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QCD at Finite temperature and density with staggered and Wilson quarks*

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One of the most challenging issues in particle physics is to study QCD in extreme conditions. Precise determination of the QCD phase diagram on temperature T and chemical potential μ plane will provide valuable information for quark-gluon plasma (QGP) and neutron star physics. We present results for phase structure on the (μ, T) plane for lattice QCD with Wilson fermions from strong coupling Hamiltonian analysis and Kogut-Susskind Fermions from Lagrangian Monte Carlo simulations at intermediate coupling.

1. Introduction

For two massless quarks, several QCD inspired models suggest the existence of a tricritical point on the (μ, T) plane separating the first order transition line at lower T and larger μ, and the second order transition line at higher T and smaller μ. Lattice gauge theory (LGT) should in principle give more reliable information on the QCD phase diagram. Heavy-ion collision experiments may determine whether the theoretical prediction is correct.

In Lagrangian formulation of SU(3) LGT, the chemical potential μ is introduced by making the replacement: $U_4(x) = e^{\mu a}U_4(x)$, $U_4^+(x) = e^{-\mu a}U_4^+(x)$. It is well known that, for real μ, the effective fermionic action is complex and the standard MC technique does not apply. The recent years have seen enormous efforts[1] on solving this problem, and some very interesting information [2,3,4] on the phase diagram for QCD with KS fermions at large T and small μ has been obtained from MC simulations. There is also a strong coupling analysis[5] of the color superconductivity phase at $T = 0$.

Nevertheless, to precisely locate the tricritical point and critical line at large μ is still an extremely difficult task. QCD at large μ is of particular importance for neutron star physics. Hamiltonian formulation of LGT doesn't encounter the notorious "complex action problem". Recently, we proposed a Hamiltonian approach[6,7] to LGT with naive fermions at finite μ, and extended it to Wilson fermions[8]. The chiral phase transition at $T = 0$ and some finite μ_C was found to be of first order. In Ref. [9], the authors studied lattice QCD with SLAC fermions at $T = 0$ in the strong coupling limit.

In this paper, we study the phase diagram on the (μ, T) plane by Lagrangian MC simulations with Kogut-Susskind quarks and Hamiltonian strong coupling analysis with Wilson fermions.

2. MC Simulation with $N_f = 2$ and 4

For small μ, some approximation can be used in simulations. If μ is purely imaginary, the effective fermionic action is real and the traditional MC methods still applies. Using Taylor expansion, the physical observables at imaginary μ can be extrapolated to real μ.

For imaginary $\mu = i\mu_I$, Roberge and Weiss[10] find that the partition function is periodic in μ_I/T with periodicity $2\pi/3$. The Polyakov loop is singular at $\mu_I/T = 2\pi(k+1/2)/3$, with k an integer. In pure gauge theory, the partition function has the universal $Z(3)$ symmetry. In full QCD, the fermionic determinant breaks the $Z(3)$ symmetry. When μ_I is increased from 0, there are phase transitions at $\mu_I/T = 2\pi(k+1/2)/3$. For $\mu_I/T < \pi/3$, the simulation is physical and for $\mu_I/T > \pi/3$, the result is only a simple copy of that for $\mu_I/T < \pi/3$, which is the artifact

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of imaginary μ. So this method works only for $\mu_I/T < \pi/3$.

We have simulated QCD with KS fermions using the standard R and HMC algorithms, for $N_f = 2$ and $N_f = 4$ respectively. We modified the MILC code to imaginary $i\mu_I$. The molecular dynamics for each configuration consists of 20 steps with step size $\delta\tau$ is 0.02. The lattice size is $8^3 \times 4$. The total trajectories are 4000. The measurements were done for every 40 trajectories. Fig. 1 and Fig. 2 show our results. The data for the chiral condensate, Polyakov loop and the its phase are consistent with Refs. [2,3]. In addition, we measured the plaquette and fermionic energy density.

3. Tricritical Point at Strong Coupling

In the Hamiltonian formulation, one can study QCD at real μ. At strong coupling, the effective Hamiltonian with four fermion interactions[8,11] is the result from integrating out the gauge fields. For convenience, we rescale the chemical potential and temperature as $\mu' = \mu/(3K/a)$ and $T' = T/(3K/a)$, with K being the effective coupling of four fermion interactions. At the second order phase transition where the chiral condensate and the dynamical mass of quark vanish continuously, there is only one global minimum in the grand thermodynamic potential. For Wilson fermions and $N_f/N_c < 1$, where $N_c = 3$ is the number of colors, we obtain an equation for the critical line

$$\begin{aligned}\mu'_C &= \left(1+r^2\right)\sqrt{1-\frac{2T'_C}{1+3r^2}} \\ &+ T'_C \ln \frac{1+\sqrt{1-\frac{2T'_C}{1+3r^2}}}{1-\sqrt{1-\frac{2T'_C}{1+3r^2}}}, \qquad (1)\end{aligned}$$

which is depicted by the dotted line for $r = 1$ in Fig. 3.

Below some finite T'_3, there is a first order chiral phase transition line

$$\mu'_C = 1 + 2r^2. \qquad (2)$$

from some finite T'_3 down to $T' = 0$. this is illustrated by the solid line for $r = 1$ in Fig. 3.

The point when two lines described by Eq. (1) and Eq. (2) join at lower T' is the tricritical point (μ'_3, T'_3). For $r = 1$, we find $(\mu'_3, T'_3) = (3, 0.4498)$, i.e. the circle in Fig. 3. The phase structure for any $r \neq 0$ is qualitatively the same. Details can be found in Ref. [12].

4. Summary

In the preceding sections, we present some preliminary MC data for QCD with KS fermions at some finite β and imaginary μ. These can be converted to some T and real μ. The results for $N_f = 2$ might indicate the existence of a second order chiral phase transition, if the data are extrapolated to the chiral limit. Those for $N_f = 4$ indicate the existence of a first order phase transition. To determine whether a tricritical point exists for $N_f = 2$, further detailed study is required.

We also investigate the QCD phase diagram in Hamiltonian lattice formulation with Wilson fermions. At the strong coupling, we find a tricritical point on the (μ, T) plane, which has not been found in previous work in the Hamiltonian formalism with KS or naive fermions. Our findings imply that on the (μ, T) plane, at least in the Hamiltonian formulation at the strong coupling, the phase structure of QCD with Wilson fermions (without species doubling) might be qualitatively different from naive or KS fermions (with species doubling).

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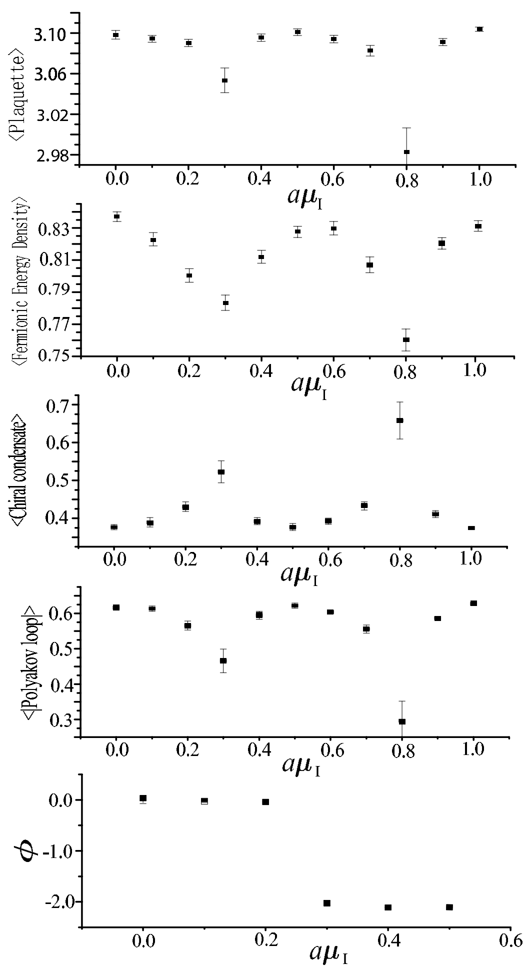

Figure 1. Results for KS quarks at $\beta = 5.1$, $am = 0.05$, and $N_f = 4$. ϕ is the phase of Polyakov loop.

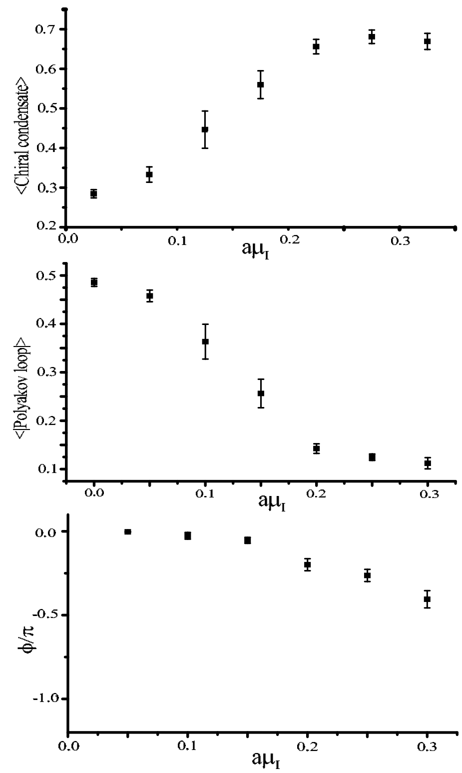

Figure 2. Results for KS quarks at $\beta = 5.3$, $am = 0.25$, and $N_f = 2$.

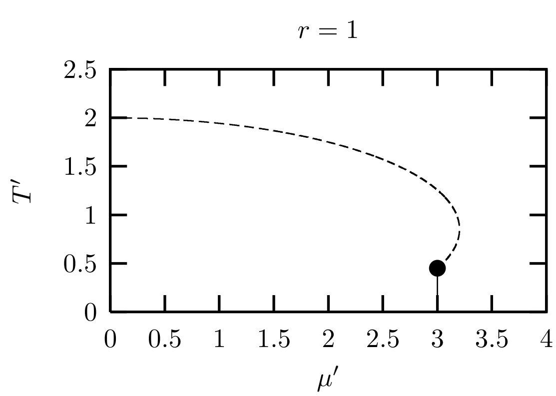

Figure 3. Phase diagram for Wilson quarks. The solid and dotted lines stand respectively for the 1st and 2nd order transitions. The circle is the tricritical point.