

Radiative origin of solar scale and U_{e3}

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Abstract

We make a general study of possibility of generating solar scale Δ_{\odot} and the CHOOZ angle U_{e3} radiatively by assuming that they are zero at some high scale. The most general neutrino mass matrix leading to this result is determined in a CP conserving theory. This matrix contains four independent parameters which can be fixed in terms of physical observables. The standard weak radiative corrections then lead to non-zero Δ_{\odot} and U_{e3} without drastically altering the other tree level results. As a consequence, both Δ_{\odot} and U_{e3} are predicted in terms of other physically observable parameters. These predictions are insensitive to specific form of the neutrino mass matrix. The solar scale and U_{e3} are strongly correlated with the effective neutrino mass m_{ee} probed in neutrinoless double beta decay. In particular, the LMA solution to the solar neutrino problem arise for m_{ee} close to the present experimental limit. An example of specific texture is presented which predicts maximal atmospheric mixing and $\tan^2 \theta_{\odot} \approx 0.5$ for the solar mixing angle θ_{\odot} .

Introduction: Neutrino oscillation experiments have provided significant information on neutrino masses and mixing [1]. The "standard" picture emerging from analysis of various experiments is the existence of two hierarchical (mass)² differences and two large and one small or zero mixing angle among three neutrinos. The overall scale of neutrino masses is not fixed directly by neutrino oscillation experiments. This complimentary information can be obtained from direct neutrino mass measurements [2] and from neutrinoless double beta decay ($0\nu\beta\beta$) experiments [3]. These types of experiments have so far provided only upper limits. The neutrino mass (assuming no mixing) is constrained by tritium β decay experiments to be less than 2.2 eV. The best limit from $0\nu\beta\beta$ decay experiments is

$$|m_{ee}| \equiv \left| \sum U_{ei}^2 m_{\nu_i} \right| \leq 0.38 \text{ h eV} \quad \text{at 95\% CL} , \quad (1)$$

where $h \sim 0.6 - 2.8$ denotes the uncertainty in nuclear matrix element [4]. U denotes here the neutrino mixing matrix and $m_{\nu_i} (i = 1, 2, 3)$ are neutrino mass values which can take either sign.

While the hierarchical neutrino masses cannot be ruled out at present, the presence of large mixing angles hints at an almost degenerate pair of neutrinos. This will become a necessity if m_{ee} would be found to be significantly larger than the atmospheric scale. This would require all three neutrinos to be nearly degenerate [5] if mixing among them is also to account for the solar and atmospheric neutrino results. The mass patterns with two [6] or all the three [5,7,8] nearly degenerate neutrinos are therefore of considerable importance.

If overall scale of neutrino masses is larger than the atmospheric scale then one would like to understand why neutrino (mass)² differences (particularly, the solar scale Δ_\odot) are much smaller? Interesting possibility is to assume degenerate neutrinos which get split [5,7,8] by radiative corrections [9,10] induced through charged current interactions in the standard model (SM) or in the minimal supersymmetric standard model (MSSM). Advantage of this scheme is its predictive power. The most general 3×3 neutrino mass matrix with completely degenerate spectrum is characterized [11] in terms of two angles and one phase. These three parameters determine all neutrino masses and (complex) mixing after the known radiative corrections are included. Unfortunately, this predictive scenario does not give [8] phenomenologically required description of neutrino masses and mixing.

The next best possibility is to assume that only two of the three neutrinos are exactly degenerate at high scale. The radiative corrections then lead to the solar splitting within this scheme. It is possible to do a quite general and fairly model independent analysis of this situation which we present in this paper. We assume that neutrino masses, the atmospheric scale and two large mixing angles are tree level effects and are already described by neutrino

mass matrix specified at a high scale. We require this mass matrix to have vanishing solar scale and vanishing CHOOZ [12] angle in flavour basis. Neutrino mass matrix with such property can be characterized by four independent parameters in a CP conserving theory. Solar scale and the CHOOZ angle U_{e3} arise after radiative corrections and represent generic prediction of this scheme. These predictions are found to be model independent and hold for all the matrices under consideration. We now present this analysis and discuss its consequences.

General analysis of pseudo Dirac neutrinos: Let us consider a CP conserving theory specified by a general 3×3 real symmetric neutrino mass matrix $M_{\nu 0}$. This matrix can always be specified in flavor basis corresponding to diagonal charged lepton masses. We adopt the following general parameterization for $M_{\nu 0}$ in the flavor basis.

$$M_{\nu 0} = \begin{pmatrix} s_1 & t & u \\ t & s_2 & v \\ u & v & s_3 \end{pmatrix} \quad (2)$$

We assume that the above $M_{\nu 0}$ describes physics at a high scale M_X . $M_{\nu 0}$ is required to yield vanishing solar scale and CHOOZ angle at M_X . Let us derive conditions on elements of $M_{\nu 0}$ for this to happen.

Vanishing of the solar scale requires that two of the eigenvalues of $M_{\nu 0}$ are degenerate with masses (m, m) or $(m, -m)$ ($m > 0$). The relative angle between the degenerate pair can be rotated away in the former case in a CP conserving theory. This is not true in case of masses differing in their sign. Thus barring possibility of radiative amplification [13], the former case will not lead to large solar angle and we concentrate on the second possibility with masses $(m, -m)$. Such a pair is equivalent to a Dirac neutrino invariant under some global $U(1)$ symmetry. The standard weak current would violate this symmetry in general [14] and the Dirac state would split into a pair of pseudo Dirac neutrinos. General conditions under which this happens in case of three generations were discussed in [15]. In particular, the $M_{\nu 0}$ should satisfy

$$tr(M_{\nu 0}) \sum_i \Delta_i = det M_{\nu 0} , \quad (3)$$

where Δ_i represents the determinant of the 2×2 block of $M_{\nu 0}$ obtained by blocking the i^{th} row and column.

When the above condition is satisfied, eigenvalues of $M_{\nu 0}$ are given by $(m, -m, T)$ where

$$m \equiv \sqrt{-\sum_i \Delta_i} \quad T \equiv tr(M_{\nu 0})$$

Let U_0 diagonalize $M_{\nu 0}$:

$$U_0^T M_{\nu 0} U_0 = \text{Diag.}(m, -m, T) \quad (4)$$

Since $M_{\nu 0}$ is specified in the flavor basis, U_0 defined above represents physical neutrino mixing matrix at tree level. The electron neutrino survival probability in reactor experiments such as CHOOZ is given by $(U_0)_{e3}$ which we require to be zero. One can show that $M_{\nu 0}$ satisfies eq.(3) and leads to $(U_0)_{e3} = 0$ provided

$$\begin{aligned} v^2 &= (s_1 + s_2)(s_1 + s_3) ; \\ t &= -\frac{uv}{s_1 + s_2} . \end{aligned} \quad (5)$$

The second equation does not hold in a special case with $v = 0$. In this case t and u are unrelated and the above two conditions uniquely lead to the following $M_{\nu 0}$:

$$M_{\nu 0} = \begin{pmatrix} s & t & u \\ t & -s & 0 \\ u & 0 & -s \end{pmatrix} \quad (6)$$

The detailed phenomenological consequences of the special solution given in eq.(6) were worked out in [16]. Here we consider rest of the the neutrino mass matrices specified by restriction given in eq.(5). Let us parameterize the mixing matrix U_0 by

$$U_0 = \begin{pmatrix} c_{\phi_0} & -s_{\phi_0} & 0 \\ s_{\phi_0} c_{\theta_0} & c_{\phi_0} c_{\theta_0} & -s_{\theta_0} \\ s_{\phi_0} s_{\theta_0} & s_{\theta_0} c_{\phi_0} & c_{\theta_0} \end{pmatrix} , \quad (7)$$

The mixing angles are determined using eq.(4) and eq.(5):

$$\begin{aligned} \tan \phi_0 &= \sqrt{\frac{m - s_1}{m + s_1}} , \\ \tan \theta_0 &= \frac{u}{t} . \end{aligned} \quad (8)$$

Here, m denotes the common mass of the Dirac pair and is given by

$$m = \sqrt{s_1^2 + t^2 + u^2}$$

We note that

- The effective neutrino mass probed in the $0\nu\beta\beta$ is given by

$$m_{ee}^0 = s_1$$

- At the tree level, there is only one $(mass)^2$ difference which provides the atmospheric scale

$$\Delta_{0A} \equiv |m^2 - T^2| . \quad (9)$$

Corresponding mixing angle ($\equiv \theta_A^0$) coincides with θ_0 and is large when $t \sim u$:

$$\sin^2 2\theta_A^0 = \sin^2 2\theta_0 . \quad (10)$$

- There is no solar splitting at this stage but would be solar mixing angle θ_\odot^0 coincides with ϕ_0 and is given by

$$\tan^2 \theta_\odot^0 = \frac{m - m_{ee}^0}{m + m_{ee}^0} \quad (11)$$

The above relations are valid for the most general $M_{\nu 0}$ with parameters satisfying eq.(5). They are derived at tree level but as we demonstrate latter, radiative corrections do not substantially change them. The major effect of radiative corrections is to generate the solar splitting and a non-zero value for U_{e3} . It turns out that these quantities are not arbitrary but are predicted in terms of other observables irrespective of the detailed form of $M_{\nu 0}$. This happens because two conditions in eq.(5) leave us with four independent parameters. They can be determined in terms of four observables namely, atmospheric scale and angle, effective mass probed in $0\nu\beta\beta$ and the solar angle. The solar splitting and U_{e3} generated radiatively then no longer remain arbitrary but are determined in terms of these observables.

We can express all parameters of $M_{\nu 0}$ in terms of observables using conditions of eq.(5):

$$\begin{aligned} m &= \left| \frac{m_{ee}^0}{\cos 2\theta_\odot^0} \right| & t^2 &= \cos^2 \theta_A^0 (m^2 - m_{ee}^{02}) \\ T^2 &= (m^2 - \Delta_A, m^2 + \Delta_A) & & \text{(for } m^2 > \Delta_A, < \Delta_A) \\ s_3 &= \cos^2 \theta_A^0 T - \sin^2 \theta_A^0 m_{ee}^0 & s_2 &= \sin^2 \theta_A^0 T - \cos^2 \theta_A^0 m_{ee}^0 \end{aligned} \quad (12)$$

The solar splitting can be obtained [15] using the relevant renormalization group equations [9,10]. The consequences of these equations have been discussed in a number of papers [7,8].

The radiatively corrected neutrino mass matrix is given by

$$M_\nu = I_g I_t \begin{pmatrix} I_e^{\frac{1}{2}} & 0 & 0 \\ 0 & I_\mu^{\frac{1}{2}} & 0 \\ 0 & 0 & I_\tau^{\frac{1}{2}} \end{pmatrix} M_{\nu 0} \begin{pmatrix} I_e^{\frac{1}{2}} & 0 & 0 \\ 0 & I_\mu^{\frac{1}{2}} & 0 \\ 0 & 0 & I_\tau^{\frac{1}{2}} \end{pmatrix}, \quad (13)$$

where

$$I_\alpha^{\frac{1}{2}} \equiv 1 + \delta_\alpha,$$

with

$$\delta_\alpha \approx c \left(\frac{m_\alpha}{4\pi v} \right)^2 \ln \frac{M_X}{M_Z}. \quad (14)$$

M_X here corresponds to a large scale and we take $M_X \sim 10^{16}$ GeV; $c = \frac{3}{2}, -\frac{1}{\cos^2 \beta}$ in respective cases of the standard model ¹ and the minimal supersymmetric standard model [7] and $\alpha = e, \mu, \tau$. $I_{g,t}$ are calculable coefficient summarizing the effect of the gauge and the top quark corrections.

Apart from the overall factor $I_g I_t$, the radiative corrections are largely dominated by the τ Yukawa couplings and it is easy to determine neutrino mixing angle and masses keeping only δ_τ corrections and working to the lowest order in δ_τ . We now have

$$U^T M_\nu U = \text{Diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}),$$

with

$$\begin{aligned} m_{\nu_1} &\approx I_g I_t (m + \delta_\tau \sin^2 \theta_A^0 (m - m_{ee}^0)) + \mathcal{O}(\delta_\tau^2), \\ m_{\nu_2} &\approx I_g I_t (-m - \delta_\tau \sin^2 \theta_A^0 (m + m_{ee}^0)) + \mathcal{O}(\delta_\tau^2), \\ m_{\nu_3} &\approx I_g I_t (T + 2\delta_\tau T \cos^2 \theta_A^0) + \mathcal{O}(\delta_\tau^2), \end{aligned} \quad (15)$$

where we have used eq.(12). The tree level mixing matrix U_0 gets modified to a general U :

$$U = \begin{pmatrix} c_\phi c_\omega & -s_\phi c_\omega & s_\omega \\ c_\phi s_\theta s_\omega + c_\theta s_\phi & c_\theta c_\phi - s_\phi s_\theta s_\omega & -s_\theta c_\omega \\ -c_\phi c_\theta s_\omega + s_\phi s_\theta & s_\phi s_\omega c_\theta + s_\theta c_\phi & c_\theta c_\omega \end{pmatrix}, \quad (16)$$

¹The value of c in case of the standard model is given by $1/2(3/2)$ according to calculations in [9] ([10]). We will use the value as in [10].

As before, the angles ϕ, θ correspond respectively to solar and atmospheric mixing angles. These are now given by

$$\begin{aligned}\tan \theta_A &= \tan \theta_A^0 \left(1 + \frac{\delta_\tau}{m^2 - T^2} (m^2 + T^2 - 2Ts_1) \right) + \mathcal{O}(\delta_\tau^2), \\ \tan^2 \theta_\odot &= \tan^2 \theta_\odot^0 + \mathcal{O}(\delta_\tau^2),\end{aligned}\tag{17}$$

where θ_A^0 (eq.(10)) and θ_\odot^0 (eq.(11)) are tree level atmospheric and the solar mixing angles respectively. Note that the solar mixing angle does not receive radiative corrections to the lowest order in δ_τ .

The effective neutrino mass probed in $0\nu\beta\beta$ is now given by

$$m_{ee} = I_g I_t m_{ee}^0 = I_g I_t s_1.\tag{18}$$

The atmospheric scale also receive radiative corrections and is now given by

$$\Delta_A \equiv \frac{1}{2}(m_{\nu_1}^2 + m_{\nu_2}^2) - m_{\nu_3}^2 = I_g^2 I_t^2 \left(\Delta_{0A} + 2\delta_\tau (m^2 \sin^2 \theta - 2T^2 \cos^2 \theta) \right) + \mathcal{O}(\delta_\tau^2).\tag{19}$$

It is seen that all the tree level predictions receive small radiative corrections. Thus all the neutrino mass matrices leading to two degenerate states and characterized by eq.(5) are stable against radiative corrections. This is to be contrasted with the case of fully degenerate neutrino spectrum which leads to radiative instability in some specific cases [7]. The non-trivial effect of the radiative corrections is generation of the solar splitting and a non-zero U_{e3} :

$$\begin{aligned}\Delta_\odot &\equiv m_{\nu_2}^2 - m_{\nu_1}^2 \approx 4m_{ee}\delta_\tau \left| \frac{m_{ee}}{\cos 2\theta_\odot} \right| \sin^2 \theta_A, \\ |U_{e3}| &= |s_\omega| \sim \left| \frac{\delta_\tau T \sin 2\theta_A \sqrt{m^2 - m_{ee}^2}}{\Delta_A} \right|.\end{aligned}\tag{20}$$

The above equations relate the solar scale and CHOOZ angle to other experimentally determined quantities as can be seen using eq.(12). Eq.(20) therefore represent basic prediction of the scheme defined by eq.(5). It is remarkable that all these matrices lead to unique predictions in eq.(20) which are insensitive to specific texture of the neutrino mass matrix.

Let us now explore consequences of eq.(20). The atmospheric mixing angle and scale are experimentally well-determined: $\Delta_A \approx (1.5 - 5) \cdot 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta_A \sim 0.8 - 1$. The solar scale Δ_\odot and mixing θ_\odot are also highly constrained [17], particularly after [18] the recent neutral current results from SNO [19]. Based on the global analysis of all the solar data, the only solutions allowed at 3σ level are the large mixing angle solution (LMA) and the LOW solution with $\Delta_\odot \sim 10^{-7} \text{ eV}^2$. The allowed ranges of parameters in these cases at 3σ

are given approximately by $\Delta_{\odot} \approx 3 \cdot 10^{-4} - 2 \cdot 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_{\odot} \sim 0.2 - 0.9$ in case of the LMA solution and $\Delta_{\odot} \approx 3 \cdot 10^{-8} - 1 \cdot 10^{-7} \text{ eV}^2$ and $\tan^2 \theta_{\odot} \sim 0.4 - 0.9$ in case of the LOW solution. Both the small mixing angle and vacuum solutions are excluded at 3σ . In particular, the solar mixing angle is found to be less than 45° in all the preferred solution a fact which plays an important role in the following.

The predicted value of Δ_{\odot} and U_{e3} are different in SM and MSSM due to different values of δ_{τ} in these two cases. In case of SM, $\delta_{\tau} \sim 10^{-5}$ while it can become larger for MSSM due to presence of $\tan \beta$. More importantly, sign of δ_{τ} is different in these two cases. The negative values of δ_{τ} in case of MSSM makes it unsuitable for the description of the solar data as we now argue.

In analyzing the solar data, Δ_{\odot} is chosen positive by convention and mixing angle θ_{\odot} is allowed to be greater than 45° . The sign of Δ_{\odot} as defined by eq.(20) is governed by the sign of m_{ee} and δ_{τ} . The sign of m_{ee} also determines magnitude of the solar angle through eq.(11). Positive (negative) values of m_{ee} gives a θ_{\odot} less (greater) than one. Since δ_{τ} is negative in case of the MSSM one needs negative m_{ee} to obtain positive Δ_{\odot} with the result that $\tan^2 \theta_{\odot}$ becomes greater than one ². Since the solar neutrino results do not allow $\tan^2 \theta_{\odot} > 1$, MSSM radiative corrections as a mechanism to generate the solar splitting is disfavoured in the present context. In contrast, the SM radiative corrections give $\tan^2 \theta_{\odot} < 1$ as required for these solutions. We discuss this case now.

The numerical value of Δ_{\odot} and U_{e3} are correlated both with m_{ee} as well as with the solar mixing angle. We show this correlation in Fig.(1) which displays variation in $10^5 \frac{\Delta_{\odot}}{\text{eV}^2}$ (solid) and $10^2 U_{e3}$ (dotted) with m_{ee} for typical values of $\tan^2 \theta_{\odot}$ needed for the LMA and LOW solutions. It is seen from the figure that LMA solution can be obtained for relatively larger value of m_{ee} typically $m_{ee} \geq 0.1 \text{ eV}$. The minimum required value of m_{ee} increases with decrease in $\tan^2 \theta_{\odot}$. The LOW solution require much smaller but experimentally accessible values [20] of m_{ee} around 0.05 eV .

The predicted values of U_{e3} are generally smaller than the present limits as well as possible detection [21] limit around ~ 0.05 for most ranges in the parameters. But if $\tan^2 \theta_{\odot}$ is $\sim 0.5 - 0.8$ then U_{e3} is predicted to be in the range $0.01 - 0.1$ and is correlated with the LMA solution.

An interesting consequence [22] of the correlation between LMA solution and large m_{ee}

²For positive m_{ee} one needs to reverse the role of ν_1 and ν_2 . The relevant $\tan^2 \theta_{\odot}$ is inverse of eq.(11) and is also greater than 1.

is as follows. For $\tan^2 \theta_\odot \sim 0.2 - 0.7$ and $m_{ee} \sim 0.3 \text{ eV}$, the common mass $m = \left| \frac{m_{ee}}{\cos 2\theta_\odot} \right|$ of the degenerate pair lies in the range

$$m \approx 0.5 - 1.8 \text{ eV} .$$

The third mass T is also required to be close to m since $\Delta_A \sim |m^2 - T^2|$. As a consequence, the LMA solution in these models is automatically correlated with almost degenerate neutrino mass spectrum with a common mass much larger than the atmospheric scale.

The above discussion is based on general class of matrices leading to pseudo-Dirac neutrino. We now supplement this with a discussion of a specific texture.

Almost degenerate neutrinos: Consider the following specific texture:

$$M_{\nu 0} = s \begin{pmatrix} 1 + \epsilon & -2 & 2 \\ -2 & 1 - \epsilon & 2 \\ 2 & 2 & 1 - \epsilon \end{pmatrix} \quad (21)$$

The above texture is determined by only two parameters s and ϵ . It satisfies conditions in eq.(5). Thus it leads to two degenerate eigenvalues and vanishing U_{e3} at a high scale. The eigenvalues of the above matrix are given by $(m, -m, s(3 - \epsilon))$ with

$$m = s\sqrt{9 + 2\epsilon + \epsilon^2} \quad (22)$$

It is seen that all neutrinos are degenerate when $\epsilon = 0$. It is known [8] that matrix with degenerate neutrinos cannot lead to the required mass pattern after radiative corrections. This is remedied here by introduction of a small ϵ which leads to the atmospheric neutrino splitting at high scale:

$$\Delta_A \approx \Delta_{A0} = 8\epsilon s^2 \quad (23)$$

The above specific texture has four predictions. As in the general case, the radiatively generated solar scale and U_{e3} are predictions of the model. In addition, both the solar and atmospheric mixing angles instead of being arbitrary are fixed here by the specific texture.

The solar splitting follows from the general expression in eq.(20):

$$\Delta_\odot \sim 2\delta_\tau s^2(1 + \epsilon)\sqrt{9 + 2\epsilon + \epsilon^2} \quad (24)$$

Two parameters s and ϵ get determined by the values of Δ_\odot and Δ_A . In particular, relatively large $\Delta_\odot \sim 10^{-5} \text{ eV}^2$ needs small ϵ and large s . The solar mixing angle is obtained using eqs.(11,22) and is given in the small ϵ limit by

$$\tan^2 \theta_{\odot} \approx 0.5 + O(\epsilon) \quad (25)$$

Thus this texture automatically predicts large mixing angle which is in the range required for the LMA or LOW solutions. The atmospheric mixing angle is predicted to be maximal. Eq.(21) therefore provides an example of the bi-large mixing patterns with almost degenerate neutrinos.

One can determine required value of s, ϵ from eqs.(23,24). Choosing $\Delta_{\odot} \sim 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta_A = 3 \cdot 10^{-3}$, one finds

$$\epsilon \sim 1.8 \cdot 10^{-3} \quad ; \quad s = 0.45 \text{ eV} .$$

Thus one can obtain Δ_{\odot} around 10^{-5} eV^2 provided the effective neutrino mass $m_{ee} = s \sim 0.4 \text{ eV}$. All the three neutrinos are almost degenerate with a common mass 1.3 eV which is not very far from the present experimental limit [2].

Summary: We discussed possibility of explaining small values of the solar scale and the angle U_{e3} through radiative corrections by assuming that these are zero at a high scale. Restrictions to be satisfied by neutrino mass matrix for this purpose in the flavor basis were determined (eq.5). Since neutrino mass matrix can always be expressed in the flavor basis, eq.(5) provides general conditions for vanishing of U_{e3} and solar scale in any model.

We showed that the standard weak radiative corrections lead to the solar splitting required on phenomenological grounds. Both Δ_{\odot} and U_{e3} are predicted in terms of other observables. These predictions are remarkably independent of detailed form of $M_{\nu 0}$ and remain true for any $M_{\nu 0}$ satisfying eq.(5).

Detailed analysis presented here shows that one can obtain the most preferred LMA solution for $m_{ee} \gtrsim 0.1 \text{ eV}$. Thus verification of LMA solution and moderate improvement in limit on m_{ee} can rule out the entire class of solutions proposed here. The LMA solution also gets correlated in these models with almost degenerate neutrino mass spectrum and measurably large $U_{e3} \sim .01 - 0.1$.

It is not possible to obtain the correct solar parameters if radiative corrections are induced in MSSM. It may be possible to make MSSM also viable by allowing some nonzero U_{e3} at high scale and/or by invoking additional sources of radiative corrections [23].

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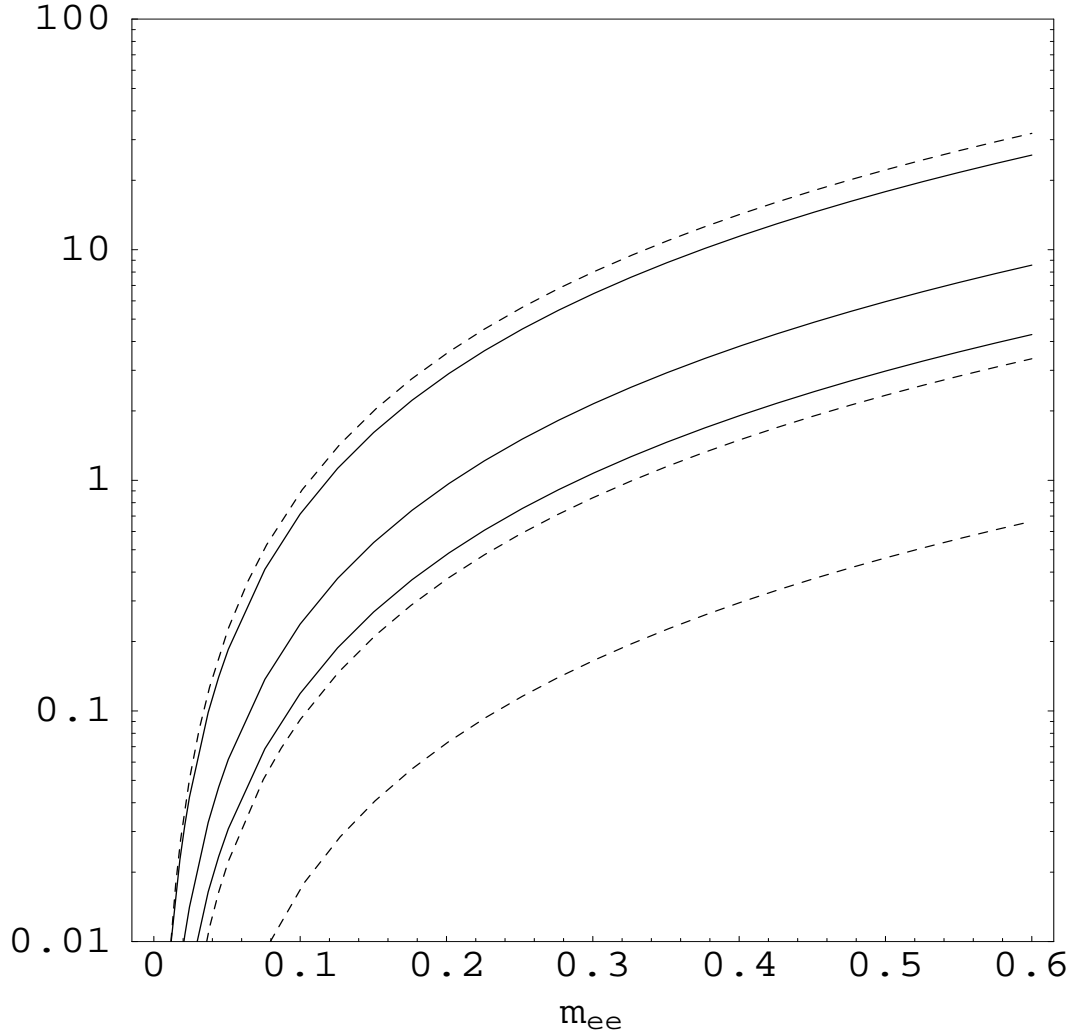


FIG. 1. $10^5 \Delta_\odot$ in (eV^2) (solid) and $10^2 |U_{e3}|$ (dotted) shown as a function of m_{ee} (in eV) for various values of $\tan^2 \theta_\odot$. The upper middle and lower curves for each quantities correspond to $\tan^2 \theta_\odot = 0.8, 0.5$ and 0.2 respectively.