

Decoupling of Photon Propagator in Compact QED

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In compact QED₂₊₁ quantum monopole fluctuations induce confinement by expelling electric flux in a dual Meissner effect. Guided by Landau-Ginzburg theory, one might guess that the inverse London penetration depth λ^{-1} —the only physical mass scale—equals the photon propagator mass pole M_γ . I show this is not true. Indeed, in the Villain approximation the monopole part of the partition function factorizes from the photon part, whose dynamical variables are Dirac strings. Since Dirac strings are gauge-variant structures, I conclude that M_γ is physically irrelevant: it is not a blood relative of λ or any other quantity in the gauge-invariant sector. This result is confirmed by numerical simulations in the full theory, where M_γ is not sensitive to monopole prohibition but essentially vanishes if Dirac strings are prohibited.

1. The Issue

In the semiclassical superconducting model of the QCD vacuum [1], in which electric flux is restricted to Abrikosov tubes of width λ , the inverse London penetration depth λ^{-1} is the mass of an effective gauge potential A_μ^{eff} . Recent calculations of λ [2] and, independently, of gluon A_μ propagator mass poles M in LGT (lattice gauge theory) [3] present the question: How, if at all, is A_μ^{eff} related to A_μ and λ to M ? In this talk I demonstrate in cQED₂₊₁ (compact QED in 2 + 1 dimensions), a QCD-like LGT, that the physically relevant quantities A_μ^{eff} and λ are unrelated to A_μ and M , which are unphysical.

2. Monopoles and Dirac Strings

In *noncompact* QED, the photon is an unbounded real field $a_\mu \in (-\infty, \infty)$ and the action² $S_0 = \frac{\beta}{4} \sum_{\mu, \nu} f_{\mu\nu}^2$ where $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$ is gauge-invariant under $\delta a_\mu = -\partial_\mu \omega_x$. Since S_0 is gaussian, the nonperturbative photon mass M_γ vanishes because Maxwell equation $\partial_\mu^* f_{\mu\nu} = 0$ in Landau gauge implies $\square a_\mu = 0$.

Nothing is wrong with noncompact QED except that S_0 does not have natural nonabelian extensions. The $U(1)$ LGT corresponding to lattice QCD is cQED. Links $U_\mu \equiv e^{-i\theta_\mu}$ in cQED

depend only on the photon A_μ part of

$$\theta_\mu \equiv A_\mu + 2\pi n_\mu \quad -\pi \leq A_\mu < \pi. \quad (1)$$

A_μ is the lattice photon whose nonperturbative propagator mass M_γ is of concern. cQED₂₊₁ has local gauge invariance, chiral symmetry breaking, and area-law electron confinement induced by quantum monopole percolation [1]. cQED photons are uncharged but they suffer confinement since, heuristically, the “adjoint” Wilson loop obeys $\langle \prod_{l \in \text{loop}} \sin \theta_l \rangle \propto \text{Re} \langle \prod_{l \in \text{loop}} e^{i\theta_l} \rangle$ where cross terms are suppressed by gauge invariance. Therefore electron confinement implies photon confinement and cQED photons, like QCD gluons, are confined.

The cQED action is $S_c \equiv \beta \sum_{\mu < \nu} (1 - \cos F_{\mu\nu})$ where the plaquette angle is

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2)$$

S_c is invariant under local gauge transformation

$$\delta \theta_\mu \equiv -\partial_\mu \omega_x, \quad (3)$$

$$\delta A_\mu \equiv (A_\mu - \partial_\mu \omega_x) \text{Mod}[-\pi, \pi) - A_\mu. \quad (4)$$

While plaquette $\exp(iF_{\mu\nu})$ is gauge-invariant, a gauge transformation inducing unequal shifts of n_μ on the four links of $F_{\mu\nu}$ shifts $F_{\mu\nu}$ by a 2π multiple. $F_{\mu\nu}$ decomposes into a gauge-invariant physical part $\Theta_{\mu\nu} \in [-\pi, \pi)$ and a gauge-variant integral kink $N_{\mu\nu} \in \mathbf{Z}$ such that

$$F_{\mu\nu} \equiv (\Theta + 2\pi N)_{\mu\nu}, \quad (5)$$

$$\delta F_{\mu\nu} = 2\pi \delta N_{\mu\nu} = \partial_\mu (\delta \theta - \delta A)_\nu - (\mu \leftrightarrow \nu). \quad (6)$$

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² $\partial_\mu h_x \equiv h_{x+\hat{\mu}} - h_x$, $\partial_\mu^* h_x \equiv h_x - h_{x-\hat{\mu}}$, and $\square \equiv \partial_\mu^* \partial_\mu$. I will ignore topological gauge transformations.

The key feature of S_c is $\cos F_{\mu\nu} = \cos \Theta_{\mu\nu}$, required by gauge invariance as $N_{\mu\nu}$ is locally gauge-variant. While any single $N_{\mu\nu}$ can be gauged away, spatial combinations of them form gauge-invariant structures which influence $\Theta_{\mu\nu}$. To see how this works, decompose according to the Hodge-DeRham theorem

$$\Theta_{\mu\nu} = \epsilon_{\mu\nu\alpha} \partial_\alpha^* \phi + \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu, \quad (7)$$

$$N_{\mu\nu} = \epsilon_{\mu\nu\alpha} \partial_\alpha^* m + \partial_\mu l_\nu - \partial_\nu l_\mu \quad (8)$$

where $\phi, \alpha_\mu \in (-\infty, \infty)$ and $m, l_\mu \in \mathbf{Z}$. ϕ and m are invariant under (3), α_μ transforms like θ_μ , and l_μ like $(A - \alpha)_\mu / 2\pi$. $\Theta_{\mu\nu}$ (and similarly $N_{\mu\nu}$) has 3 independent polarizations while ϕ and α_μ are 4 functions because $\Theta_{\mu\nu}$ is invariant under $\delta \alpha_\mu = -\partial_\mu \omega_x$.

In vector notation Eq. (5) becomes

$$\vec{B} = \vec{H} + 2\pi \vec{\eta} \quad (9)$$

where the total \vec{B} and physical \vec{H} magnetic (actually electromagnetic) fields are

$$\vec{B} \equiv \nabla \times \vec{A}, \quad \vec{H} \equiv \nabla \phi + \nabla \times \vec{\alpha}. \quad (10)$$

It will be advantageous to recast Dirac string field

$$\vec{\eta} = \nabla m + \nabla \times \vec{l} \quad (11)$$

in terms of its divergence and curl

$$q \equiv \nabla \cdot \vec{\eta} = \square m, \quad (12)$$

$$\vec{\rho} \equiv \nabla \times \vec{\eta} = \nabla(\nabla \cdot \vec{l}) - \square \vec{l}. \quad (13)$$

Since $\nabla \cdot \vec{B} = 0$ by (10), $\nabla \cdot \vec{H} = -2\pi q$, that is, q causes dislocations in the physical field \vec{H} . For example, let $s(t)$ be the step function. A monopole at the origin attached to a string along the positive \hat{t} -axis corresponds to $\eta_\mu = \delta_{\mu,0} \delta_{x,0} \delta_{y,0} s(t)$, $q_x = \delta_{x,0} \delta_{y,0} \delta_{t,0}$ and $\vec{\rho} = (\delta'_{x,0} \delta_{y,0} \hat{y} - \delta_{x,0} \delta'_{y,0} \hat{x}) s(t)$. By tautology, q is the magnetic monopole density, gauge invariant since m is gauge invariant. In contrast $\vec{\rho}$, a continuous current wrapping around $\vec{\eta}$, is gauge-variant.

In general, kinks occur either in monopoles, Dirac strings connecting a monopole anti-monopole pair, or Dirac string loops. Loops can either be homologically trivial or toroidally

wind around the periodic boundaries.³ Monopole charge density q is gauge-invariant but the number of string loops and the length and shape of all strings vary with gauge. Segments of string $\vec{\eta}$ are characterized by $\vec{\rho} = \nabla \times \vec{\eta}$, continuous flows winding around $\vec{\eta}$.

3. Difference Between \vec{A}^{eff} and \vec{A}

Upon adopting a condition such as $\nabla \cdot \vec{l} = 0$ and ignoring Laplacian zero modes, Eqs. (5)-(13) constitute 1-to-1 variables changes

$$\{N\} \rightarrow \{m, \vec{l}\} \rightarrow \{q, \vec{\rho}\} \rightarrow \{\phi, \vec{\alpha}\} \quad (14)$$

where, if $\square \Delta_{x,y} = -\delta_{x,y}$, $\partial_\mu^* \Delta^{\mu\nu} = 0$, and $\square \Delta_{x,y}^{\mu\nu} = -\delta_{\mu,\nu} \delta_{x,y}$, then

$$\phi = 2\pi \int_y \Delta_{x,y} q_y, \quad \alpha_\mu = \vec{A}_\mu - 2\pi \int_y \Delta_{x,y}^{\mu\nu} \vec{\rho}_{\nu y}. \quad (15)$$

In Villain's periodic gaussian approximation⁴ $S_c \rightarrow S_c^V$ where following (5) and (7)

$$Z_c \equiv \int_A e^{-S_c^V} \equiv \sum_{\{N\}} \int_A e^{-\frac{\beta}{4} \sum_x (F[A] - 2\pi N)^2} \quad (16)$$

$$= \sum_{\{q, \vec{l}\}} \int_A e^{-\frac{\beta}{4} \sum_x F^2[A - 2\pi l] + 2(\nabla \phi)^2} \quad (17)$$

$$= Z_m[0] \times Z_{Al}[0], \quad (18)$$

$$Z_m[\xi] \equiv \sum_{\{q\}} e^{\sum_x \xi_x q_x - 2\pi^2 \beta \sum_{x,y} q_x \Delta(x-y) q_y}, \quad (19)$$

$$Z_{Al}[0] = \int_\alpha e^{-\frac{\beta}{4} \sum_x F^2[\alpha]}, \quad \alpha_\mu \in (-\infty, \infty). \quad (20)$$

The sum over $\{N\}$ in (16) maintains gauge invariance under (6). (18) follows from

$$\sum_x \epsilon_{\mu\nu\lambda} F_{\mu\nu} \partial_\lambda^* \phi = - \sum_x \phi \epsilon_{\mu\nu\lambda} \partial_\lambda F_{\mu\nu} = 0, \quad (21)$$

(19) from (15), and (20), which says $\vec{\alpha}$ is a massless noncompact photon, from calculus identity

³While my numerical gauge configurations have many string loops, I have found no homologically nontrivial ones.

⁴I will employ shorthand such as $F_{\mu\nu}[\alpha] \equiv \partial_\mu \alpha_\nu - (\mu \leftrightarrow \nu)$, $F \cdot G \equiv \sum_{\mu,\nu} F_{\mu\nu} G_{\mu\nu}$ and $\sum_{\{N\}} \equiv \prod_{\mu,\nu} \sum_{N_{\mu\nu}=-\infty}^{\infty} \delta(-N_{\nu\mu}, N_{\mu\nu})$.

$\sum_{l=-\infty}^{\infty} \int_{A=-\pi}^{\pi} h(A - 2\pi l) = \int_{\alpha=-\infty}^{\infty} h(\alpha)$.
Eq. (18) also relies on the quadratic character of the Villain approximation; keeping $\mathcal{O}(\Theta^4)$ terms in the $\cos \Theta_{\mu\nu}$'s of S_c would destroy factorization.

Let us make contact with A_μ^{eff} . Following Polyakov [1] the dilute gas expansion and occupation number resummation over $q \in \{0, \pm 1\}$ of Z_m in (19) yields

$$Z_m[\xi] \propto \int_{\Phi} e^{-\frac{1}{4\pi^2\beta} \sum_x (\nabla(\Phi - \xi))^2 - 2\lambda^{-2} \cos \Phi} \quad (22)$$

where $\lambda^2 = 2\pi^2 \beta e^{-2\pi^2 \beta \Delta(0)}$. Dummy scalar Φ is semiclassically identified with ϕ in (7) via (15) because $\sum_x \xi_x q_x = \sum_{x,y} \nabla \xi \cdot \nabla \cdot \Delta_{x,y} q_y$. Comparing (19) to (22) implies for $\xi \rightarrow 0$

$$\sum_y \nabla \cdot \Delta_{x,y} \langle q_y \rangle_m = \frac{1}{Z_m} \frac{\delta Z_m}{\delta \nabla \xi} = \frac{\langle \nabla \Phi \rangle_{\Phi}}{2\pi^2 \beta} \quad (23)$$

where $\langle \rangle_S$ refers to the expectation associated with partition function Z_S . Hence with $\vec{V} \equiv \nabla \times \vec{\alpha}$,

$$\langle \nabla \cdot \vec{H} \rangle_{Alm} = -2\pi \langle q \rangle_m = 0, \quad (24)$$

$$\langle \vec{H}_y \vec{H}_x \rangle_{Alm} = \langle \vec{V}_y \vec{V}_x \rangle_{Al} + \frac{4\pi^2}{Z_m} \frac{\delta^2 Z_m}{\delta \nabla \xi_y \delta \nabla \xi_x}. \quad (25)$$

If $\beta/\lambda \ll 1$, (24) and (25) are reproduced by an $M_\gamma = \lambda^{-1}$ free photon \vec{A}^{eff} with $\vec{H}^{\text{eff}} \equiv \nabla \times \vec{A}^{\text{eff}}$, that is,

$$\nabla \cdot \vec{H}^{\text{eff}} = 0, \quad \langle \vec{H}_y^{\text{eff}} \vec{H}_x^{\text{eff}} \rangle_{\text{eff}} \approx \langle \vec{H}_y \vec{H}_x \rangle_{Alm}. \quad (26)$$

The second relation in (26) relies on masslessness of $\vec{\alpha}$ in (25), shown in (20), and $\cos \Phi \rightarrow -\Phi^2/2$ in (22). \vec{A}^{eff} is the massive Landau-Ginzburg photon and λ the London penetration depth.

The \vec{A} propagator is generated by $Z_{Al}[J]$, defined by adding $J \cdot A$ to the action in (17), which does not affect factorization result (18). Thus the \vec{A} mass M_γ has nothing to do with monopoles q and, hence, nothing to do with \vec{A}^{eff} or λ . In contrast to the mass of $\vec{\alpha}$, M_γ may be nonzero since $J \cdot A$ breaks the pure $\vec{\alpha}$ -dependent form of $Z_{Al}[0]$. Manipulations like those leading to (19) yield $Z_{Al}[J] =$

$$\int_A \sum_{\{\vec{\rho}\}} e^{\sum_x (J + \pi \beta \rho) \cdot A - \frac{\beta}{4} F^2 - \pi^2 \beta \sum_y \rho \cdot \Delta \cdot \rho}. \quad (27)$$

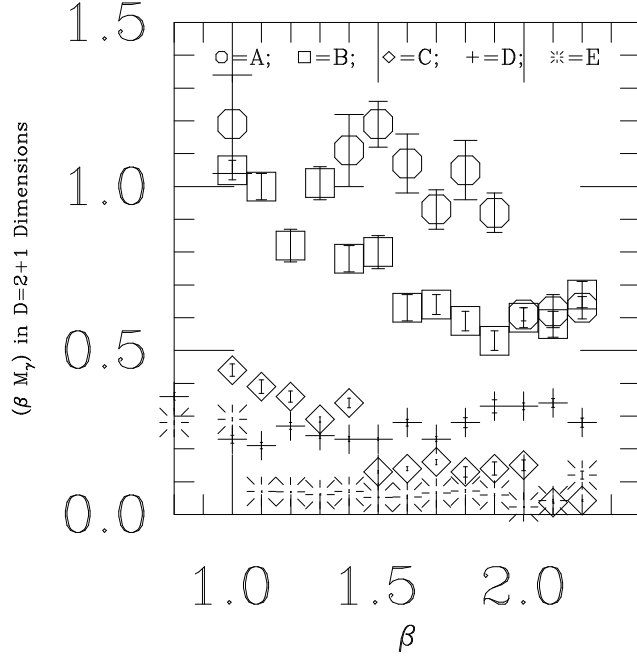


Figure 1. βM_γ in five QED₂₊₁ variations.

Summing $\{\vec{\rho}\}$ is equivalent to summing Dirac string configurations. In Landau gauge $\nabla \cdot \vec{\rho} = 0$ and Z_{Al} is the partition function of a Coulombic $\vec{\rho}$ loop gas. Interestingly $\vec{\rho}$ is a mixed state in the gas since for M_γ to be nonzero

$$\int_{y,y'} \Delta_{x,y}^{\mu\alpha} \Delta_{0,y'}^{\nu\beta} \langle \rho_{y,\alpha} \rho_{y',\beta} \rangle_{\vec{\rho}} \quad (28)$$

must have a negative norm massless mode to cancel the α pole and an independent M_γ mode.

In conclusion the \vec{A} propagator decouples from monopoles q in the Villain approximation and, accordingly, M_γ is independent of the London penetration depth. Numerical experiments described in Section 4 support this result in full cQED.

4. Numerical Experiments

Figure 1 shows that Landau gauge M_γ in cQED (“A”) is relatively insensitive to monopole prohibition (“B”) but dramatically reduced by kink prohibition (“C” and “E”). Kinks are prohibited either by inserting a delta function in the link measure (“C”) or by replacing $\cos F_{\mu\nu}$ in S_c with $\sim F_{\mu\nu}^2$ (“E”). The action for E is not invari-

ant under kink-creating gauge transformations, which are also prohibited. Restoring the possibility of such gauge transformations during Landau gaugefixing (“D”) does not affect M_γ much. This indicates that the kinks responsible for M_γ in A are from the pre-gaugefixing configurations and not specifically created during Landau gaugefixing. (I suspect gaugefixing gives smaller string loops than those responsible for the bulk of M_γ .) At $\beta = 1.8$ the kink number density for cases A-E are $\rho_A = .41(.01)$, $\rho_C \equiv 0$, $\rho_D = .23(.004)$, and $\rho_E \sim 10^{-5}$. Since the $\beta = 1.8$ monopole number density is $8.0(1.1)10^{-3}$, forbidding monopoles doesn’t change the kink density and $\rho_B = \rho_A$.

$\beta \times M_\gamma$ in the Figure, a dimensionless number in $D = 2 + 1$, is the log of the ratio of successive $\vec{p} = 0$ photon propagator timeslices. The central value of A is from 500 S_c -based configurations on $17^2 \times 19$ lattices. The first configuration is thermalized by 500 forty-hit, 40%-acceptance Metropolis sweeps and 5000 checkerboard gaugefixing sweeps. Configurations thereafter are separated by 5 forty-hit Metropolis sweeps and 5000 checkerboard gaugefixing sweeps. Errors are jackknife sigmas based on 10 450-configuration subaverages. Configurations 1 – 50 are omitted from the first subaverage, 51–100 from the second, The numerical photon operator and gauge condition are $S_\mu \equiv \sin A_\mu$ and $\partial_\alpha^* S_\mu = 0$. S_μ corresponds to the gluon operator used in QCD simulations [3]. Since $\sin A_\mu = \sin(\pi - A_\mu)$, S_μ leaves A_μ ambiguous in reflections about $\pm\pi/2$.

B, Landau gauge cQED with monopoles q prohibited, refers to configurations generated according to S_c with the insertion of delta function $\prod_{\{x\}} \delta_{q,0}$ into the link measure. This is implemented starting with the $\theta_\mu = 0$ configuration and linkwise forbidding updates which create monopoles. Landau gaugefixing, which cannot change q , proceeds normally. C refers to S_c configurations with the insertion of kink-forbidding delta function $\prod_{\{N\}} \delta_{N,0}$ into the measure. This insertion affects Landau gaugefixing by forbidding kink-creating gauge transformations. Due to this restriction, a good Landau gauge is not achieved but the photon propagator signal is strong. The tiny residual mass is due to $\mathcal{O}(\Theta_{\mu\nu}^4)$ terms in S_c which ruin factorization (18).

D and E are based on the action $S_E = \frac{\beta}{4} \sum_x F^2$ where (2) defines $F_{\mu\nu}$. Unlike S_c , S_E is invariant only under gauge transformations which preserve $N_{\mu\nu}$. E refers to S_E configurations put as close as possible to Landau gauge with kink-changing gauge transformations forbidden. D refers to S_E configurations fixed to Landau gauge by the *full set* of cQED gauge transformations. From the S_E standpoint D, corrupted by action-changing kink creation and annihilation, is gauge inequivalent to E. The difference between M_γ in D and E, gauge equivalent from the S_c viewpoint, indicates how much kinks generated by the Landau gaugefixing algorithm contribute to M_γ .

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