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We present the results of a precision computation of  $B_K$  with Wilson fermions. Simulations are performed at different lattice spacings, enabling continuum limit extrapolations. Two different twisted mass QCD (tmQCD) regularisations are considered for the computation of bare matrix elements. In both cases the relevant four-fermion operator renormalises multiplicatively. In one regularisation it is possible to perform the computation directly at the physical kaon mass value, thus avoiding extrapolations in the mass. Nonperturbative renormalisation is carried out using available Schrödinger Functional results.

### 1. tmQCD for $B_K$

The kaon mixing parameter  $B_K$  is currently the dominant source of uncertainty in unitarity triangle analyses of CP violation. It is therefore of utmost importance to have as precise a lattice QCD determination of this quantity as possible. Our project aims at a computation of  $B_K$  with Wilson fermions that brings all the systematics (apart from quenching) under control, in order to obtain a final result with an uncertainty of at most a few percent.

The main sources of uncertainty in existing quenched computations with Wilson fermions are the limits to simulations imposed by the presence of exceptional configurations and the mixing under renormalisation of the parity-even part of the relevant four-fermion operator

$$O^{\Delta S=2} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L) . \quad (1)$$

Both problems can be eliminated with the use of tmQCD Wilson regularisations.<sup>1</sup> A framework

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<sup>1</sup>For a review of tmQCD and recent developments related to it, see [1].

to compute  $B_K$  without the need of determining mixing coefficients, via a tmQCD regularisation with a fully twisted  $(u, d)$  doublet and an untwisted  $s$  quark, was introduced in [2]. The first results obtained within this approach have been presented in previous Lattice conferences [3]. Here we present our first analysis of the full set of results, and supply a preliminary value for  $B_K$  in the continuum limit.

We always work within a Schrödinger Functional (SF) framework. Our action is nonperturbatively  $O(a)$  improved in the bulk of the SF cylinder, and one-loop  $O(a)$  improved at the time boundaries. For the nonperturbative renormalisation of the four-fermion operator we use the results in [4].

### 2. Results

Simulations have been performed at three values of the lattice coupling  $\beta$  and a number of different quark masses in the limit of unbroken  $SU(3)_V$  symmetry  $M_s = M_d$ , where  $M_i$  are the physical renormalised quark masses. The parameters employed in the runs from which our final

value for  $B_K$  is derived are summarised in Table 1, together with the values of the pseudoscalar meson mass and the bare value of  $B_K$  obtained in each case. The latter is extracted from a suitable ratio of correlation functions, as described in [3].

To obtain the value of  $B_K$  at the physical point we have extrapolated the results in Table 1 to  $(r_0 M_{\text{PS}})^2 = 1.591$ , assuming a linear dependence of  $B_K$  on  $M_{\text{PS}}^2$  (which actually leads to our best fits). We stress that an independent simulation has been performed at each value of the mass, and therefore the results are fully uncorrelated.

After nonperturbative renormalisation, we can extrapolate our results to the continuum limit as depicted in Fig. 1. Since the matrix element is not fully  $\mathcal{O}(a)$  improved, the renormalisation group invariant parameter  $\hat{B}_K$  is expected to approach the continuum limit linearly in  $a/r_0$ ; however, we observe no cutoff dependence within the statistical accuracy of the results, and therefore take a fit to a constant as our best result. As a preliminary value in the continuum limit we finally quote

$$\begin{aligned} \hat{B}_K &= 0.817(22) , \\ \overline{B}_K^{\overline{\text{MS}},\text{NDR}}(2 \text{ GeV}) &= 0.592(16) . \end{aligned} \quad (2)$$

We have performed a number of tests to ensure that the quoted error adequately takes into account a number of systematic uncertainties. First of all, simulations on larger lattices (corresponding to  $L \approx 2.2$  fm) at  $\beta = 6.0$  have shown that the value of  $B_K$  at the lightest pseudoscalar meson mass does not depend on the physical volume within its statistical uncertainty. We also performed a study, working at  $\beta = 6.0$  and  $r_0 M_{\text{PS}} \simeq 1.777$ , of the dependence of  $B_K$  on the breaking of the  $\text{SU}(3)_V$  symmetry, parameterised by  $\epsilon = (M_s - M_d)/(M_s + M_d)$ . Fig. 2 shows the results: no significant dependence is observed up to  $\epsilon \sim 0.4$ .

### 3. Eliminating mass extrapolations

The main remaining source of systematic uncertainty in the previous method comes from the fact that using an untwisted  $s$  quark prevents us from computing at physical values of the kaon mass in the  $M_s = M_d$  limit. There are various tmQCD regularisations that allow to avoid

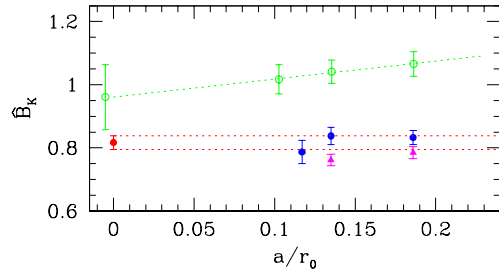


Figure 1. Extrapolation (solid points, band) to the continuum limit of the renormalisation group invariant  $B$ -parameter  $\hat{B}_K$ . To allow for an easy comparison with a reference computation employing Wilson fermions, we include the results of the method without subtractions of [5] (empty points), which display a linear extrapolation in  $a/r_0$ . Also displayed (triangles) are the results for  $B_K$  obtained with  $(\pi/4)$ -twisted quarks (see text).

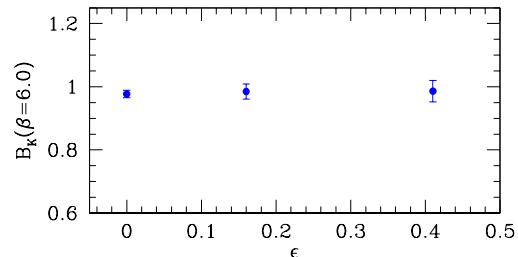


Figure 2. Dependence of the bare  $B_K$  at  $\beta = 6.0$  on the  $\text{SU}(3)_V$  breaking parameter  $\epsilon$ .

this problem. One simple possibility consists in considering a regularisation in which the doublet  $(s, d)$  is twisted with angle  $\alpha = \pi/4$ . It can be easily shown that the renormalisation properties of the relevant four-fermion operator are exactly the same as in the previous case. This approach is designed for mass degenerate strange and down quark masses, and, since both quarks are twisted, the physical kaon mass can now be reached without any problem. An alternative that uses only fully twisted ( $\alpha = \pi/2$ ) quarks has been proposed in [6]. An attractive feature of this latter proposal

$\beta$	lattice	# config.	$\kappa_s$	$\kappa_d$	$a\mu_d$	$aM_{\text{PS}}$	$B_K$
6.0	$16^3 \times 48$	402	0.1335	0.135169	0.03186	0.3895(11)	1.025(8)
		398	0.1338	0.135178	0.03152	0.3551(12)	0.993(11)
		402	0.1340	0.135183	0.02708	0.3320(11)	0.977(11)
		400	0.1342	0.135187	0.02261	0.3054(11)	0.943(10)
6.2	$24^3 \times 64$	200	0.1346	0.135780	0.028324	0.2815(8)	0.980(7)
		201	0.1347	0.135783	0.025985	0.2685(9)	0.952(8)
		214	0.1349	0.135787	0.021290	0.2432(10)	0.934(8)
6.3	$24^3 \times 72$	204	0.1348	0.135771	0.023639	0.2385(10)	0.970(14)
		212	0.1349	0.135773	0.021254	0.2251(11)	0.937(12)
		205	0.1351	0.135776	0.016467	0.1964(12)	0.885(15)

Table 1

Parameters for the runs from which our main data have been extracted. The bare twisted  $d$  quark mass has been denoted by  $\mu_d$ . The mass parameters are tuned so as to have equal renormalised  $s$  and  $d$  quark masses up to  $O(a^2)$  cutoff effects.

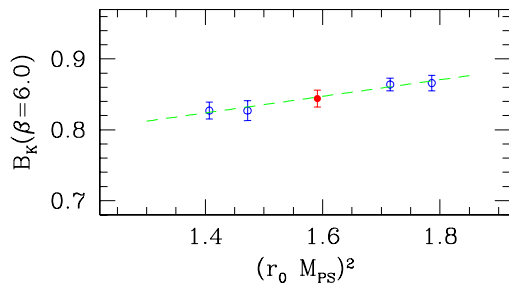


Figure 3. Interpolation to obtain  $B_K(\beta = 6.0)$  at the physical kaon mass in the  $(\pi/4)$ -twisting approach (solid point) from the results for four different pseudoscalar masses (open points).

is that it is expected to bring in automatic  $O(a)$  improvement of the (bare) matrix element.

As a feasibility study of the  $(\pi/4)$ -twisting approach we have performed simulations at  $\beta = 6.0, 6.2$  and a number of different pseudoscalar meson masses, on a box of physical length  $L \approx 1.5$  fm. The value of  $B_K$  at the physical kaon point is then obtained by linear interpolation in  $M_{\text{PS}}^2$ , as shown e.g. in Fig. 3. The results are also shown in Fig. 1. They exhibit remarkably small uncertainties, and suggest a good agreement with the outcome from the previous approach. How-

ever, preliminary results with the same run parameters on a larger physical volume hint at the presence of a small but, given the small statistical uncertainties, still relevant finite volume effect. This issue has therefore to be settled by more extensive simulations.

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