Form Factors for Semileptonic Decays of Heavy-Light Hadrons

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The strong coupling lattice QCD solution [1] for the Isgur-Wise functions, parametrising the semileptonic decay form factors of hadrons containing a single heavy quark, is reviewed. Several useful features connected with the result are pointed out.

In the quark mass $M \to \infty$ limit, QCD has an exact $SU(2N_f)$ spin-flavour symmetry. This symmetry relates many matrix elements and form factors of hadrons containing heavy quarks [2,3]. In particular, the semileptonic weak decay form factors of s-wave hadrons containing a single infinitely heavy quark can be reduced to two unknown functions, one for mesons (ξ) and another for baryons (ζ). The knowledge of these Isgur-Wise functions is of practical importance in a precise determination of the quark mixing matrix element V_{cb} : the experimentally measured $B \to D$ semileptonic decay rates can be fitted to a leading term involving ξ plus phenomenological estimates of corrections suppressed by powers of 1/M.

In the $M \to \infty$ limit, it is convenient to scale all variables so as to explicitly separate the factors of heavy quark mass, e.g. a heavy hadron state is characterised by its four-velocity v_{μ} and the Isgur-Wise functions depend only on the Lorentz invariant $v \cdot v'$. In the static geometry, the light QCD degrees of freedom decouple from the dynamics of the heavy quark, and flavour independence of QCD fixes the absolute normalisation of the form factors: $\xi(v \cdot v' = 1) = 1 = \zeta(v \cdot v' = 1)$.

As an explicit case, consider the form factor for semileptonic weak decay $\overline{B} \to D l \overline{\nu}$, and its behaviour as m_b and m_c go to infinity. Only the partially conserved vector current contributes:

$$\langle D(p') | \bar{c} \gamma_{\mu} b | \bar{B}(p) \rangle = (p+p')_{\mu} f_{+}(q^{2}) + \frac{M_{B}^{2} - M_{D}^{2}}{q^{2}} q_{\mu} [f_{0}(q^{2}) - f_{+}(q^{2})] \xrightarrow{M \to \infty} \sqrt{M_{B} M_{D}} (v+v')_{\mu} \xi(v \cdot v') .$$
 (1)

Here q = p - p' is the momentum transfer, and $v \cdot v' = (M_B^2 + M_D^2 - q^2)/2M_BM_D$. To eliminate the spurious pole at $q^2 = 0$, we must have $f_0(0) = f_+(0)$. The divergence of Eq.(1) gives

$$\frac{m_b - m_c}{M_B - M_D} \langle D(p') | \bar{c}b | \bar{B}(p) \rangle = (M_B + M_D) f_0(q^2)$$
$$\stackrel{M \to \infty}{\longrightarrow} \sqrt{M_B M_D} (1 + v \cdot v') \xi(v \cdot v') \quad . \tag{2}$$

This result contains several noteworthy points: (a) A partially conserved current can undergo an ultraviolet finite renormalisation. Thus in general a renormalisation constant $Z_0(M_B, M_D, q^2)$ appears multiplying the matrix element on the l.h.s. of these equations. According to the Ademollo-Gatto theorem [4], the deviation from 1 of the form factor for the charge operator is of second order in the symmetry breaking parameter:

$$f_+(M_B \approx M_D, q^2 = 0) = 1 + \mathcal{O}((m_b - m_c)^2)$$
 (3)

Since only first order deviation from the heavy quark flavour symmetric limit appears in Eq.(2), it follows that the associated renormalisation constant obeys $Z_0(M_B = M_D, q^2 = 0) = 1$. In a mass independent renormalisation scheme, e.g. dimensional regularisation, the renormalisation constants are independent of the momentum insertion. In such a case, we can forget about Z_0 altogether provided we use Eq.(2) in the heavy quark flavour symmetric limit only. On the lattice, with an explicit momentum cutoff, a q^2 dependent renormalisation constant is not ruled out. But the fact that the quark mass operator is a local point operator on the lattice (note that Fourier transform of a δ -function is a constant) may still allow one to "infer" the continuum result, particularly when the lattice result has a simple structure.

(b) Due to an enhanced kinematic factor on the r.h.s., the quark-hadron duality analysis of Eq.(2) produces a sum rule yielding a tighter upper bound for ξ than Bjorken's [5]:

$$\xi(v \cdot v') \leq 2/(1 + v \cdot v') \quad . \tag{4}$$

An implication is that the sum rules containing the scalar operator would have less contamination from excited states and saturate faster compared to sum rules containing other operators.

(c) The 3-point function appearing on the l.h.s. obeys a Ward identity, which relates it to 2-point functions [6]:

$$(m_b - m_c) \langle D(y) | \bar{c}b(q_\mu = 0) | B(x) \rangle$$

= $\langle D(y) | D(x) \rangle - \langle B(y) | B(x) \rangle$. (5)

This identity guarantees that $Z_0(M_B = M_D, q^2 = 0) = 1$. Moreover, it is exact even for distinct m_b and m_c , and therefore contains enough information to determine ξ (even though $q_{\mu} = 0, v \cdot v'$ can be varied by changing M_B and M_D). In practice, the difference of 2-point functions may be a simpler object to study than the 3-point function, for example while applying QCD sum rules.

In Ref.[1], ξ was evaluated in strong coupling lattice QCD, and then the result was converted to continuum language. The logic of this analysis was inspired by Wilson's renormalisation group based solution of the Kondo problem [7]—the weak and strong coupling results for the same quantity can be related to each other provided one can evaluate the RG connection between the two with high precision. It must be kept in mind that the weak and the strong coupling fixed points describe quite different physics; the scaling relations which hold at one fixed point may not hold at the other fixed point. Generically, we can write

$$f(\lambda) = f^*[1 + \mathcal{O}(a/\lambda)] , \quad f^* \equiv f(\lambda = \infty) , \quad (6)$$

where λ is the correlation length. Scaling is exact only on the critical surface defined by $\lambda = \infty$. The $\mathcal{O}(a/\lambda)$ terms are non-universal, i.e. they can be different for different quantities and hence violate scaling. As a consequence, even along the



Figure 1. Strong coupling diagrams for the scalar form factor in $\overline{B} \to D l \overline{\nu}$ decays: (a) the leading tree level contribution, (b) an example of the next order loop contribution.

renormalised trajectory (i.e. for an action containing no lattice artifacts, yet having a finite cutoff), there exist scaling violations. For example, Wilson's calculation demonstrates how the spectrum of the Kondo problem changes under RG evolution. (Another example is the ratio of two particle binding energy to a single particle mass. It vanishes at the strong coupling fixed point, while it can be non-zero at the weak coupling fixed point of an interacting theory.) Thus scaling relations of the theory must be extracted only in the weak coupling region, after individually converting each quantity calculated at strong coupling to its weak coupling analogue.

In QCD scaling violations are $\mathcal{O}(a\Lambda_{QCD})$, once the coupling g^2 is traded off for a dynamically generated scale. To keep them under control, it is of paramount importance to select a quantity which has a simple RG connection between the strong and weak coupling fixed points, and which still contains the physics of interest. The choice of the quark mass operator happens to be crucial for this reason—in a lattice regularisation respecting the quark flavour symmetry, it is a local point operator with a non-perturbatively constrained renormalisation constant.

The scalar form factor is easily evaluated in the strong coupling limit, the leading term being a simple pole contribution from the tree level hadron diagram of Fig.1a. The subleading corrections arise from diagrams containing light quark loops. The diagrams where the loops are part of the external \overline{B} and D legs merely redefine the external parameters. The modification of the form factor arises from diagrams of type Fig.1b, where the loops interact with the scalar hadron state. All such diagrams involve at least two more heavy quark propagators compared to the tree level diagram, and hence are suppressed in the $M \to \infty$ limit given the mass independence of quark-gluon interactions.

Explicitly, with staggered fermions (their chiral symmetries are essential in restricting the renormalisation of the quark mass operator) in the strong coupling limit, the scalar form factor is:

$$f_0(q^2) = \frac{1}{1 - (4/M_{sc}^2) \sum_{\alpha} \sin^2(q_{\alpha}/2)} + \mathcal{O}(\Lambda_{QCD}^2/M^2) , \qquad (7)$$

where M_{sc} is the mass of the scalar meson \overline{cb} . The normalisation of this form factor at $q^2 = 0$ confirms the expected non-renormalisation constraint. The "sin" –function in the denominator is a reflection of the nearest neighbour lattice action used in the analysis; a more general lattice action would give rise to a different function. (As a matter of fact the explicit structure of the strong coupling lattice action generated by RG evolution is not known for QCD.) What would remain unchanged for any lattice action, however, is the simple pole structure of $f_0(q^2)$. In the continuum language, this simple pole corresponds to

$$\xi(v \cdot v') = 4/(1 + v \cdot v')^2 \quad , \tag{8}$$

where we have set $M_B = M_D$ and then taken the $M \to \infty$ limit (which implies $M_{sc} \to M_B + M_D$). A similar analysis for the baryon form factor yields $\zeta(v \cdot v') = 2/(1 + v \cdot v')$.

The result of Eq.(8) fits the experimental data for $\overline{B} \to D^* l \overline{\nu}$ decays reasonably well [8,9]. It is reassuring to find that the agreement is within $\approx 10\%$, which is the precision of the experimental data as well as the estimated magnitude (e.g. from QCD sum rule calculations) of the $\mathcal{O}(1/M)$ difference between the actual form factor and ξ .

Encouraged by this agreement, we turn the problem around and construct a phenomenologi-

cal model of hadron wavefunctions which reproduces Eq.(8) exactly. Such wavefunctions can be useful for studying other hadronic properties. The scalar form factor at $q^2 = 0$ is nothing but the overlap of the initial and final state light-cone wavefunctions. Eqs.(2) and (8) require

$$\int_{0}^{\infty} du \ \phi_{D}^{*}(u) \ \phi_{B}(u) = \left(\frac{2}{1+v \cdot v'}\right)_{q^{2}=0}^{3/2} , \quad (9)$$

where e^{-u} is the fraction of longitudinal lightcone momentum carried by the heavy quark. A simple choice for the wavefunction is

$$\phi(u) = 2(M/m_0)^{3/2} \ u \ e^{-uM/m_0} \ , \tag{10}$$

with an M-independent parameter m_0 . By replacing the factor of u with $1 - e^{-u}$ and choosing m_0 to be two-thirds the constituent quark mass in the chiral limit, the structure of this wavefunction can be symmetrised between the two constituent quarks of the meson. It would be interesting to explore how accurately this wavefunction describes many other properties of hadrons.

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