

DECONFINEMENT AND NON-ZERO BARYON DENSITY

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I discuss a few issues related with deconfinement at finite baryon density by considering lattice results for two colors QCD, and “toy” studies of three colors QCD.

Substantial progress has been achieved in the understanding of the phase diagram of models that share the same global symmetries of QCD¹. Confinement, however, is not handled in a completely satisfactory way by these studies. Among the approaches which include confinement, the monomer–dimer–polymer representation of the partition function² is limited to the infinite gauge coupling limit, where the dynamics is far from realistic, and the nature of the phase transition is completely dominated by lattice artifacts. Polyakov loop models³, on the other hand, recover the quenched limit of QCD and incorporate confinement also in the continuum. Unfortunately, as these studies rely on large quark masses they cannot describe the chiral transition. Approaches based on the Dyson-Schwinger equation, which in principle can treat both chiral symmetry breaking and confinement, are intrinsically approximate⁴.

Summing up (and oversimplifying), at finite density chiral symmetry and mechanisms of confinement have been studied independently, but little is known about their interplay, at least not from first principles. Such interplay lie at the very core of the physics of the problem – after all, deconfinement and chiral symmetry restoration at finite density are intimately related with asymptotic freedom, and long distance screening. In addition, it has also been suggested that the exact realisation of confinement might be crucial for a successful algorithm⁵.

A so far unique possibility to study a gauge model at finite density is afforded by two colors QCD, which can be studied via numerical simulations even at non-zero chemical potential⁷. I shall first discuss the interplay of thermodynamics, deconfinement and chiral symmetry in this model, then I shall try to assess the chances to address similar questions in real (three colors) QCD.

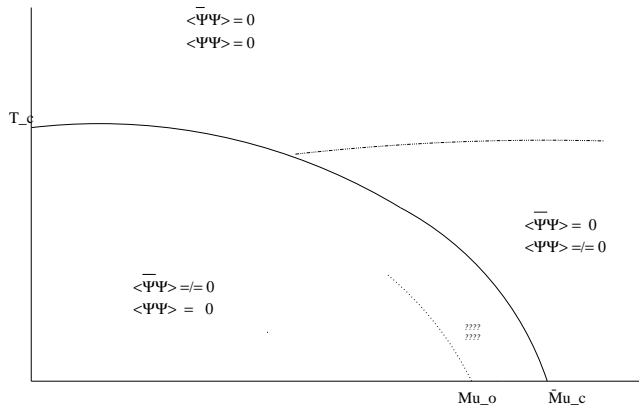


Figure 1. A sketchy view of the phase diagram of two colors QCD for a non-zero quark mass

1 Two colors

In Fig. 1 I sketch a possible (i.e. consistent with the numerical results described below) phase diagram of two colors QCD, in the chemical potential-temperature plane, for an arbitrary, non-zero bare quark mass.

The $\mu = 0$ axis has been investigated in ⁶, while the $T \simeq 0$ axis has been studied in ⁷, and discussed at this meeting ⁸. I stress that both axes, even the $\mu = 0$ one ⁹, present a number of interesting open problems. The study presented here, which attempts at reconstructing the complete phase diagram, is even more exploratory. None the less, I feel that some interesting aspects are already emerging at this stage.

Let me start by considering what happens while increasing chemical potential at fixed temperature. According to the standard scenario, at zero temperature, for a chemical potential μ_o comparable with the mass of the lightest baryons, such baryons start to be produced thus originating a phase of cold, dense matter. For $SU(2)$ baryons (diquarks) are bosons (as opposed to the fermionic baryons of real QCD). There are then important differences between the dense phase in $SU(2)$ and $SU(3)$, as, obviously, the thermodynamics of interacting Bose and Fermi gases is different. In particular, diquarks might well condense, however partial quark liberation is possible as well. We might then expect a rather complicated “mixed” nuclear matter phase, perhaps characterised by both types of condensates – the one marked with question marks in Fig. 1 (such mixed phases are also predicted by more detailed instanton stud-

ies ¹¹). Infact, on the cold lattice (Fig. 2, left) a pure cubic term, expected of a cold phase of massless free quarks, does not describe the behaviour of the number density. We do not have pure Bose condensate either, thought. For $\mu > \mu_c$ our results ⁷ suggest $\langle \bar{\psi}\psi \rangle = 0$, i.e. pure diquark condensation. At the same time (see below) indications of long distance screening and deconfinement become more pronounced, so the behaviour might get closer to a pure free quark phase, which, however, does not seem supported by the data: as the diquark states appear to be bound ⁷, their condensates can still influence the thermodynamics. A direct measure of diquark condensate should completely clarify this point ⁸. A “pure” free quark phase, with complete restoration of chiral symmetry (i.e. $\langle \bar{\psi}\psi \rangle = \langle \psi\psi \rangle = 0$) could be reached at even larger μ – as this region is dominated by lattice saturation artifacts, improved/perfect actions ¹² might be necessary to explore it.

Despite these uncertainties, it seems anyway clear that screening and deconfinement compete against condensation, and this is better seen on a “warmer” lattice, close to T_c : here, $n \propto \mu^3$, consistent with a free, massless quark gas (with a somewhat surprisingly small temperature contribution ¹⁰), suggesting the existence of a critical temperature for diquark condensation (i.e. a temperature beyond which diquarks will not condense at any value of the chemical potential) smaller than T_c .

To obtain a more direct probe of deconfinement, we can look at the interquark potential by calculating the correlations $\langle P(O)P^\dagger(z) \rangle$ of the zero momentum Polyakov loops, averaged over spatial directions. This quantity is related to the string tension σ via $\langle P(O)P^\dagger(z) \rangle \propto e^{-\sigma z}$.

We show the results for the Polyakov loop correlators in Fig 3, where we compare the behaviour at various temperature with that at various chemical potentials. In both cases the trend suggests increased fermion screening, string breaking and the passage to a deconfined phase.

2 Two and three colors

To gain further insight into the “dynamical” role of the chemical potential, and its effect on the gauge fields, we can take a look at a Toy ¹³ version of the model, obtained by inverting the Dirac operator at nonzero chemical potential in a background of gauge fields generated at zero chemical potential. For these configurations the potential is always the same as at $\mu = 0.0$. Interestingly, we have found that in this case the behaviour of the diquarks propagators resembles that of the infamous quenched $SU(3)$ “baryonic pions” measured in ¹⁴. This confirms that the nature of the interquark forces and the deconfinement transition might well play a major role in $SU(2)$ and $SU(3)$ alike.

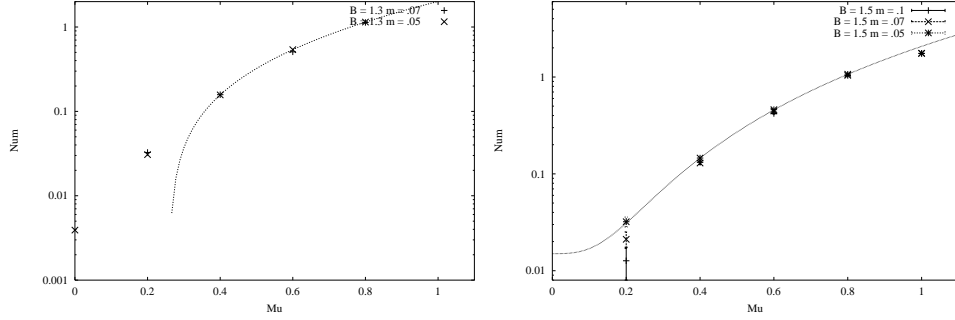


Figure 2. Number density as a function of the chemical potential, for different masses, on a cold (warm) lattice on the left(right) hand side. The cubic fits to $a\mu^3 + b\mu^2 + c$ are superimposed. $b, c \simeq 0$ on the warm lattice, consistent with a pure massless quark gas, while a 'mixed' phase is possible on the cold lattice (see text)

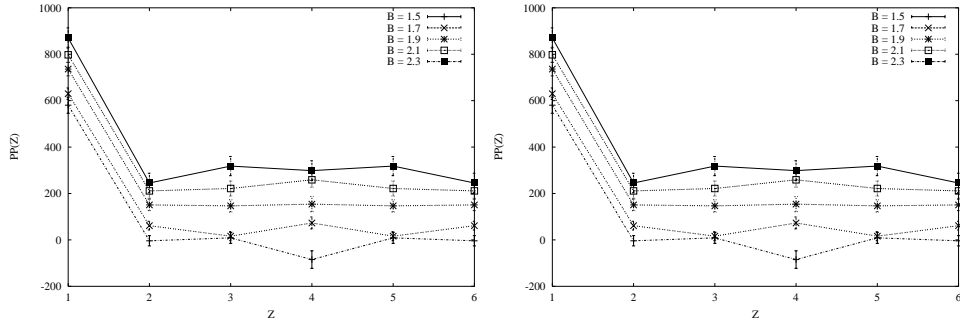


Figure 3. Correlations of the zero momentum Polyakov loop as a function of the space separation. The left diagram is for $\mu = 0$, and β as indicated. The righthand part is for $\beta = 1.5$ and μ as indicated. In both cases we observe long range ordering possibly associated with deconfinement

It is essential to have the correct quark–quark and quark–antiquark forces, since they control and soften diquark condensations, including the pathological ones. The inclusion of the chemical potential into the dynamics seems mandatory.

This brings us back to the necessity of first principle calculations of QCD at finite density, hence to the problems with complex actions.

3 Three colors

Attempts at beating such problems fall in two main categories. Firstly, simulations at $\mu = 0$ ¹⁵, or imaginary chemical potential (which does not systematically bias the ensemble)¹⁶, combined with reweighting and/or analytic continuation. The main problem encountered here is that the ensemble does not overlap with the non-zero density state of interest. Because of this, *physical* transitions might *disappear*. Other approaches (as in the quenched approximation, or calculations which use the modulus of the determinant¹⁷) include conjugate quarks so to keep the action real when the chemical potential is included. The main problem here is the generation of light particles with baryonic number (baryonic pions). Because of this, *unphysical* transition might *appear*.

There are reasons to believe that both problems might be alleviated in the proximity of the $\mu = 0$ deconfinement phase transition. Firstly, baryons will become lighter, then easier to fluctuate. Secondly, baryonic pions will become heavier, and possibly decouple or dissolve (clearly, one will have to pay attention to the counting of degrees of freedom).

Consider the partition function in the $g = \infty$ limit of QCD¹⁸:

$$Z(\lambda, \mu) = 2\cosh(N_t N_c \mu) + \sinh[(N_t + 1)N_c \lambda] / \sinh(N_t \lambda)$$

where the variational parameter λ (essentially, $\langle \bar{\psi}\psi \rangle$) is to be determined by a minimum condition, N_t is the number of points in time direction, N_c the number of colors. When N_t grows large $Z \simeq e^{N_t N_c \mu} + e^{N_t N_c \lambda}$: the critical chemical potential is a measure of the strength of symmetry breaking.

For a purely imaginary chemical potential, $\cosh(N_t N_c \mu) \rightarrow \cos(N_t N_c \mu)$. We see the expected periodicity $2\pi/(N_t N_c)$, and we note that the chemical potential term can be ignored for large N_t (zero temperature). Indeed, we have verified that in this limit any dependence on the chemical potential is lost¹⁹: clearly, $\langle \bar{\psi}\psi \rangle$ as a function of (complex) μ is a constant in the half plane $\Re(\mu) < \mu_c$, μ_c being a real number. By increasing the temperature, the effective potential changes with imaginary μ , and the chiral condensate *increases* with imaginary chemical potential, as it should. However, for this effect to be appreciable, one needs to be very close to the critical point as the reader can easily check by exploiting the above formula.

Can we guess where we should work in real QCD for imaginary μ to be useful? Clearly, we need a region with large $\mu = 0$ derivatives. The fluctuations of baryons are measured by the baryon number susceptibility²⁰

$$\chi(T, \mu) = \partial \rho(\mu, T) / \partial \mu = \partial^2 \log Z(\mu, T) / \partial^2 \mu$$

Lattice results indicate the range where $\chi(T, \mu = 0)$ is significantly different from zero. This is the candidate region for performing imaginary μ calculation below T_c – a narrow, but not minuscule interval. Perhaps it is also worth considering that, as $\log Z(T, \mu) = K + \chi(T, 0)\mu^2 + O(\mu^4)$, the baryon number susceptibility obtained by numerical differentiation of the number density with imaginary chemical potential should be the negative of the one measured with standard methods. It should then be possible to assess and use imaginary chemical potential calculations without use of the Laplace transform, or other analytic continuation techniques. This should open the possibility to study the effect of a small chemical potential on the deconfinement transition and to address issues such as spontaneous parity violation near T_c ²¹.

In the same “hot” region reweighting too might prove useful. To check that, we have tested the Glasgow method in one dimensional QCD²², a solvable model without SSB, but with baryons, whose partition function is easily related to that of four dimensional QCD. At large “temperature” the Glasgow method reproduces the exact results ²³, thus supporting the idea that reweighting methods can be successfully used in that regime. It should be noticed, however, that an analysis of the Lee Yang zeros shows that in this case there is no pathological onset ²⁴: the Glasgow method only rearranges zeros on circular patterns. It is reasonable to expect that the same happens in high temperature, four dimensional QCD.

The simple discussions presented here support qualitative arguments suggesting that imaginary chemical potential and reweighting might be successful at large temperature, close to the deconfining transition. However, they also confirm the feeling that the existing algorithms will not be adequate to study the low temperature, finite density regime no matter how big the statistics is. Perturbative approaches to QCD which use composite nucleons as fundamental variables ²⁵ might well offer a promising avenue for finite density calculations in the low temperature, confined phase.

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