

LATTICE QCD STUDY OF THE PENTAQUARK BARYONS

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We study the spin $\frac{1}{2}$ hadronic state in quenched lattice QCD to search for a possible $S = +1$ pentaquark resonance. Simulations are carried out on $8^3 \times 24$, $10^3 \times 24$, $12^3 \times 24$ and $16^3 \times 24$ lattices at $\beta=5.7$ at the quenched level with the standard plaquette gauge action and Wilson quark action. We adopt two independent operators with $I = 0$ and $J^P = \frac{1}{2}^-$ to construct a 2×2 correlation matrix. After the diagonalization of the correlation matrix, we successfully obtain the energies of the ground-state and the 1st excited-state in this channel. The volume dependence of the energies suggests the existence of a possible resonance state slightly above the NK threshold in $I = 0$ and $J^P = \frac{1}{2}^-$ channel.

1. Introduction

After the discovery ¹ of $\Theta^+(1540)$, the property of the particle is one of the central issues in hadron physics. While it is very likely that the isospin of Θ^+ is zero, the spin and the parity of Θ^+ and the origin of its tiny width of a few MeV still remain open questions. ² In spite of many theoretical studies on Θ^+ , ² the nature of the exotic particle is still controversial.

As for the theoretical approaches, the lattice QCD is considered as one of the most reliable tools for studying the properties of hadron states from first principle, which has been very successful in reproducing the non-exotic hadron mass spectra. ³ Up to now, several lattice QCD studies have been reported, which mainly aim to look for pentaquarks. However, the results are unfortunately contradictory with each other. On one hand, in refs. ^{4,5}, the authors conclude the parity of Θ^+ is negative. On the other hand, in ref. ⁶, Θ^+ in the positive parity channel is reported. In refs. ^{7,8}, even the absence of Θ^+ is suggested.

A difficulty in the study of Θ^+ using lattice QCD comes from the contaminations of the scattering states of nucleon and Kaon, since the mass of Θ^+ is above the NK threshold. In order to verify the existence of a resonance state, we need to properly discriminate the first few lowest states,

pick up one state as a candidate, and determine whether it is a resonance state or not.

In this paper, we study $I = 0$ and $J = \frac{1}{2}$ channel in quenched lattice QCD. With the aim to extract the possible resonance state slightly above the NK threshold in this channel, we adopt two independent operators with $I = 0$ and $J = \frac{1}{2}$ and diagonalize the 2×2 correlation matrix of the operators. After the successful separation of the states, we investigate the volume dependence of the energy so that we can distinguish the resonance state from simple scattering states.

2. Formalism to separate the states

In lattice QCD calculations, the energies of states can be measured using the operator correlations. Let us consider the correlation $\langle O(T)O(0)^\dagger \rangle$ with $O(T)$ an interpolating operator on the time plane $t = T$ with a certain quantum number. The correlation $\langle O(T)O(0)^\dagger \rangle$ physically represents the situation in which the states with the same quantum number as $O(t)$ are created at $t = 0$ and propagate into Euclidean time direction and finally are annihilated at $t = T$. Taking into account the fact that the created state $O^\dagger|vac\rangle$ can be expressed by the complete set of QCD eigenstates as $O^\dagger|vac\rangle = c_0|0\rangle + c_1|1\rangle + \dots = \sum_i c_i|i\rangle$, we can write down the correlation $\langle O(T)O(0)^\dagger \rangle$ as $\langle O(T)O(0)^\dagger \rangle = \sum_i \sum_j \bar{c}_i c_j \langle i|e^{-\hat{H}T}|j\rangle = \sum_i |c_i|^2 e^{-E_i T}$ in terms of the QCD transfer matrix $e^{-\hat{H}T}$, with QCD Hamiltonian \hat{H} and n -th excited-state $|n\rangle$ normalized as $\langle m|n\rangle = \delta_{mn}$. We can extract the ground-state energy E_0 by taking large T so that the correlation $\langle O(T)O(0)^\dagger \rangle$ behaves as a single-exponential function $\langle O(T)O(0)^\dagger \rangle \sim |c_0|^2 e^{-E_0 T}$. It is however rather difficult to obtain the excited-state energy $E_i (i > 0)$ from the multi-exponential function of T , which is a problem for extracting the possible pentaquark signal above the threshold. To overcome this difficulty, we adopt the variational method using correlation matrices of independent operators.⁹ In this method, we need a set of independent operators $\{O^I\}$. We define the correlation matrix \mathbf{C}_{IJ} as $\mathbf{C}_{IJ}(T) \equiv \langle O^I(T)O^{J\dagger}(0) \rangle$. Then, from the product $\mathbf{C}^{-1}(T+1)\mathbf{C}(T)$, we can extract the energies $\{E_i\}$ as the logarithm of eigenvalues $\{e^{E_i}\}$ of the matrix $\mathbf{C}^{-1}(T+1)\mathbf{C}(T)$. In the case when we prepare N independent operators, the correlation matrix is an $N \times N$ matrix and we can obtain N eigenenergies $\{E_i\}$ ($0 \leq i \leq N-1$).

3. Lattice set up

As interpolating operators $\{O^I\}$, we take two independent operators; $\Theta^1(x) \equiv \varepsilon^{abc}[u_a^T(x)C\gamma_5 d_b(x)]u_e(x)[\bar{s}_c(x)\gamma_5 d_e(x)] - (u \leftrightarrow d)$ which is expected to have a larger overlap with Θ^+ state, and $\Theta^2(x) \equiv \varepsilon^{abc}[u_a^T(x)C\gamma_5 d_b(x)]u_e(x)[\bar{s}_e(x)\gamma_5 d_c(x)] - (u \leftrightarrow d)$ which we expect to have larger overlaps with NK scattering states. Both of them have the quantum number of $(I, J) = (0, \frac{1}{2})$. Here, Dirac fields $u(x)$, $d(x)$ and $s(x)$ are up, down and strange quark field, respectively and the Roman alphabets $\{a, b, c, e\}$ denote color indices. In this case, the correlation matrix is a 2×2 matrix. Furthermore, for a source, we construct and use ‘‘wall’’ operators $\Theta_{\text{wall}}^1(t)$ and $\Theta_{\text{wall}}^2(t)$ defined using spatially spread quark fields $\sum_{\vec{x}} q(x)$ as $\Theta_{\text{wall}}^1(t) \equiv \left(\sqrt{\frac{1}{V}}\right)^5 \sum_{\vec{x}_1 \sim \vec{x}_5} \varepsilon^{abc}[u_a^T(x_1)C\gamma_5 d_b(x_2)]u_e(x_3)[\bar{s}_c(x_4)\gamma_5 d_e(x_5)] - (u \leftrightarrow d)$. We fix the source operator $\bar{\Theta}_{\text{wall}}(t)$ on $t=6$ plane to reduce the effect of the Dirichlet boundary on $t=0$ plane,¹⁰ and calculate $C^{IJ}(T) = \sum_{\vec{x}} \langle \Theta^I(\vec{x}, T+6) \bar{\Theta}_{\text{wall}}^J(6) \rangle$.

We adopt the standard plaquette (Wilson) gauge action at $\beta = 5.7$ and Wilson quark action with the hopping parameters as $(\kappa_{u,d}, \kappa_s) = (0.1600, 0.1650)$, $(0.1625, 0.1650)$, $(0.1650, 0.1650)$, $(0.1600, 0.1600)$ and $(0.1650, 0.1600)$, which correspond to the current quark masses $(m_{u,d}, m_s) \sim (240, 100)$, $(170, 100)$, $(100, 100)$, $(240, 240)$ and $(100, 240)$, respectively in the unit of MeV. We perform lattice QCD calculations at $\beta = 5.7$ employing four different sizes of lattices, $8^3 \times 24$, $10^3 \times 24$, $12^3 \times 24$ and $16^3 \times 24$ with 2900, 2900, 1950 and 950 gauge configurations. At $\beta = 5.7$, the lattice spacing a is set to be 0.17 fm so that the ρ meson mass is properly reproduced. Then, in the physical unit, the lattice sizes are $1.4^3 \times 4.0 \text{ fm}^4$, $1.7^3 \times 4.0 \text{ fm}^4$, $2.0^3 \times 4.0 \text{ fm}^4$ and $2.7^3 \times 4.0 \text{ fm}^4$.

We take periodic boundary conditions in the directions for the gauge field, whereas we impose periodic boundary conditions on the spatial directions and the Dirichlet boundary condition on the temporal direction for the quark field. Many lattice studies about Θ^+ adopted a periodic or antiperiodic boundary condition on the temporal direction for quarks. In the case when the temporal length N_t is not long enough, we have to be careful about the contaminations by particles which go beyond the temporal boundary. For example, with the periodic boundary condition, the correlation $\langle \Theta(T) \bar{\Theta}(0) \rangle$ contains the unwanted contributions such as $\langle K | \Theta(T) | N \rangle \langle N | \bar{\Theta}(0) | K \rangle \sim e^{-E_N T + E_K(T - N_t)}$. While no quark can go over the boundary on $t=0$ in the temporal direction, the boundary is transparent for the particles composed only by gluons; *i.e.* glueballs, due to the

periodicity of the gauge action. Taking into account that these particles are rather heavy, it would be however safe to neglect these gluonic particles going beyond the boundary. Then, the correlation $\langle \Theta(T) \bar{\Theta}(0) \rangle$ mainly contains such terms as $\langle vac | \Theta(T) | NK \rangle \langle NK | \bar{\Theta}(0) | vac \rangle$ and we can use the exactly same prescription mentioned in the previous section.

4. Ground-state and 1st Excited-state in $(I, J^P) = (0, \frac{1}{2}^-)$ channel

In this section, we investigate the ground-state and the 1st excited-state in $(I, J^P) = (0, \frac{1}{2}^-)$ channel. We focus on the volume dependence of the energy of each state, to distinguish a possible resonance state from NK scattering states. We expect a resonance state to have almost no volume dependence, while the energies of NK scattering states are expected to scale as $\sqrt{M_N^2 + |\frac{2\pi}{L}\vec{n}|^2} + \sqrt{M_K^2 + |\frac{2\pi}{L}\vec{n}|^2}$ according to the relative momentum $\frac{2\pi}{L}\vec{n}$ between N and K on finite periodic lattices on the assumption that the NK interaction is negligible. Note here that there are other possible finite volume effects due to the finite size of N and K or possible pentaquark states, than the volume dependence arising from $\frac{2\pi}{L}\vec{n}$. We then need to take account of this fact in the following analysis.

Now we show the lattice QCD results of the ground-state and the 1st excited-state in $I = 0$ and $J^P = \frac{1}{2}^-$ channel. In Fig. 1, we plot the ground-

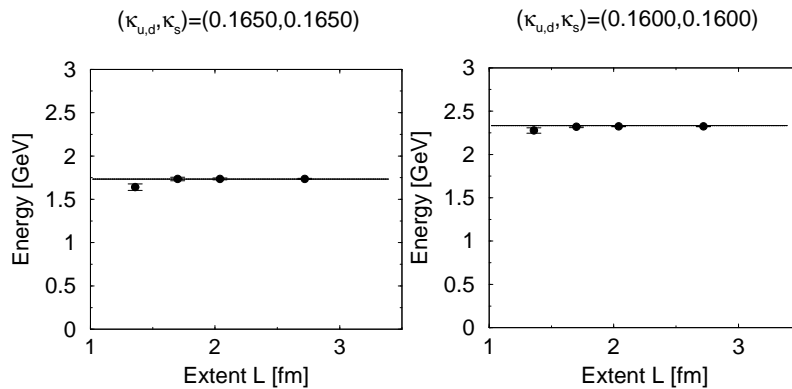


Figure 1. The lattice QCD data of the ground-state in $(I, J^P) = (0, \frac{1}{2}^-)$ channel are plotted against the lattice extent L . The left figure is for $(\kappa_{u,d}, \kappa_s) = (0.1650, 0.1650)$ and the right figure is for $(\kappa_{u,d}, \kappa_s) = (0.1600, 0.1600)$. The solid line denotes the simple sum $M_N + M_K$ of the masses of nucleon M_N and Kaon M_K . We use the central values of M_N and M_K obtained on the lattice with $L = 16$

state energies in $I = 0$ and $J^P = \frac{1}{2}^-$ channel on four different volumes. Here the horizontal axis denotes the lattice extent L in the physical unit and the vertical axis is the energy of this state. The symbols with error-bars are lattice data and the solid line denotes the sum $M_N + M_K$ of the nucleon mass M_N and Kaon mass M_K . At a glance, we find that the energy of this state takes almost constant value against the volume variation and coincides with the simple sum $M_N + M_K$. We can then conclude the ground-state in $I = 0$ and $J^P = \frac{1}{2}^-$ channel is the NK scattering state with the relative momentum $|\mathbf{p}| = 0$.

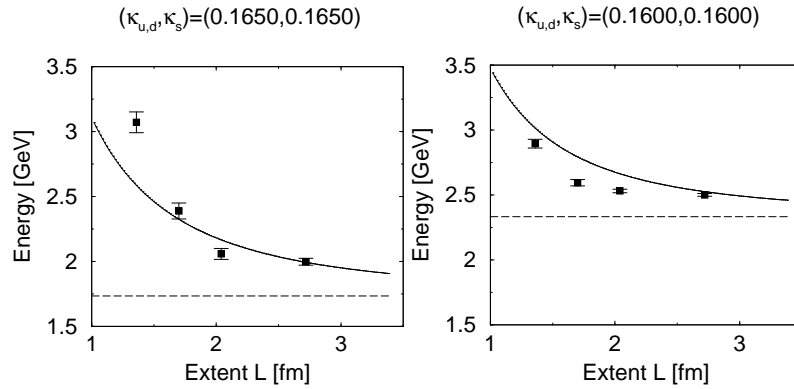


Figure 2. The lattice QCD data of the 1st excited-state in $(I, J^P) = (0, \frac{1}{2}^-)$ channel are plotted against the lattice extent L . The solid line represents $\sqrt{M_N^2 + |\mathbf{p}|^2} + \sqrt{M_K^2 + |\mathbf{p}|^2}$ with $|\mathbf{p}| = 2\pi/L$ the smallest relative momentum on the lattice. The dashed line denotes $M_N + M_K$.

Fig. 2 shows the 1st excited-state energies in this channel again in terms of L . The solid line shows $\sqrt{M_N^2 + (2\pi/L)^2} + \sqrt{M_K^2 + (2\pi/L)^2}$ as the function of L . In this case, the lattice data exhibit clearly some volume dependence. One would naively consider this dependence to be that of the 2nd lowest NK scattering state, which scales as $\sqrt{M_N^2 + (2\pi/L)^2} + \sqrt{M_K^2 + (2\pi/L)^2}$, and would take the deviations as from other possible finite size effects. However, if it is the case, the significant deviations in $1.5 \lesssim L \lesssim 3$ fm in the right figure may not be justified. In the range of L where the lattice data in the left figure scale just as the solid line, we should observe the same behavior in the right figure. Because, in the case when quarks are heavy, composite particles will be rather compact and we expect smaller finite size effects.

We can understand this behavior on the assumption that this state is

a resonance state rather than a scattering state. In fact, while the data in the left figure rapidly go above the solid line as L is decreased, which can be considered to arise due to the finite size of a resonance state, the lattice data exhibit almost no volume dependence in the right figure especially in $1.5 \lesssim L \lesssim 3$ fm, which can be regarded as the characteristic of resonance states.

5. $(I, J^P) = (0, \frac{1}{2}^+)$ channel

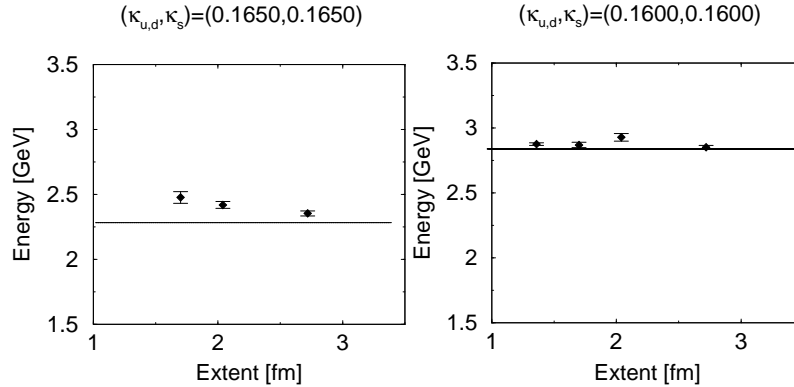


Figure 3. The lattice QCD data in the $(I, J^P) = (0, \frac{1}{2}^+)$ channel are plotted against the lattice extent L . The solid line denotes the simple sum $M_{N^*} + M_K$ of the masses of the ground-state negative-parity nucleon M_{N^*} and Kaon M_K .

In the same way as $(I, J^P) = (0, \frac{1}{2}^-)$ channel, we have attempted to diagonalize the correlation matrix on $(I, J^P) = (0, \frac{1}{2}^+)$ channel using the wall-sources $\bar{\Theta}_{\text{wall}}(t)$. In this channel, the diagonalization is rather unstable and we find only one state except for tiny contributions of possible other states. We plot the lattice data in Fig. 3. One finds that they have almost no volume dependence and that they coincide with the solid line which represents the simple sum $M_{N^*} + M_K$ of M_{N^*} and M_K , with M_{N^*} the mass of the ground-state of the *negative-parity* nucleon. From this fact, the state we observe is concluded to be the N^*K scattering state with the relative momentum $|\mathbf{p}| = 0$. It is rather strange because the p-wave state of N and K with the relative momentum $|\mathbf{p}| = 2\pi/L$ will be lighter than the N^*K scattering state with the relative momentum $|\mathbf{p}| = 0$. We miss this lighter state in our analysis. This failure would be due to the wall-like operator $\Theta_{\text{wall}}(t)$: The operator $\Theta_{\text{wall}}(t)$ is constructed by the spatially spread quark fields $\sum_{\vec{x}} q(\vec{x})$ with zero momentum. This may lead to the

large overlaps with the scattering state with zero relative momentum. It is also desired to try another operator which couples to p-wave NK scattering state.

6. Summary

We have performed the lattice QCD study of the $S=+1$ pentaquark baryons on $8^3 \times 24$, $10^3 \times 24$, $12^3 \times 24$ and $16^3 \times 24$ lattices at $\beta=5.7$ at the quenched level with the standard plaquette gauge action and Wilson quark action. With the aim to separate states, we have adopted two independent operators with $I = 0$ and $J^P = \frac{1}{2}$ so that we can construct a 2×2 correlation matrix. From the correlation matrix of the operators, we have successfully obtained the energies of the ground-state and the 1st excited-state in $(I, J^P) = (0, \frac{1}{2}^-)$ channel. The volume dependence of the energies shows that the 1st excited-state in this channel is rather different from a simple NK scattering state and is likely to be the resonance state located slightly above the NK threshold, and also indicates that the ground-state is the NK scattering state with the relative momentum $|\mathbf{p}| = 0$. As for the $(I, J^P) = (0, \frac{1}{2}^+)$ channel, we have observed only one state in the present analysis, which is likely to be a N^*K scattering state of the negative-parity nucleon N^* and Kaon with the relative momentum $|\mathbf{p}| = 0$.

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