THE STATIC PENTA-QUARK POTENTIAL IN LATTICE QCD*

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We perform the first study for the static penta-quark (5Q) potential in lattice QCD with β =6.0 and $16^3 \times 32$ at the quenched level. Accurate results of the 5Q potential are extracted from the 5Q Wilson loop using the smearing method, which enhances the ground-state component. The tetra-quark potential for the QQ- $\bar{Q}\bar{Q}$ system is also studied in lattice QCD. The multi-quark potentials are found to be well described as a sum of the one-gluon-exchange Coulomb term and the multi-Y linear confinement term based on the flux-tube picture.

1. Introduction

The recent experimental discoveries of $\Theta^+(1540)$,^{1,2} $\Xi^{--}(1862)^3$ and $\Theta_c(3099)^4$ as the candidates of penta-quark (5Q) baryons are expected to reveal new aspects of QCD and hadrons. Such experiments were motivated by the theoretical prediction by Diakonov *et al.*⁵, and many theoretical studies have been done to clarify the 5Q baryons,^{2,6-17} However, there are so many open problems for several remarkable features of the 5Q baryons. Experimental data indicate extremely narrow decay widths and small masses of the 5Q baryons, and the parity determination is also an open problem. For the physical understanding of these features, theoretical analyses are necessary as well as the experimental studies. In particular, to clarify the inter-quark force in the multi-quark system based on QCD is required for the realistic modeling of the multi-quark system.

For this purpose, we perform the static penta-quark potential V_{5Q} in lattice QCD. We here investigate QQ-Q-QQ type configurations for the 5Q

^{*}The lattice QCD simulation has been done on NEC-SX5 at Osaka University.

system^{8,9} where the two QQ clusters belong to $\mathbf{3}^*$ representation in SU(3)_c, since the $\mathbf{3}^*$ diquark has a smaller energy than the $\mathbf{6}$ diquark.⁶ Similarly, we investigate QQ- $\bar{\mathbf{Q}}\bar{\mathbf{Q}}$ type configurations for the 4Q system where the QQ and the $\bar{\mathbf{Q}}\bar{\mathbf{Q}}$ clusters belong to $\mathbf{3}^*$ and $\mathbf{3}$ representation, respectively.

2. Theoretical Ansatz: OGE Coulomb plus multi-Y Ansatz

To begin with, we give a theoretical consideration for the multi-quark potential. From recent lattice QCD studies, the static three-quark (3Q) potential for baryons is found to obey the Coulomb plus Y-type linear potential, *i.e.*, the Y-Ansatz. ¹⁸ In fact, the lattice data for the $Q\bar{Q}$ and the 3Q potentials can be well described as

$$V_{Q\bar{Q}} = -\frac{A_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}}r + C_{Q\bar{Q}}, \tag{1}$$

$$V_{3Q} = -A_{3Q} \sum_{i \le j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3Q} L_{\min} + C_{3Q},$$
 (2)

where A denotes the Coulomb coefficient, σ the string tension, C a constant and L_{\min} the minimal length of the flux tube linking the valence quarks. These lattice QCD results indicate the flux-tube picture for hadrons.

We generalize the Y-Ansatz to the static 5Q potential V_{5Q} , and conjecture the one-gluon-exchange (OGE) Coulomb plus multi-Y linear potential^{8,9} for V_{5Q} , *i.e.*, OGE Coulomb plus multi-Y Ansatz,

$$V_{5Q} = \frac{g^2}{4\pi} \sum_{i < j} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{5Q} L_{\min} + C_{5Q}$$

$$= -A_{5Q} \left\{ \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{2} \left(\frac{1}{r_{15}} + \frac{1}{r_{25}} + \frac{1}{r_{35}} + \frac{1}{r_{45}} \right) + \frac{1}{4} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) \right\} + \sigma_{5Q} L_{\min} + C_{5Q}, \tag{3}$$

where the two quarks at $(\mathbf{r}_1, \mathbf{r}_2)$ and those at $(\mathbf{r}_3, \mathbf{r}_4)$ belong to $\mathbf{3}^*$ representation, respectively, and the antiquark locates at \mathbf{r}_5 . Here, L_{\min} is defined as the minimal length of the flux tube linking all the valence (anti-)quarks.

For the 4Q system, OGE Coulomb plus multi-Y Ansatz is expressed as

$$V_{4Q} = -A_{4Q} \left\{ \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{2} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) \right\} + \sigma_{4Q} L_{\min} + C_{4Q}, \quad (4)$$

where two quarks locate at $(\mathbf{r}_1, \mathbf{r}_2)$ and two antiquarks at $(\mathbf{r}_3, \mathbf{r}_4)$.

We theoretically expect $\frac{1}{2}A_{Q\bar{Q}} = A_{3Q} = A_{4Q} = A_{5Q}$ as the OGE result, and $\sigma_{Q\bar{Q}} = \sigma_{3Q} = \sigma_{4Q} = \sigma_{5Q}$ as the universality of the string tension.

3. Multi-quark Wilson loops and multi-quark potentials

In QCD, the static potential V is derived from the Wilson loop W as

$$V = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W \rangle. \tag{5}$$

The static multi-quark potentials can be also obtained from the corresponding multi-quark Wilson loops. As shown in Figs.1 and 2, we define the 5Q Wilson loop $W_{5\mathrm{Q}}{}^{8}$ and the 4Q Wilson loop $W_{4\mathrm{Q}}$ as

$$W_{5Q} \equiv \frac{1}{3!} \epsilon^{abc} \epsilon^{a'b'c'} \tilde{M}^{aa'} (\tilde{L}_3 \tilde{L}_{12} \tilde{L}_4)^{bb'} (\tilde{R}_3 \tilde{R}_{12} \tilde{R}_4)^{cc'}, \tag{6}$$

$$W_{4Q} \equiv \frac{1}{3} \text{tr}(\tilde{M}_1 \tilde{L}_{12} \tilde{M}_2 \tilde{R}_{12}),$$
 (7)

where $\tilde{L}_i, \tilde{R}_i, \tilde{M}, \tilde{M}_j$ (i=1,2,3,4,j=1,2) are given by

$$\tilde{L}_i, \tilde{R}_i, \tilde{M}, \tilde{M}_j \equiv P \exp\{ig \int_{L_i, R_i, M, M_j} dx^{\mu} A_{\mu}(x)\} \in SU(3)_c,$$
 (8)

i.e., \tilde{L}_i , \tilde{R}_i , \tilde{M} , \tilde{M}_j (i=3,4,j=1,2) are line-like variables and \tilde{L}_i , \tilde{R}_i (i=1,2) are staple-like variables, and \tilde{L}_{12} , \tilde{R}_{12} are defined by

$$\tilde{L}_{12}^{a'a} \equiv \frac{1}{2} \epsilon^{abc} \epsilon^{a'b'c'} \tilde{L}_{1}^{bb'} \tilde{L}_{2}^{cc'}, \quad \tilde{R}_{12}^{a'a} \equiv \frac{1}{2} \epsilon^{abc} \epsilon^{a'b'c'} \tilde{R}_{1}^{bb'} \tilde{R}_{2}^{cc'}. \tag{9}$$

Note that both the 4Q Wilson loop $W_{\rm 4Q}$ and the 5Q Wilson loop $W_{\rm 5Q}$ are gauge invariant.

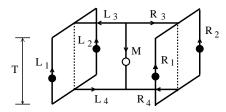


Figure 1. The penta-quark (5Q) Wilson loop $W_{5\mathrm{Q}}$. A gauge-invariant 5Q state is generated at t=0 and annihilated at t=T. Four quarks and an antiquark are spatially fixed for 0 < t < T.

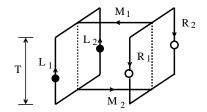
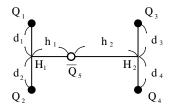


Figure 2. The tetra-quark (4Q) Wilson loop $W_{4\mathrm{Q}}$. A gauge invariant 4Q state is generated at t=0 and annihilated at t=T. Two quarks and two antiquarks are spatially fixed for 0 < t < T.

In general, the multi-quark operator in the multi-quark Wilson loop contains excited-state components. In order to extract the ground-state potential, we use the smearing method.¹⁸ By this procedure, we obtain the quasi-ground-state operators for the static multi-quark systems, and thus perform the accurate calculations for the multi-quark potentials with them.

4. Lattice QCD results and Concluding Remarks

The lattice QCD simulation is performed at $\beta = 6.0$, i.e., $a \simeq 0.1$ fm, on the $16^3 \times 32$ lattice at the quenched level.^{8,9} In this paper, we investigate the planar and twisted configurations for the 5Q system as shown in Figs.3 and 4, and demonstrate the case of $d_1 = d_2 = d_3 = d_4 \equiv d$ and $h_1 = h_2 \equiv h/2$.



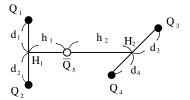
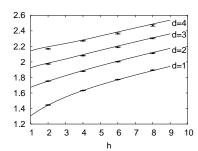


Figure 3. A planar configuration of the penta-quark system.

Figure 4. A twisted configuration of the penta-quark system with $Q_1Q_2 \perp Q_3Q_4$.

In Fig.5, we show the lattice QCD results of the 5Q potential $V_{5\rm Q}$. The lattice data denoted by the symbols are found to be well reproduced by the theoretical curves of the OGE plus multi-Y Ansatz^{8,9a} with $(A_{5\rm Q}, \sigma_{5\rm Q})$ fixed to be $(A_{3\rm Q}, \sigma_{3\rm Q}) \simeq (0.1366, 0.046a^{-2})$ in the 3Q potential.^{18b}



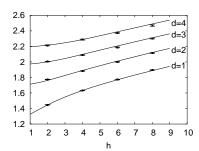
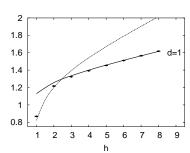


Figure 5. Lattice QCD results of the 5Q potential $V_{\rm 5Q}$ in the lattice unit for the planar configurations (left) and the twisted configurations (right). The symbols denote the lattice data. The theoretical curves of the OGE plus multi-Y Ansatz are added.

Figure 6 shows the lattice results of the 4Q potential V_{4Q} . For large h, V_{4Q} coincides with the energy V_{c4Q} of the connected 4Q system. For small h, V_{4Q} coincides with the energy $2V_{Q\bar{Q}}$ of the "two-meson" system composed of two flux-tubes. Thus, we get the relation of $V_{4Q} = \min(V_{c4Q}, 2V_{Q\bar{Q}})$, and find the "flip-flop" between the connected 4Q system and the "two-

^a For the extreme case, e.g., $d > \sqrt{3}h_1$, we here assume that the flux-tube is formed on the straight lines of Q_1Q_5 and Q_2Q_5 .

^bDue to this fixing, there is no adjustable parameter except for an irrelevant constant.



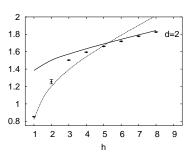


Figure 6. The 4Q potential V_{4Q} for d=1 (left) and d=2 (right) for the planar 4Q configuration similar to Fig.3. The horizontal axis h corresponds to h_1+h_2 . The symbols denote the lattice QCD results. The theoretical curves are added for the connected 4Q system (the solid curve) and for the "two-meson" system (the dashed curve).

meson" system around the level-crossing point where these two systems are degenerate as $V_{\rm c4Q}=2V_{\rm Q\bar{Q}}$. This result also indicates the flux-tube picture.

To summarize, we have performed the static 5Q and 4Q potentials in lattice QCD, and have found that the multi-quark potentials are well described with the OGE Coulomb plus multi-Y linear potential except for extreme cases. For the static 4Q potential, we have found the "flip-flop" between the connected 4Q system and the "two-meson" system. The present lattice QCD results for the multi-quark potentials provide a guiding principle in modeling the multi-quark system.

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