COLOR SUPERCONDUCTIVITY IN TWO- AND THREE-FLAVOR SYSTEMS AT MODERATE DENSITIES

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Abstract. Basic features of color superconductivity are reviewed, focusing on the regime of "moderate densities", which is not accessible by perturbation theory. We discuss the standard picture of two- and three flavor color superconductors and study the color-flavor unlocking phase transition within an NJL-type model.

1. Introduction

The structure of the QCD phase diagram is one of the most exciting topics in the field of strong interactions (For reviews see, e.g. [1, 2, 3, 4, 5]). For a long time the discussion was restricted to two phases: the hadronic phase and the quark-gluon plasma (QGP). The former contains "our" world, where quarks and gluons are confined to color-neutral hadrons and chiral symmetry is spontaneously broken due to the presence of a non-vanishing quark condensate $\phi = \langle\bar\psi\psi\rangle$. In the QGP quarks and gluons are deconfined and chiral symmetry is (almost) restored, $\phi \simeq 0$.

Although color-superconducting phases were discussed already in the '70s [6, 7, 8] and '80s [9], until quite recently not much attention was payed to this possibility. This changed dramatically after it was discovered that due to non-perturbative effects, the gaps which are related to these phases could be of the order of $\Delta \sim 100$ MeV [10, 11], much larger than expected from the early perturbative estimates. Since in standard weak-coupling BCS theory the critical temperature is given by $T_c \simeq 0.57\,\Delta(T=0)$ [12], this also implies a sizable extension of the color-superconducting phases into the temperature direction [13]. It was concluded that color-superconducting phases could be relevant for neutron stars [14, 15] and – in very optimistic cases – even for heavy-ion collisions [16].

Rather soon after the beginning of this new era, it was noticed that there is probably more than one color superconducting phase in the QCD phase diagram. At large chemical potential, where up, down, and strange quarks can condense, matter is expected to be in the so-called color-flavor locked phase [17], whereas at intermediate densities, just above the deconfinement phase transition, we might have a two-flavor color superconductor (2SC). Other phases have been suggested more recently, like crystalline phases in a small window between the 2SC phase and the CFL phase [18, 19] or a CFL phase accompanied by a kaon condensate (CFL + K) [20, 21]. In addition, those color or flavor degrees of freedom which do not participate in the "standard" condensates could pair in different, usually more fragile, channels, thus forming additional phases [22, 23, 24].

In this talk we review some of the basic features of color superconductivity, mainly focusing on the "standard" phases for two and three flavors and the transition from the 2SC to the CFL phase. Very recently (after the Stara Lesna workshop) it has been argued that in neutron stars there might be no 2SC phase at all, because of the rather different Fermi surfaces of u- and d- quarks in charge neutral matter [25]. We will briefly comment on this possibility in the end of this article.

2. Diquark condensates

According to Cooper's theorem any arbitrarily weak interaction leads to an instability at the Fermi surface which is cured by the formation of Cooper pairs. At very large densities, where asymptotic freedom allows to perform the analysis in terms of a single gluon exchange it can easily be shown that there are indeed attractive channels and hence QCD matter must be a color superconductor at these densities. However, because of the large number of possible channels related to the quantum numbers of spin, flavor and color, we can almost be sure that also in the nonperturbative regime just above the deconfinement phase transition some of them will be attractive.

In general, a diquark condensate may be written as $\langle\psi^T \mathcal{O} \psi\rangle$, where ψ is a quark field and $\mathcal{O}$ an operator, acting in color, flavor and Dirac space. It can also contain derivatives, but we will not consider this possibility here. A priori, the only restriction to $\mathcal{O}$ is provided by the Pauli principle, which requires that $\mathcal{O}$ must be totally antisymmetric. This still leaves many possibilities, and thus the interaction must decide about the actual condensation pattern.

As already mentioned, at very large chemical potentials, $\mu \gg \Lambda_{QCD}$, $\alpha_s(\mu)$ is small and the problem can be (and has been) attacked from first principles [13, 16, 26, 27, 28]. To estimate the range of validity of these calculations we assume (quite optimistically) that the perturbative regime begins at $\mu \approx 1.5$ GeV. For two massless flavors this corresponds to a baryon density $\rho_B = 2/(3\pi^2)\mu^3 \approx 30$ fm^{-3}, which is about 175 times nuclear saturation density. It turns out that the situation is even worse: In a numerical study Rajagopal and Shuster [29] found that (gauge dependent) higher-order terms can only be neglected if $\mu \gg 10^5$ GeV!

Hence, asymptotic studies, although interesting by themselves, cannot be trusted down to densities which are present, e.g., in the interior of neutron stars. In this regime one has to rely on effective interactions, like instanton interactions [30], or (local or nonlocal) 4-point interactions ("NJL-type models") whose structure is also abstracted from the instanton vertex [10] or purely phenomenological. This is quite analogous to the Landau-Migdal interaction used to describe nuclear matter. However, we should be aware of the fact that there are presently no data to constrain the parameters in the deconfined phase itself. They are therefore usually fixed in vacuum, which is clearly a source of big uncertainties.

Nevertheless, there are good reasons to believe, that a dominant role might be played by the Lorentz-invariant scalar ($J = 0^+$) condensate,

$$s_{AA'} \;=\; \langle \psi^T C\gamma_5\, \tau_A \lambda_{A'} \psi \rangle \;, \tag{1}$$

which corresponds to the most attractive channel, both for interactions with the quantum numbers of a single gluon exchange as well as for instanton induced interactions. Here C is the matrix of charge conjugation, and τ_A and $\lambda_{A'}$ are the antisymmetric generators of flavor-$SU(N_f)$ and color-$SU(N_c)$, respectively. Throughout this article, we will restrict ourselves to the physical number of colors, $N_c = 3$. Then the $\lambda_{A'}$ denote the three antisymmetric Gell-Mann matrices, λ_2, λ_5 and λ_7, i.e., $s_{AA'}$ is a color anti-triplet. Concerning the number of flavors we begin with $N_f = 2$ in the next section and we will dicuss $N_f = 3$ later on.

3. Two flavors

For two flavors ($N_f = 2$), the flavor index in Eq. (1) is restricted to $A = 2$, describing the pairing of an up quark with a down quark. In the limit of massless up and down quarks $s_{2A'}$ is invariant under chiral $SU(2)_L \times SU(2)_R$ transformations. The three condensates s_{22}, s_{25}, and s_{27}, form a vector in color space, which always can be rotated into the $A' = 2$-direction. Hence the two-flavor superconducting state (2SC) state can be characterized by

$$s_{22} \neq 0 \quad \text{and} \quad s_{AA'} = 0 \quad \text{if} \quad (A, A') \neq (2, 2)\,. \tag{2}$$

Since only the first two colors ("red" and "green") participate in the s_{22}, while the third one ("blue") does not, color $SU(3)$ is spontaneously broken down to $SU(2)$. As a result five of the eight gluons acquire a mass [31].

Like in ordinary BCS theory the pairing produces a gap in the spectrum of the corresponding quasiparticles, which in the case of massless quarks is characterized by the dispersion laws

$$E_{\mp}(\vec{p}) \;=\; \sqrt{(p \mp \mu)^2 + |\Delta|^2}\;, \tag{3}$$

where $p = |\vec{p}|$. The gap Δ is proportional to s_{22} and is determined by a gap equation. For local 4-point intercations the latter takes the form

$$\Delta \;=\; const.\Delta \int \frac{d^3p}{(2\pi)^3}\,(\frac{1}{E_-}\tanh\frac{E_-}{2T} + \frac{1}{E_+}\tanh\frac{E_+}{2T})\;, \tag{4}$$

where *const.* contains a coupling constant in the scalar diquark channel and degeneracy factors. The result is typically of the order of 100 MeV [10, 11].

Until this point we have neglected all other possible condensates, which might compete or coexist with s_{22}. Because of the empirical fact that the (approximate) chiral SU(2) symmetry of the QCD Lagrangian is not respected by the QCD vacuum, it is natural to ask whether the quark (-antiquark) condensate

$$\phi \;=\; \langle\bar{\psi}\,\psi\rangle\;, \tag{5}$$

persists also in the ground state of QCD matter at finite baryon density. This question has been addressed first by Berges and Rajagopal [32] within a phenomenological NJL-type model. For massless quarks at $T = 0$ they found a first-order phase transition from the vacuum state with $\phi \neq 0$ and $s_{22} = 0$ to a high-density phase with $s_{22} \neq 0$ and $\phi = 0$. This is different if there is a small quark mass m which explicitly breaks chiral symmetry. In this case ϕ cannot exactly vanish above the phase transition and coexists with the diquark condensate. In fact, just above the phase transition the gaps related to the two condensates can be of similar magnitude [32].

However, this is not yet the whole story: At finite density the existence of Lorentz non-invariant expectation values becomes possible. The most obvious example is of course the density itself,

$$\rho \;=\; \langle\bar{\psi}\,\gamma^0\,\psi\rangle\;, \tag{6}$$

which transforms like the time component of a 4-vector. Together with s_{22} and ϕ this means that color-$SU(3)$, chiral symmetry and Lorentz invariance are broken in the system. Therefore a fully selfconsistent description requires to take into account further condensates which are no longer prohibited by one or more of the above symmetries.

It turns out that in general three more condensates should be considered: First, after color-$SU(3)$ is broken, there is no need for the scalar and vector densities to be the same for "red" and "blue" quarks. Hence, in addition to Eqs. (5) and (6) there could be condensates of the form

$$\phi_8 \;=\; \langle\bar\psi\,\lambda_8\,\psi\rangle \;=\; \frac{2}{\sqrt{3}}\,(\phi_r - \phi_b) \tag{7}$$

and

$$\rho_8 \;=\; \langle\bar\psi\,\gamma^0\,\lambda_8\,\psi\rangle \;=\; \frac{2}{\sqrt{3}}\,(\rho_r - \rho_b)\;. \tag{8}$$

Note that all green quantities are equal to the red ones. Since the scalar densities are closely related to the constituent quark masses, a non-vanishing ϕ_8 would mean that the constituent masses of red and blue quarks, M_r and M_b, can differ from each other.

Finally, there could be another diquark condensate of the form [9, 33, 37]

$$s'_{22} \;=\; \langle\psi^T\,C\gamma^0\gamma_5\,\tau_2\,\lambda_2\,\psi\rangle\;, \tag{9}$$

which breaks all three symmetries, i.e., Lorentz invariance, color-$SU(3)$, and chiral symmetry, at the same time. It is related to a second gap parameter Δ'.

The simultaneous treatment of these six condensates leads to a set of six coupled gap equations which we have analyzed in Ref. [34] within an NJL-type model. The dispersion laws for the paired quarks now take the form

$$E_\mp(p) \;=\; \sqrt{(\sqrt{\vec p^{\,2} + M_{eff}^2} \mp \mu_{eff})^2 + |\Delta_{eff}|^2}\;, \tag{10}$$

where M_{eff}, μ_{eff} and Δ_{eff} are functions of the six condensates [34]. It is interesting, that under certain circumstances Δ_{eff} can vanish, even if the gap parameters Δ and Δ' both are non-zero. However, there are indications that these effectively gapless modes might be always unstable [34].

Some numerical results of our analysis at $T = 0$ are displayed in Fig. 1. On the r.h.s. we show the constituent mass M_r of the red quark and the diquark gap Δ as functions of μ. The bahavior is quite similar to the results of Ref. [32], with a phase transition into a color-superconducting phase at $\mu \simeq 400$ MeV. On the r.h.s. the second diquark gap Δ' and the difference between red and blue constituent quark masses, $M_r - M_b$ are shown. We see that – at least for our model interaction – these quantities are relatively small, which a posteriori justifies their usual negligence.

On the l.h.s. of Fig. 2 the number densities of red and blue quarks are displayed as functions of μ. As one can see, the density of the paired quarks is about 10 - 20% larger than the density of the unpaired quarks in the regime which is shown. One might therefore ask, how matter should arrange

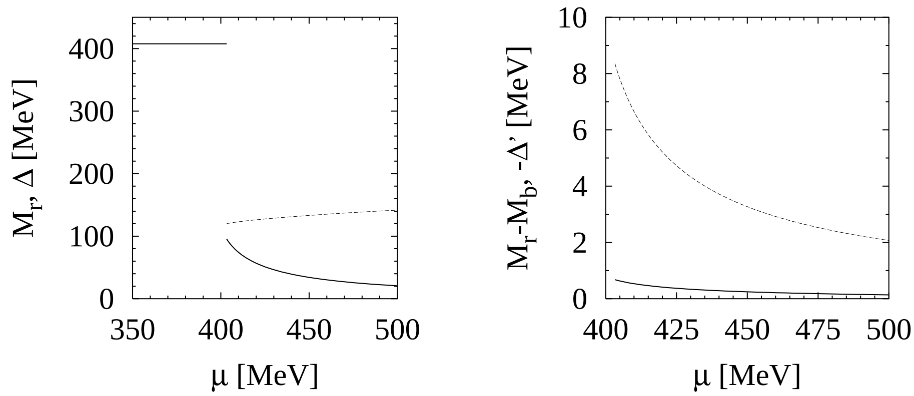

Figure 1. Various quantities as functions of the quark chemical potential μ [34]. Left: M_r (solid), Δ (dashed). Right: $M_r - M_b$ (solid), $-\Delta'$ (dashed).

itself to be color neutral. A possible scenario could be that several domains emerge in which the symmetry is broken into different directions, such that the total number of red, green and blue quarks is equal. Alternatively we could construct a uniform phase with equal densities of all three colors. In this case the chemical potential has to be larger for the unpaired quarks than for the paired ones. In order to compare these two possibilities the energy per quark as function of the total quark number density is shown on the r.h.s. of Fig. 2. The dashed line corresponds to quark matter with equal densities of paired and unpaired quarks, the solid line corresponds to quark matter with equal chemical potentials. Obviously the latter is energetically favored, but the difference is small and the situation might change, if surface effects are included.

So far we have assumed that one color (the "blue" quarks) does not participate in a condensate. However, because of Cooper's theorem we should expect that they will also condense if there is attraction in an appropriate channel. Since only quarks of a single color are involved, the pairing must take place in a channel which is symmetric in color. Assuming s-wave condensation in an isospin-singlet channel, a possible candidate is spin-1 [10]. This interesting possibility has recently been analyzed in Ref. [23] and will be discussed in more detail in Jiří Hošek's contribution [35].

4. Three flavors

For two flavors the flavor index in Eq. (1) was restricted to $A = 2$. This is different for $N_f = 3$, where the flavor operators τ_A denote the three antisymmetric Gell-Mann matrices, i.e., $A = 2, 5, 7$, describing ud-, us-, and ds-pairing, respectively. The two-flavor condensation pattern, Eq. (2)

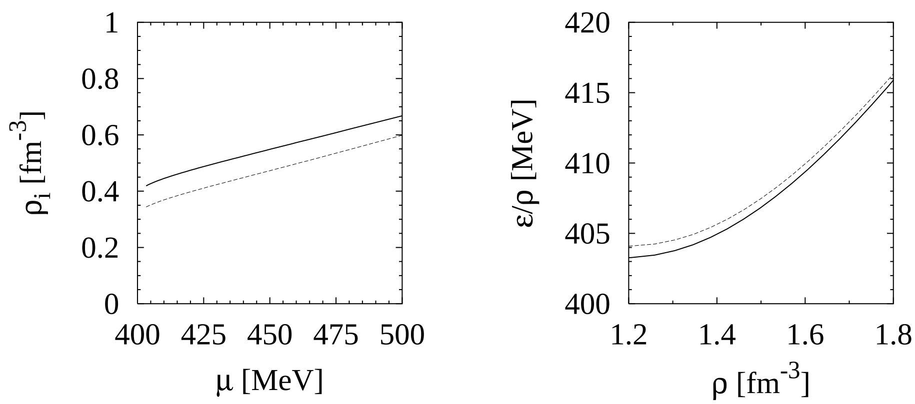

Figure 2. Left: Number densities of red quarks (solid) and blue quarks (dashed) as functions of the quark chemical potential μ. Right: Energy per quark as function of the total quark number density for a color superconducting system with equal densities of gapped and ungapped colors (dashed) and with unequal densities as given in the left panel (solid).

is still possible, but now there are several other combinations which cannot be transformed into s_{22} via color or flavor rotations.

In the case of three degenerate light flavors, dense matter is expected to form a so-called color-flavor locked (CFL) state [17], characterized by the situation

$$s_{22} = s_{55} = s_{77} \neq 0 \quad \text{and} \quad s_{AA'} = 0 \quad \text{if} \quad A \neq A' \,. \tag{11}$$

In this state color $SU(3)$ as well as the chiral $SU(3)_L \times SU(3)_R$ and the $U(1)$-symmetry related to baryon-number conservation are broken down to a common $SU(3)_{color+V}$ subgroup where color and flavor rotations are locked. As a consequence all gluons receive a mass and there is a gap in the disperison laws of all nine (3 flavors, 3 colors) quark quasiparticle states.

The situations discussed so far are idealizations of the real world, where the strange quark mass M_s is neither infinite, such that strange quarks can be completely neglected, as in the previous section, nor degenerate with the masses of the up and down quarks. For sufficiently large quark chemical potentials $\mu \gg M_s$, the s quark mass becomes of course almost negligible against μ and matter is expected to be in the CFL phase. It is not clear, however, whether this CFL phase is directly connected to the hadronic phase [36] at low densities, or whether an intermediate 2SC phase exists, where only up and down quarks are paired. It is obvious that the answer to this question depends on the strange quark mass. This has first been analyzed by Alford, Berges and Rajagopal [37] who have studied the color-flavor unlocking phase transition in a model calculation with different values of M_s. Assuming that the region below $\mu \simeq 400$ MeV belongs to the hadronic phase, these authors came to the conclusion that a 2SC-phase exists if $M_s \gtrsim 250$ MeV. Here M_s is the constituent mass of the strange quark, which could be considerably larger than the current quark mass $m_s \sim 100$ to 150 MeV in the Lagrangian. Similar to the nonstrange constituent quark masses, dicussed in the previous section, it is in general T- and μ-dependent and can depend on the presence of quark-antiquark and diquark condensates. In particular, it can be discontinous along a first-order phase transition line. This means, not only the phase structure depends on the effective quark mass, but also the quark mass depends on the phase.

Recently, we have studied these interdependencies [38, 39] within an NJL-type model defined by the Lagrangian

$$\mathcal{L}_{eff} \;=\; \bar\psi(i\partial\!\!\!/ - \hat m)\psi \;+\; \mathcal{L}_{q\bar q} \;+\; \mathcal{L}_{qq}\,. \qquad (12)$$

The mass matrix $\hat m$ has the form $\hat m = diag(m_u, m_u, m_s)$ in flavor space, where we have assumed isospin symmetry, $m_u = m_d$. To study the interplay between the color-superconducting diquark condensates $s_{AA'}$ and the quark-antiquark condensates ϕ_u and ϕ_s we consider an NJL-type interaction with a quark-quark part

$$\mathcal{L}_{qq} \;=\; H \sum_{A=2,5,7}\;\sum_{A'=2,5,7} (\bar\psi\, i\gamma_5 \tau_A \lambda_{A'}\, C \bar\psi^T)(\psi^T C\, i\gamma_5 \tau_A \lambda_{A'}\, \psi)\,. \qquad (13)$$

and a quark-antiquark part

$$\begin{aligned}\mathcal{L}_{q\bar q} \;=\; & \quad G \sum_{a=0}^{8} \Big[(\bar\psi \tau_a \psi)^2 \;+\; (\bar\psi i\gamma_5 \tau_a \psi)^2 \Big] \\ & - \; K \Big[\mathrm{det}_f \Big(\bar\psi(1+\gamma_5)\psi \Big) \;+\; \mathrm{det}_f \Big(\bar\psi(1-\gamma_5)\psi \Big) \Big]\,. \qquad (14)\end{aligned}$$

As before $\tau_a, a = 1, ..., 8$, denote Gell-Mann matrices acting in flavor space, while $\tau_0 = \sqrt{\frac{2}{3}}\,\mathbb{1}_f$ is proportional to the unit matrix. Eq. (14) corresponds to a typical 3-flavor NJL-model Lagrangian. It consists of a $U(3)_L \times U(3)_R$-symmetric 4-point interaction and a 't Hooft-type 6-point interaction which breaks the the $U_A(1)$ symmetry. The latter has been neglected in Ref [38].

Starting from this Lagrangian it is tedious, but straight forward to calculate the mean-field thermodynamic potential Ω at temperature T and quark chemical potential μ (for details see Ref. [38]) and to determine the selfconsistent solutions for the expectation values $\phi_u = \phi_d$, ϕ_s, s_{22}, and $s_{55} = s_{77}$ by minimizing Ω with respect to these expectation values. In this context it is convenient to introduce the constituent quark masses

$$M_u = m_u - 4G\phi_u + 2K\phi_u\,\phi_s\;, \qquad M_s = m_s - 4G\phi_s + 2K\phi_u^2\;, \qquad (15)$$

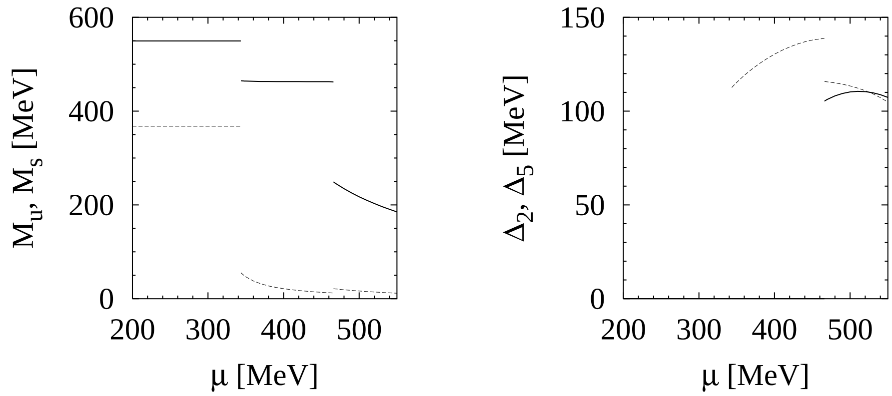

Figure 3. Gap parameters at $T = 0$ as functions of the quark chemical potential μ. Left: Constituent masses of up and down quarks (dashed), and of strange quarks (solid). Right: Diquark gaps Δ_2 (dashed) and Δ_5 (solid).

and the diquark gaps

$$\Delta_2 = -2Hs_{22} \quad \text{and} \quad \Delta_5 = -2Hs_{55}\ . \tag{16}$$

To determine the values of the various condensates we first have to specify the parameters of the interaction. We take the parameter values of Ref. [40] which were obtained by fitting vacuum masses and decay constants of pseudoscalar mesons. The coupling constant H which cannot be fixed in this way was chosen to yield "typical" values for Δ_2 in the 2SC phase [39].

Our results for the constituent quark masses (left) and the diquark gaps (right) at $T = 0$ as functions of μ are displayed in Fig. 3. Obviously, one can distinguish three phases. At low μ, the diquark gaps vanish and the constituent quark masses stay at their vacuum values. Hence, in a very schematic sense, this phase can be identified with the "hadronic phase" (although there are of course no hadrons in our model).

At a critical $\mu = \mu_1$ a first-order phase transition to the 2SC phase takes place: The diquark gap Δ_2 has now a non-vanishing value, whereas Δ_5 remains zero. At the same time the mass of the light quarks drops from the vacuum value to about 50 MeV and the baryon number density jumps from zero to about 2.5 nuclear matter density. At $\mu = \mu_2$ the system undergoes a second first-order phase transition, this time from the 2SC phase into the CFL phase, which is characterized by $\Delta_5 \neq 0$ (and $\Delta_2 \neq 0$).

We now extend our analysis to. $T \neq 0$. The resulting phase diagram in the μ-T plane is shown in Fig. 4. We can distinguish four different regimes: At low T we find (with increasing μ) the "hadronic phase", the 2SC phase, and the CFL phase. Similar to $T = 0$ these three phases are well

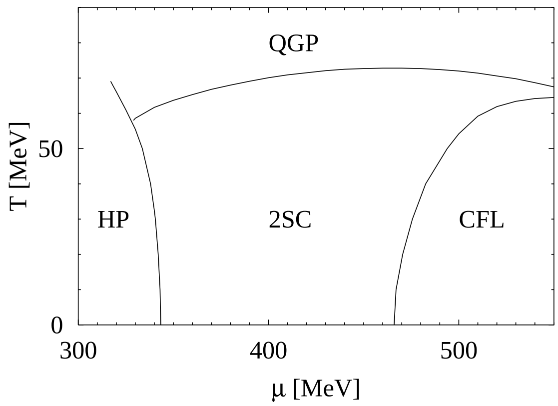

Figure 4. Phase diagram in the $\mu - T$ plane.

separated by first-order phase transitions. The high temperature regime is governed by the QGP phase, which is characterized by vanishing diquark condensates and small values of ϕ_u and (for large enough μ) ϕ_s. There we find smooth crossovers with respect to ϕ_u and ϕ_s instead of the first-order phase transitions. The transition from the 2SC phase to the QGP phase is of second order and the critical temperature is in almost perfect agreement with the well-known BCS relation $T_c = 0.57\Delta_2(T = 0)$.

It has been argued [37] that the color-flavor-unlocking transition has to be first order because pairing between light and strange quarks can only occur if the gap is of the same order as the mismatch between the Fermi surfaces. Moreover, the phase transition corresponds to a finite temperature chiral restoration phase transition in a three-flavor theory, and therefore the universality arguments of Ref. [41] should apply [3]. At low T our results are in agreement with these predictions. However, above a critical point we find a second order unlocking transition. In fact, the above arguments are not as stringent as they seem to be on a first sight: First the Fermi surfaces are smeared out due to thermal effects and secondly the 2SC phase is not a three-flavor chirally restored phase, but only $SU(2) \times SU(2)$ symmetric.

5. Discussion: charge neutral matter

In this article we discussed general features of two- and three-flavor color superconductors. For simplicity, we restricted our studies to a common chemical potential for all flavors. However, for many applications, e.g., to the decription of quark cores of neutron stars, one has to consider color and charge neutral matter in β-equilibrium. Very recently, it was argued by Alford and Rajagopal that these constraints could completely rule out the existence of a 2SC phase in compact stars [25]. This could give rise to a much larger window for crystalline phases than expected earlier [42].

To this end, we consider a system of massless u and d quarks together with electrons, but – in a first step – with no strange quarks. Since the density of electrons is small (see, e.g., [43]), to achieve charge neutrality the density of d-quarks must be almost twice as large as the density of u-quarks, and hence $\mu_d \approx 2^{1/3}\mu_u$. This means that, e.g., for $\mu_u = 400$ MeV, the Fermi momenta of u and d differ by about 100 MeV, making ud BCS-pairing very difficult. Alford and Rajagopal approached the problem from the opposite side, performing an expansion in terms of the strange quark mass. They found that, whenever the 2SC phase is more favorite than no pairing at all, the CFL phase is even more favorite. However, this analysis did not include selfconsistently calculated quark masses, and should be redone including these effects. Work in this direction is in progress.

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