime. Specifically, we employ an ansatz introduced by Becirevic & Kaidalov (BK) [7],

$$f_{+}(q^{2}) = \frac{C_{B}(1-\alpha_{B})}{(1-\tilde{q}^{2})(1-\alpha_{B}\tilde{q}^{2})}$$
 (7)

$$f_0(q^2) = \frac{C_B \left(1 - \alpha_B\right)}{\left(1 - \tilde{q}^2 / \beta_B\right)} \tag{8}$$

 $(\tilde{q}^2 \equiv q^2/M_{B^*}^2)$. This ansatz satisfies the kinematic constraint $f_+(0) = f_0(0)$, HQET scaling laws and the requirement of a pole in $f_+(q^2)$ at $q^2 = M_{B^*}^2$. We find an excellent fit to this BK ansatz using the physical M_{B^*} mass and this is shown as the full curves in Fig.1 (a satisfactory BK parametrization was not possible before the chiral extrapolation). The fit parameters are

$$C_B = 0.42(3), \quad \alpha_B = 0.41(7), \quad \beta_B = 1.18(5),$$

which translates into

$$f_0(0) = f_+(0) = 0.251(15)$$

and an effective pole in $f_0(q^2)$ at $q^2 = (M_{f0}^{pole})^2 = 33.35(1.36) \text{GeV}^2$. Both $f_{0,+}(0)$ and M_{f0}^{pole} are in good agreement with a recent semileptonic B decay analysis based on Sum Rules [8]. The data

points in Fig.1 and the $f_{0,+}(0)$ quoted above include only statistical and fitting errors. Further systematic errors are discussed in the next section. A comparison of our new dynamical form factor results with old quenched data is given, for instance, in reference [9].

4. Estimating $|V_{ub}|$

The CLEO collaboration has published branching fractions for exclusive semileptonic B decays, including binning into several q^2 ranges [10]. We combine these experimental inputs with lattice results for $f_+(q^2)$ to extract values for the CKM matrix element $|V_{ub}|$. The differential decay rate for $B^0 \to \pi^-$, $l^+\nu$,

$$\frac{1}{|V_{ub}|^2} \frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} \, p_\pi^3 \, |f_+(q^2)|^2 \tag{9}$$

can be integrated to give $\frac{\Gamma}{|V_{ub}|^2}$ and the partial width Γ can be determined from CLEO's branching fraction and the Particle Data Group's B^0 lifetime of $1.542 \pm 0.016 \ ps$. Our <u>preliminary</u> estimate for $|V_{ub}|$ is then

$$|V_{ub}| = \begin{cases} 3.86(32)(58) \times 10^{-3} & 0 \le q^2 \le q_{max}^2 \\ 3.52(73)(44) \times 10^{-3} & 16 \,\text{GeV}^2 \le q^2 \end{cases}$$

where the two values correspond to either using the entire allowed q^2 range or restricting both experiment and theory to the $q^2 \geq 16~{\rm GeV}^2$ region. The first error is experimental and the second is our current best estimate of lattice statistical and systematic errors added in quadrature. In addition to $4 \sim 6\%$ statistical errors, we estimate $\sim 9\%$ higher order perturbative matching, $\sim 5\%$ chiral extrapolation, $\sim 5\%$ relativistic and discretization errors. This adds up to $\sim 12.5\%$ lattice errors for $|V_{ub}|$ obtained from the $q^2 \geq 16~{\rm GeV}^2$ region. For $|V_{ub}|$ based on the full q^2 range we increase the lattice errors to 15% (an additional 8% added in quadrature) taking into account the need to rely on the BK parametrization to enter the low q^2 region.

5. Summary

Unquenched simulations of heavy meson semileptonic decays are now feasible and we re-

port here on the first such calculations using NRQCD heavy and Asqtad light quarks (see talk by Okamoto for results using Fermilab heavy quarks [11]). The use of the improved staggered light quark action has allowed for significantly smaller statistical and chiral extrapolation errors than in the past. Combining lattice results for $f_+(q^2)$ with experimental branching fraction data has led to preliminary estimates of $|V_{ub}|$.

Many improvements are planned: inclusion of all dimension 4 $(1/M, \alpha_s/M \text{ and } a\alpha_s)$ current corrections, more sophisticated chiral extrapolations [12], and simulations at other dynamical quark masses and lattice spacings. Use of "Moving NRQCD" [5] will also allow us to simulate directly at lower q^2 .

Acknowledgements: This work was supported by the DOE, PPARC and NSF. Simulations were carried out at NERSC.

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