Atmospheric Neutrino Constraints on Lorentz Violation

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Sensitive tests of Lorentz invariance can emerge from the study of neutrino oscillations, particularly for atmospheric neutrinos where the effect is conveniently near-maximal and has been observed over a wide range of energies. We assume these oscillations to be described in terms of two neutrinos with different masses and (possibly) different maximal attainable velocities (MAVs). It suffices to examine limiting cases in which neutrino velocity eigenstates coincide with either their *mass* or *flavor* eigenstates. We display the modified ν_μ-ν_τ transition probability for each case. Data on atmospheric neutrino oscillations at the highest observed energies and pathlengths can yield constraints on neutrino MAV differences (*i.e.,* tests of special relativity) more restrictive than any that have been obtained to date on analogous Lorentz-violating parameters in other sectors of particle physics.

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It seems unlikely that Lorentz violation *per se,* rather than neutrino mass, can explain observed neutrino oscillations. In particular, atmospheric neutrinos such as are observed at SuperKamiokande and MACRO, as well as K2K data, appear to be well described by nearly maximal conventional (*i.e.,* mass-associated) two-flavor $\nu_\mu - \nu_\tau$ oscillations. [1] where the transition probability of muon neutrinos or antineutrinos (neglecting matter effects) is given by:

$$P\big(\nu_\mu(\overline{\nu}_\mu) \to \nu_\tau(\overline{\nu}_\tau)\big) \simeq \sin^2\big(\delta m^2 L/4E\big), \tag{1}$$

with $\delta m^2 \simeq 2 \times 10^{-3}$ eV^2. The 2-family approximation suffices because of the near degeneracy of two neutrino masses and the smallness of the subdominant PMNS angle θ_{13}.

Here we ask whether small Lorentz-violating effects involving atmospheric neutrinos can reveal themselves as departures from Eq.(1). We retain the two-family formalism, presuming that any MAV difference between the neutrinos relevant to solar neutrino oscillations plays no significant role for atmospheric neutrinos. We also assume CPT conservation. To determine the modified neutrino transition probabilities, we recall Eqs. 11 and 12 of our 1997 paper* [3], with $\theta_m = \theta_{23} = \pi/4$ so as to reproduce the observations of maximal atmospheric neutrino oscillations at low energy. We obtain:

$$P\big(\nu_\mu(\overline{\nu}_\mu) \to \nu_\tau(\overline{\nu}_\tau)\big) = \sin^2 2\Theta\,\sin^2\big(\Delta L/4E\big), \tag{2}$$

where

$$\Delta(E)\sin 2\Theta(E) = \left|\delta m^2 + 2ae^{i\eta}E^2\sin 2\theta_v\right|, \tag{3a}$$

$$\Delta(E)\cos 2\Theta(E) = 2aE^2\cos 2\theta_v. \tag{3b}$$

The tiny positive parameter a is the fractional difference between the maximal attainable velocities (MAVs) of the two neutrinos; the phase η is unconstrained; the angle θ_v determines the neutrino velocity eigenstates. To interpret this result, I define the critical neutrino energy:

$$E_c \equiv \sqrt{\frac{\delta m^2}{2a}}\,. \tag{4}$$

For $E \ll E_c$, Lorentz violation is ineffective. Oscillations are maximal with their conventional phase $\delta m^2 L/4E$. For $E > E_c$, the neutrino mass difference becomes ineffective. The

* Several subsequent studies of Lorentz and CPT violation in the neutrino sector [2] offer relevant commentary, although they are largely directed toward other issues.

oscillation amplitude is no longer maximal but approaches $\sin^2 2\theta_v$ as its phase approaches $aEL/2$.

It is sufficient to examine the precise consequences of Eq.(2) in two extreme limits. For Case A we set $\theta_v = 0$, thus taking as the neutrino states with definite (and different) MAVs the *flavor* eigenstates, ν_μ and ν_τ. The conventional oscillation formula Eq.(1) is changed as follows:

$$P\big(\nu_\mu(\overline{\nu}_\mu) \to \nu_\tau(\overline{\nu}_\tau)\big) = \frac{1}{f^2(E)} \sin^2\big(f(E)\, \delta m^2 L/4E\big), \tag{5}$$

where

$$f(E) \equiv \sqrt{1 + 4a^2\, E^4/(\delta m^2)^2} = \sqrt{1 + (E/E_c)^4}. \tag{6}$$

At energies above the critical energy E_c, oscillations rapidly decline in amplitude while decreasing in oscillation length. Loosely speaking, oscillations remain maximal or nearly maximal at energies below E_c, but they wash out above E_c.

For Case B we set $\theta_v = \pi/4$, thus taking the neutrino states with definite MAVs to be their *mass* eigenstates. The neutrino oscillation probability becomes:

$$P\big(\nu_\mu(\overline{\nu}_\mu) \to \nu_\tau(\overline{\nu}_\tau)\big) = \sin^2\big(g(E)\, \delta m^2 L/4E\big), \tag{7}$$

where

$$g(E) \equiv \big|1 + 2ae^{i\eta}E^2/\delta m^2\big| = \big|1 + e^{i\eta}(E/E_c)^2\big|. \tag{8}$$

The oscillations remain maximal at all energies, but their phase increases rapidly as neutrino energies exceed E_c.

Eqs. (5) and (7) show that the most sensitive tests of Lorentz invariance may be obtained from atmospheric neutrino observations at the highest observed energies and longest baselines, *i.e.,* at $E \sim 100$ GeV and $L \sim 10^4$ km. We suspect that in Cases A and B — and indeed, for any intermediate situation described by Eq.(2)— limits as severe as $E_c > 100$ GeV, or equivalently $|a| < 10^{-25}$, can be set through dedicated analyses of currently available Super-K or MACRO data. These tests of special relativity should be of wide interest because they are considerably more restrictive than any that have been reported on analogous Lorentz-violating parameters (MAV differences) in other sectors of particle physics.[†]

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[†] See, for example, the constraints summarized in [4], and more recently, in [5].

References

[1] *E.g.,* The SuperKamiokande Collaboration, `hep-ex/0404034`;
A. Giacomelli, A. Margiotta, `hep-ex/0406037`;
The K2K Collaboration, `http://neutrino.kek.jp/news/2004.06.10/index-e.htm`.

[2] A. Kostelecy, M. Mewes, `hep-ph/0308300, 0406255`;
A. Kostelecky, `hep-ph/0403088`.

[3] S.R. Coleman, S.L. Glashow, Phys. Lett. B403:229,1997.

[4] S.R. Coleman, S.L. Glashow, Phys.Rev.D59:116008,1999.

[5] O. Gagnon, G.D. Moore, `hep-ph/0404196`.